Time dependence of correlation functions in homogeneous and isotropic turbulence

Diane Wright, US

Léonie Canet

NCTRV Les Houches 10/04/2019

Presentation outline

1. Introduction: why Renormalisation Group ?
   (blackboard)

2. Navier-Stokes field theory and extended symmetries
   (blackboard)

3. Time-dependence of generic *n*-point correlation functions
   (blackboard)

4. Illustration for the two-point correlation function

Renormalisation Group

perturbative RG approaches

- *early works* Forster, Nelson, Stephen PRL **36** (1976),de Dominicis, Martin, PRA **19** (1979), Fournier, Frisch, PRA **28** (1983), Yakhot, Orszag, PRL **57** (1986) · · ·
- *reviews* Zhou, Phys. Rep. **488** (2010), Adzhemyan *et al.*, *The Field Theoretic RG in Fully Developed Turbulence*, Gordon Breach, 1999

Functional and Non-Perturbative RG

▷ RG fixed point for physical forcing

Tomassini, Phys. Lett. B **411** (1997)
Mejía-Monasterio, Muratore-Ginnaneschi, PRE **86** (2012)
LC, Delamotte, Wschebor, PRE **93** (2016)

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LC, V. Rossetto, N. Wschebor, G. Balarac, Phys. Rev. E **95** (2017)
M. Tarpin, LC, N. Wschebor, Phys. Fluids **30**, 055102 (2018)

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Navier-Stokes field theory

forced Navier Stokes equation for incompressible flows

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\frac{1}{\rho} \vec{\nabla} p + \nu \vec{\nabla}^2 \vec{v} + \vec{f}$$

- incompressibility condition $\vec{\nabla} \cdot \vec{v} = 0$
- $\vec{f}(\vec{x}, t)$ gaussian stochastic stirring force with variance

$$\left\langle f_\alpha(t, \vec{x}) f_\beta(t', \vec{x}\,') \right\rangle = 2\delta_{\alpha\beta}\delta(t - t') N_L(|\vec{x} - \vec{x}\,'|).$$

with N_L peaked at the integral scale (energy injection)

Navier-Stokes field theory

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{v} + \vec{f} \qquad \text{with} \qquad \vec{\nabla}\cdot\vec{v} = 0$$

$$\left\langle f_\alpha(t,\vec{x}) f_\beta(t',\vec{x}\,') \right\rangle = 2\delta_{\alpha\beta}\delta(t-t')N_L(|\vec{x}-\vec{x}\,'|).$$

MSR Janssen de Dominicis formalism: NS field theory

Martin, Siggia, Rose, PRA **8** (1973), Janssen, Z. Phys. B **23** (1976), de Dominicis, J. Phys. Paris **37** (1976)

$$\mathcal{Z} = \int \mathcal{D}\vec{v}\,\mathcal{D}\bar{\vec{v}}\,\mathcal{D}p\,\mathcal{D}\bar{p}\,e^{-\mathcal{S}_{\rm NS}}$$

$$\mathcal{S}_{\rm NS} = \int_{t,\vec{x}} \bar{v}_\alpha \left[\partial_t v_\alpha + v_\beta\partial_\beta v_\alpha + \frac{1}{\rho}\partial_\alpha p - \nu\nabla^2 v_\alpha\right] + \bar{p}\left[\partial_\alpha v_\alpha\right]$$
$$- \int_{t,\vec{x},\vec{x}'} \bar{v}_\alpha \left[N_L(|\vec{x}-\vec{x}'|)\right] \bar{v}_\alpha$$

Navier-Stokes field theory: extended symmetries

- time-gauged Galilean invariance: $\mathcal{G} = \begin{cases} \vec{x} \to \vec{x} + \vec{\epsilon}(t) \\ \vec{v} \to \vec{v} - \dot{\vec{\epsilon}}(t) \end{cases}$
  - well-known
- time-gauged shift symmetry: $\mathcal{R} = \begin{cases} \delta \bar{v}_\alpha(t,\vec{x}) & = \bar{\epsilon}_\alpha(t) \\ \delta \bar{p}(t,\vec{x}) & = v_\beta(t,\vec{x})\bar{\epsilon}_\beta(t) \end{cases}$
  - not identified yet!

LC, B. Delamotte, N. Wschebor, Phys. Rev. E **91** (2015)

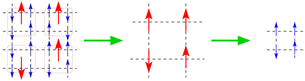
infinite set of *local in time* exact Ward identities
for all vertices with one $\vec{q} = 0$

$$\Gamma^{(m,n)}_{\alpha_1\cdots\alpha_{n+m}}(\omega, \vec{q} = \vec{0}; \{\nu_i, \vec{p}_i\}) = \mathcal{D}_{\alpha_1}(\omega)\Gamma^{(m-1,n)}_{\alpha_2\cdots\alpha_{n+m}}(\{\nu_i, \vec{p}_i\})$$

$$\Gamma^{(m,n)}_{\alpha_1\cdots\alpha_{m+n}}(\nu_1, \vec{p}_1, \cdots, \nu_{m+1}, \vec{q} = 0, \cdots) = 0$$

Non-perturbative Renormalization Group

▶ based on Wilson idea of the RG
$\longrightarrow$ progressive coarse-graining of fluctuations

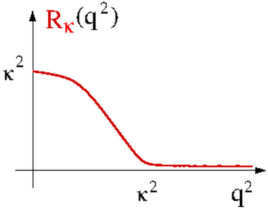


▶ exact RG equation for $\mathcal{W} = \ln \mathcal{Z}$

Polchinski, Nucl. Phys. B **231** (1984), Wetterich, Phys. Lett. B **301** (1993)

$$\partial_\kappa \mathcal{W}_\kappa = -\frac{1}{2} \int_{\mathbf{x},\mathbf{y}} \partial_\kappa [\mathcal{R}_\kappa]_{ij}(\mathbf{x}-\mathbf{y}) \left\{ \frac{\delta^2 \mathcal{W}_\kappa}{\delta j_i(\mathbf{x}) \delta j_j(\mathbf{y})} + \frac{\delta \mathcal{W}_\kappa}{\delta j_i(\mathbf{x})} \frac{\delta \mathcal{W}_\kappa}{\delta j_j(\mathbf{y})} \right\},$$

- $\mathcal{R}_\kappa$: separates fluctuations
- j_i : sources



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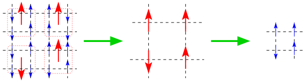

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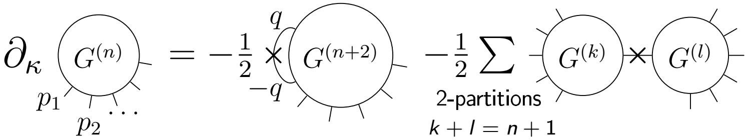
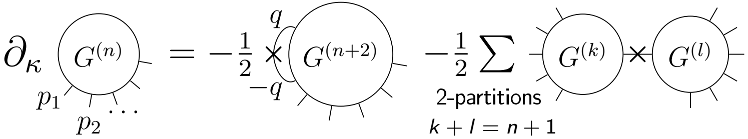
exact but infinite hierarchy for flow of connected correlation functions...



Non-perturbative Renormalization Group

closed flow equation for all $G^{(n)}(\{t_i, \vec{p}_i\})$ in the limit $|\vec{p}_i| \gg L^{-1}$



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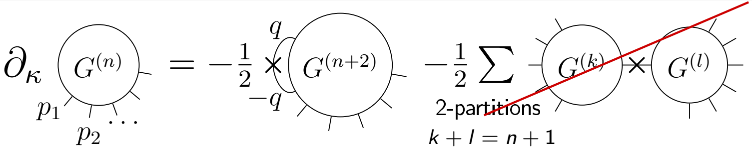
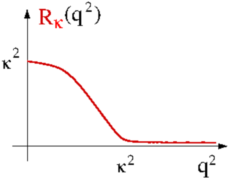


▷ key point: large wave-number expansion $|\vec{p}_i| \gg L^{-1}$



▷ $\partial_\kappa R_\kappa(\vec{p}_i) = 0$

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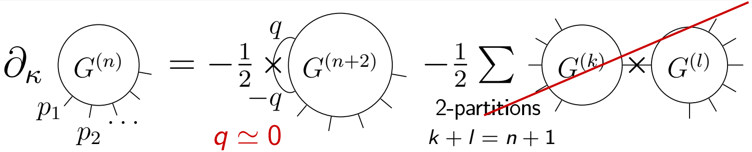

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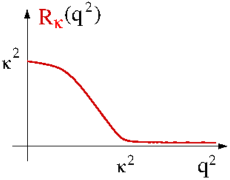

▷ $\partial_\kappa R_\kappa(\vec{p}_i) = 0$

▷ $\partial_\kappa R_\kappa(\vec{q})$: $|\vec{q}| \ll |\vec{p}|$

$\Longrightarrow$ set $\vec{q} = 0$ in all vertices

- asymptotically exact in the limit $|\vec{p}_i| \gg \kappa \sim L^{-1}$
- close with Ward identities vertices with $\vec{q} = 0$

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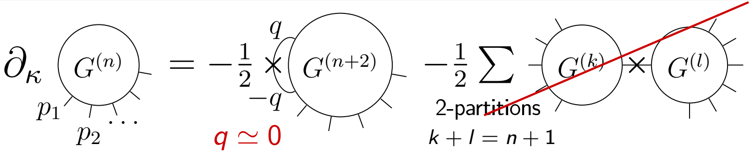

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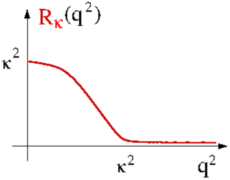

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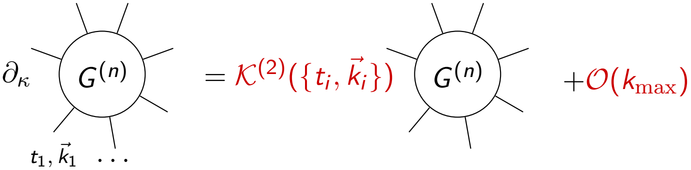

$$\mathcal{K}^{(2)}(\{t_i, \vec{k}_i\}) = \frac{1}{3}\int_\omega J^{(2)}(\omega) \sum_{k,\ell} \frac{\vec{k}_k \cdot \vec{k}_\ell}{\omega^2}\left(e^{i\omega(t_k - t_\ell)} - e^{i\omega t_k} - e^{-i\omega t_\ell} + 1\right)$$

with the non-linear part hidden in

$$J^{(2)}(\omega) = -\int_{\vec{q}} \Big\{ 2\kappa\partial_\kappa N_\kappa(\vec{q})\,|G_\kappa(\nu,\vec{q})|^2 - 2\kappa\partial_\kappa R_\kappa(\vec{q})\; C_\kappa(\nu,\vec{q})\Re G_\kappa(\nu,\vec{q}) \Big\}$$

M. Tarpin, LC, N. Wschebor, Phys. Fluids **30**, 055102 (2018)

Time dependence of n-point correlation functions

solution at the fixed point: universal behaviour

- standard critical phenomena: decoupling at large $\vec{k}_i$

$$\mathcal{K}^{(2)}(\{t_i, \vec{k}_i\}) \to 0$$

- solution:

  fixed point + decoupling $\Longrightarrow$ scaling form (Family-Wilczek)

  with K41 scaling: $z = 2/3$, $d_v = -1/3$

$$G^{(n)}_{\alpha_1 \ldots \alpha_n}(\{t_i, \vec{k}_i\}) = k_1^{-d_G} H^0_{\alpha_1 \ldots \alpha_n}\left(\left\{k_1^{2/3} t_i, \vec{k}_i / k_1\right\}\right)$$

$\Longrightarrow$ standard scale invariance

Time dependence of n-point correlation functions
Small time delays

solution at the fixed point: non-decoupling !

- limit of small time delays $t_i \to 0$

$$\mathcal{K}^{(2)}(\{t_i, \vec{k}_i\}) \to \mathcal{K}_0(\{t_i, \vec{k}_i\}) = I_0^* \Big| \sum_\ell \vec{k}_\ell t_\ell \Big|^2$$

- solution ($\vec{\rho}_i$ appropriately rotated wave-vectors):

$$G^{(n)}_{\alpha_1 \dots \alpha_n}(\{t_i, \vec{k}_i\}) = \overbrace{\rho_1^{-d_G} H^0_{\alpha_1 \dots \alpha_n}\big(\{\rho_1^{2/3} t_i, \hat{\rho}_i\}\big)}^{\text{standard scale invariance}} \times \exp\Big(\underbrace{-\alpha_0 L^{2/3} \Big| \sum_\ell \vec{k}_\ell t_\ell \Big|^2}_{\text{violation}} + \mathcal{O}(\vec{k}_{\max} L) \Big)$$

$\Longrightarrow$ breaking of standard scale invariance

Time dependence of n-point correlation functions
Small time delays

solution at the fixed point: non-decoupling !

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$\Longrightarrow$ intermittency corrections at $t = 0$ not captured at this order

Time dependence of n-point correlation functions
Large time delays

solution at the fixed point: non-decoupling !

- limit of large time delays $t_i \to \infty$

$$\mathcal{K}^{(2)}(\{t_i, \vec{k}_i\}) \to \mathcal{K}_\infty(\{t_i, \vec{k}_i\}) = I_\infty^* \sum_{k,\ell} \vec{k}_k \cdot \vec{k}_\ell \big(|t_k| + |t_\ell| - |t_\ell - t_k|\big)$$

- solution ($\vec{\varrho}_i$ appropriate linear combination of wave-vectors):

$$G^{(n)}_{\alpha_1 \dots \alpha_n}(t, \{\vec{k}_i\}) = \varrho_1^{-d_G} H^\infty_{\alpha_1 \dots \alpha_n}\big(\varrho_1^{2/3} t, \{\hat{\varrho}_i\}\big) \times \exp\left(-\alpha_\infty L^{4/3} \, |t| \sum_{k\ell} \vec{k}_k \cdot \vec{k}_\ell + \mathcal{O}(\vec{k}_{\max} L)\right)$$

$\Longrightarrow$ breaking of scale invariance, crossover in the time dependence

Two-point correlation function at large wave numbers
Small delays: random sweeping effect

$$C(t,\vec{k}) = \underbrace{\frac{\epsilon^{2/3}}{k^{11/3}} H(\epsilon^{1/3} k^{2/3} t)}_{\text{scaling form (z=2/3)}} \underbrace{\exp\left(-\alpha_0 (\epsilon L)^{2/3} k^2 t^2\right)}_{\text{scale dependence (z=1)}}$$

random sweeping effect

- early predictions:

Kraichnan (1959), Tennekes (1975)

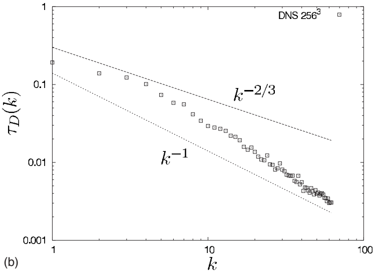
- frequency energy spectrum

$$E(\omega) \propto \omega^{-5/3}$$

$\neq$ standard scaling theory

with $z = 2/3 \Longrightarrow E(\omega) \propto \omega^{-2}$

Chevillard *et al*, Phys. Rev. Lett. **95** (2005)

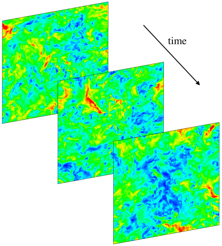


Favier, Godeferd, Cambon,

Phys. Fluids **22** (2010)

Two-point correlation function at large wave numbers
Small delays: random sweeping effect

numerical data



- our simulations

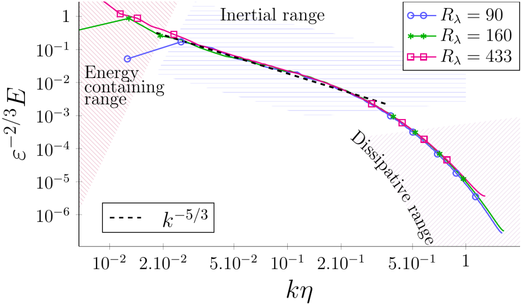
based on pseudo-spectral code

Lagaert, Balarac, Cottet,
J. Comp. Phys. **260** (2014)

- JHTBD

Johns Hopkins TurBulence Database

http://turbulence.pha.jhu.edu/



LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

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Small delays: random sweeping effect

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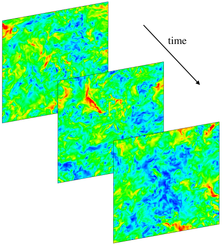
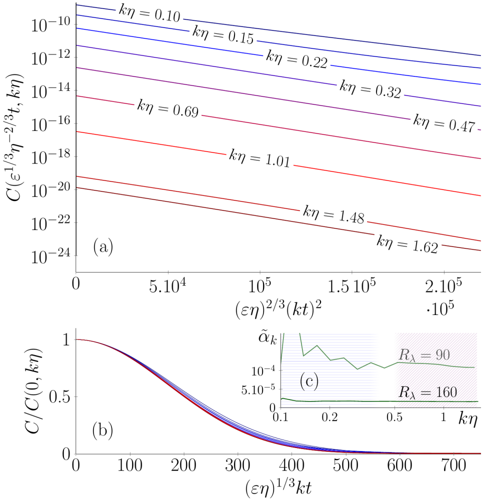


analytical prediction

$$C(t,k) \propto \frac{\exp(-\alpha_0 k^2 t^2)}{k^{11/3}}$$



LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

Two-point Correlation function at large wave numbers
Large delays: another breaking of scale invariance

$$C(t,\vec{k}) = \underbrace{\frac{\epsilon^{2/3}}{k^{11/3}} H(\epsilon^{1/3}k^{2/3}t)}_{\text{scaling form}} \underbrace{\exp\left(-\alpha_\infty \epsilon^{1/3} L^{4/3} k^2|t|\right)}_{\text{scale dependence}}$$

breaking of scale invariance

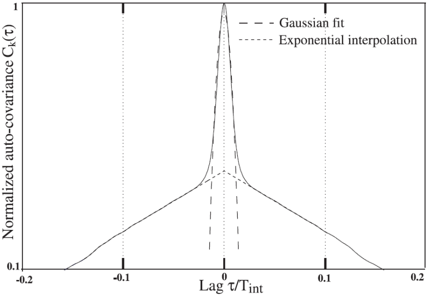
- different form than random sweeping

crossover from k^2t^2 to $k^2|t|$

- hints of this crossover in experiments

Poulain, Mazellier, Chevillard, Gagne, Baudet, Eur. Phys. J. B **53** (2006)

turbulent air jet



Time dependence of generic n-point functions

at small time delays

$$G^{(n)}_{\alpha_1\ldots\alpha_n}(\{t_i,\vec{k}_i\}) = \rho_1^{-d_G} H^0_{\alpha_1\ldots\alpha_n}\big(\{\rho_1^{2/3} t_i, \hat{\rho}_i\}\big) \times \exp\Big(-\alpha_0 L^{2/3} \big|\sum_\ell \vec{k}_\ell t_\ell\big|^2 + \mathcal{O}(\vec{k}_{\max} L)\Big)$$

at large time delays

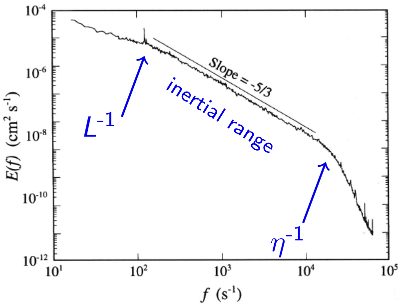
$$G^{(n)}_{\alpha_1\ldots\alpha_n}(t,\{\vec{k}_i\}) = \varrho_1^{-d_G} H^\infty_{\alpha_1\ldots\alpha_n}\big(\varrho_1^{2/3} t, \{\hat{\varrho}_i\}\big) \times \exp\Big(-\alpha_\infty L^{4/3} |t| \sum_{k\ell} \vec{k}_k \cdot \vec{k}_\ell + \mathcal{O}(\vec{k}_{\max} L)\Big)$$

Energy spectrum in the dissipative range : theory

universal behaviour of the solution in the dissipative range

- kinetic energy spectrum

$$E(k) \propto \frac{\epsilon^{2/3}}{k^{5/3}} F(\eta k)$$



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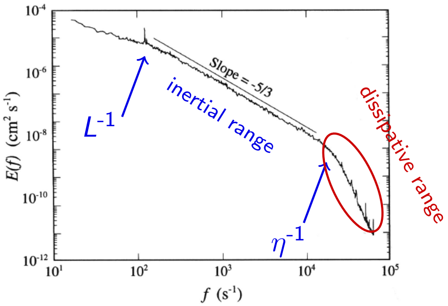


Energy spectrum in the dissipative range : theory

universal behaviour of the solution in the dissipative range

regime of $k \gg \kappa$, $t \to 0$, but existence of a finite scale η

assume that scaling variable saturates $tk^{2/3} \to \epsilon^{1/3}\tau_K/L^{2/3} = (\eta/L)^{2/3}$

- kinetic energy spectrum

$$E(k) \propto \frac{\epsilon^{2/3}}{k^{5/3}} \exp\left[-\mu(\eta k)^{2/3}\right]$$

▶ valid for large $k \gg L^{-1}$ but controlled by the fixed point

at very small scales, regularisation by the viscosity
$\Longrightarrow$ simple exponential decay

Energy spectrum in the dissipative range : theory

▷ several empirical propositions $\exp[-ck^\gamma]$ with $\gamma = 1/2, 1, 3/2,\ 4/3,\ 2, \ldots$

Monin and Yaglom, *Statistical Fluid Mechanics: Mechanics of Turbulence* (1973)

▷ early theoretical arguments advocated $\gamma = 1$

Kraichnan, Fluid Mech. **5** (1958), Foias et al. Phys. Fluids A **2** (1990) She, Jackson Phys. Fluids A **5** (1992)

▷ numerical studies

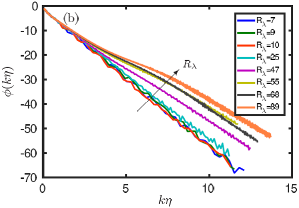
Martinez, Kraichnan et al., J. Plasma Phys. **57** (1997)

Sreenivasan, Antonia, Annu. Rev. Fluid Mech. **29** (1997)

Ishihara, Gotoh, Kaneda, Annu. Rev. Fluid Mech. **41** (2009)

Schumacher, EPL **80** (2007)

Khurshid, Donzis, Sreenivasan, Phys. Rev. Fluids **3** (2018)

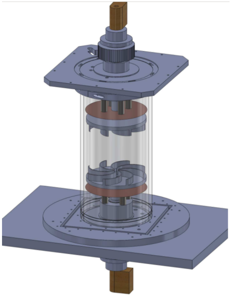


two regimes : $\begin{cases} \text{Near Dissipative Range with} & \exp[-ck^\gamma], \quad \gamma < 1 \\ \text{Far Dissipative Range with} & \exp[-bk] \end{cases}$

Energy spectrum in the dissipative range : experiments

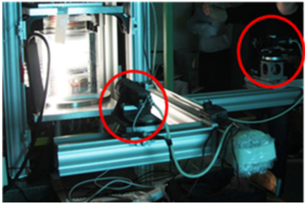
SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow



PhD Brice Saint-Michel (2013)

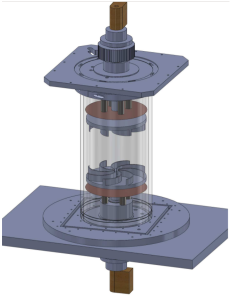
PIV: particle image velocimetry



Energy spectrum in the dissipative range : experiments

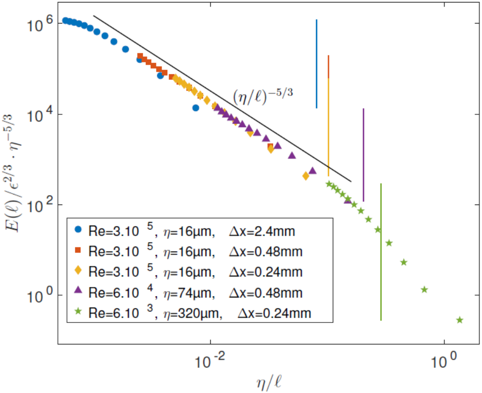
SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow



PhD Brice Saint-Michel (2013)

kinetic energy spectrum



PhD Paul Dubue (in preparation)

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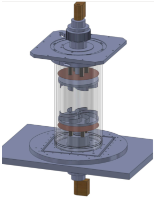
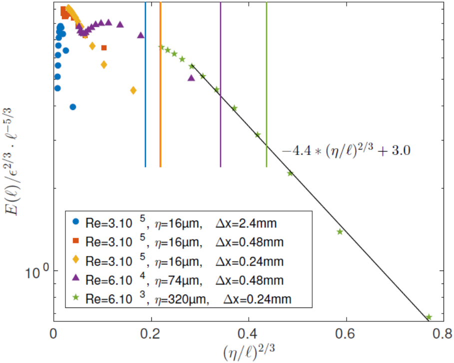
SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow



PhD Brice Saint-Michel (2013)

kinetic energy spectrum



Dubue, Kuzzay, Saw, Daviaud, Dubrulle, LC, Rossetto (2017)

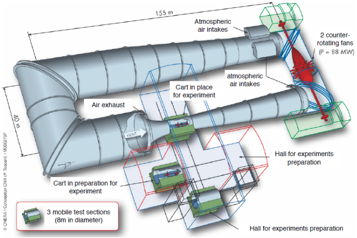
analytical prediction

$$E(k) \propto \frac{\exp(-\mu(\eta k)^{2/3})}{k^{5/3}}$$

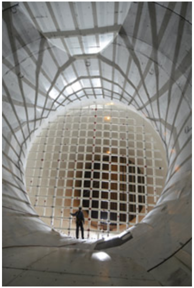
Energy spectrum in the dissipative range : experiments

ONERA S1MA wind tunnel Modane (ESWIRP European project)

ONERA wind tunnel facility



grid from the ESWIRP project

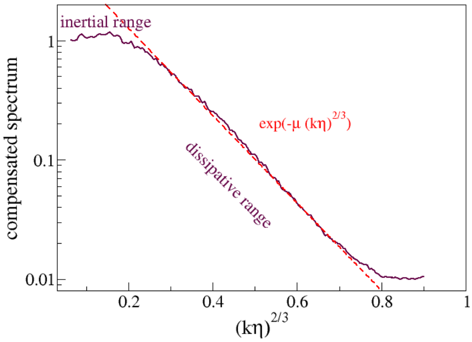


from ONERA website

M. Bourgoin *et al.*, CEAS Aeronautical Journal (2017)

Energy spectrum in the dissipative range : experiments

ONERA S1MA wind tunnel Modane (ESWIRP European project)



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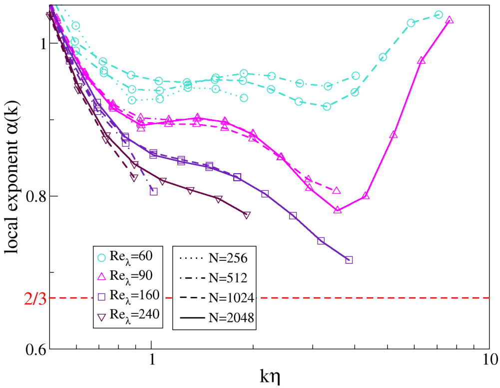
A. Gorbunova, G. Balarac, M. Bourgoin, LC, N. Mordant, V. Rossetto, *in preparation* (2019)

Energy spectrum in the dissipative range : DNS

theoretical prediction: $E(k) \propto \frac{\exp(-\mu k^{\alpha})}{k^{5/3}}$ with $\alpha = 2/3$.

- local determination of the exponent: $\alpha = \frac{d \ln}{d \ln k} \left| \frac{d \ln E(k)}{d \ln k} \right|$



Energy spectrum in the dissipative range : experiments

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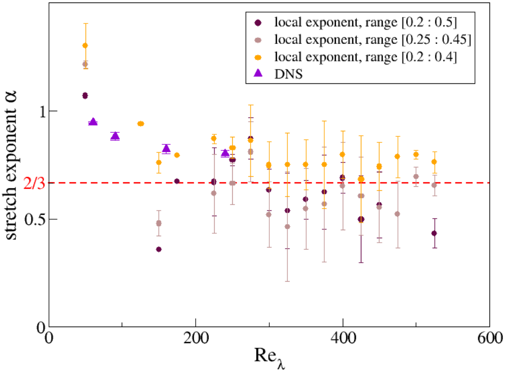

Energy spectrum in the dissipative range : experiments

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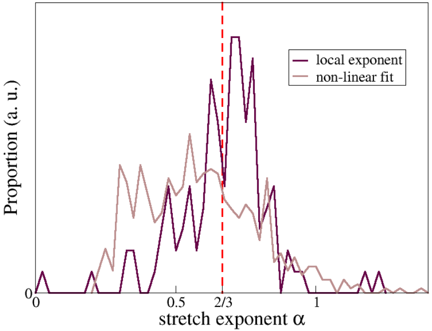

$\alpha = 0.68 \pm 0.09$

Summary and perspectives

Summary

- closure of NPRG flow equations based on symmetries exact in the limit of large wave numbers
- analytical form of *n*-point correlation functions

$\longrightarrow$ leading time-dependence in 3D
$\longrightarrow$ violation of scale invariance

Other results

- kinetic energy spectrum in the dissipative range
- 2D: leading time-dependence of *n*-point correlation functions
- 2D: next-to-leading order in the direct cascade

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Summary and perspectives

Summary

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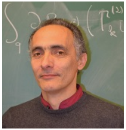
Perspectives

- test of NPRG predictions in simulations and experiments
- intermittency exponents
  $\longrightarrow$ calculation of NLO terms at large wave-numbers
  $\longrightarrow$ passively advected scalars (Kraichnan model)
  $\longrightarrow$ Burgers' turbulence

In collaboration with ...



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Thank you for attention !