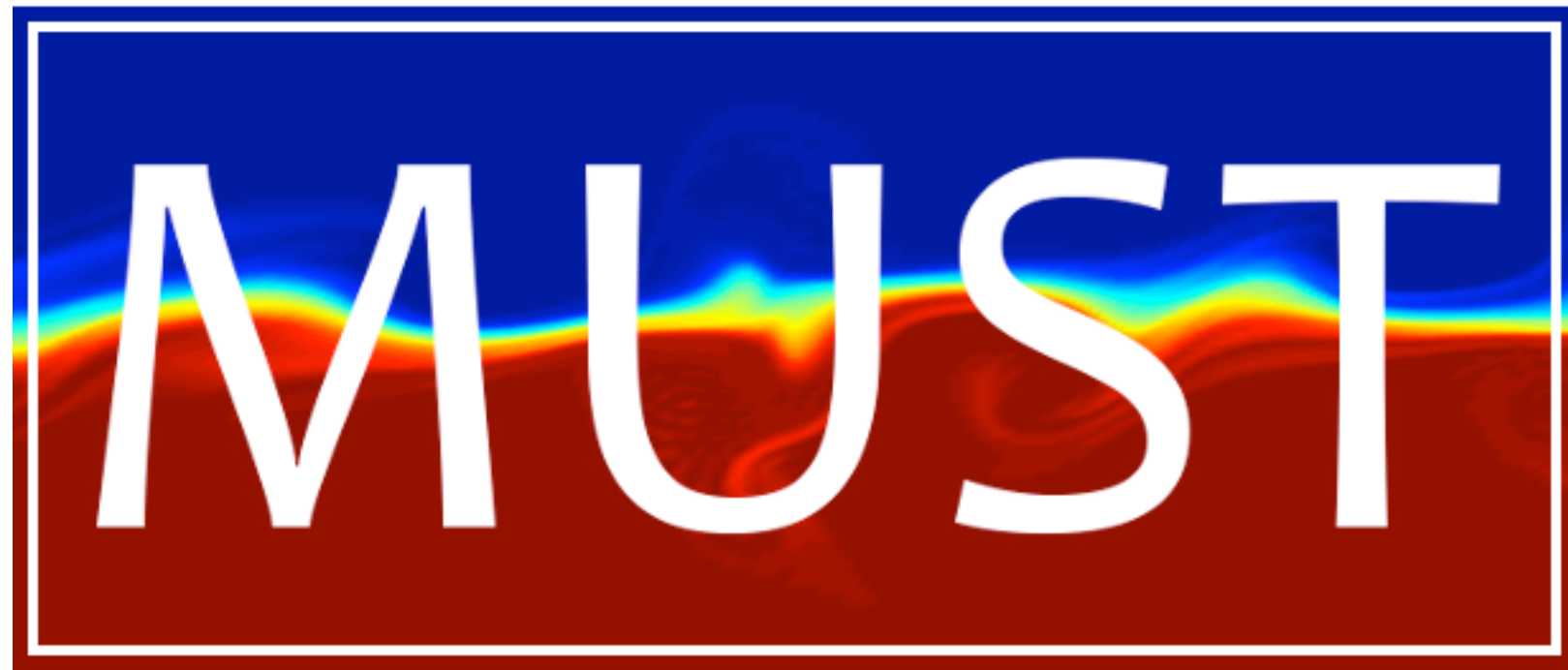


Turbulence in Stratified Fluids

Do we even know what we don't know?

C. P. Caulfield

BP Institute & DAMTP, University of Cambridge



Les Houches April 2019



Or...

(At least) 5 ways in which
Stratified Turbulence is like Brexit

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I. Nobody understands what is happening

(At least) 5 ways in which **Stratified Turbulence** is like **Brexit**

1. Nobody understands what is happening
2. Brexit means Brexit = ST means ST...

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(At least) 5 ways in which **Stratified Turbulence** is like **Brexit**

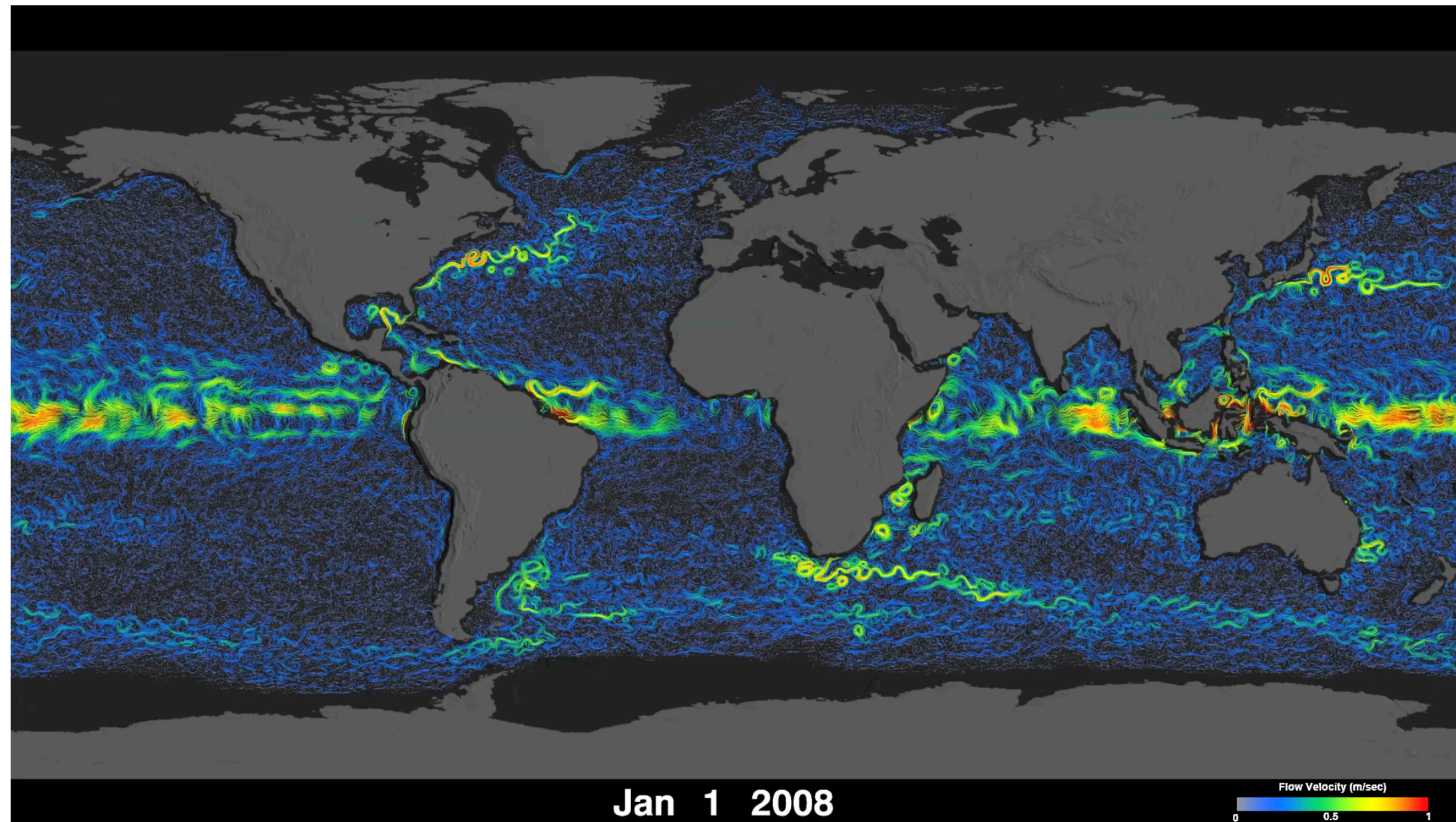
1. Nobody understands what is happening
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3. What's happening near boundaries is most important
4. Being free is very different from being forced

(At least) 5 ways in which **Stratified Turbulence** is like **Brexit**

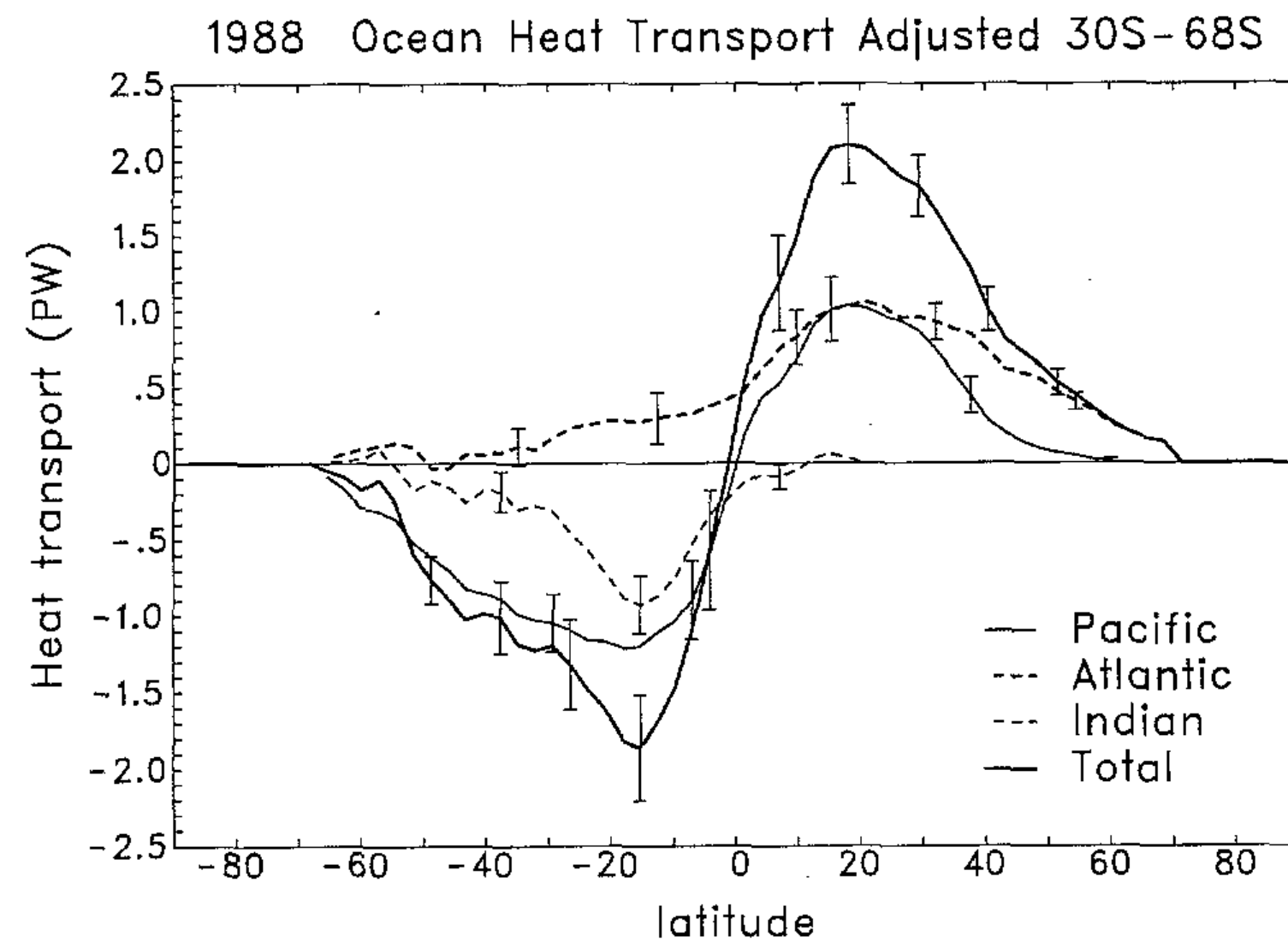
1. Nobody understands what is happening
2. Brexit means Brexit = ST means ST...
3. What's happening near boundaries is most important
4. Being free is very different from being forced
5. History REALLY matters

Motivation: Saving the Planet?

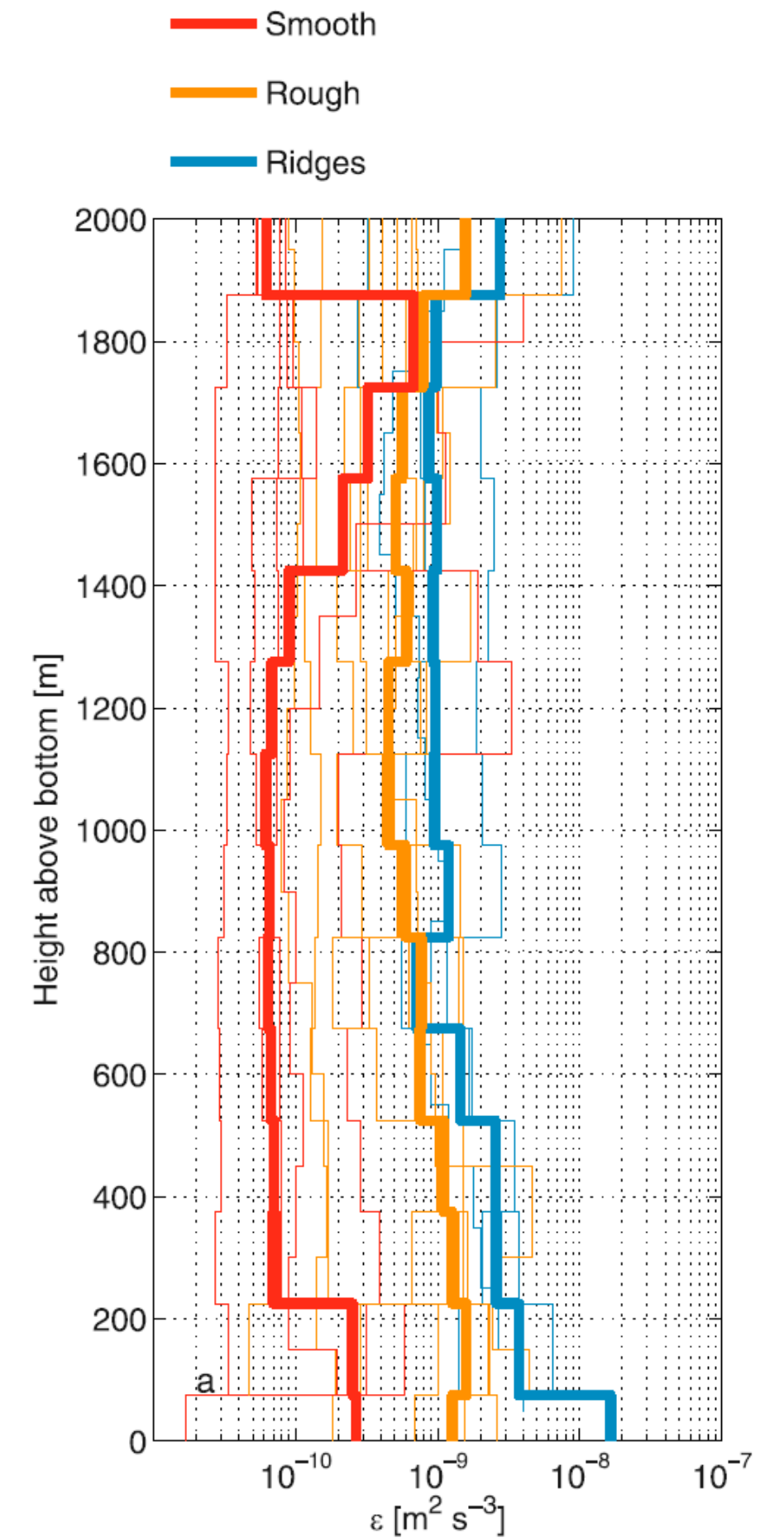
- (Vertical) heat transport is (in the top 3 of) the most uncertain parameterisation(s) in climate science
- Models @1-10km scale: mixing @ 1cm scale: 5-6 orders of magnitude...



OSCAR (Dohan)



Trenberth 1994



Waterhouse et al 2016

- 1-2 PW of poleward heat transport...turbulence in stratified fluids key
- Parameterisation and relevance to MOC?

Motivation: Saving the Planet?

- Although the parameterisation is uncertain in truth, (up/down?) it is a very hot and urgent topic...

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Oceans

Global warming of oceans equivalent to an atomic bomb per second

Seas absorb 90% of climate change's energy as new research reveals vast heating over past 150 years



▲ An Argo float is deployed into the ocean. Photograph: CSIRO

Global warming has heated the oceans by the equivalent of one atomic bomb explosion per second for the past 150 years, according to analysis of new research.

Damian Carrington
Environment editor

@dpcarrington

Mon 7 Jan 2019 20.00 GMT

f t e

17k

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Climate Consensus - the 97% Climate change

Our oceans broke heat records in 2018 and the consequences are catastrophic

Rising temperatures can be charted back to the late 1950s, and the last five years were the five hottest on record



▲ Bleached coral in Guam. The heating of oceans is causing tremendous problems for sea life. Photograph: David Burdick/AP

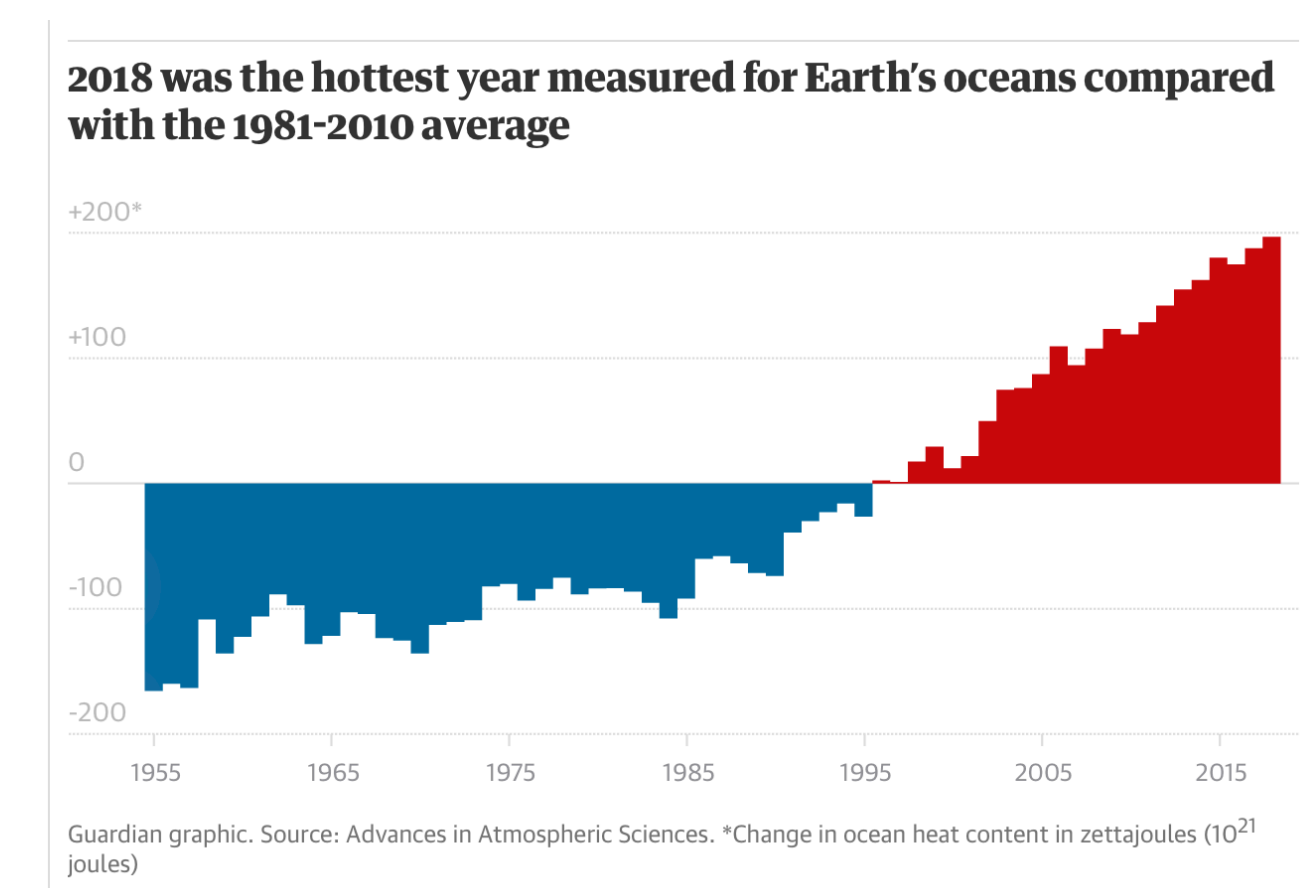
Last year was the hottest ever measured, continuing an upward trend that is a direct result of manmade greenhouse gas emissions.

John Abraham

Wed 16 Jan 2019 11.00 GMT

f t e

3804



Effects over 50 years:

1. Expansion
2. Ice loss
3. Extreme events
4. Fishing...

Billions globally potentially affected

(Stratified) fluid equations

- Remember the Navier-Stokes equations (remembering all forces) for an incompressible fluid:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P - g\rho \hat{\mathbf{z}} + \mu \nabla^2 \mathbf{u}; \quad \nabla \cdot \mathbf{u} = 0$$

- Usually** we just say the density is constant, and absorb the **hydrostatic** component into the pressure, divide across by the density and carry on serenely without worrying about density:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \left(\frac{p}{\rho} \right) + \nu \nabla^2 \mathbf{u}; \quad \nabla \cdot \mathbf{u} = 0 \quad P = P_h(\mathbf{z}) + p(\mathbf{x}, t); \quad \frac{dP_h}{dz} = -g\rho$$

- When the density is constant, the hydrostatic pressure gradient is constant
- But we want to consider a situation where the density is a function of space and time
- We can still subtract off a hydrostatic component if we decompose the density field into a horizontally (and temporally) averaged hydrostatic part (which can still depend on z if req.):

$$P = P_h(\mathbf{z}) + p'(\mathbf{x}, t); \rho = \rho_h(\mathbf{z}) + \rho'(\mathbf{x}, t); \frac{dP_h}{dz} = -g\rho_h; \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p' - \frac{g\rho'}{\rho} \hat{\mathbf{z}} + \nu \nabla^2 \mathbf{u}; \quad \nabla \cdot \mathbf{u} = 0$$

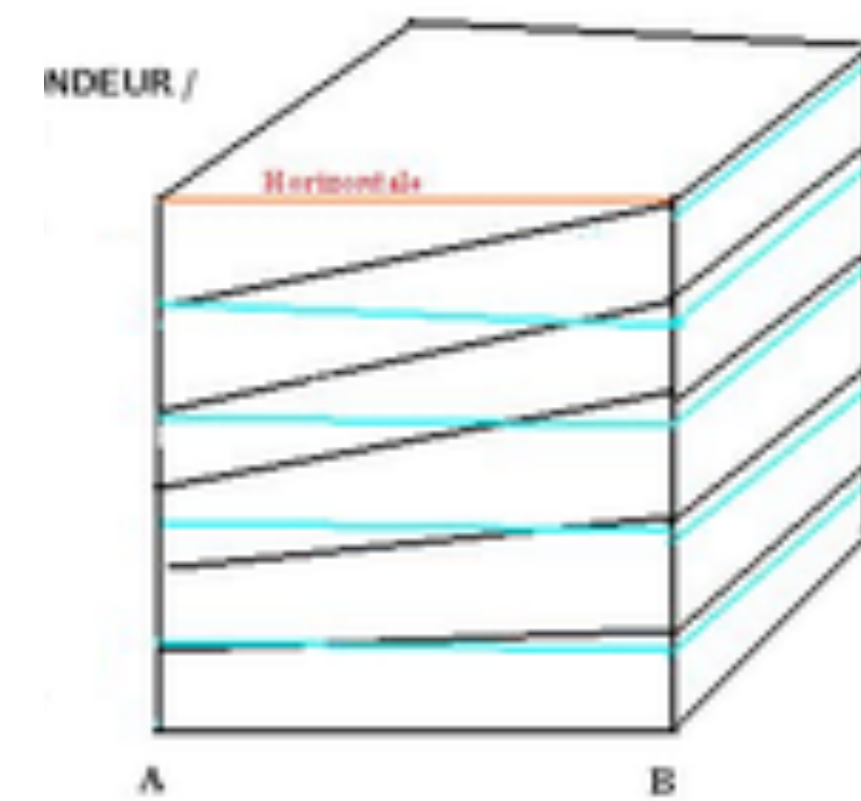
- Now we have a **buoyancy force**: eg locally relatively dense fluid will be accelerated downwards!

(Stratified) vorticity equation

- If the density is not constant, the vorticity equation is also very different:

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \omega \cdot \nabla \mathbf{u} + \frac{1}{\rho^2} \nabla \rho \times \nabla P + \nu \nabla^2 \omega; \quad \nabla \times \mathbf{u} = \omega$$

- Extra** source of vorticity if pressure and density gradients are not parallel
- This **baroclinic torque** has a particularly simple form under the **Boussinesq approximation**
- Because density differences in the atmosphere and ocean are often of the order of a few percent (e.g salt water is roughly 3% more dense than fresh water, and every 10 degrees celsius changes the density of air by approximately 4%) the effect of density variations on a fluid's inertia is relatively small (while the buoyancy force remains important).



wikipedia

- Assume the density is constant except in the buoyancy force, equivalent to a distinguished limit:

$$g \rightarrow \infty; \quad \rho' \rightarrow 0; \quad g\rho' \text{ remains finite} \quad \rho = \rho_0[1 - \alpha(T - T_0)] \rightarrow \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = \kappa \nabla^2 \rho$$

- Also linear equation of state implies density satisfies advection **diffusion** equation...

The Boussinesq approximation

- Valid when the scales of the motion \ll scales the density of the fluid varies substantially
- Valid over scales of 100s of metres in the ocean, and $< O(\text{km})$ in the atmosphere
- Important exceptions are: fires, explosions, avalanches, volcanic eruptions etc...

- But under the Boussinesq approximation, the N-S, vorticity/density equations become:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_0} \nabla p' - g' \hat{\mathbf{z}} + \nu \nabla^2 \mathbf{u}; \quad g' = \frac{g \rho'}{\rho_0};$$

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \omega \cdot \nabla \mathbf{u} + \frac{g}{\rho_0} \hat{\mathbf{z}} \times \nabla \rho + \nu \nabla^2 \omega; \quad \frac{\partial(\rho \mathbf{z})}{\partial t} + \mathbf{u} \cdot \nabla(\rho \mathbf{z}) = \rho w + \kappa \mathbf{z} \nabla^2 \rho$$

- Here ρ_0 is some reference density and g' is called the **reduced gravity**: $\rho_0 \gg \rho'$
- Notice symmetry of new terms involving density and vertical velocity: coupling KE and PE
- Lead to key term the **buoyancy flux** and extra sink “tax” of energy in stratified flow: $\mathcal{B} \equiv \frac{g}{\rho_0} \langle w' \rho' \rangle$

Mixing parameterisation

- Central question: how to parameterise vertical diffusivity of heat:

- Classic model due to Osborn (1980):
$$\Gamma \equiv \frac{\frac{g}{\rho_0} \langle w' \rho' \rangle}{2\nu s'_{ij} s'_{ij}} = \frac{\mathcal{B}}{\mathcal{E}} \leq 0.2$$

- Determines eddy diffusivity:

$$\kappa_T = \frac{\langle w' \rho' \rangle}{|\partial \bar{\rho} / \partial z|} = \frac{\mathcal{B}}{N^2} = \Gamma \frac{\mathcal{E}}{N^2}$$

- Also how PE changes **tax** KE changes

$$= \nu \Gamma \left(\frac{\mathcal{E}}{\nu N^2} \right) = \nu \Gamma Re_B$$

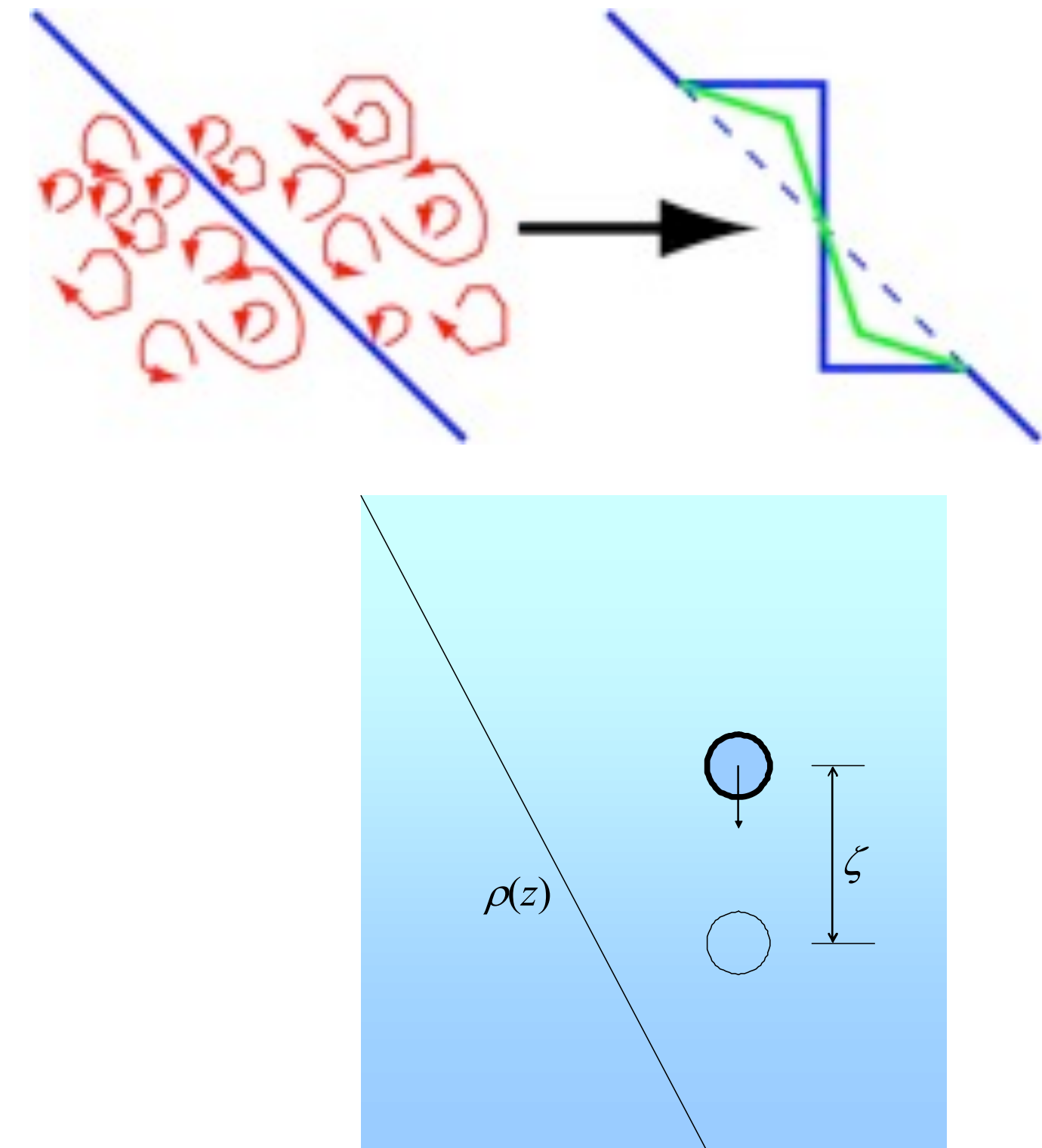
- Turbulent kinetic energy equation: $\frac{d}{dt} \mathcal{K} = \mathcal{P} - \mathcal{B} - \mathcal{E}; \mathcal{P} \equiv -\langle u' w' \rangle \frac{\partial \bar{u}}{\partial z}$

- How is production (due to forcing/IVP etc...) partitioned between buoyancy flux and dissipation?

- Many, many open questions (order one not agreed):

1. Can anything generic be said about κ_T that improves modelling?

2. What does Γ depend on?



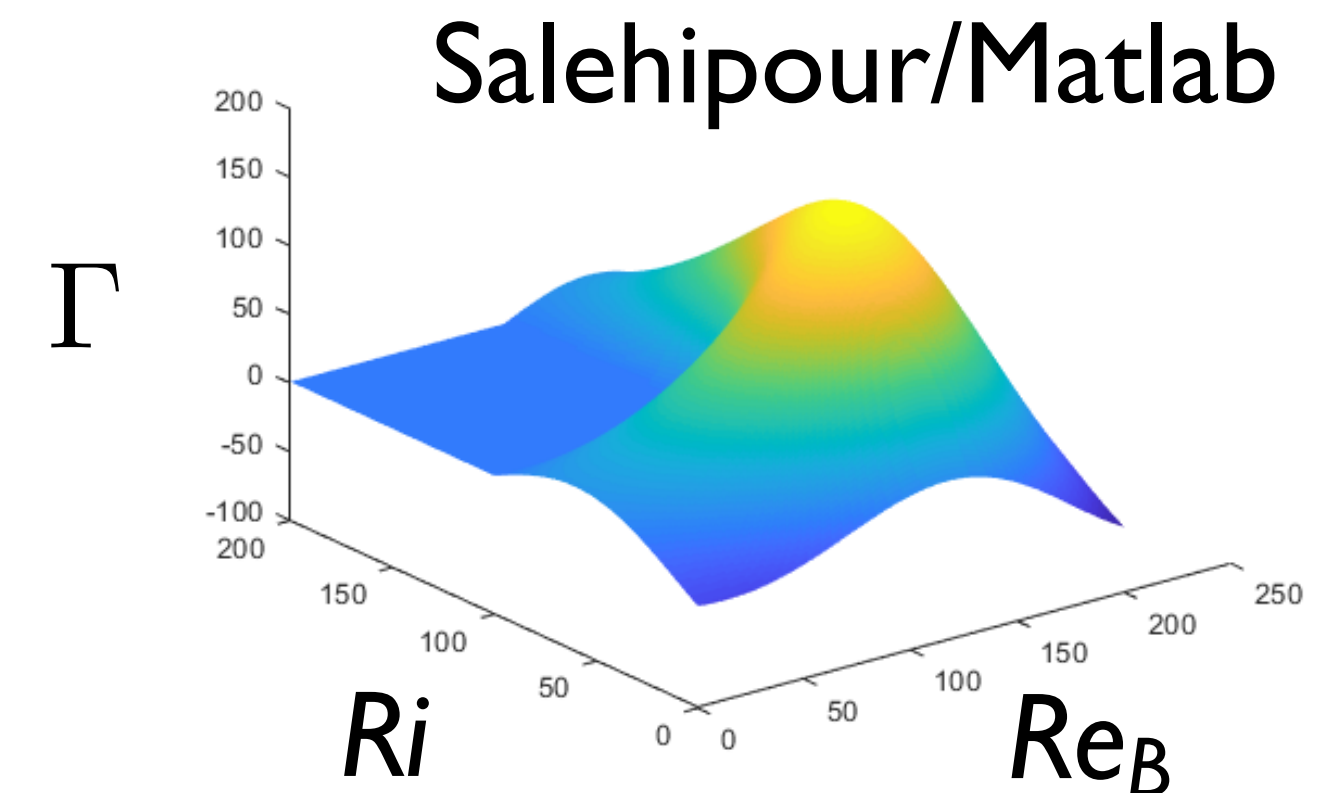
$$\mathcal{K} = \frac{1}{2} \langle u^2 + v^2 + w^2 \rangle, \quad \mathcal{P} = \frac{g}{\rho_0} \langle \rho z \rangle$$

$$\begin{aligned} \frac{d}{dt} \mathcal{K} &= \mathcal{F} - \frac{g}{\rho_0} \langle w' \rho' \rangle - \mathcal{E} \\ &= \mathcal{F} - \mathcal{B} - \mathcal{E} \end{aligned}$$

$$\frac{d}{dt} \mathcal{P} = +\mathcal{B} + \Phi_D$$

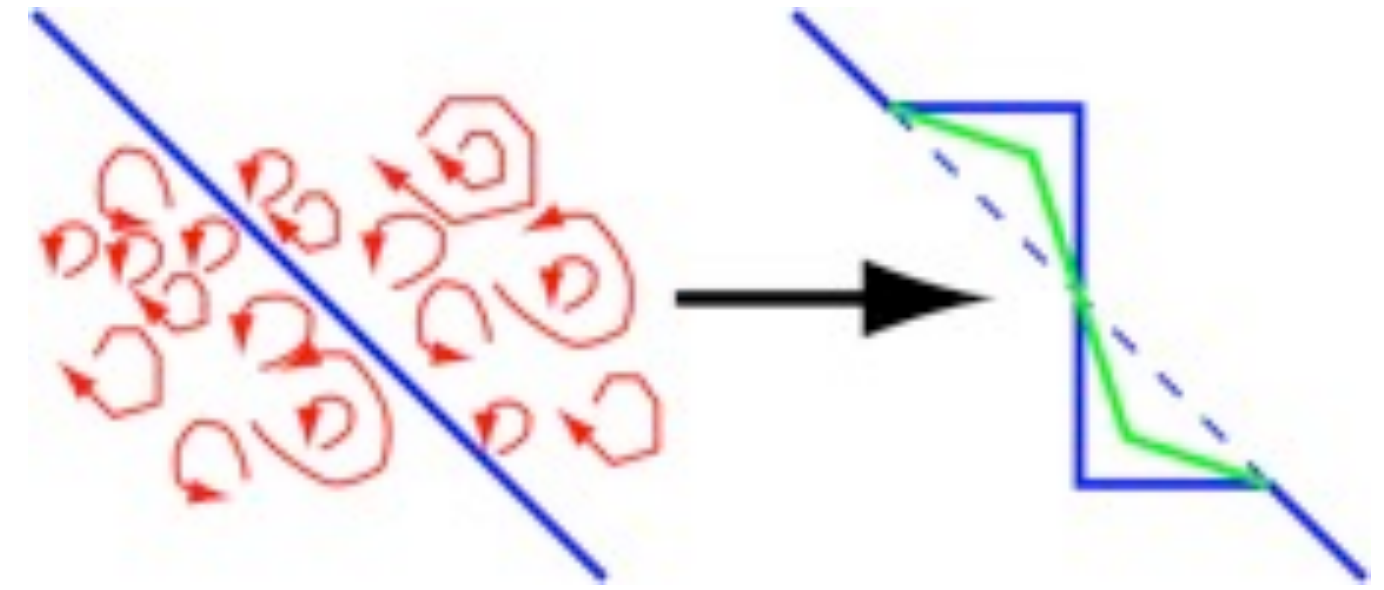
Stratified Turbulence: Length Scales

- Fundamentally, there are (at least) two levels of challenge:
 1. In a “perfect” world (e.g. in numerical simulations/lab) what is needed to describe ST?
 2. In the “real” world (e.g. in the ocean/lab) can what is measurable be useful?
- Central concerns:
 1. What is meant by ST needs to be defined carefully
 2. What is used to parameterise ST needs to be defined carefully
- Gathering evidence (e.g. Ellison/Linden/Thorpe/Smyth/Ivey/Maffioli/Venayagamoorthy/Venaille etc)
- Length and time scales play key roles (as do ratios of scales: dimensionless parameters)
- Critical issues involve definitions, and also whether quantities are correlated...



Properties

- (At least) four classes of properties for a stratified turbulent fluid



1. Properties of the **fluid**: ν, κ (also double diffusion, nonlinear eq of state etc etc...)

2. Properties of the **background**: $N^2 \equiv \frac{-g}{\rho_0} \frac{\partial \bar{\rho}}{\partial z}$, $S \equiv \frac{\partial \bar{u}}{\partial z}$ (what does overline mean? Spatio-temporal?)

3. Properties of the **turbulence**: $\mathcal{K} \equiv \frac{1}{2} \langle u'_i \cdot u'_i \rangle$, $\mathcal{E} \equiv 2\nu \langle s'_{ij} s'_{ij} \rangle$, $s'_{ij} \equiv \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)$ (u fluctuation field)

4. Properties of the **density fluctuation field** $\mathcal{K}_\rho \equiv \frac{1}{2} \left\langle \frac{g^2}{\rho_0^2 N^2} (\rho')^2 \right\rangle$, $\chi \equiv \kappa \left\langle \frac{g^2}{\rho_0^2 N^2} |\nabla \rho'|^2 \right\rangle$

- These quantities can be formed into length scales/time scales/nondimensional parameters...

- Is ST a snake/spear/rope/fan/wall/tree? Yes...and no...sometime/where

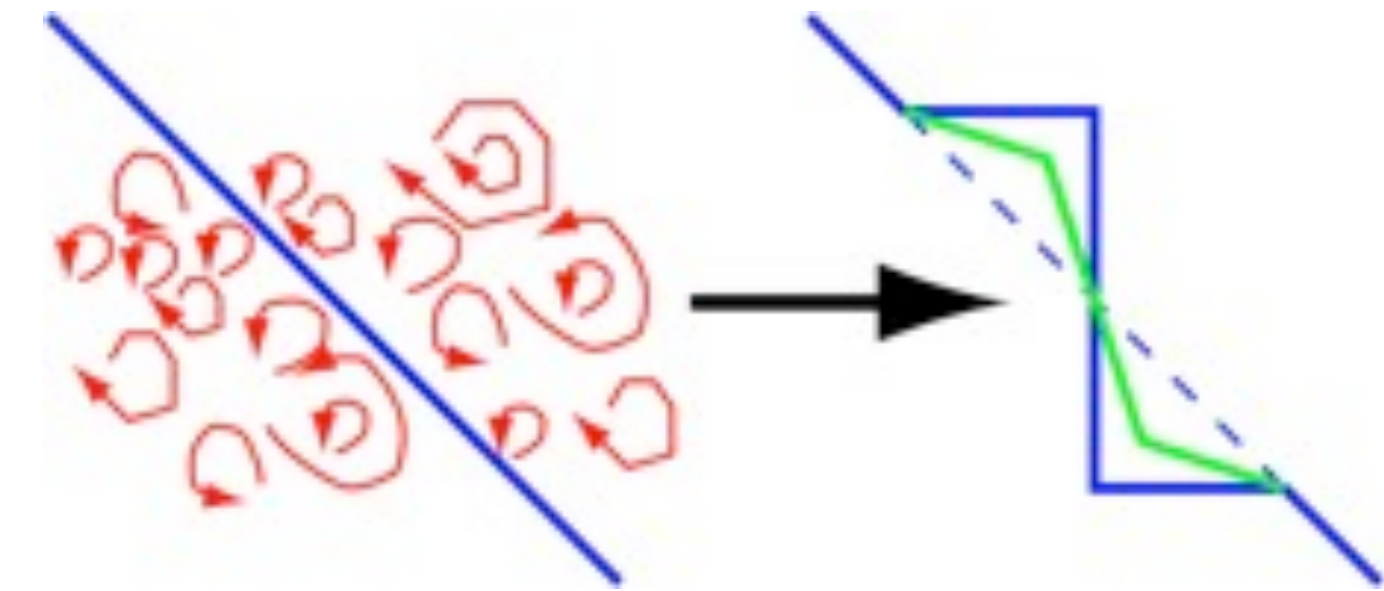
- Can unique parameters be assigned? Are such quantities correlated?

- Does mechanism matter? Does history matter?

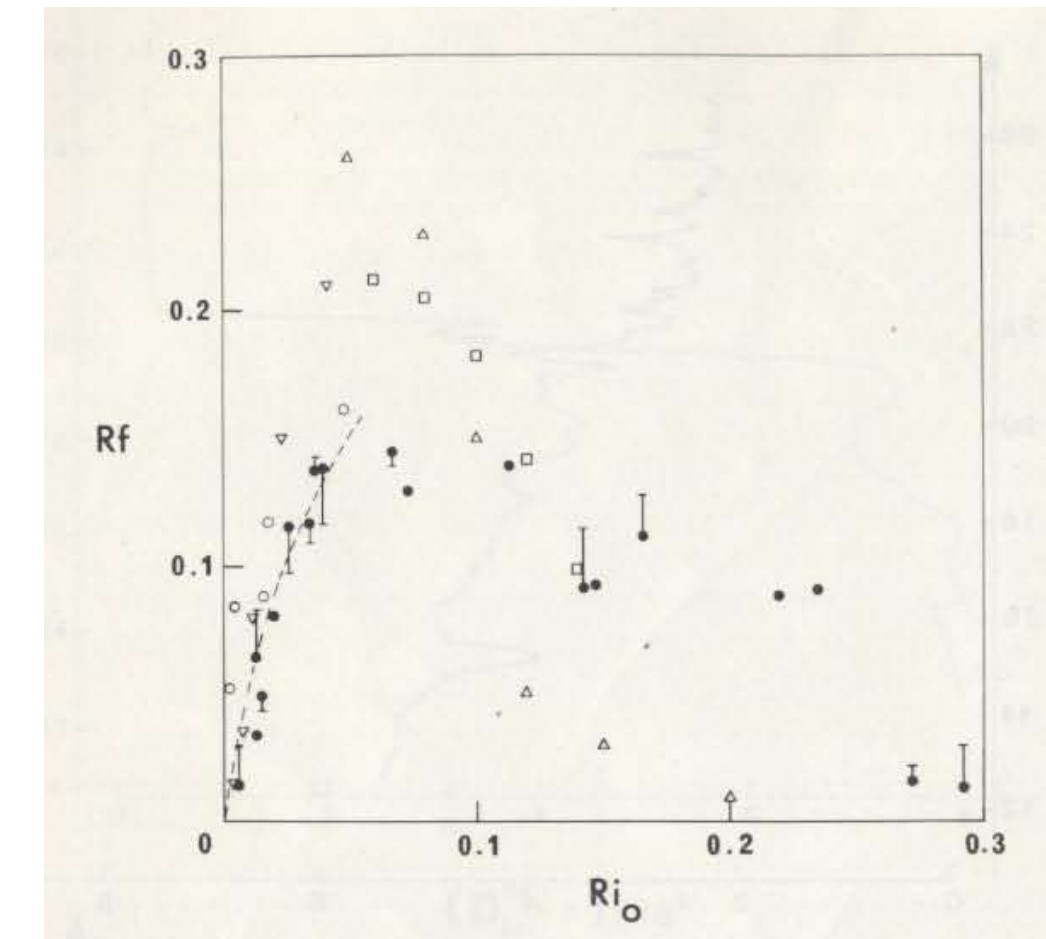


(Some of the) Open Questions

- There are a huge number of open issues (if one is honest)
- Very interesting developments (though not answers) in at least seven interconnected areas:



1. Does Γ vary with parameters and/or mechanism and/or time?
2. If Γ (and flux) does vary with parameters, is it possible to access **right flank**?
3. Does Layered Anisotropic Turbulence of Lindborg/Riley/Chomaz/Billant exist?
4. Is layering (deep well-mixed layers separated by thin sharp interfaces) generic?
5. Is stratified turbulence ever “generic” or does it always remember its time history?
6. Is linear stability theory relevant to turbulent dynamics?
7. Can observational data ever be connected to more complicated models for turbulent flow?



Linden 1979

Or...

3 other ways in which
Stratified Turbulence is like Brexit

3 other ways in which Stratified Turbulence is like Brexit

1. System is unstable IF background is sufficiently weak

3 other ways in which Stratified Turbulence is like Brexit

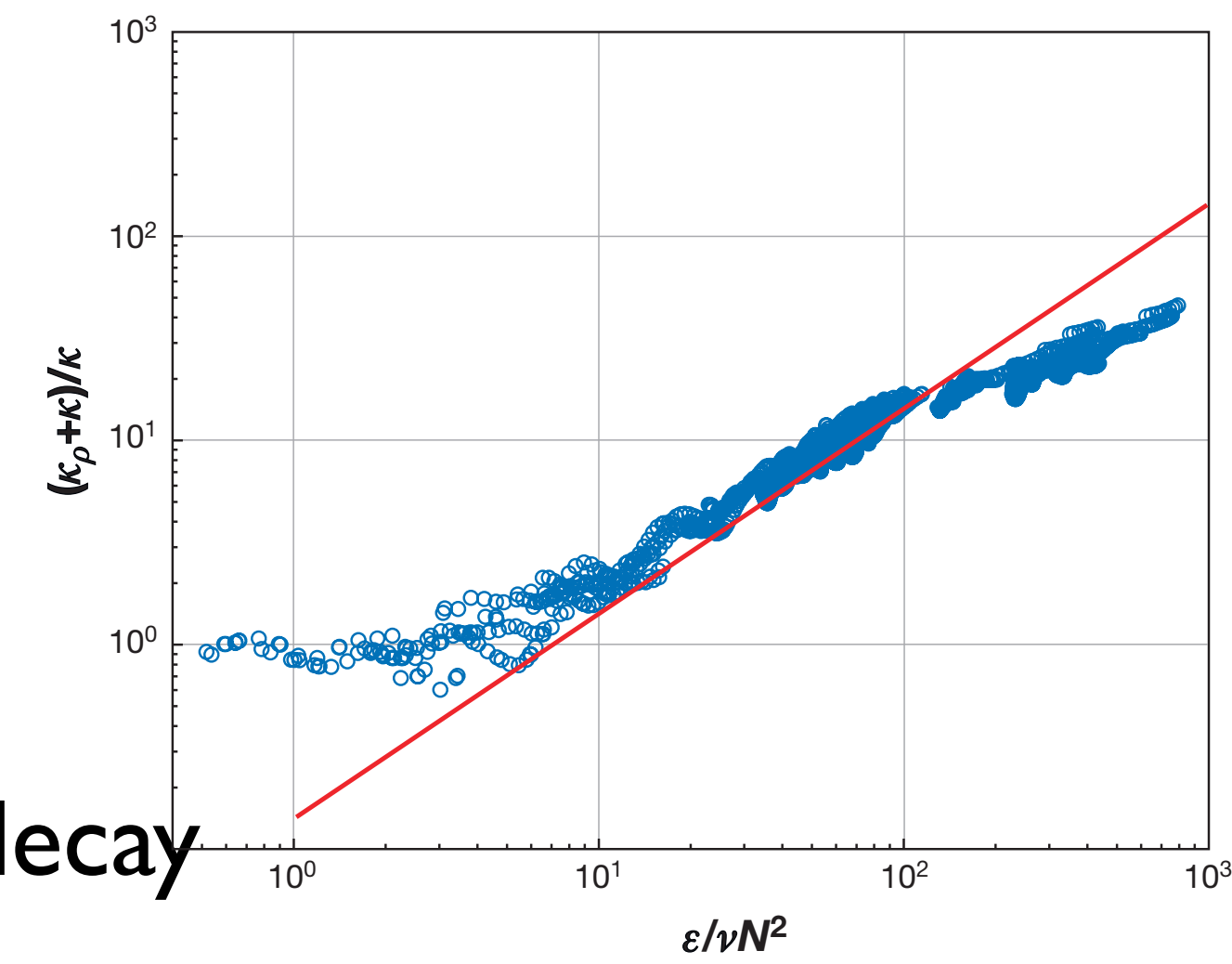
1. System is unstable IF background is sufficiently weak
2. Once it starts, no way to know how long it will last

3 other ways in which Stratified Turbulence is like Brexit

1. System is unstable IF background is sufficiently weak
2. Once it starts, no way to know how long it will last
3. The background changes qualitatively and irreversibly

Variation of Gamma (or not...)

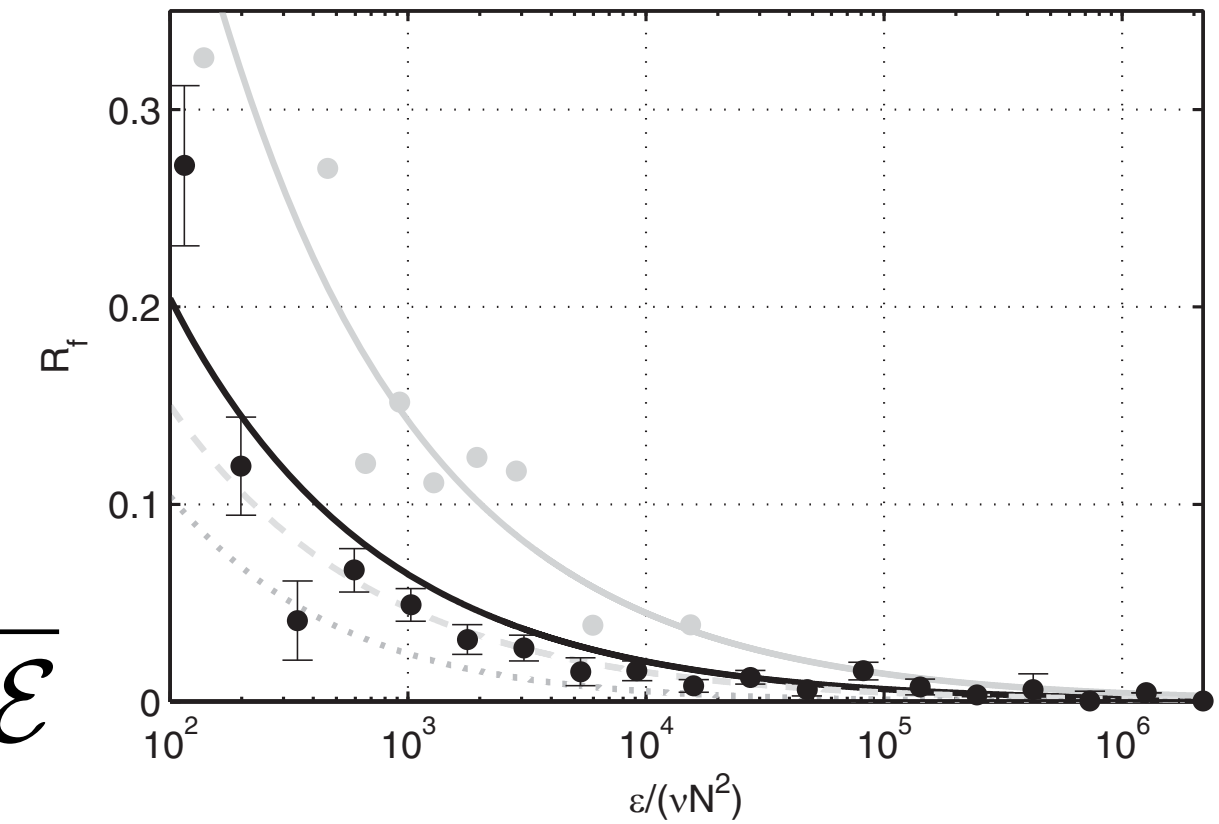
- Deeply influential simulations of Shih et al (2005): **Stanford School**
- Shows: intermediate regime of constant Γ then $\Gamma \propto Re_b^{-1/2} \rightarrow \kappa_T \propto Re_B^{1/2}$
- Also recent observational evidence Monismith et al 2018 consistent with decay



- But remember major issues with averaging/reversibility and:

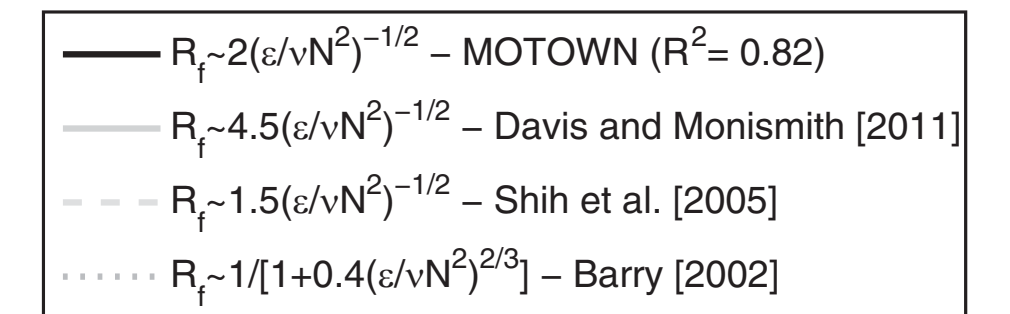
$$\Gamma \equiv \frac{\frac{g}{\rho_0} \langle w' \rho' \rangle}{2\nu s'_{ij} s'_{ij}} = \frac{\mathcal{B}}{\mathcal{E}}, \quad \kappa_T = \frac{\langle w' \rho' \rangle}{|\partial \bar{\rho} / \partial z|} = \frac{\mathcal{B}}{N^2} = \Gamma \frac{\mathcal{E}}{N^2} = \nu \Gamma \left(\frac{\mathcal{E}}{\nu N^2} \right) = \nu \Gamma Re_B$$

- Flow in steady state: $\frac{d}{dt} \mathcal{K} = 0 \rightarrow \mathcal{P} = \mathcal{B} + \mathcal{E} \rightarrow Ri_f \equiv \frac{\mathcal{B}}{\mathcal{P}} = \frac{\Gamma}{1 + \Gamma} \simeq \frac{\chi}{\chi + \mathcal{E}}$



- But remember definition of turbulent Prandtl number:

$$\kappa_T = \frac{\langle \rho' w' \rangle}{-\frac{\partial \bar{\rho}}{\partial z}} = \frac{\mathcal{B}}{N^2}; \quad \nu_T = \frac{-\langle uw \rangle}{\frac{\partial \bar{u}}{\partial z}} = \frac{\mathcal{P}}{(\partial \bar{u} / \partial z)^2} \rightarrow Pr_T \equiv \frac{\nu_T}{\kappa_T} = \frac{Ri}{Ri_f}; \quad Ri \equiv \frac{N^2}{S^2}$$



- So...is $\Gamma(Ri, Re_b)$? Or are Re_b and Ri correlated? Or are there naturally different regimes with Re_b ?

Buoyancy Reynolds number and length scales

- Large buoyancy Reynolds number ensures wide separation between Ozmidov & Kolmogorov scales:

- Gives some chance of isotropic inertial range $Re_b \equiv \frac{\epsilon}{\nu N^2} = \left[\left(\frac{\mathcal{E}}{N^3} \right)^{1/2} \left(\frac{\mathcal{E}}{\nu^3} \right)^{1/4} \right]^{4/3} \equiv \left(\frac{L_0}{L_K} \right)^{4/3}$

- Particularly if Ozmidov scale is ALSO forcing injection scale

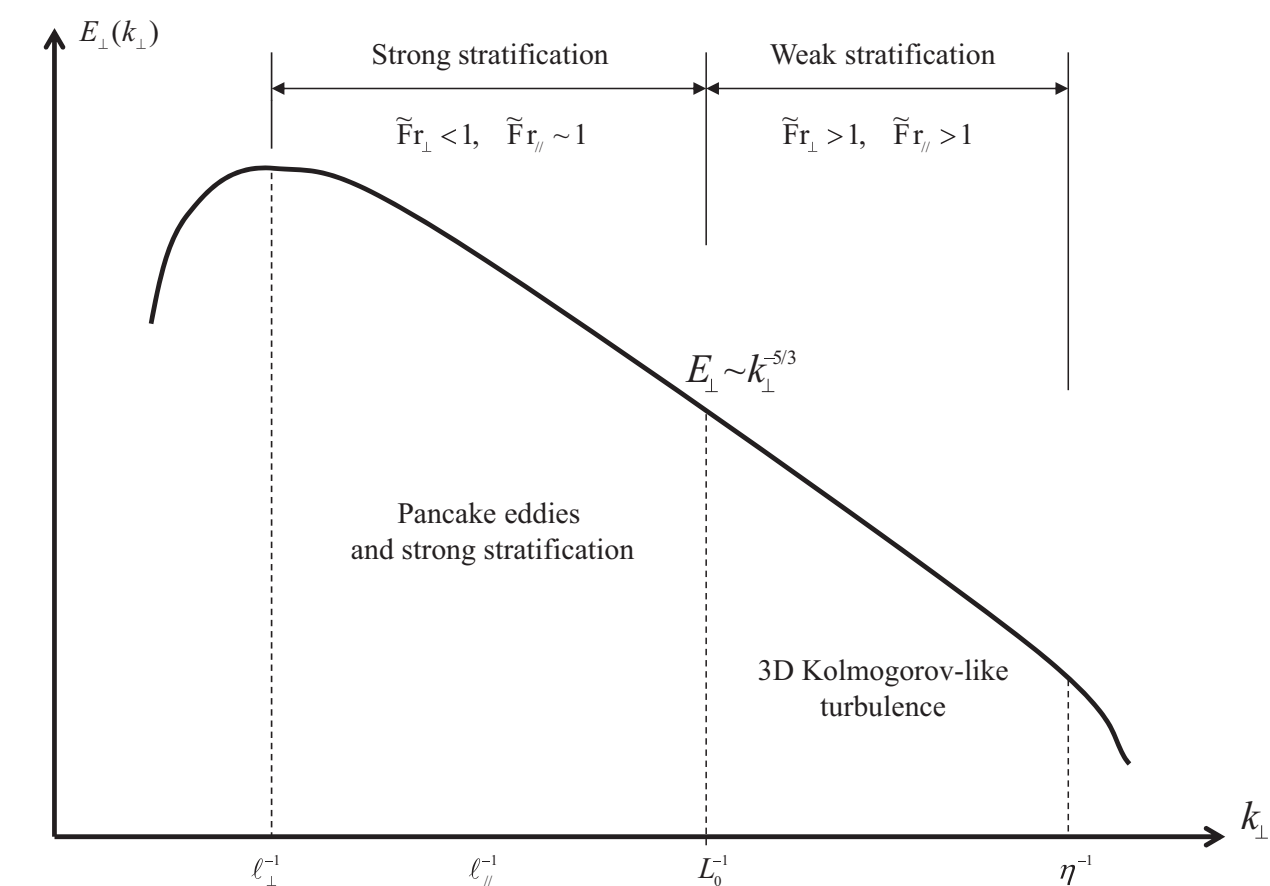
- So two kinds of right flank: strong stratification and/or strong turbulence...

- Layered Anisotropic Stratified Turbulence (LAST) regime: both!

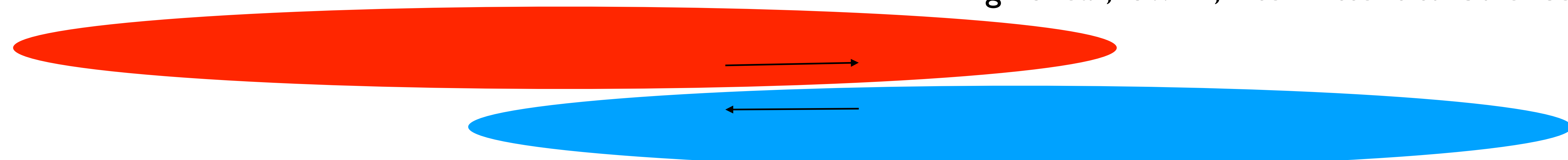
- Scaling arguments of Billant/Chomaz/Lindborg: $L_H \gg L_V \gg L_0 \gg L_K$

$$Re_H \equiv \frac{U_H L_H}{\nu} \gg 1; Fr_H \equiv \frac{U_H}{N L_H} \ll 1; Fr_V \equiv \frac{U_H}{N L_V} \sim 1; \mathcal{E} \sim \frac{U^3}{L_H} \rightarrow Re_H Fr_H^2 \gg 1 \leftrightarrow Re_b \gg 1$$

High shear, low Ri, intermittent turbulence



Davidson 2013

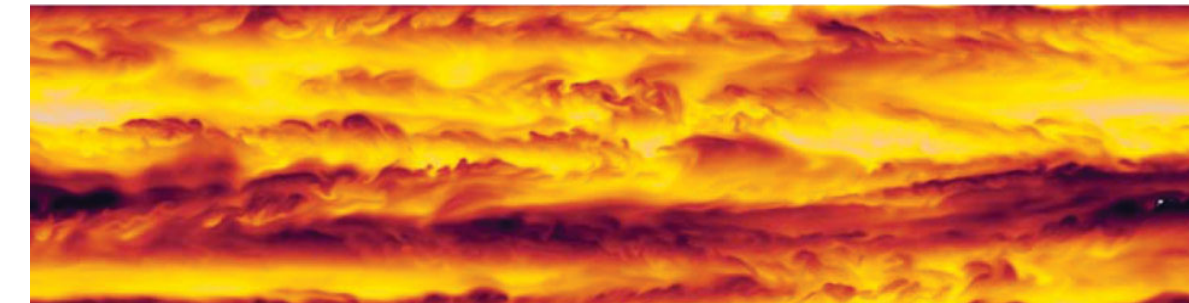


LAST regime?

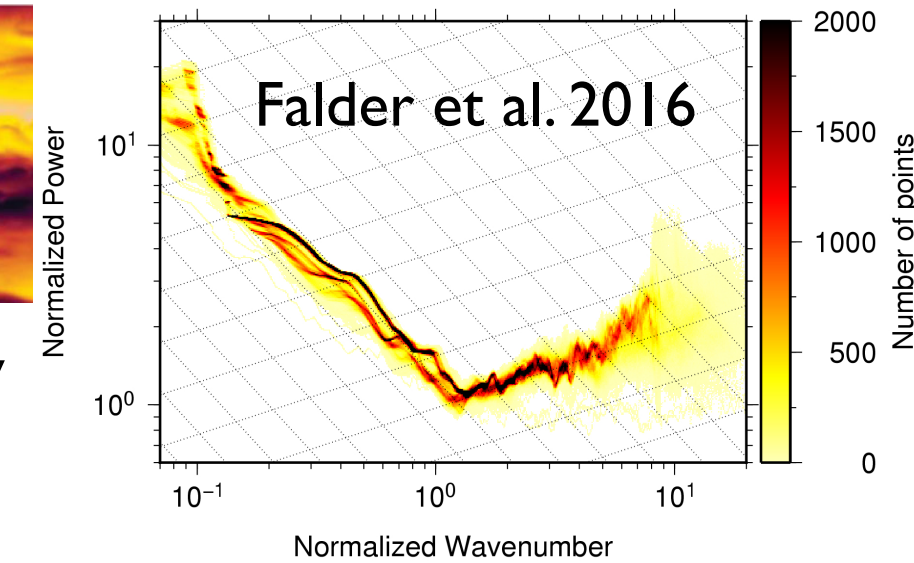
- **Suggestive** numerical evidence that this regime can occur: Brethouwer/Maffioli/Bartello/Tobias etc

- Seismic oceanography gives anisotropic $E(k_H) \sim k_H^{-5/3}$

- But how can such a flow be born/sustained?



Brethouwer et al. 2007



- Numerically can add an artificial body force...

- Instability? Miles-Howard $Ri < 1/4$ somewhere

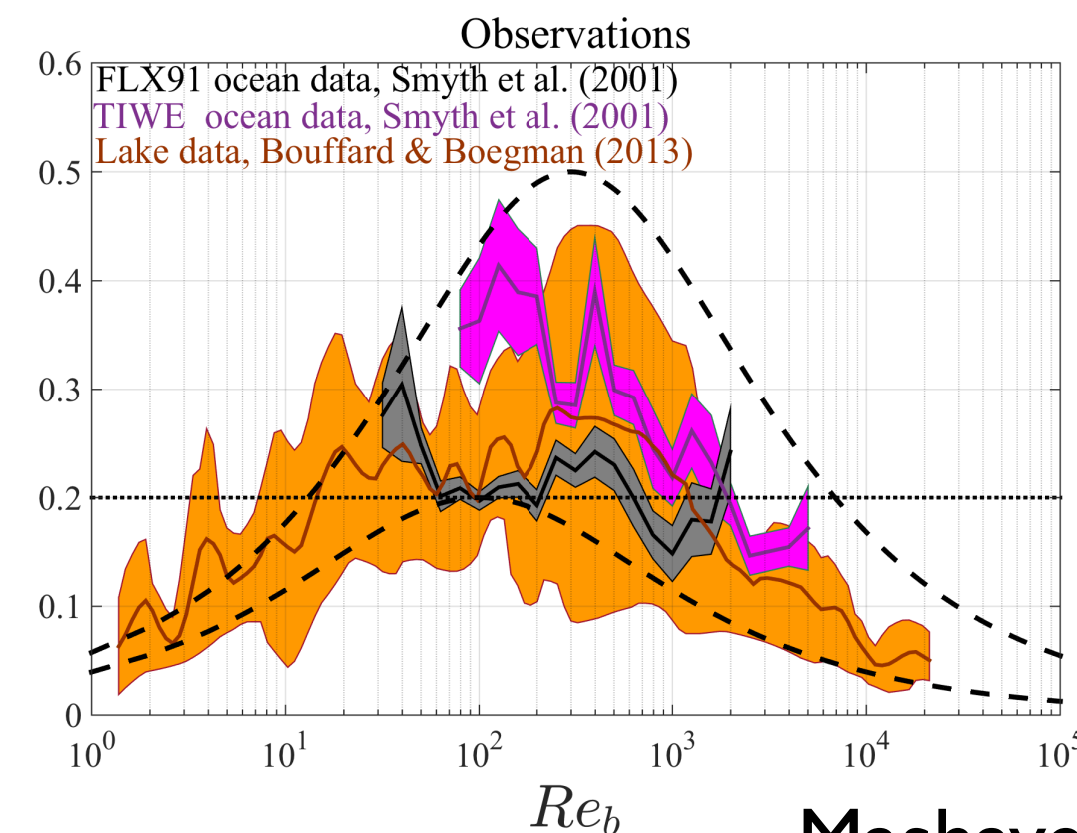
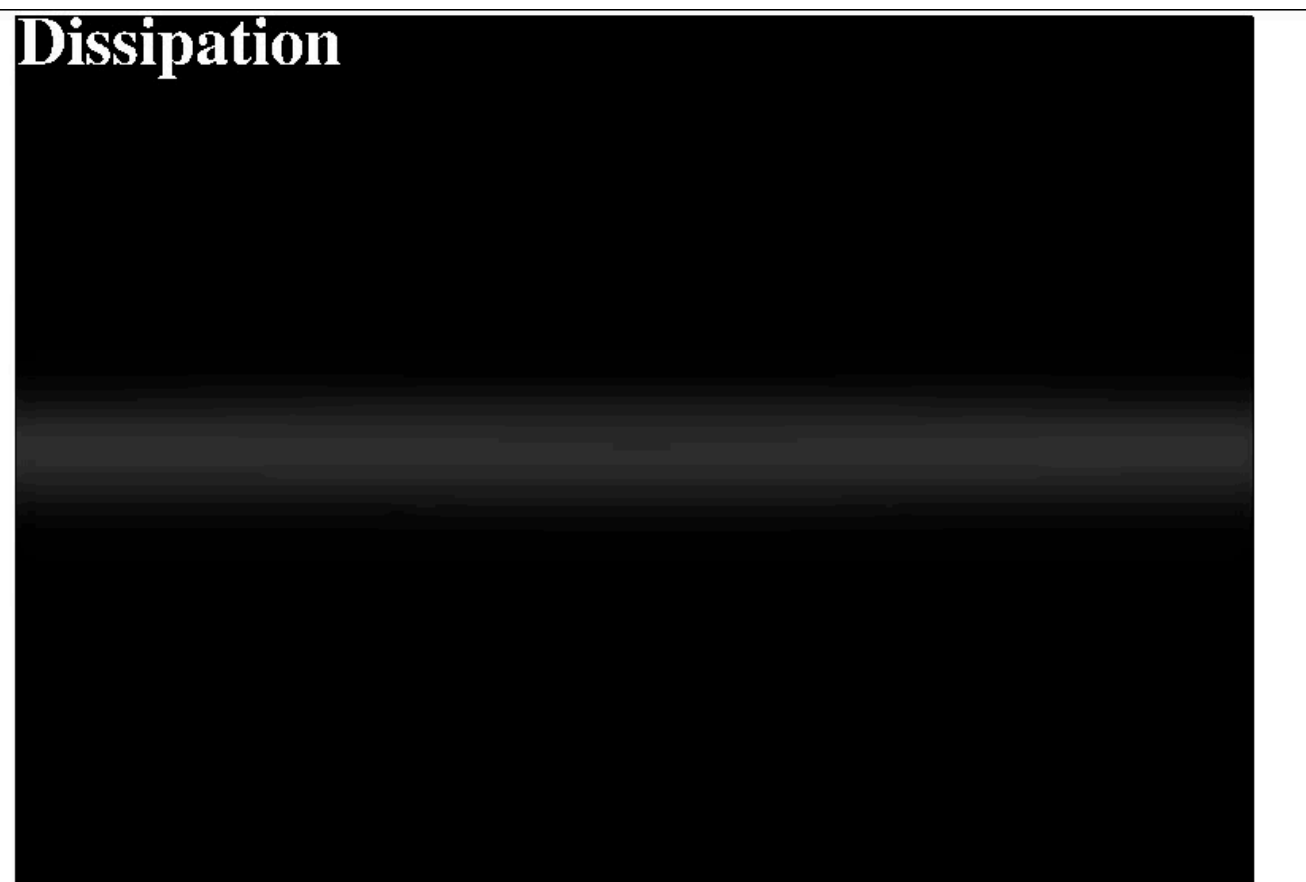
- Simplest KH overturning instability, high Re_b always low Ri ...

Both laboratory and DNS work indicate that at these extremes, when either $\varepsilon/\nu N^2 \sim O(1)$ or $\varepsilon/\nu N^2 \sim O(10^5)$, the mixing efficiency $R_f \rightarrow 0$ and the use of large $R_f \approx 0.2$ in field situations in these limits cannot be justified. This is not simply a matter of curiosity. There is a fundamental inconsistency between the results from the laboratory and DNS experiments and the inference of diffusivity from microstructure in the field that remains unresolved. Ivey et al. 2008

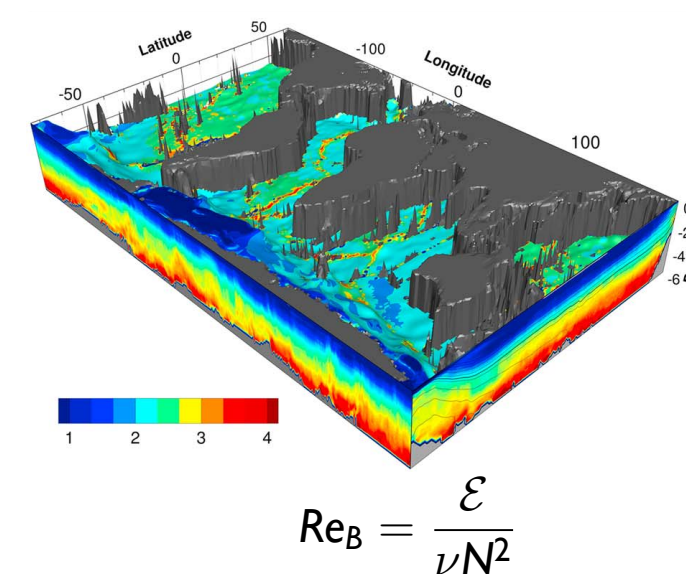
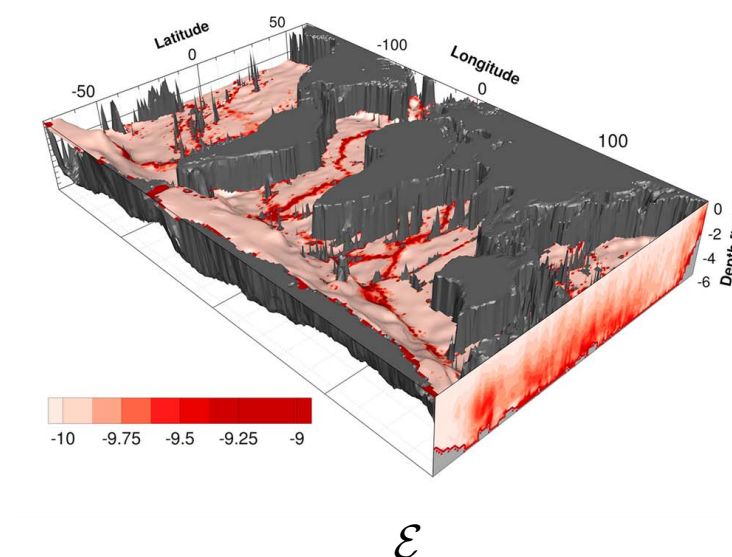
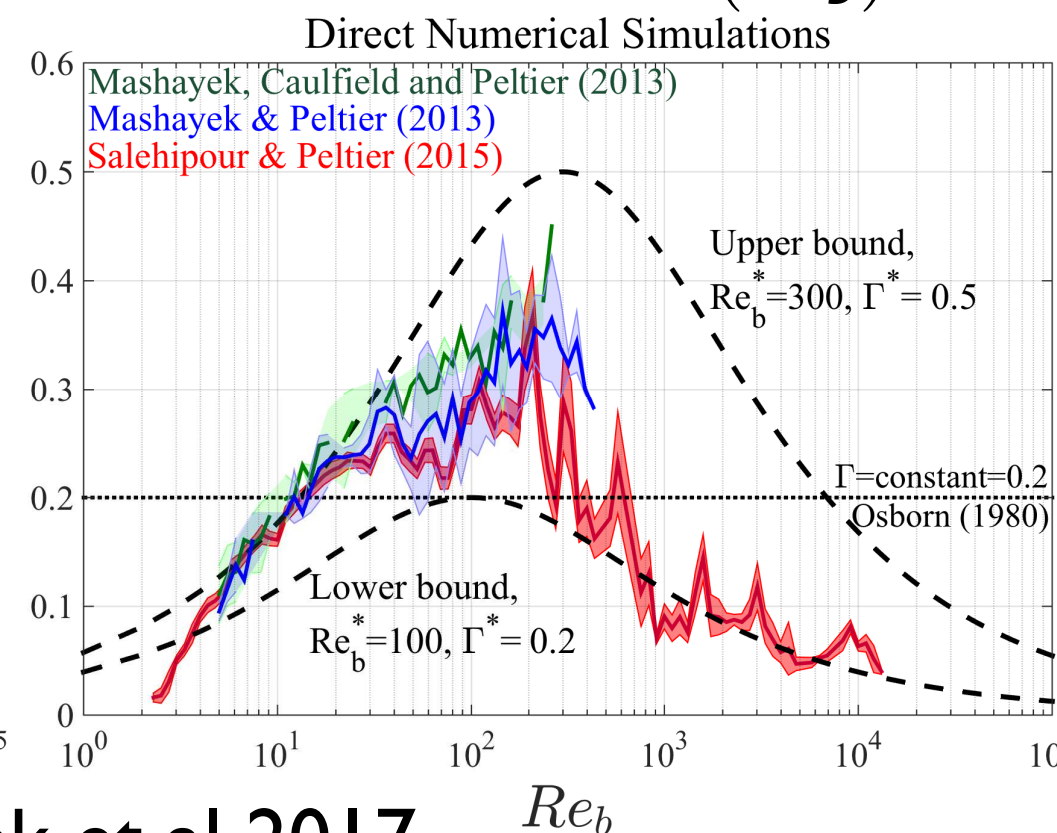
$$\Gamma(Re_b) = \frac{2\Gamma^* \left(\frac{Re_b}{Re_b^*}\right)^{1/2}}{1 + \left(\frac{Re_b}{Re_b^*}\right)}$$

Huge spatial variability

Dissipation



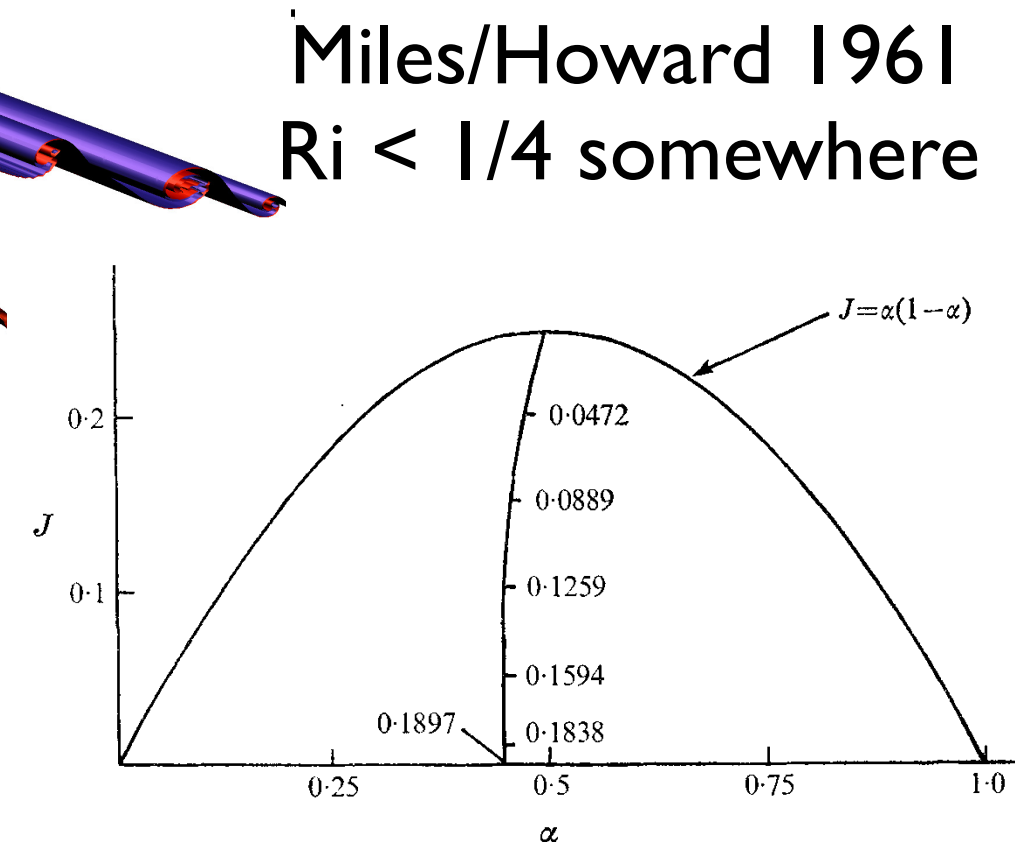
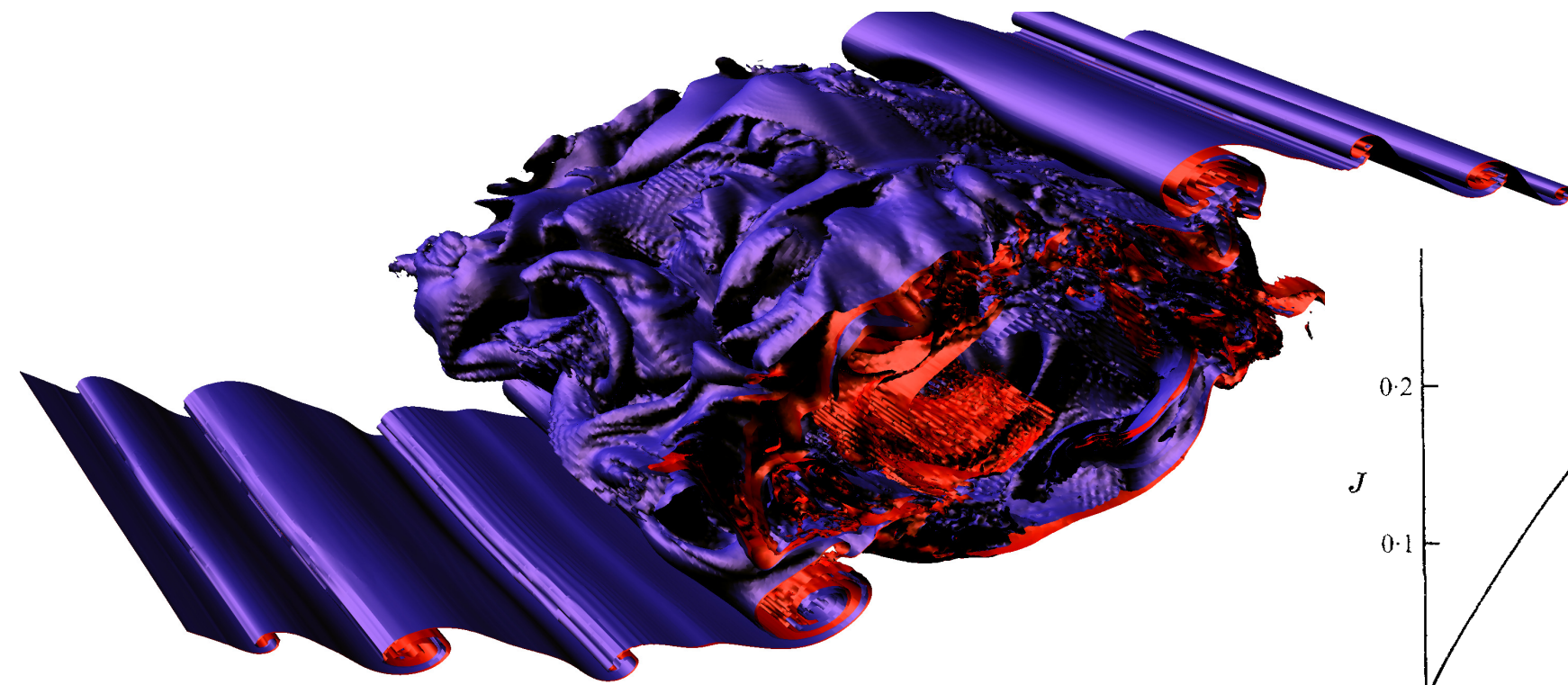
Mashayek et al 2017



$$Re_B = \frac{\varepsilon}{\nu N^2}$$

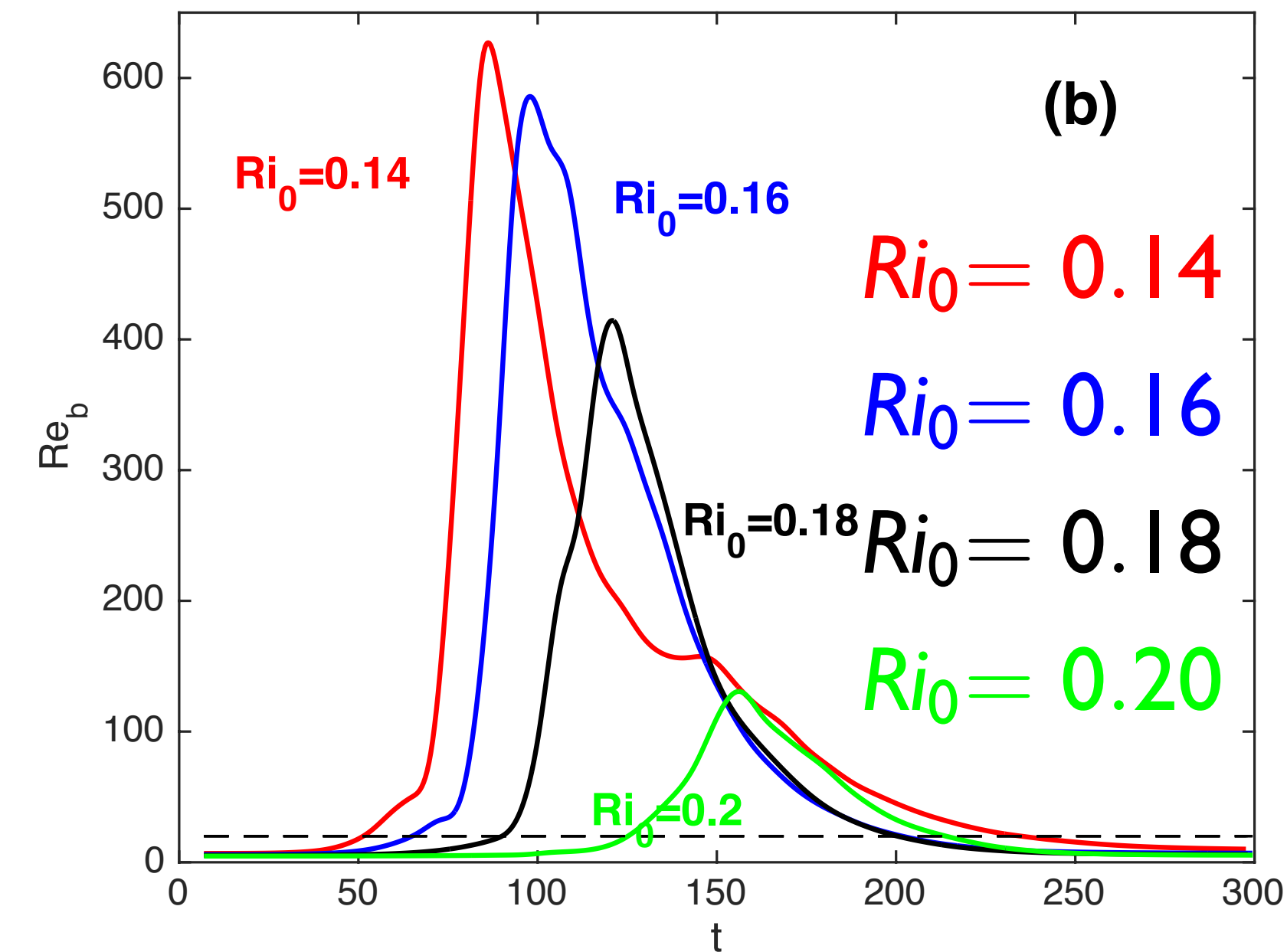
“Optimal” mixing by overturning?

- Controlled mixing simulations: KH instability at $Re=6000$, $Pr=1$



$$Re_B = \left(\frac{\mathcal{E}}{\nu N^2} \right)$$

$$Re_B = \left(\frac{L_0}{L_K} \right)^{4/3}$$



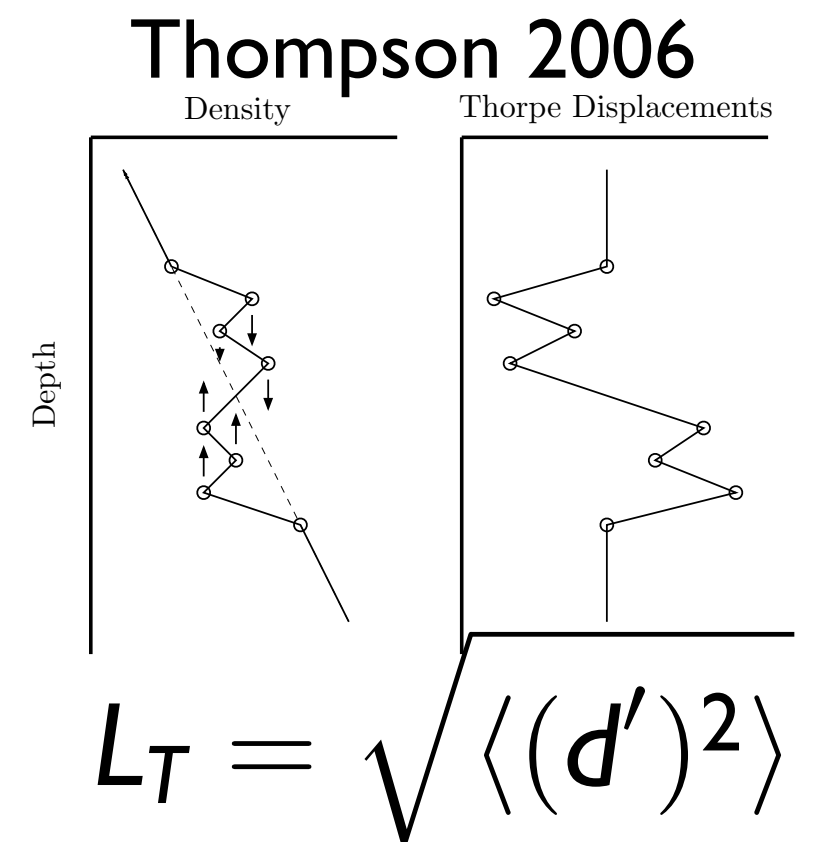
- Inherently unsteady, transient mixing process stabilised as Ri increases...

- Very high peak Re_b with a slow decay: $50-100 < t < 200$ turbulent

- How do length scales evolve? Proxies for age/mixing Dillon (1982)

- A lot easier to measure **Thorpe** scale and N ...

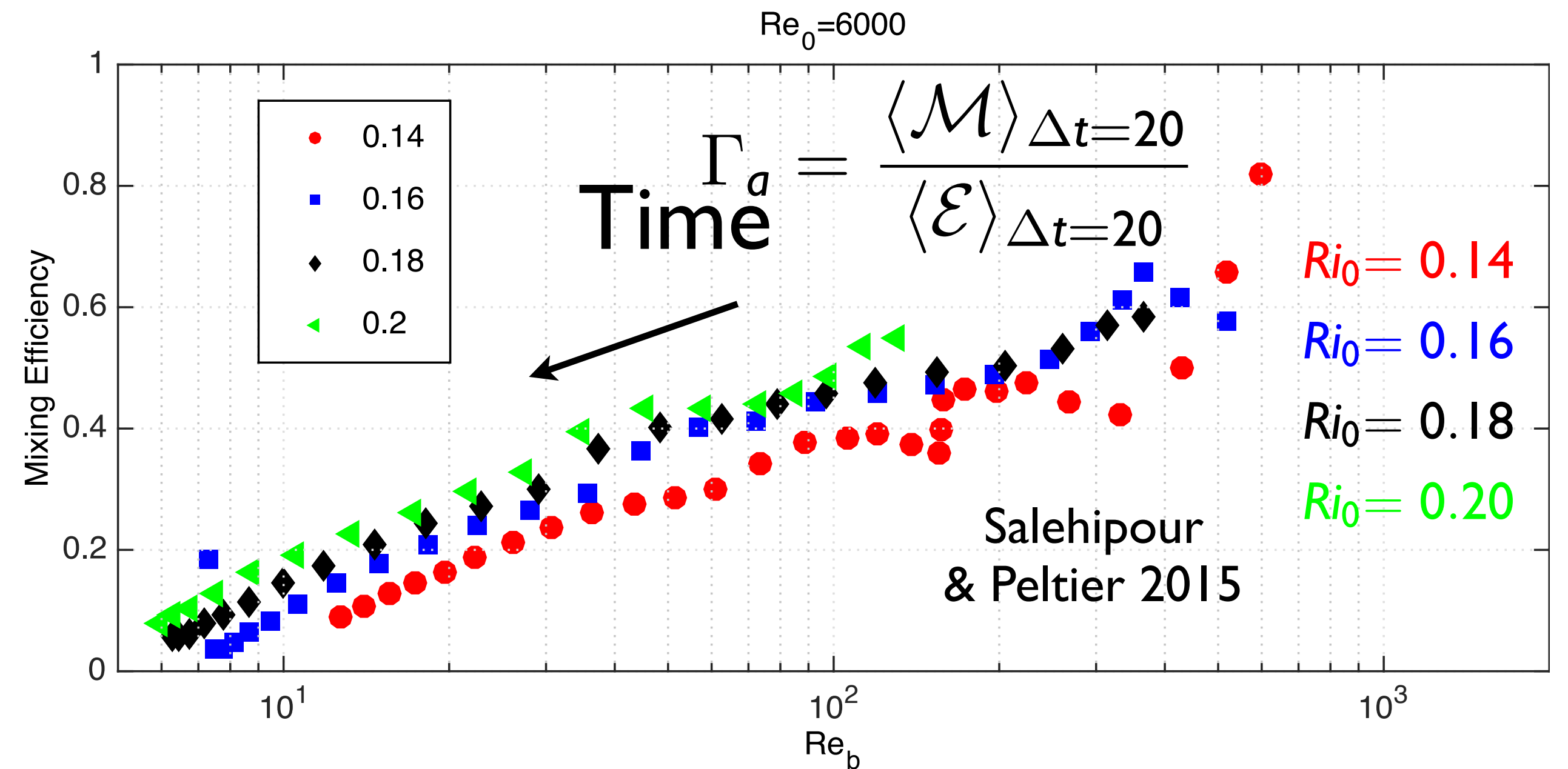
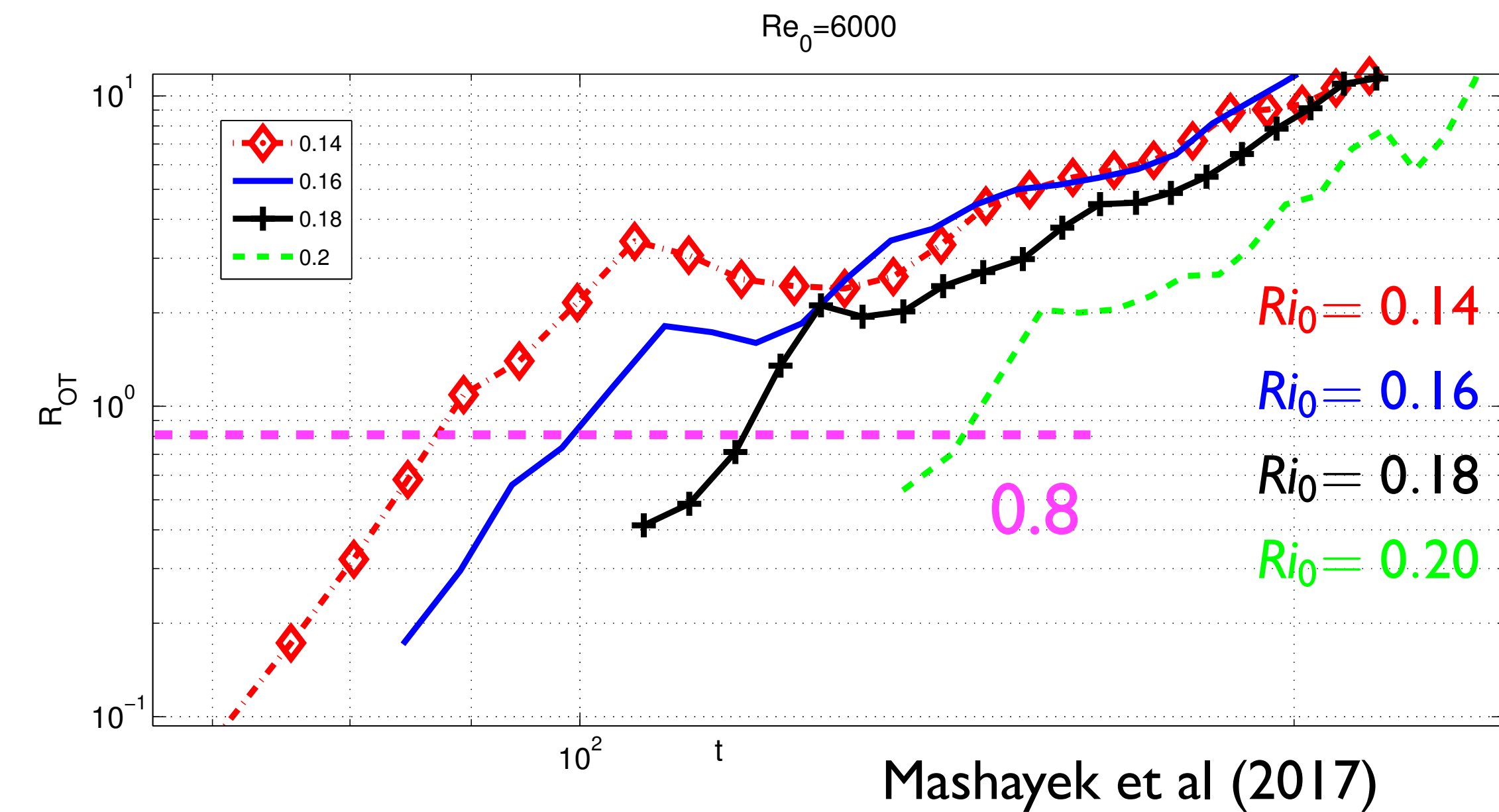
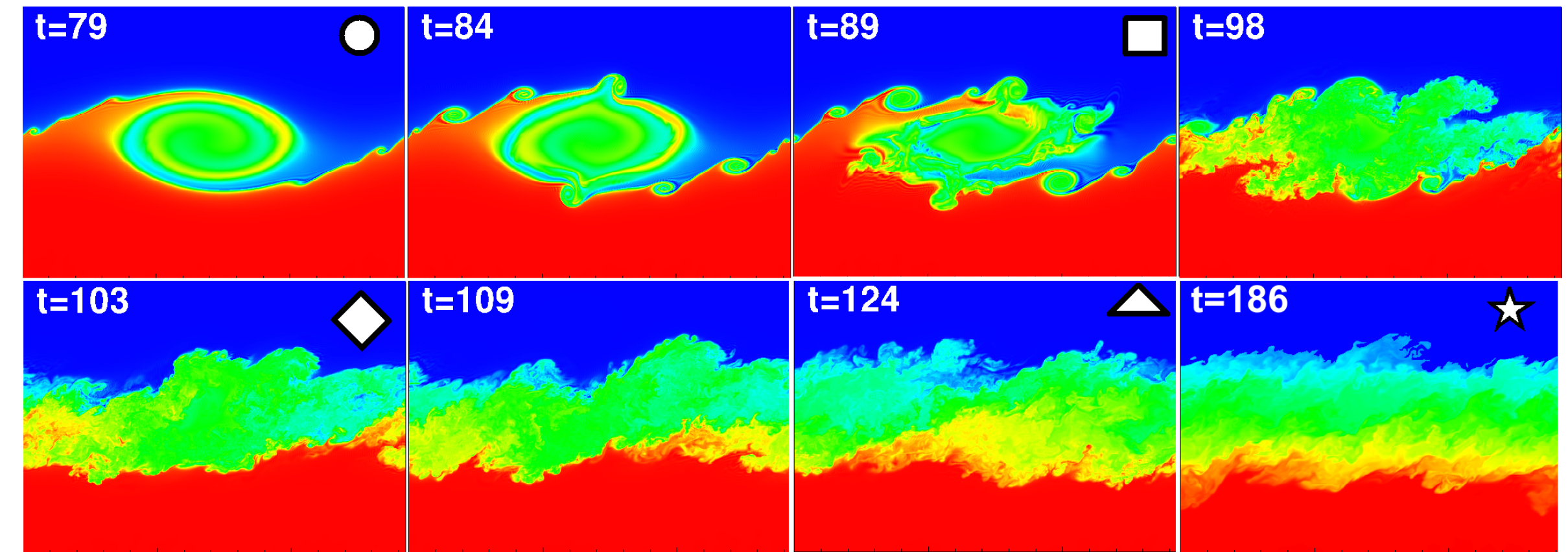
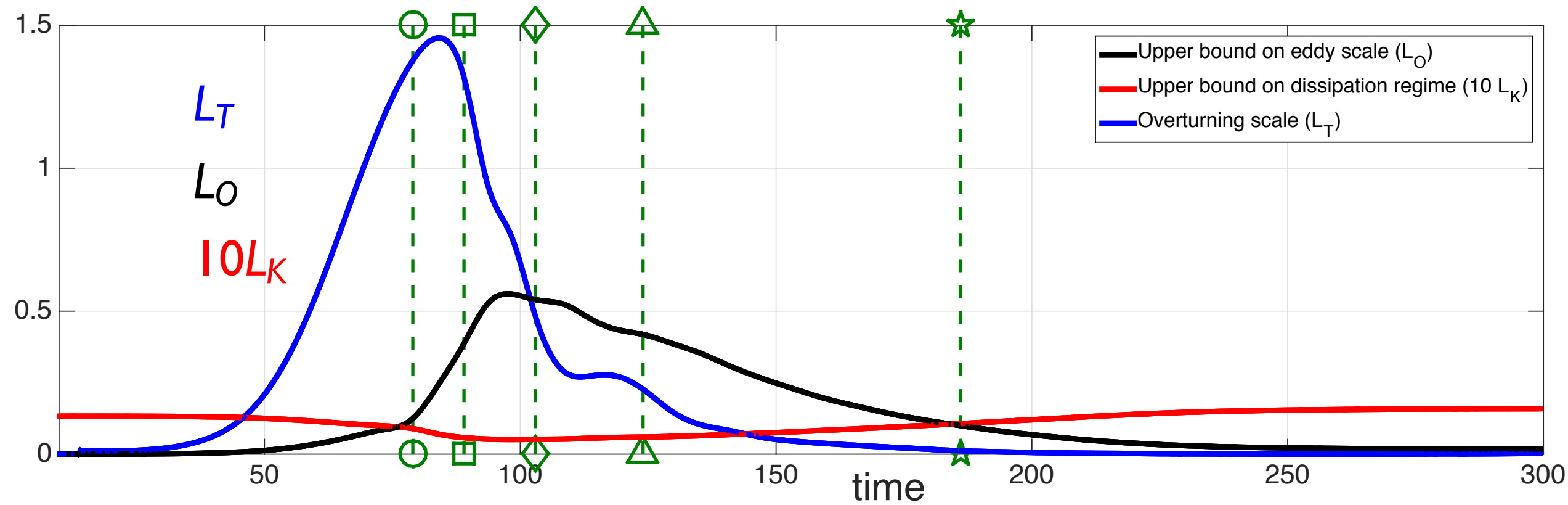
- (eg Mater et al/Scotti etc)



$$\kappa_T = \Gamma \frac{\mathcal{E}}{N^2} = \Gamma L_0^2 N = \Gamma \frac{L_0^2}{L_T^2} L_T^2 N = \Gamma R_{OT}^2 L_T^2 N$$

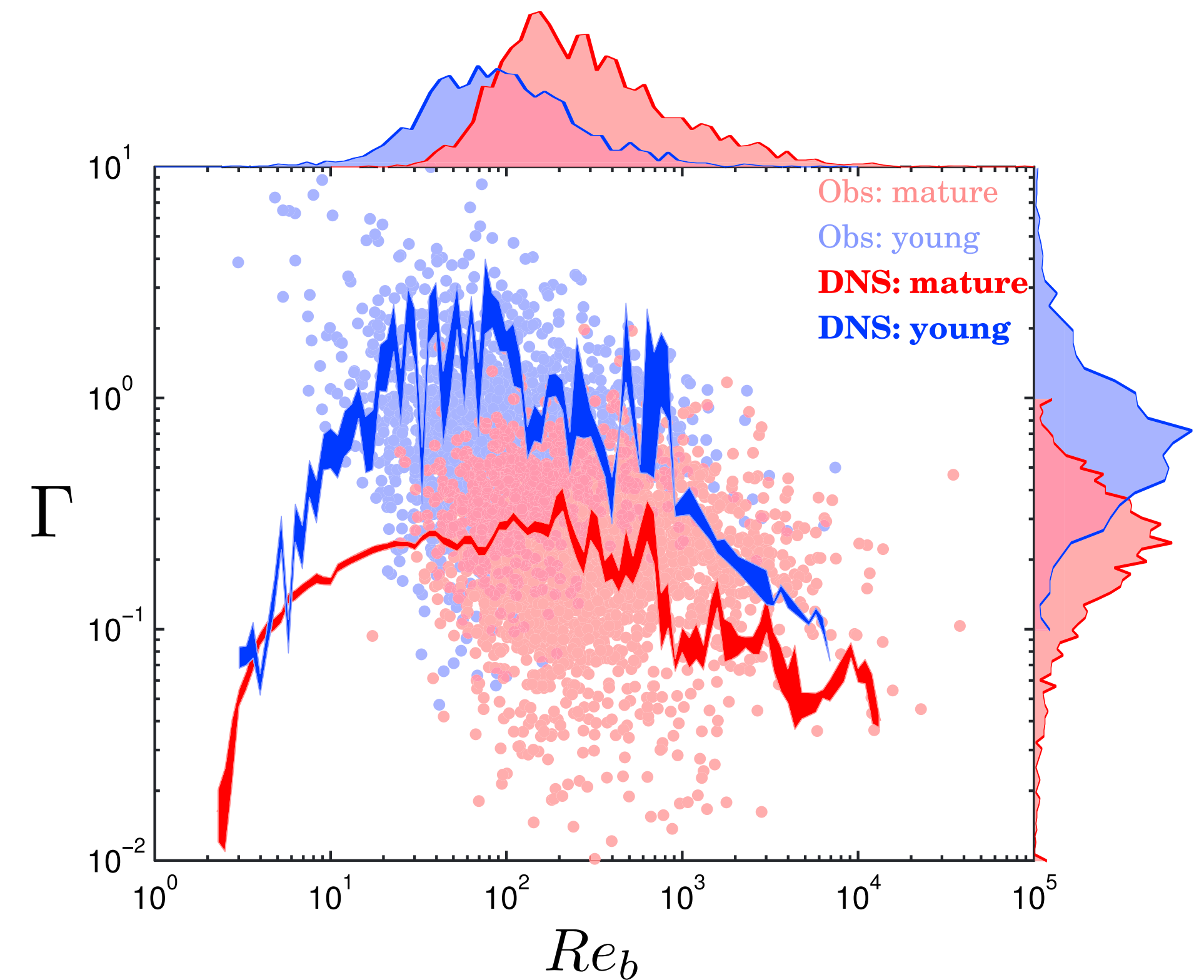
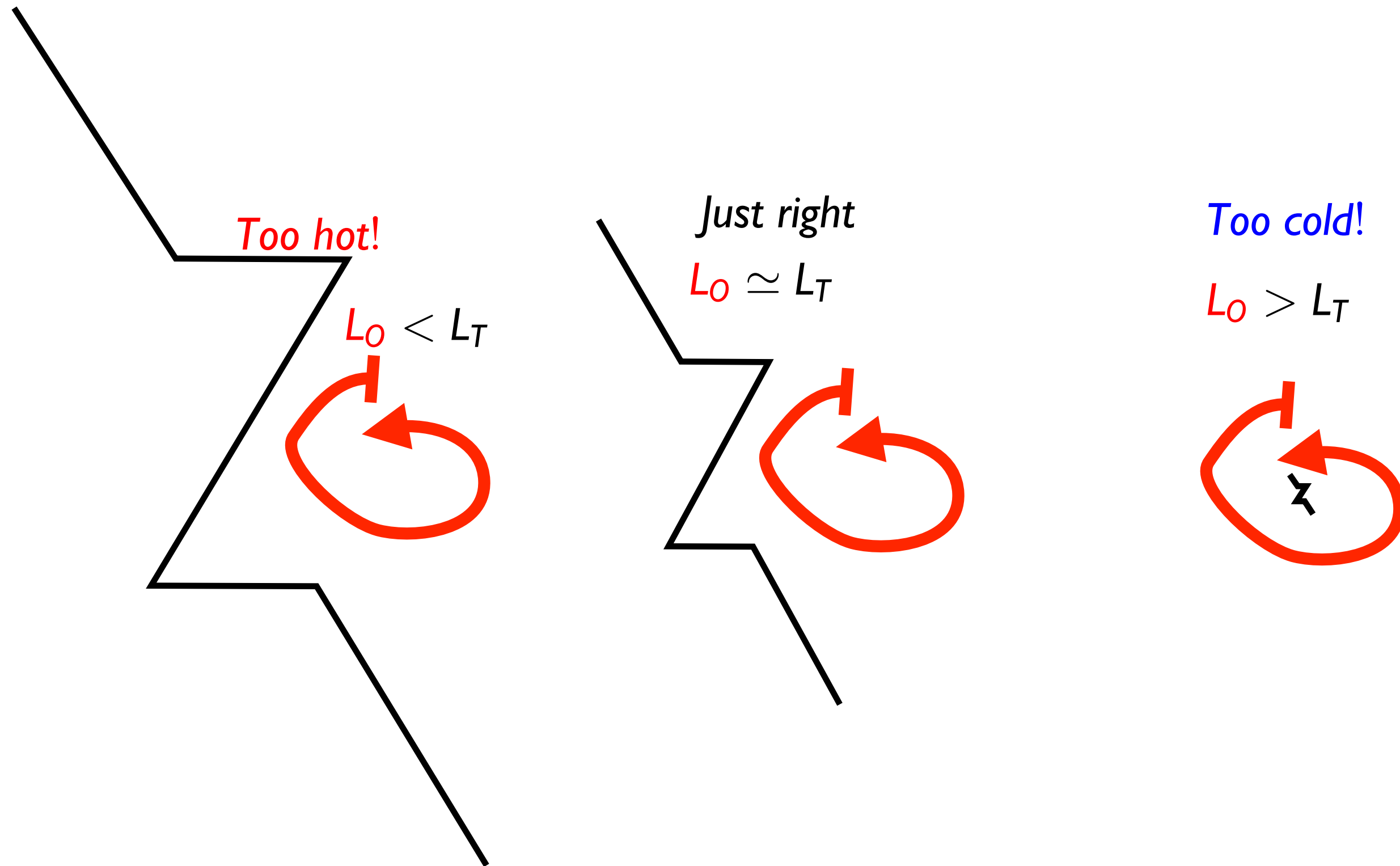
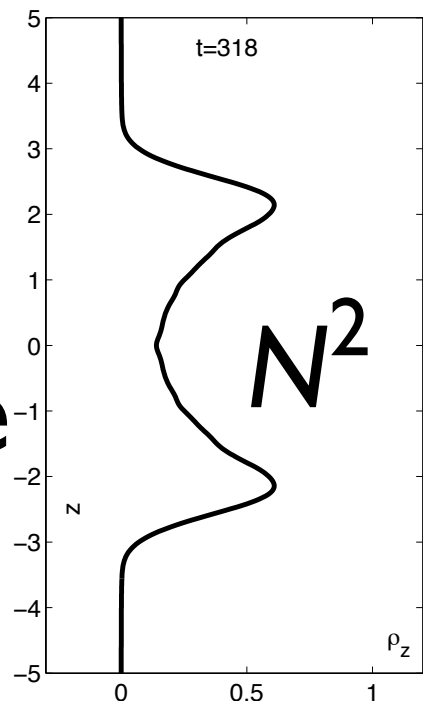
Overturning memory: Goldilocks mixing?

- $Re=6000$, $Ri(0)=0.16$: How does $R_{OT} = \frac{L_O}{L_T}$ vary? L_T flares, L_O burns



Optimal Goldilocks Mixing

- Very high values: maximum **generically** when $L_0 = L_T \leftrightarrow R_{OT} = 1$ early in **turbulent** life cycle
- $\kappa_T = \nu \Gamma Re_B$ maximum because **both** Γ and Re_B maximum at $L_0 = L_T$
- Overturning: layer scale of stratified turbulence? Optimal if **precisely** at top of unaffected range



Scouring or Overturning?

- In stratified shear flows, can have either scouring or overturning

- Woods et al (2010): Scouring has “sharp” interfaces and layers...

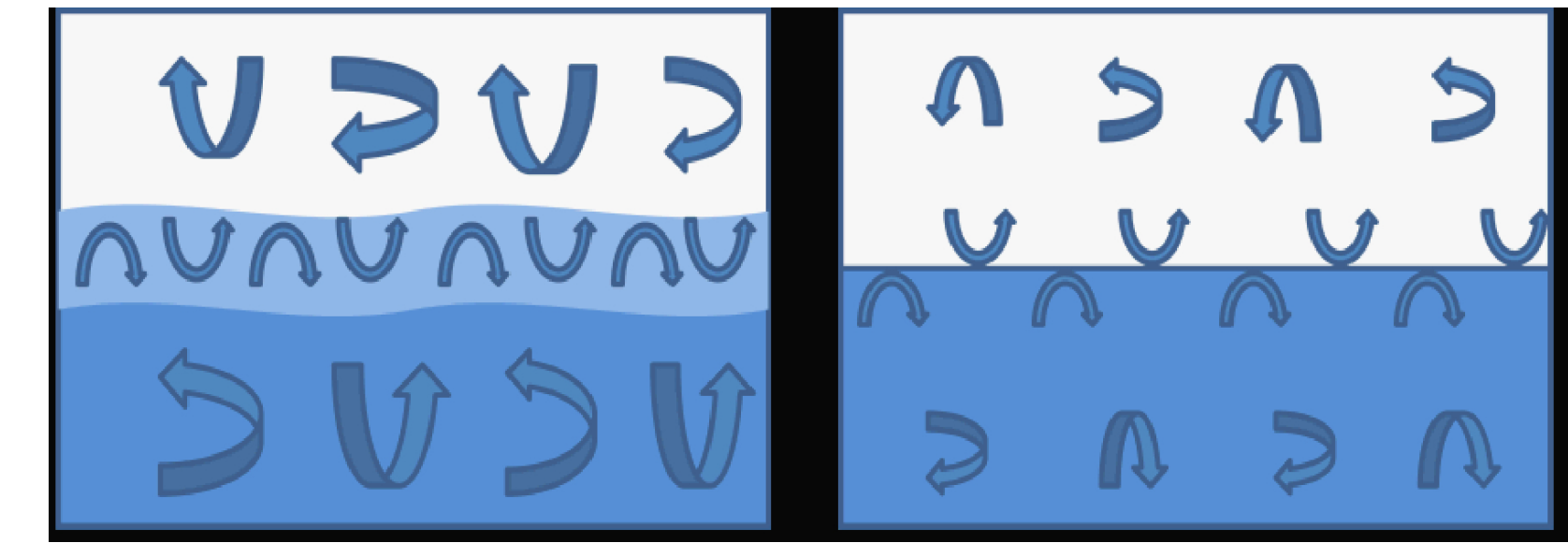
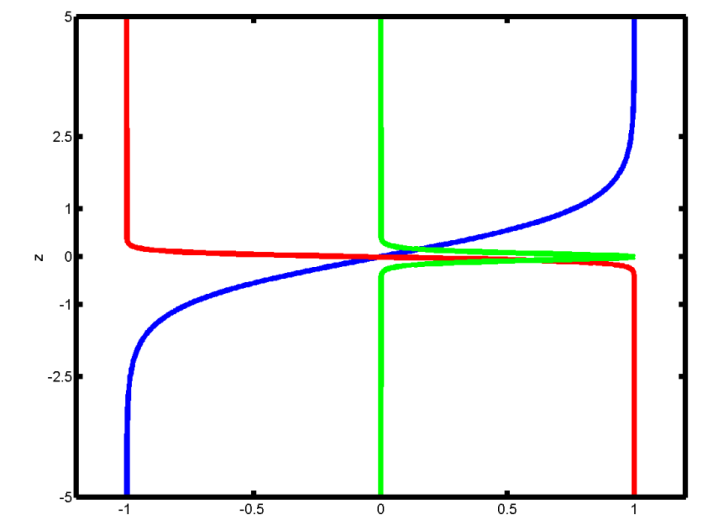
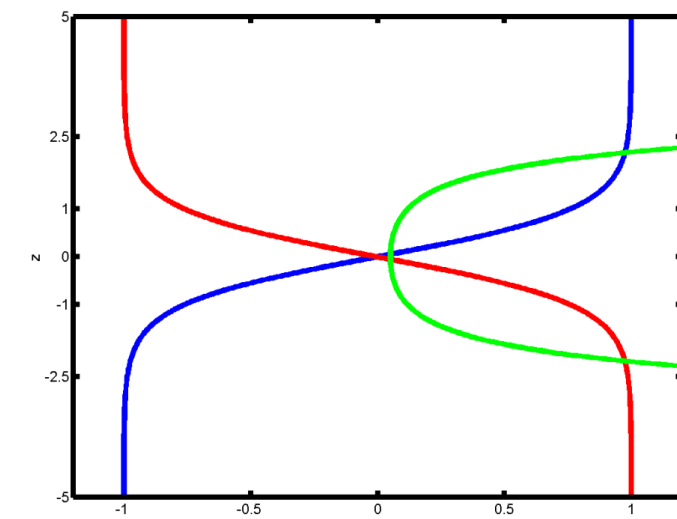
- Ocean is principally stratified with heat $Pr = \frac{\nu}{\kappa} \sim O(10)$

- High Pr : interfaces/maximum Ri at middle of shear layers:

Velocity blue

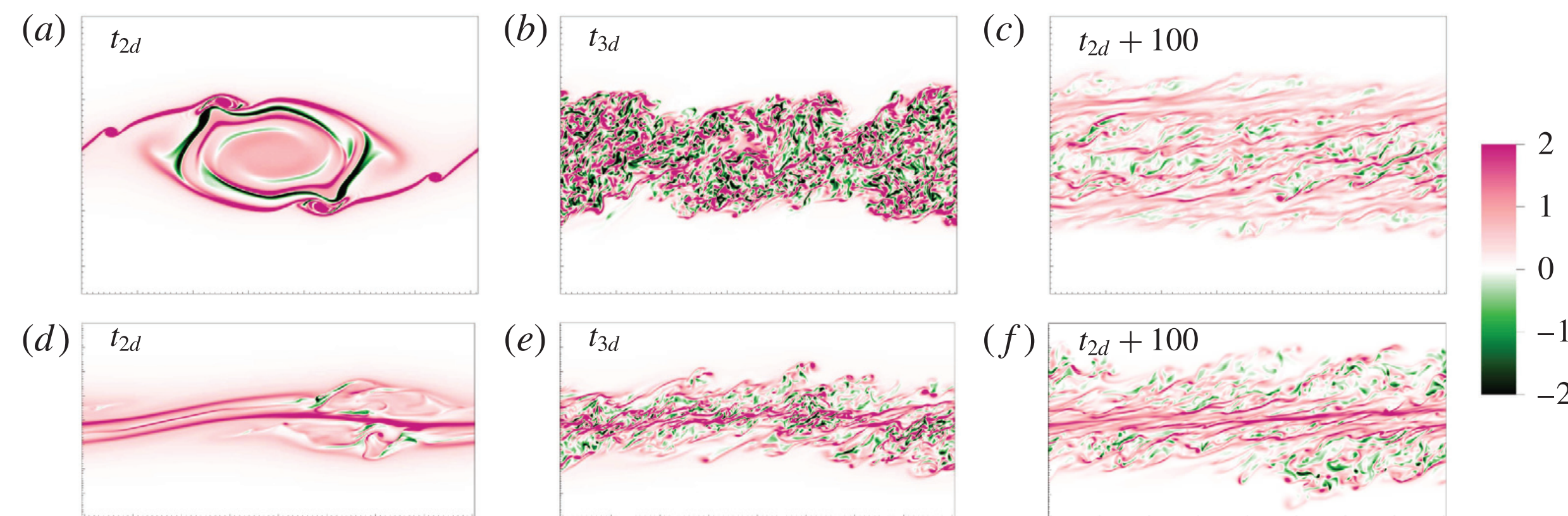
Density red

$Ri(z)$ green



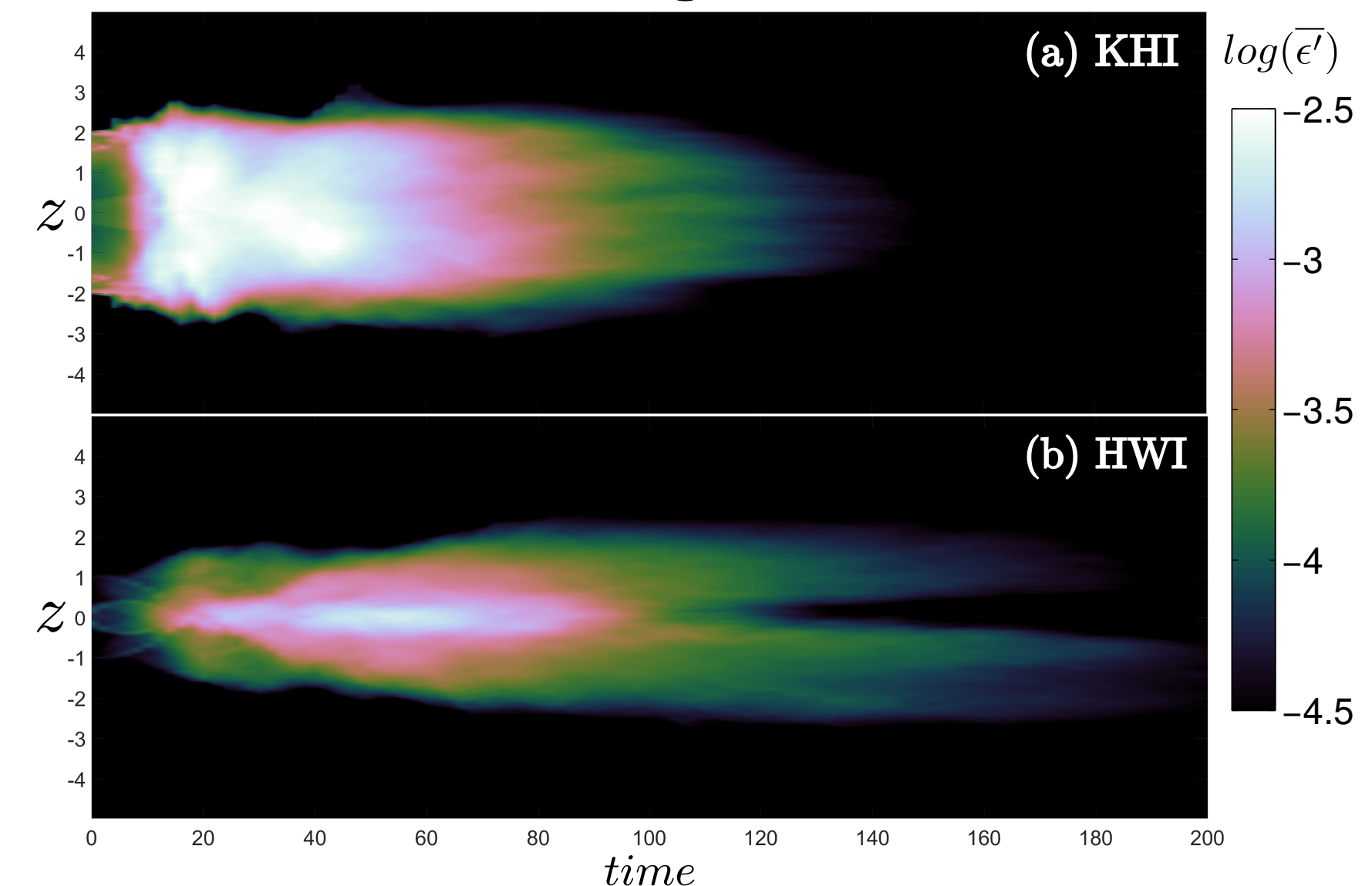
weak: **overturning** strong: **scouring**

- Flows have qualitatively different types of instability...how about turbulence/mixing?



KHI: flares
& overturns

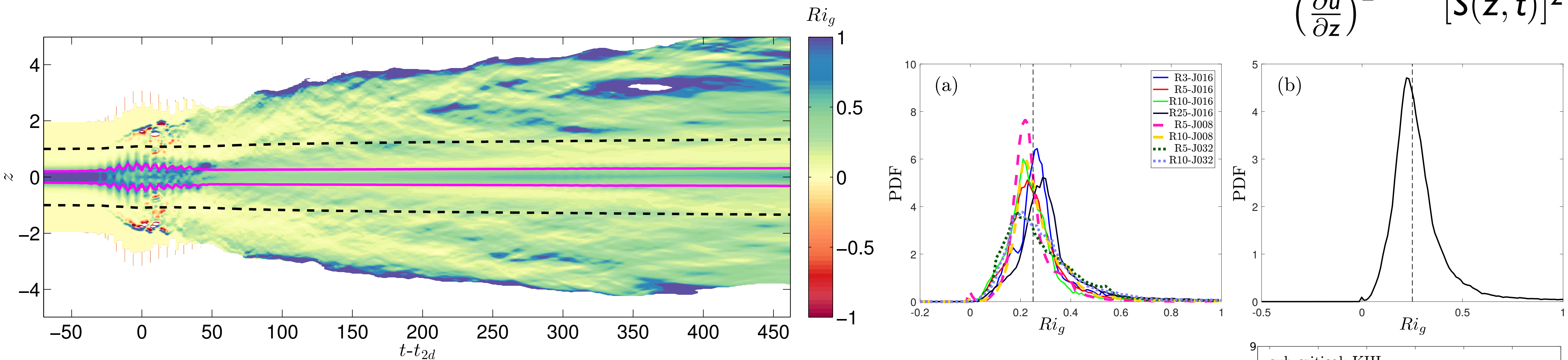
HWI: burns
& scours



Scouring not really diffusive process at all...

Is Holmboe-induced turbulence different? (Salehipour 2018)

- Shear flows become turbulent: horizontally average velocity & density: $Ri_g(\mathbf{z}, t) = \frac{-\frac{g}{\rho_a} \frac{\partial \bar{\rho}}{\partial \mathbf{z}}}{\left(\frac{\partial \bar{u}}{\partial \mathbf{z}}\right)^2} = \frac{\overline{N^2}(\mathbf{z}, t)}{[\overline{S}(\mathbf{z}, t)]^2}$

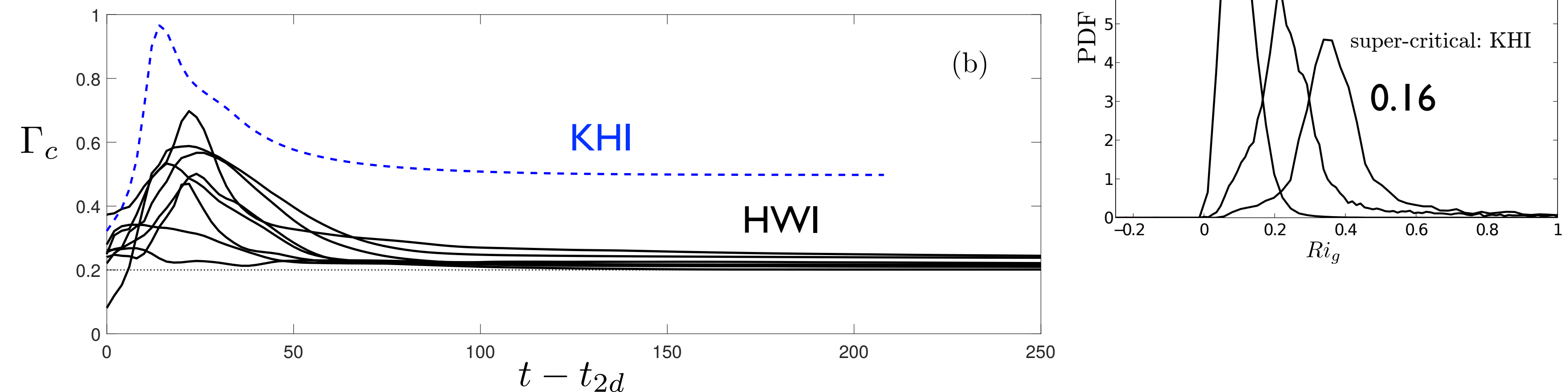


- HWI (not KHI) has PDF strongly peaked generically $Ri_g(\mathbf{z}, t) \sim 1/4 \leftrightarrow \Gamma \sim 0.2$



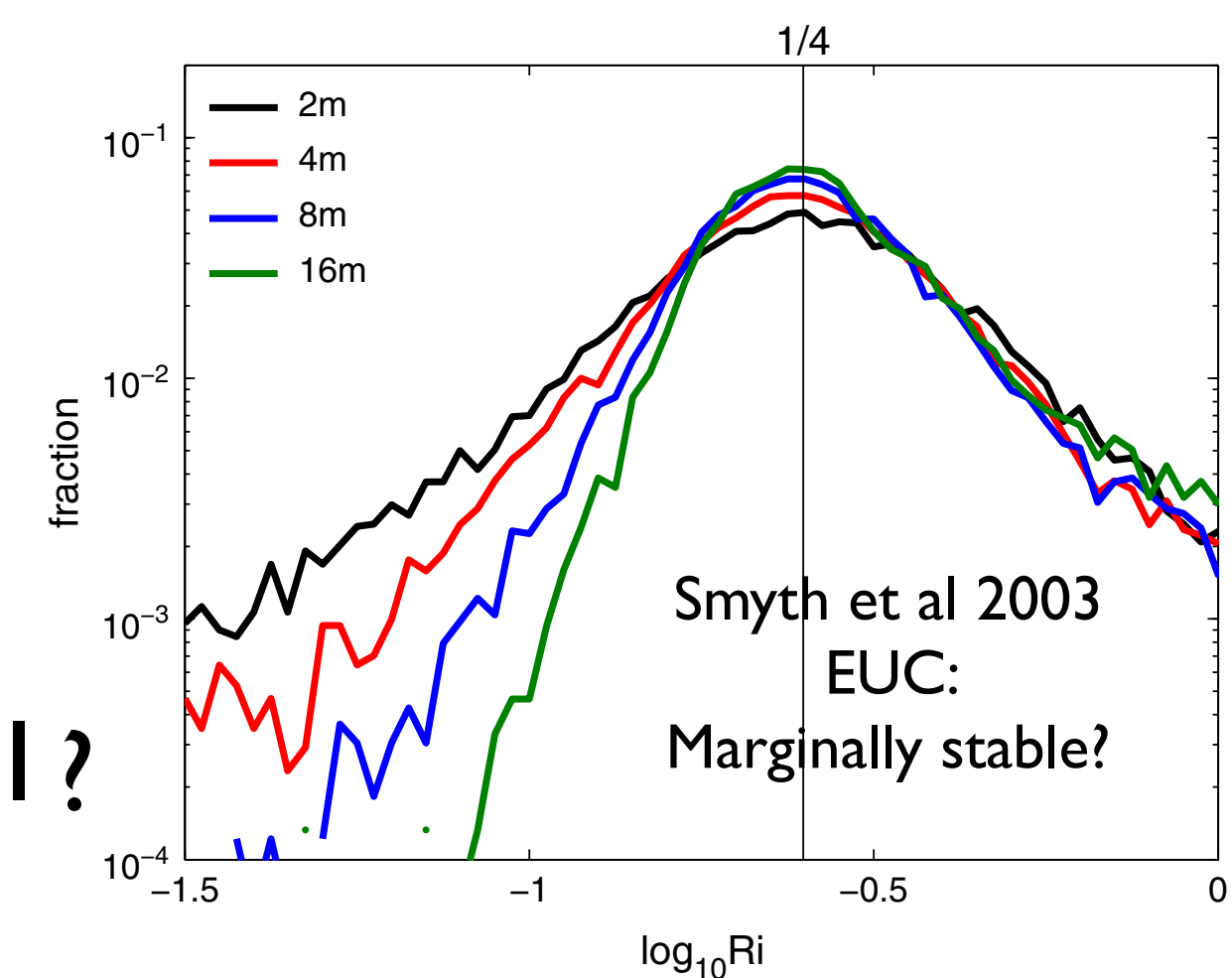
Bak et al (1987) Hesse & Gross (2014)

Self-Organised Criticality?

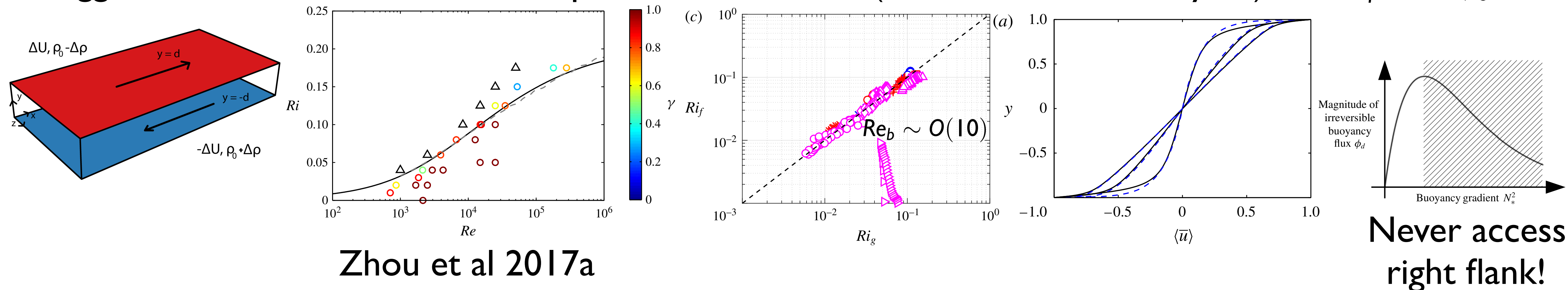


Issues...(though see Kaminski et al. 2019...)

- Why should marginal linear stability be relevant to such a turbulent flow?
- Is $\Gamma \sim 0.2$ just a property of quasi-steady high Re turbulence with $Pr_T \sim 1$?



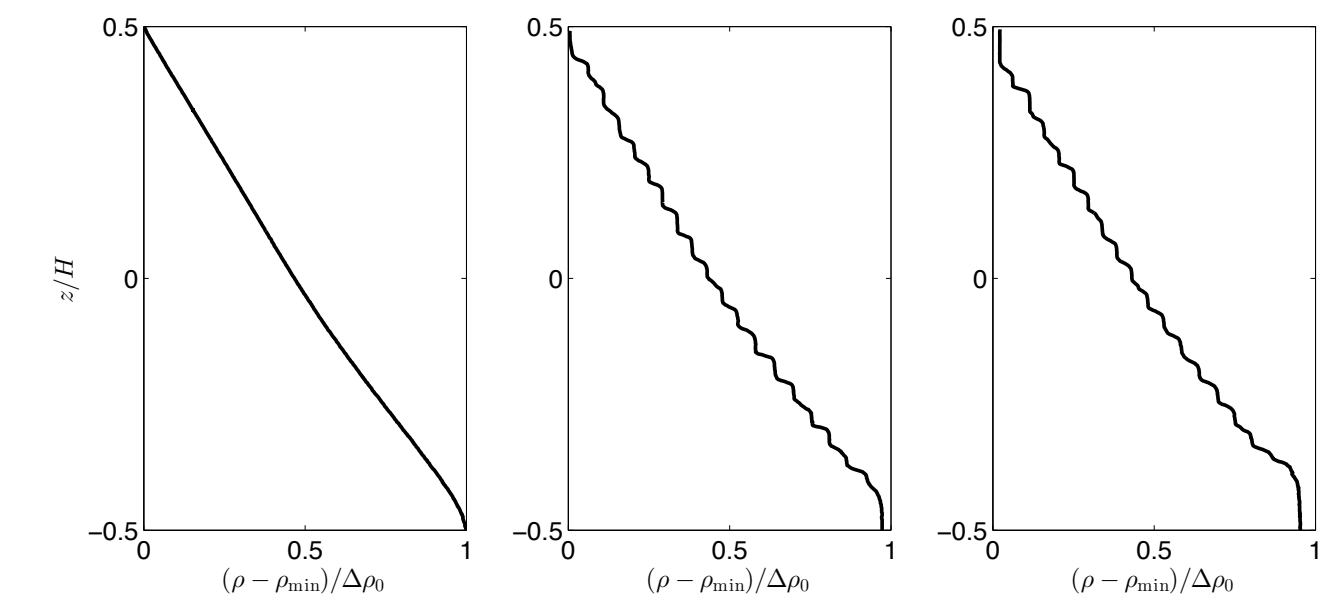
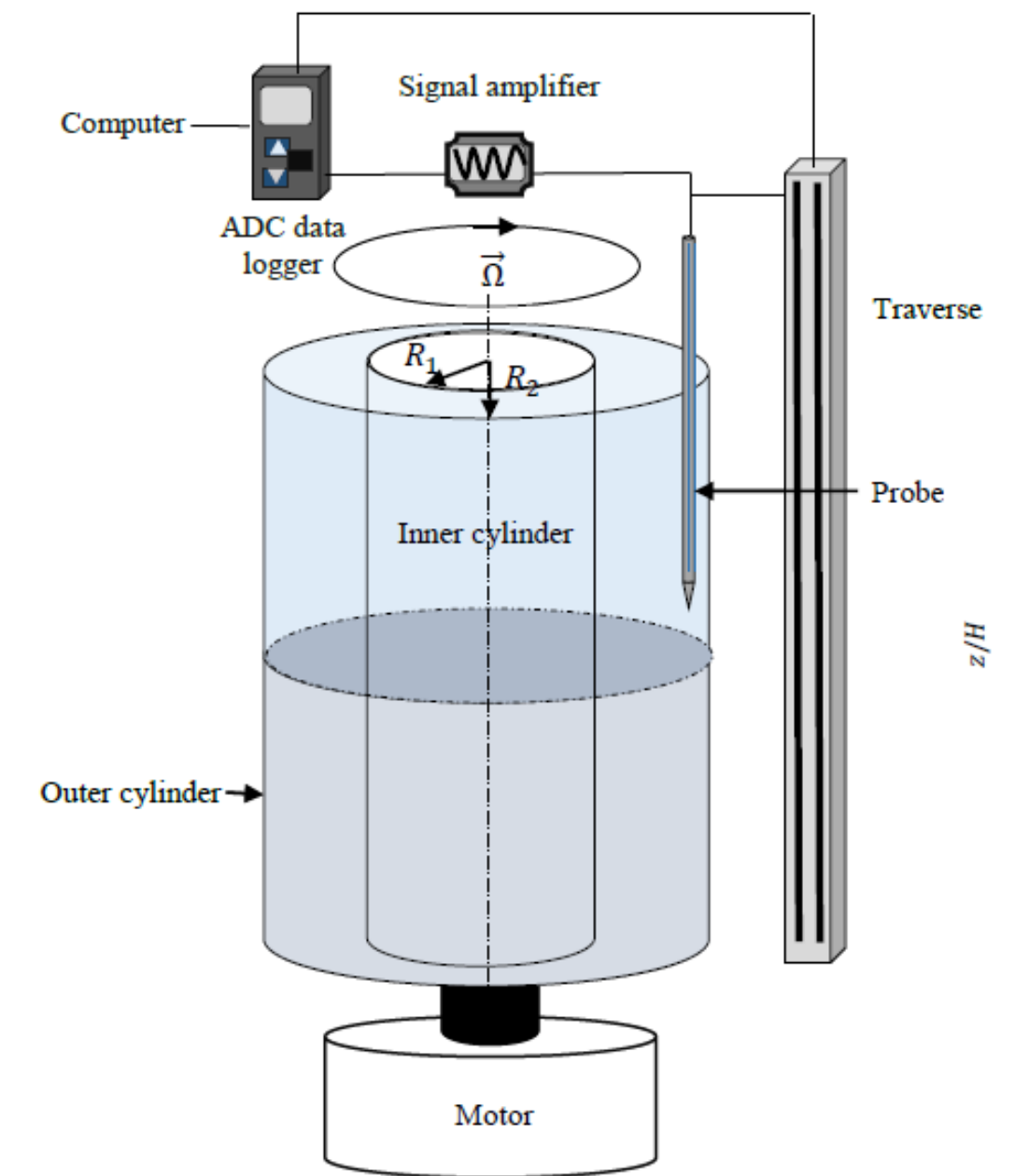
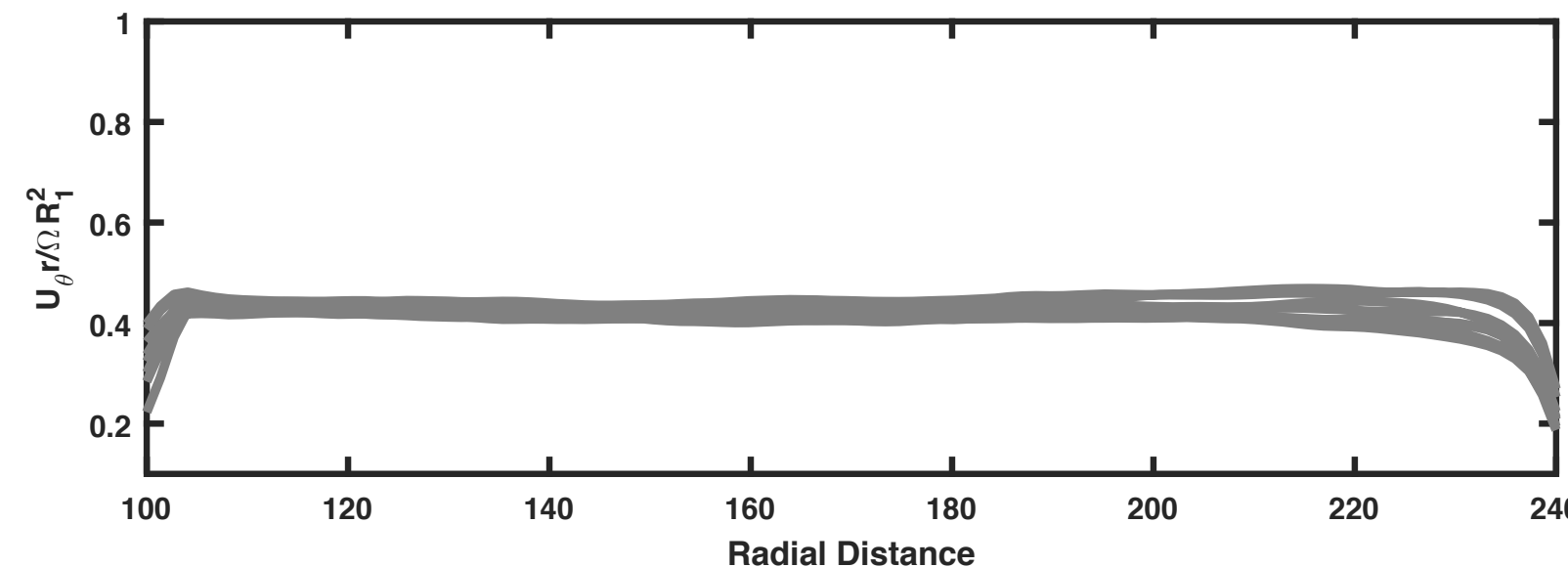
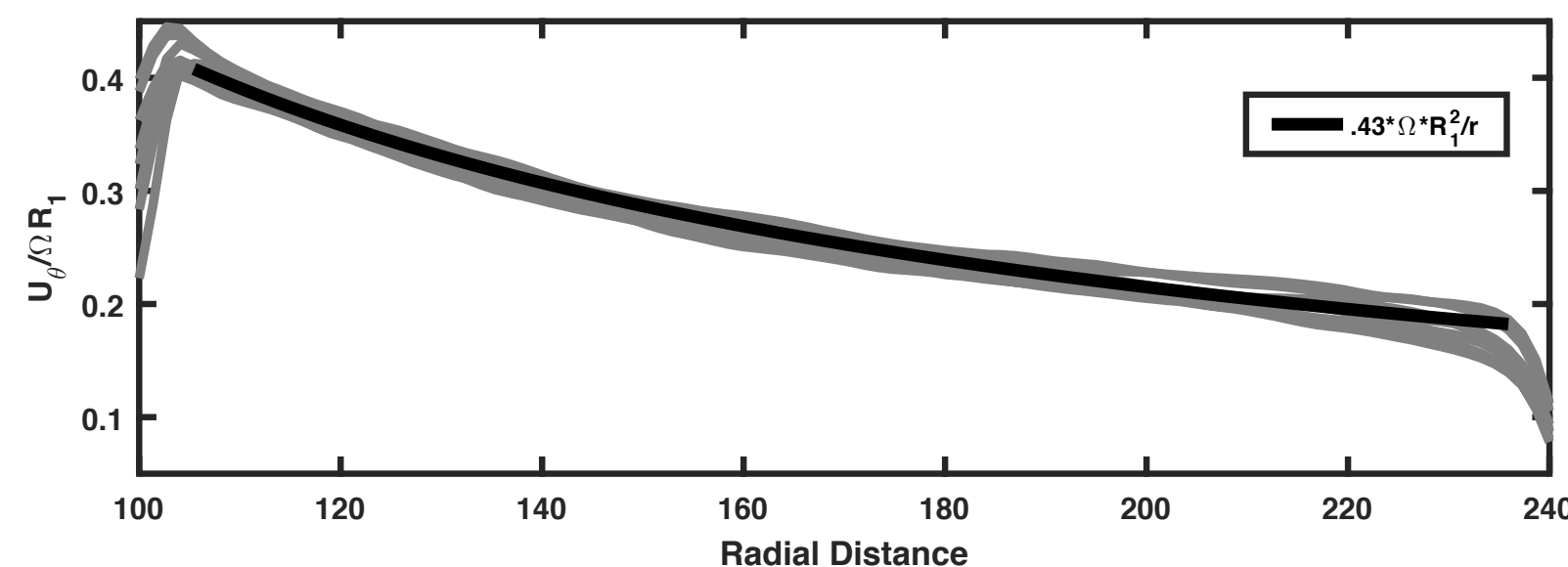
- Suggestive evidence from stratified plane Couette flow (M-O constant flux layers) that $Ri_f \simeq Ri \lesssim 0.2$



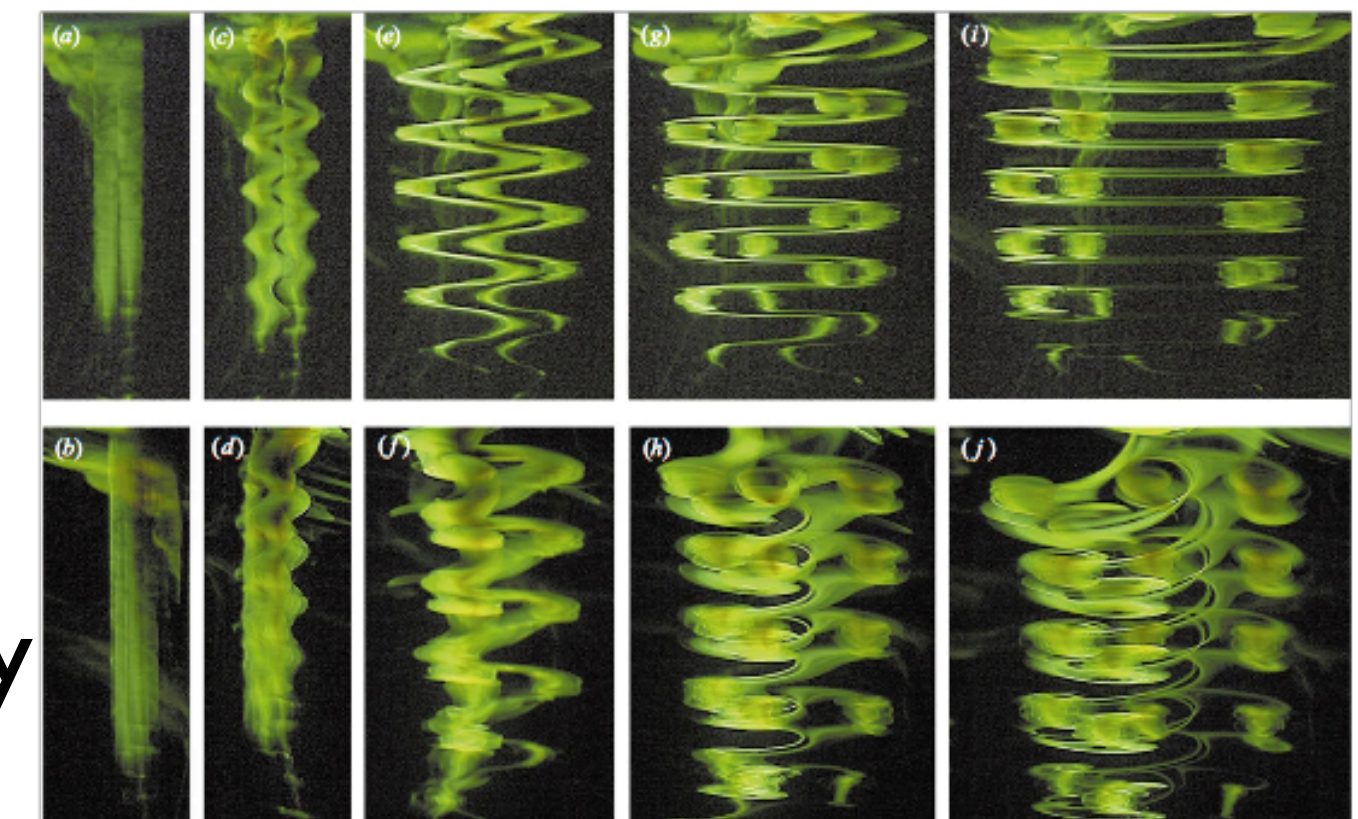
- Velocity and density couple: unique value of Ri: $Pr_T \sim 1$ until turbulence can no longer be sustained
- Mixing is essentially passive: “left flank” with memory: can still support layers (Zhou et al 2017b)
- Turbulence switches off as boundary layers stabilised...fundamentally boundary forced...

(Vertically) Stratified Taylor-Couette Flow

- Stratify Taylor-Couette in the vertical: Horizontal (boundary) forcing
- Outer cylinder 24.7cm; inner: 5, 10, 15 cm; $\eta = \frac{R_i}{R_o} = 0.208, 0.417, 0.625$
- Concentrate on stationary outer cylinder: $Re = \frac{\Omega R_o (R_o - R_i)}{\nu} > 10^4$
- Close to constant angular momentum: inherently 3D from start...

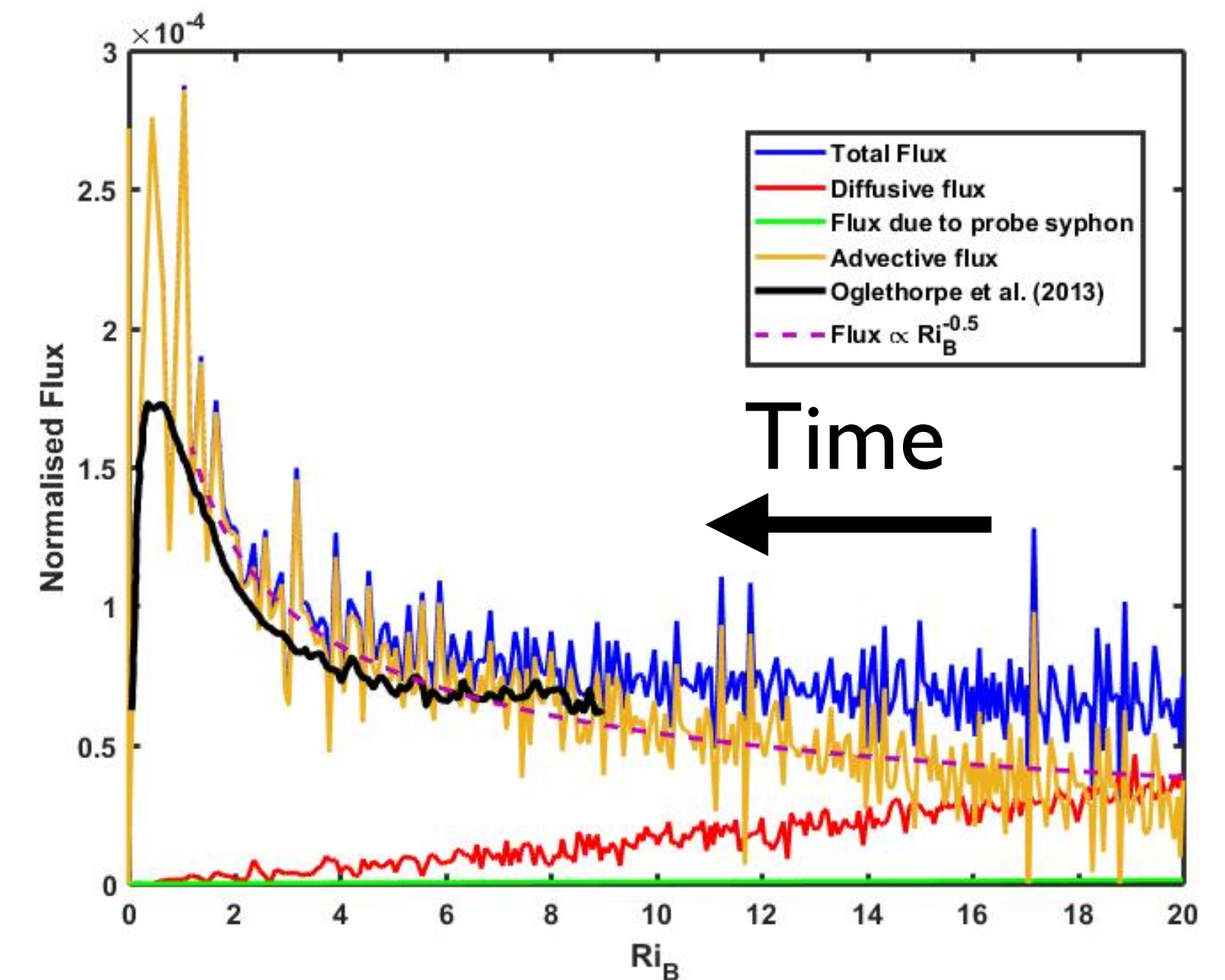
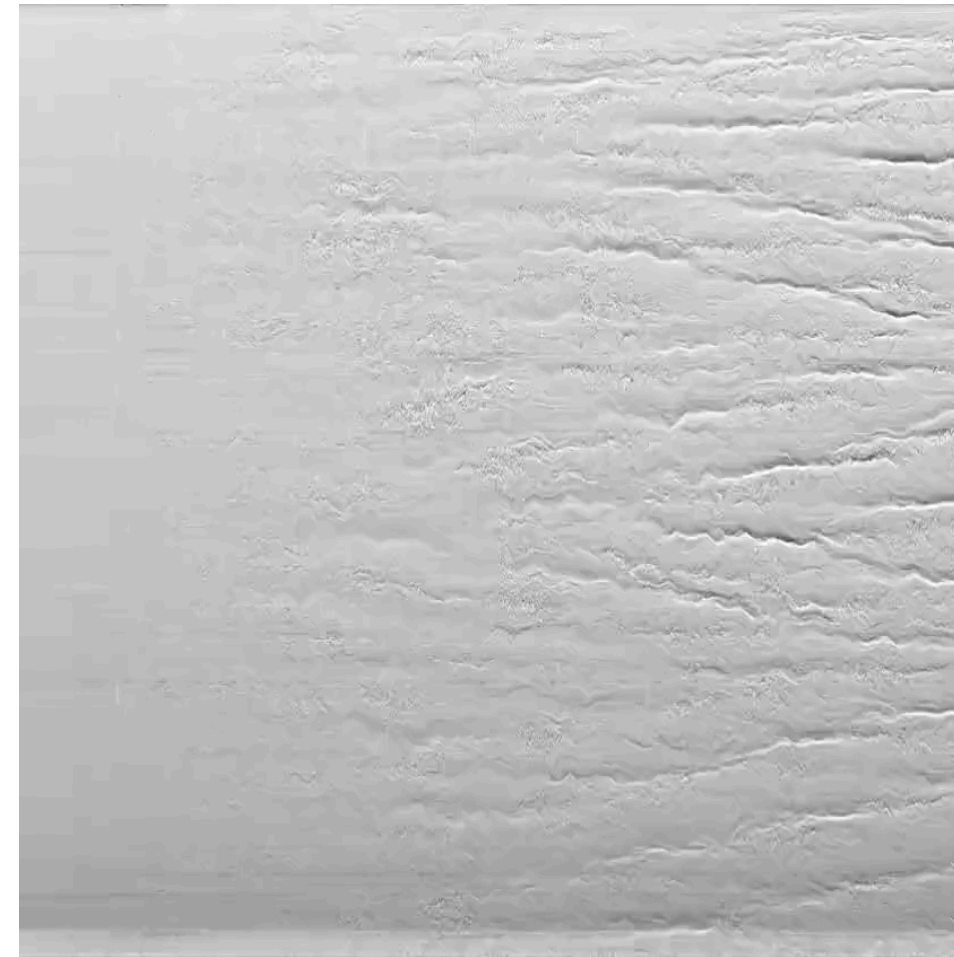


- Spontaneous layer formation, even though there is no fixed scale
- Is layering generic, and what role do interface/layer systems play?
- “Zig-zag” instability of Billant/Chomaz: horizontal shear/vertical vorticity



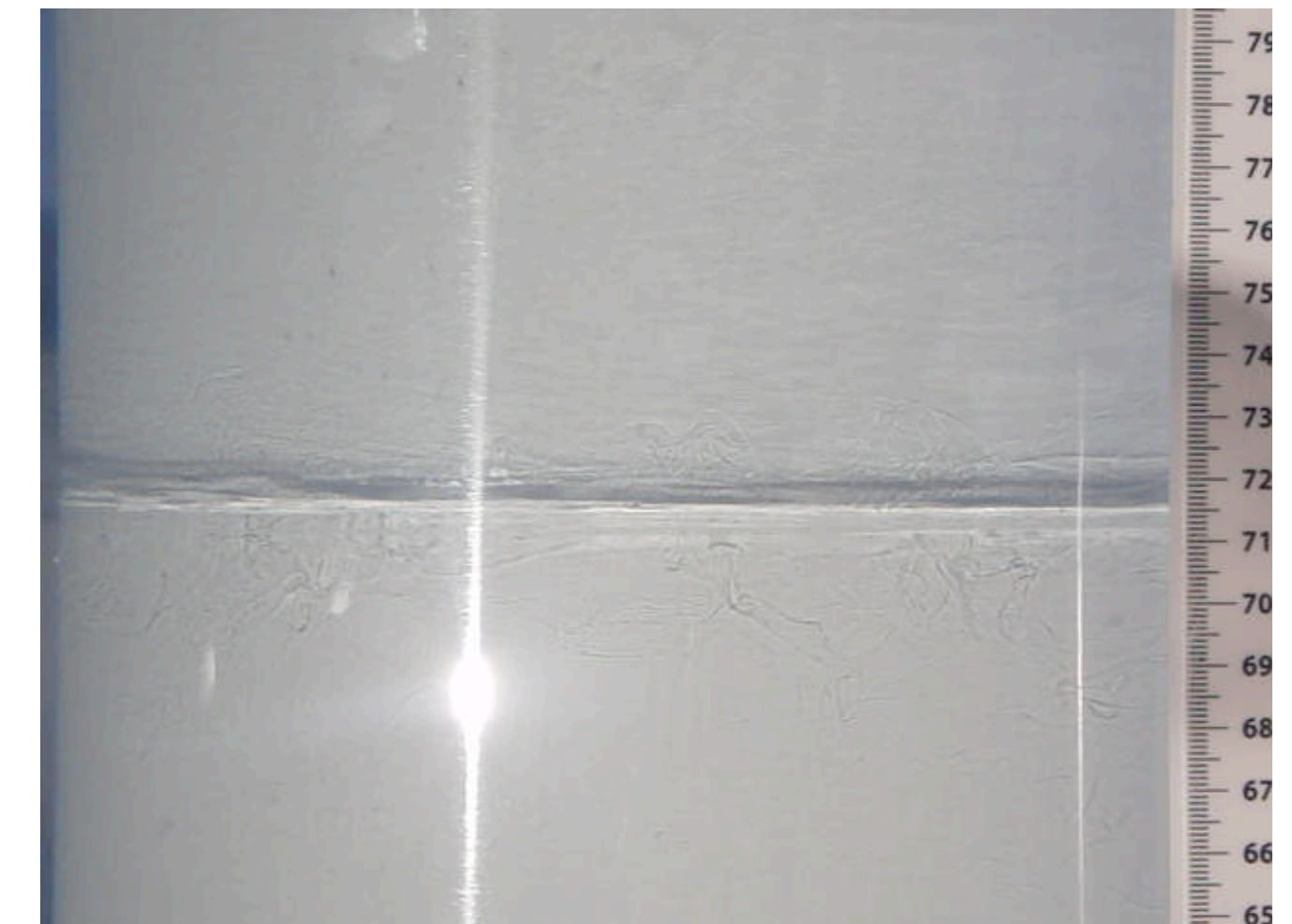
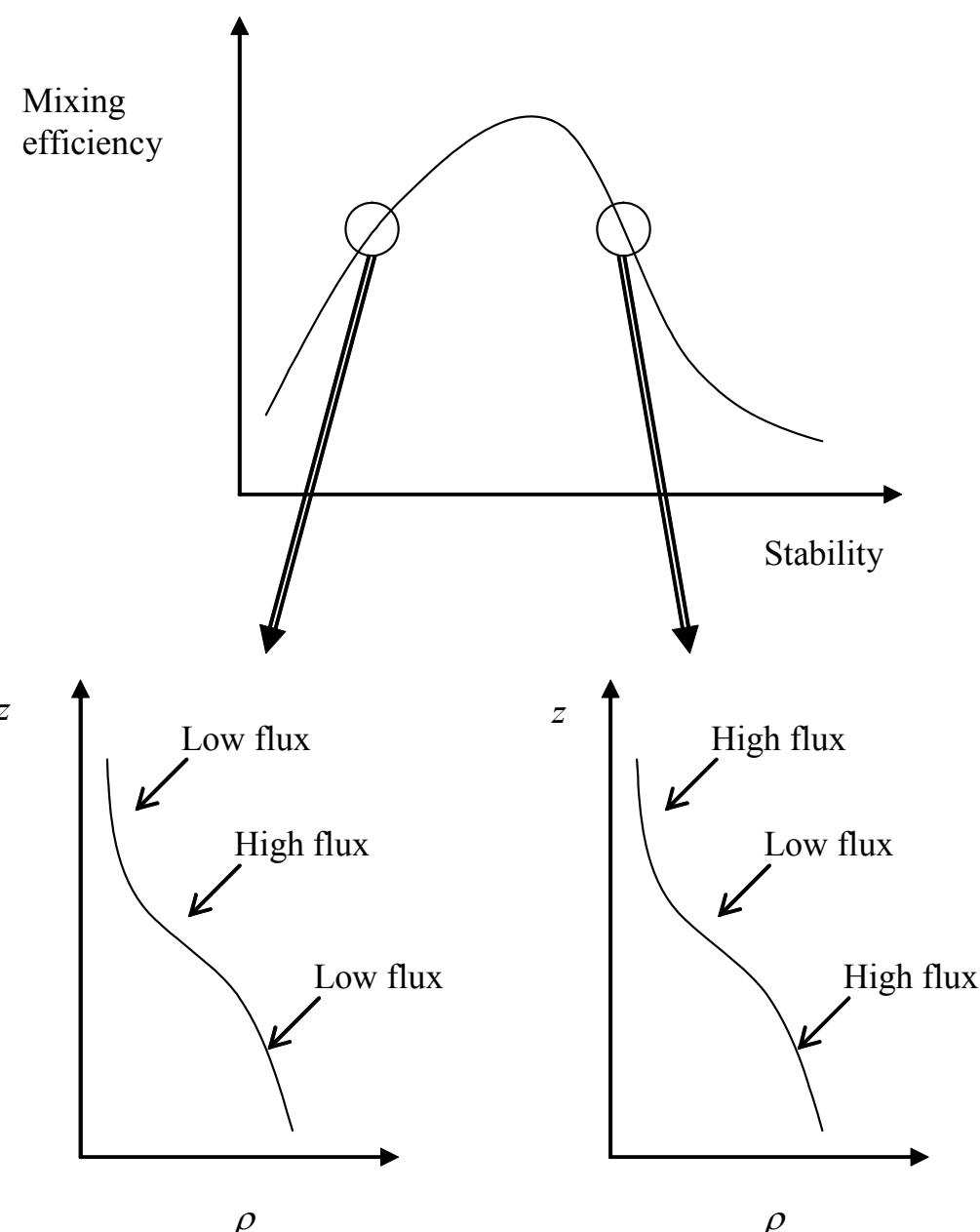
Universal Flux Law & the Phillips Mechanism?

- Directly measured vertical flux of salt/buoyancy follows universal flux law: (Oglethorpe et al 2013)



- Non-monotonicity consistent with Phillips 1972:

- Initially observed by Guyez et al 2002
- Layers very long-lived...
- Mixing independent of structure
- Focus on one interface
- Intermittent: strong/weak turbulence
- Both strong & weak stratification
- (Generic) role in mixing?
- Are curves showing intermittency?

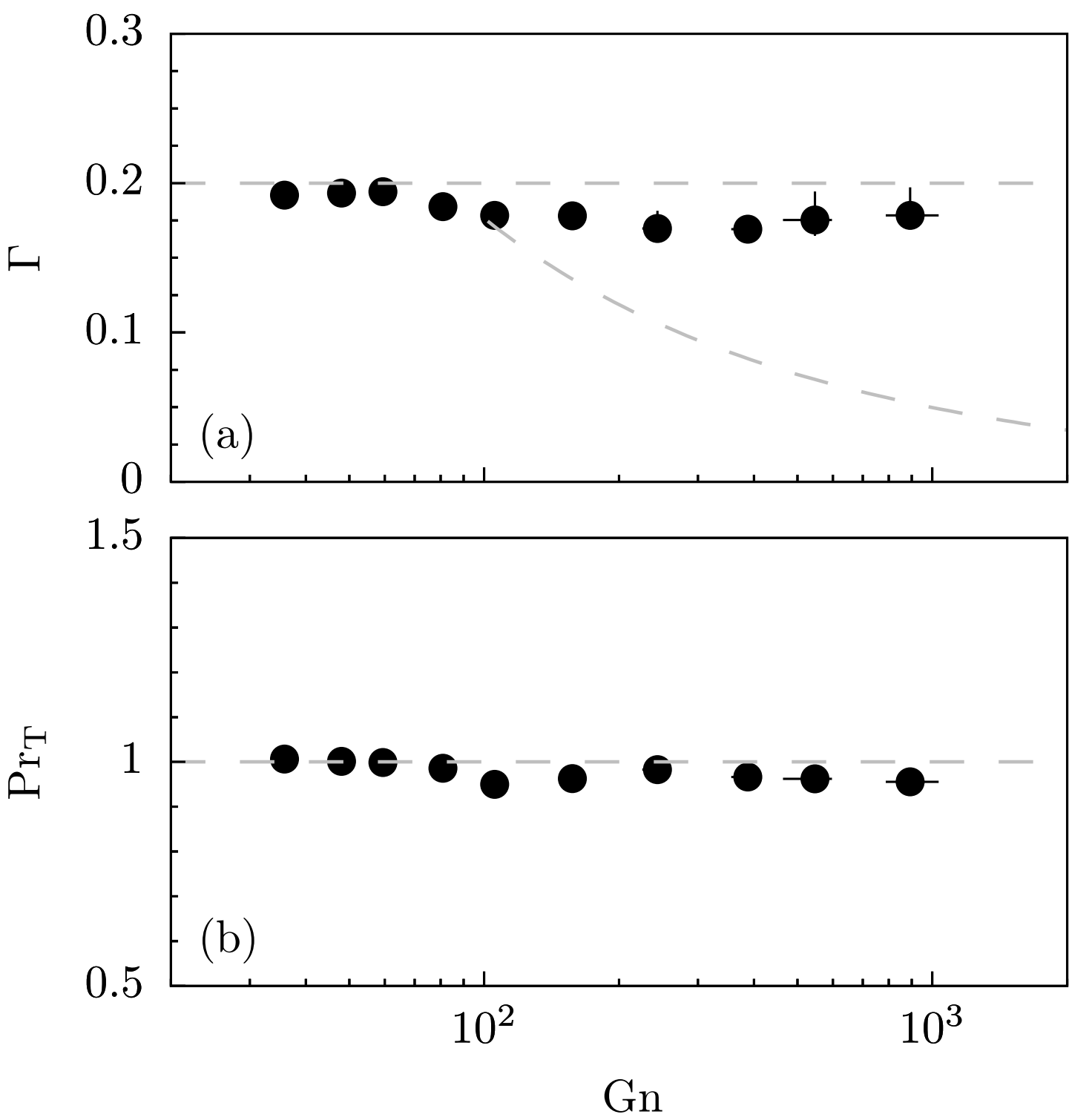


Forced statistically steady flow: Osborn regime?

- Can force uniformly sheared and stratified flow to be statistically steady (with up to 10^4 gridpoints)
- Choose \mathcal{K} ; choose ν ; fix S and vary g so that flow is steady: Ri, \mathcal{E}, χ **emerge** as consequence
- Emergent quantities have **fixed** $Ri \simeq 0.16$; $Pr_T \simeq 1 \rightarrow \Gamma \simeq 0.2, D \simeq 1/\Gamma, L_\rho \simeq \sqrt{1 + \Gamma} L_s \quad Fr \equiv \frac{\mathcal{E}}{N\mathcal{K}}$

Case	Gn	Ri	Fr	N_x
SHSST-R1	36	0.163	0.46	1024
R2	48	0.159	0.47	1280
R3	59	0.162	0.48	1536
R4	81	0.154	0.50	1792
R5	110	0.155	0.52	2048
R6	160	0.157	0.48	3072
R7	240	0.156	0.48	4096
R8	390	0.146	0.46	6144
R9	550	0.163	0.45	8192
R10	900	0.152	0.42	9600

$$L_x = 2L_y = 4L_z$$



Hypothesis

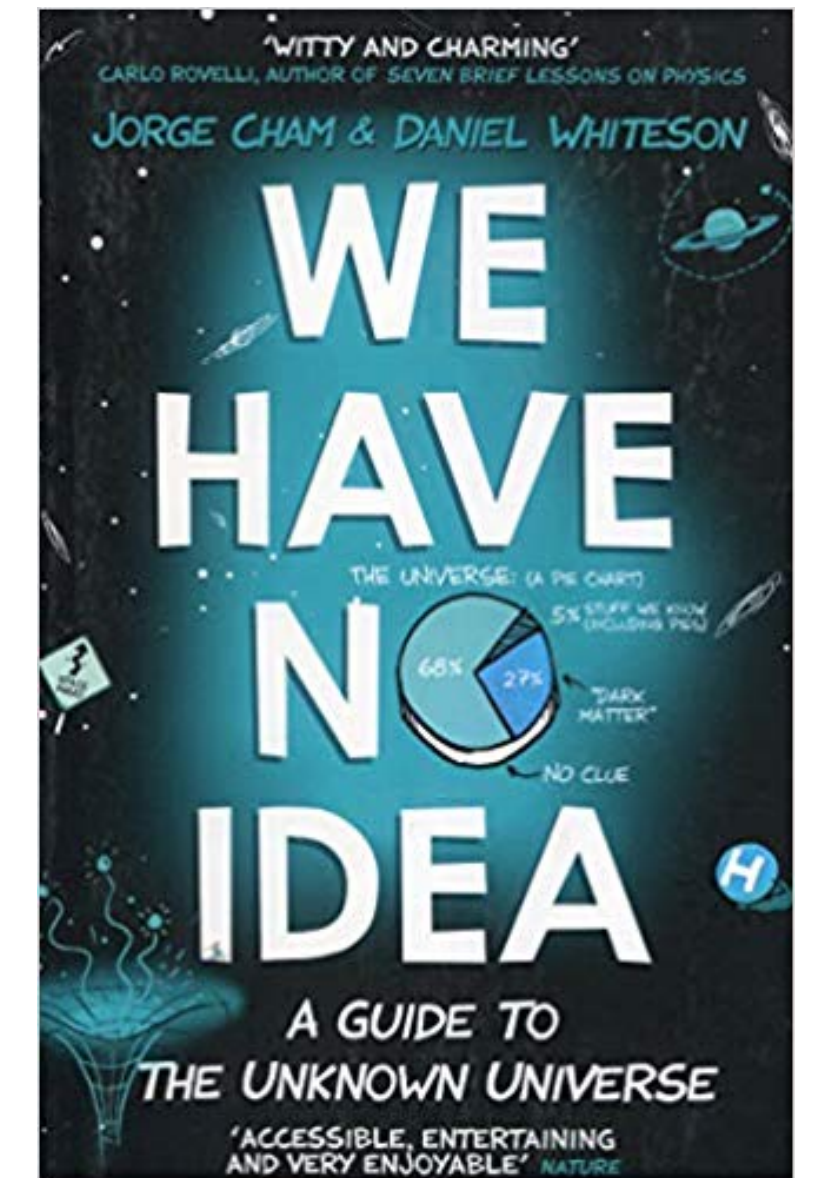
Osborn parameterisation characteristic of

1. steady
2. shear-forced
3. weakly stratified
4. equivalent to Osborn-Cox
5. $Pr_T = 1$ turbulence

that's not really the ocean...
Q: Is $M-H \sim 1/4$ a coincidence?

(Some of the) Open Questions

- What does Ri mean in a turbulent or spatio-temporally varying flow?
- Does stability theory have any relevance at all?
- Do forced flows have any connection with freely evolving flows?
- How can the history/memory/advection of a flow be captured in a parametric description?
- Are layered states generic or even accessed?
- Do non-monotonic flux laws have any meaning , particularly on their (unconfirmed?) right flanks?
- Can boundaries ever be ignored or modelled appropriately?
- Is there any hope to use deterministic “physics” models to describe mixing in stratified turbulence?
- Is the future data-driven/statistical with a census of “all” possible processes required to **deep-learn**?



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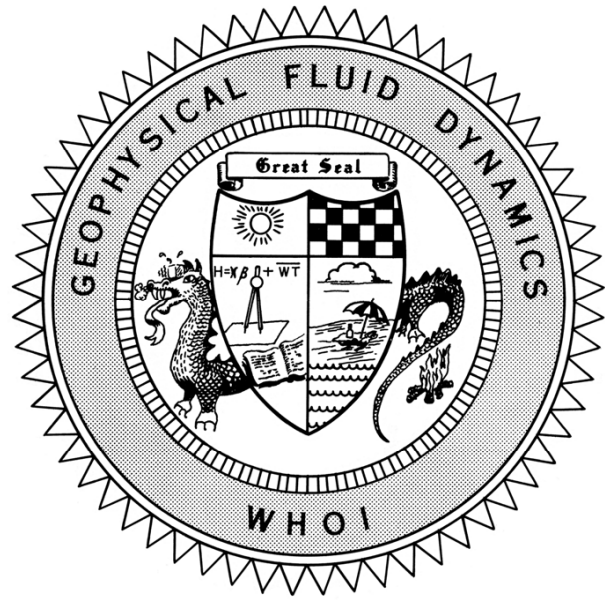
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Fellowships in Geophysical Fluid Dynamics at Woods Hole Oceanographic Institution

June 17 to August 22, 2019

Since 1959 the GFD program has promoted an exchange of ideas among researchers in the many distinct fields that share a common interest in the nonlinear dynamics of fluid flows in oceanography, meteorology, geophysics, astrophysics, applied mathematics, engineering and physics. Each year, the program is organized around a ten-week course of study and research for a small group of competitively selected graduate-student fellows. The overall philosophy is to bring together researchers from a variety of backgrounds to provide a vigorous discussion of concepts that span different disciplines, and thereby to create an intense research experience. For the student fellows, the centerpiece of the program is a research project, pursued under the supervision of the staff. At the end of the program, each fellow presents a lecture and a written report for the GFD proceedings volume. Over its history, the GFD Program has produced numerous alumni, many of whom are prominent scientists at universities throughout the world. The interdisciplinary atmosphere of the Program is the ideal place for young scientists to learn the habits of broad inquiry, of speaking to others with very different backgrounds and viewpoints, and of seeking answers in unfamiliar places.

The Program commences with two weeks of Principal Lectures focusing on a particular theme in GFD. For 2019, the theme is **"Stratified Turbulence and Oceanic Mixing Processes"** and the lecturers will be **Colm-cille Caulfield** (University of Cambridge), who will concentrate on recent theoretical developments in the fluid dynamical description of turbulent stratified mixing, and **Stephanie Waterman** (University of British Columbia), who will concentrate on the observational evidence of such mixing in the world's oceans.

Up to ten competitive fellowships are available for graduate students. Successful applicants will receive stipends of \$7,205 and an allowance for travel expenses within the United States. A small number of unpaid fellowships may also be available for strongly qualified students who can support themselves financially. Fellows are expected to be in residence for the full ten weeks of the program. The application deadline is February 15, 2019. Awards will be announced by April 1, 2019. We seek applicants from all areas of Geophysical Fluid Dynamics, and particularly encourage applications from women and members of underrepresented groups. Further information and application forms may be obtained at <http://gfd.whoi.edu>, or by writing to: gfd@whoi.edu

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