Particle motion and irreversibility of turbulent flows

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Preamble

- What is at stake in turbulence: many problems of crucial importance in the natural sciences (geo-, astro-physics, etc) or in engineering.

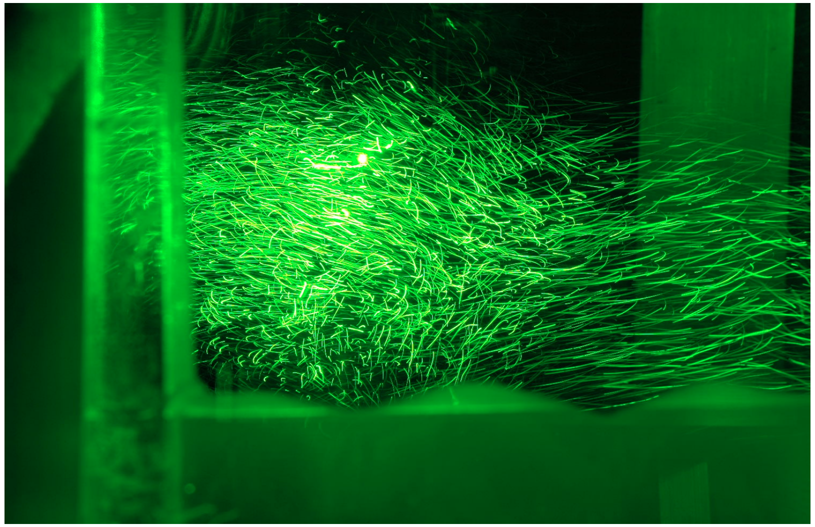
- From the point of view of fundamental physics: turbulence is a challenging non-equilibrium statistical mechanics problem.

~ one needs to find the right concepts/methods to cope with a highly non-equilibrium, nonlinear system.

- Here: explore the problem from the point of view of the motion of individual particles (the "lagrangian approach").

The Lagrangian point of view

Accurate measurements of particles moving (very fast) in a turbulent flow (Cornell/Göttingen; Lyon; Zürich; Copenhagen …). DNS results of similar quality.



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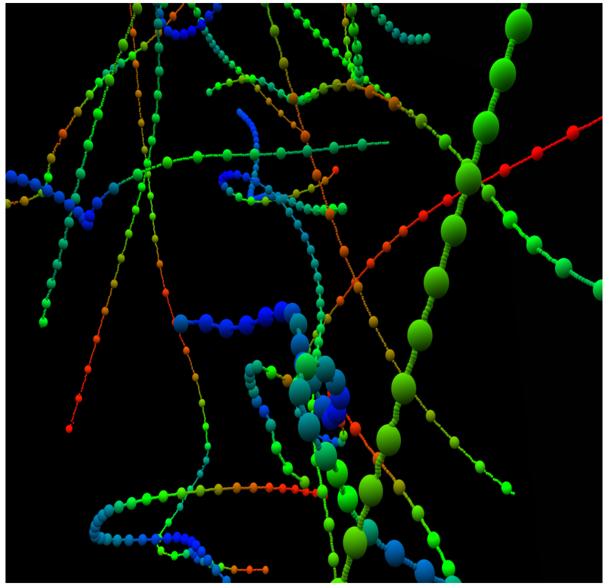


What can we learn about turbulence by following particles?

Turbulent motion from a particle perspective

- Classical results:

- *At long times*, GI Taylor (1922) showed that the motion is diffusive, with a coefficient of diffusion:

$$D = \int_0^\infty \langle u(x(t),t)u(x(0),0)\rangle dt$$

- Dispersion of a set of particles by a turbulent flow (Richardson, 1928):

$$\langle R^2(t)\rangle \propto \varepsilon\, t^3$$

Turbulent motion from a particle perspective

New perspective:

With the newly available possibilities in experiments or numerical simulations, describe the motion of particles at small spatial and temporal scales.

- How does turbulence affect the motion of one particle ?
- … of several particles ?
- Any connection with “stochastic thermodynamics” ?

Time irreversibility

Turbulent dynamics (Navier-Stokes eqns)

~ *energy flux through scale*

~ ***time irreversible process.***

"A trained eye viewing a movie of turbulence run backwards can tell that something is going wrong…"

(Falkovich & Sreenivasan, Physics Today 2006)

Can one really distinguish "the arrow of time" ?
(see e.g. Pomeau, 1982).

"training the eye":

Two-particle statistics

Training the eyes by following pairs of particles

Deduce the equation for the evolution of the relative evolution of two particle relative distance:

$$R(t) = r_2(t) - r_1(t);\ \ \delta R(t) = R(t) - R(0)$$

Introduce: $\delta u = u_1(t) - u_2(t);\ \ \delta a = a_1(t) - a_2(t)$

=> $\left\langle \delta R(t)^2 \right\rangle = \left\langle \delta u(0)^2 \right\rangle t^2 + \left\langle \delta u(0).\ \delta a(0) \right\rangle t^3 + O(t^4)$

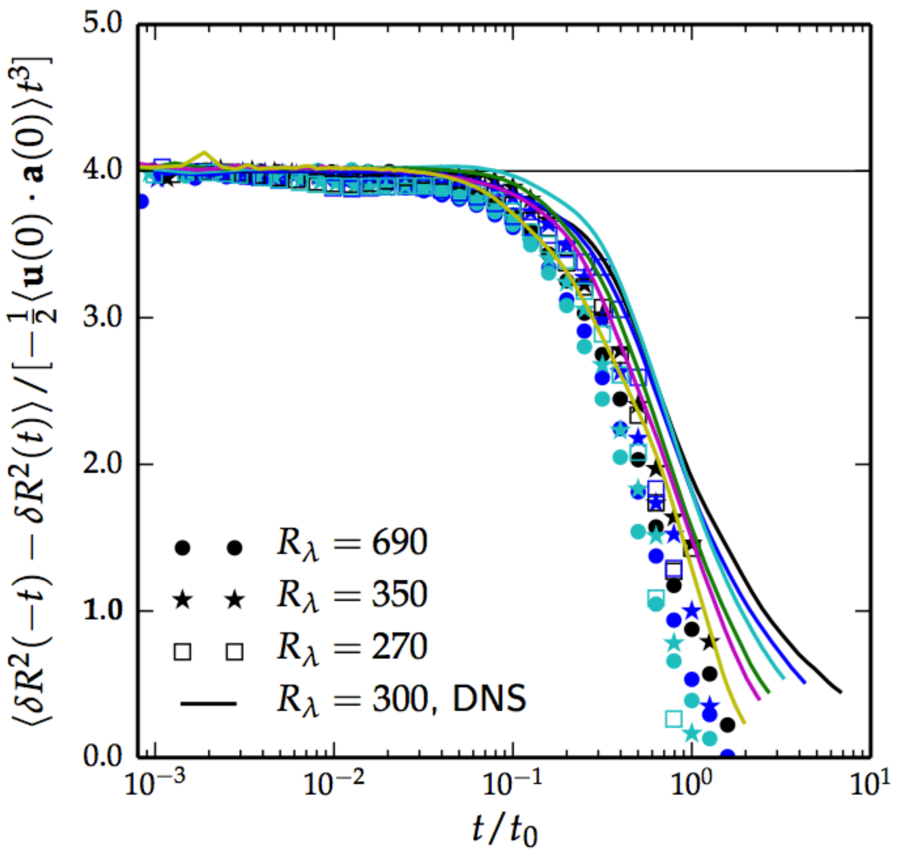
Identity: (Ott and Mann 2000, Pumir et al 2001, Falkovich et al 2001)

$$\frac{1}{2}\frac{d}{dt}\left\langle (u_2 - u_1)^2 \right\rangle = \left\langle \delta u(0).\ \delta a(0) \right\rangle = -2\varepsilon$$

Training the eyes by following pairs of particles

Consequence:

$$\left\langle \delta R(-t)^2 \right\rangle - \left\langle \delta R(t)^2 \right\rangle = -2\left\langle \delta u(0).\ \delta a(0) \right\rangle t^3 + O(t^5)$$



Use different initial separations, R_0, different Reynolds numbers + rescale time with $t_0=(R_0^2/\varepsilon)^{1/3}$

Good data collapse !

(Jucha et al, PRL 2014)

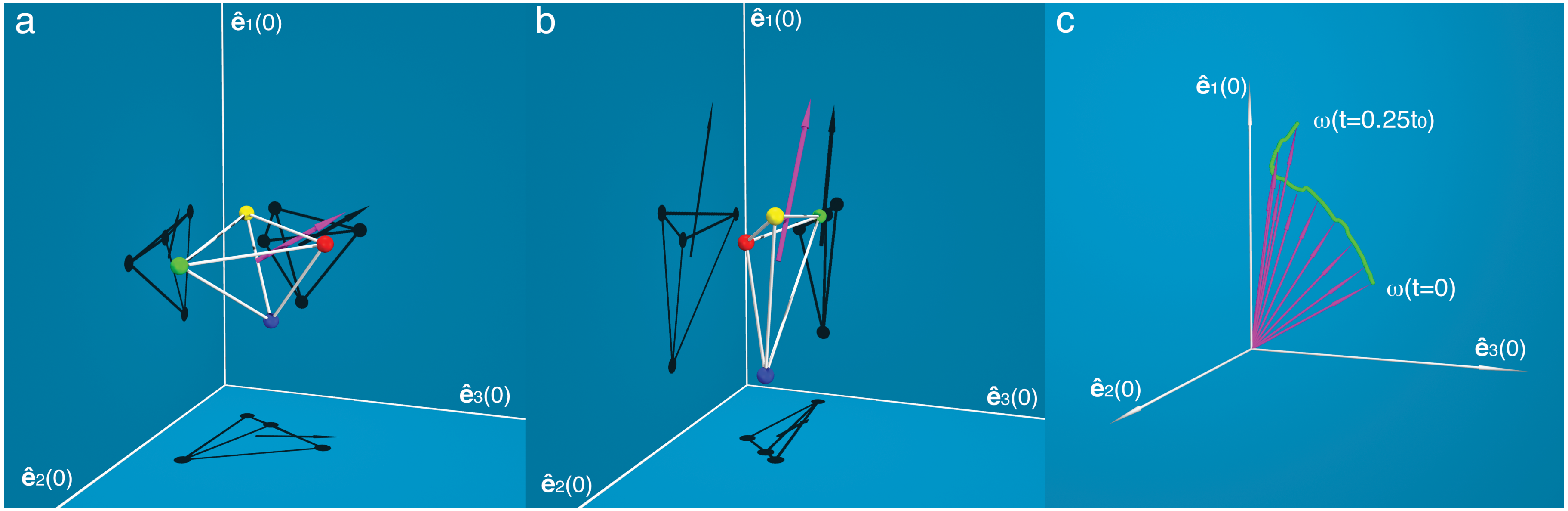
Training the eyes by following pairs of particles

Physical content: particles separate faster backward in time than they separate forward in time !

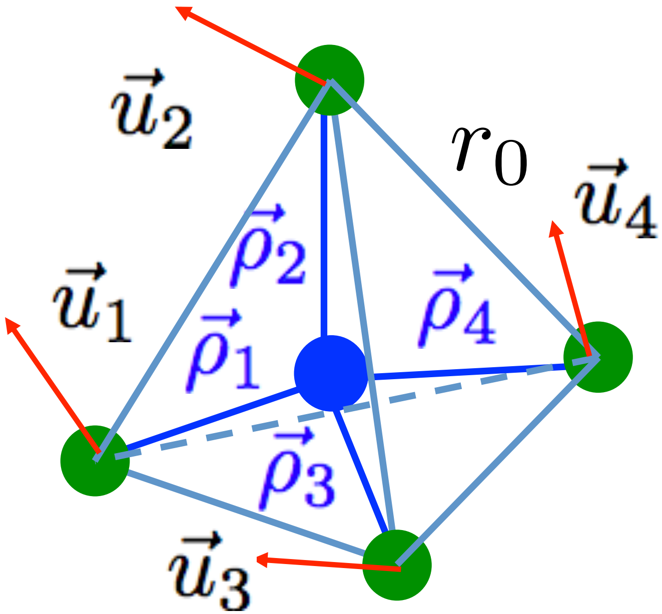
=> Measuring how particles separate does provide a way of distinguishing forward- and backward- evolution by looking at pairs of particles.

Using more particles to "train the eye"

More particles => more info !



Analysis: relate the observed time-asymmetry to fundamental properties of the velocity gradient tensor.



Velocity gradient perceived by tetrads:

$$\mathbf{M}: \quad \mathbf{M}\vec{\rho}_i = \vec{u}_i - \frac{1}{4}\sum_i \vec{u}_i$$

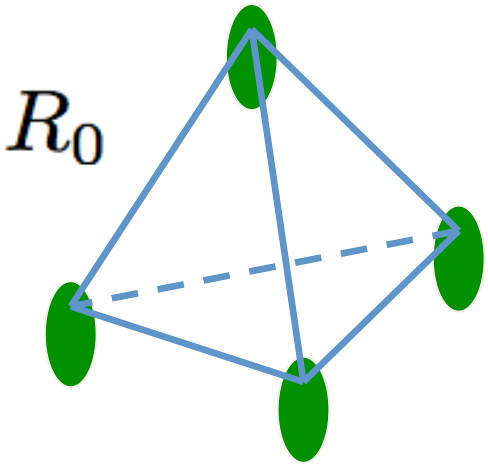
$$\mathbf{M} = \mathbf{S} + \mathbf{\Omega}$$

Perceived strain: $$\mathbf{S} = \frac{1}{2}(\mathbf{M} + \mathbf{M}^T)$$

Perceived vorticity: $$\mathbf{\Omega} = \frac{1}{2}(\mathbf{M} - \mathbf{M}^T)$$

Chertkov, Pumir, Shraiman (**Phys. Fluids**, 1999)

Quantifying the shape evolution

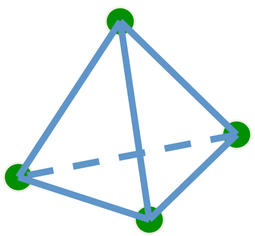


Eigenvalues g_i of the moment of inertia tensor (r^G = center of mass):

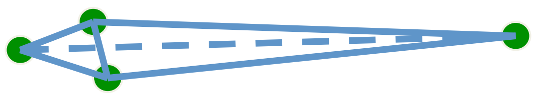
$$g_{ij} = \sum_{a=vertices} (r_i^a - r_i^G)(r_j^a - r_j^G)$$

Define: $$I_i = \frac{g_i}{g_1 + g_2 + g_3}$$

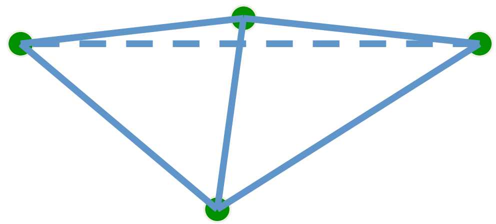
$$I_1 = I_2 = I_3 = \frac{1}{3}$$



$$I_1 \approx 1, I_2 \approx I_3 \approx 0$$



$$I_1 \geq I_2 > I_3 \approx 0$$



Pumir, Shraiman, Chertkov (**Phys. Rev. Lett.**, 2000)

Shape deformation

Evolution for the shape of the set of particles (disregard the motion of the center of mass):

$$d\rho/dt = M \, . \, \rho$$

Take as an initial condition a regular tetrahedron of size R_0

Expand $\boldsymbol{S}$ in Taylor series: $\boldsymbol{S} = \boldsymbol{S}_0 + t\,\boldsymbol{S}_1 + t^2/2\,\boldsymbol{S}_2 + \ldots$

Work in the eigen-basis of $\boldsymbol{S}_0$: eigenvalues of $\boldsymbol{S}_0$:

$S_{0,i}$, with $\qquad S_{0,1} > S_{0,2} > S_{0,3}$

Obtain the following expression for $<g_i(t)>$:

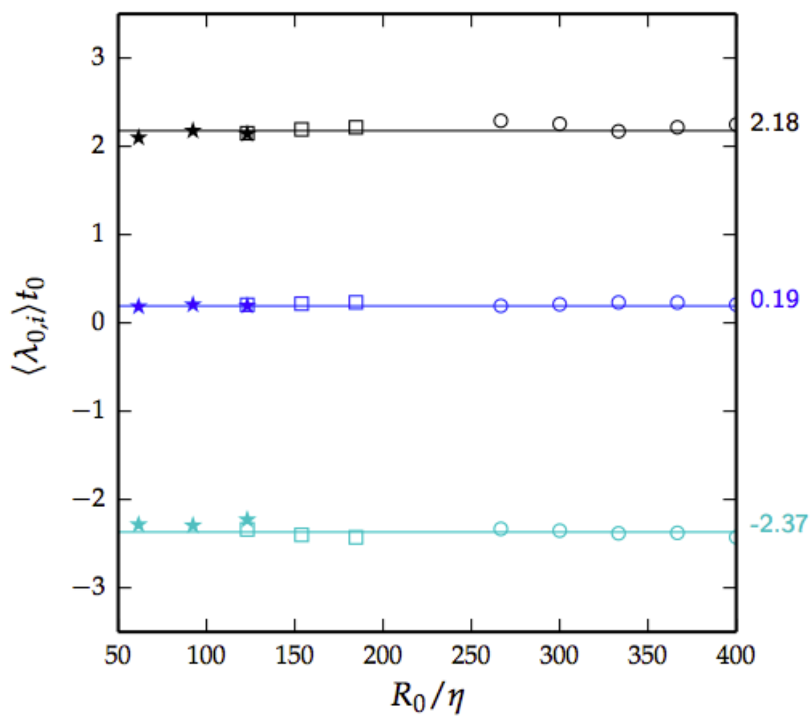
$$<g_i(t)> = 1/2\, R_0^2\, [\, 1 + 2 <S_{0,i}> t$$
$$+ <2\, S_{0,i}^2 + S_{1,i}> t^2 + O(t^3)\,]$$

Shape deformation and time asymmetry t -> -t

The distribution of the eigenvalues of the $\boldsymbol{S_0}$, the strain tensor based on tetrads, is skewed; such that

$$<S_{0,2}> \; > 0$$

A fundamental property *of turbulence (Betchov 1956, Siggia 1981, Ashurst et al, 1987)..*

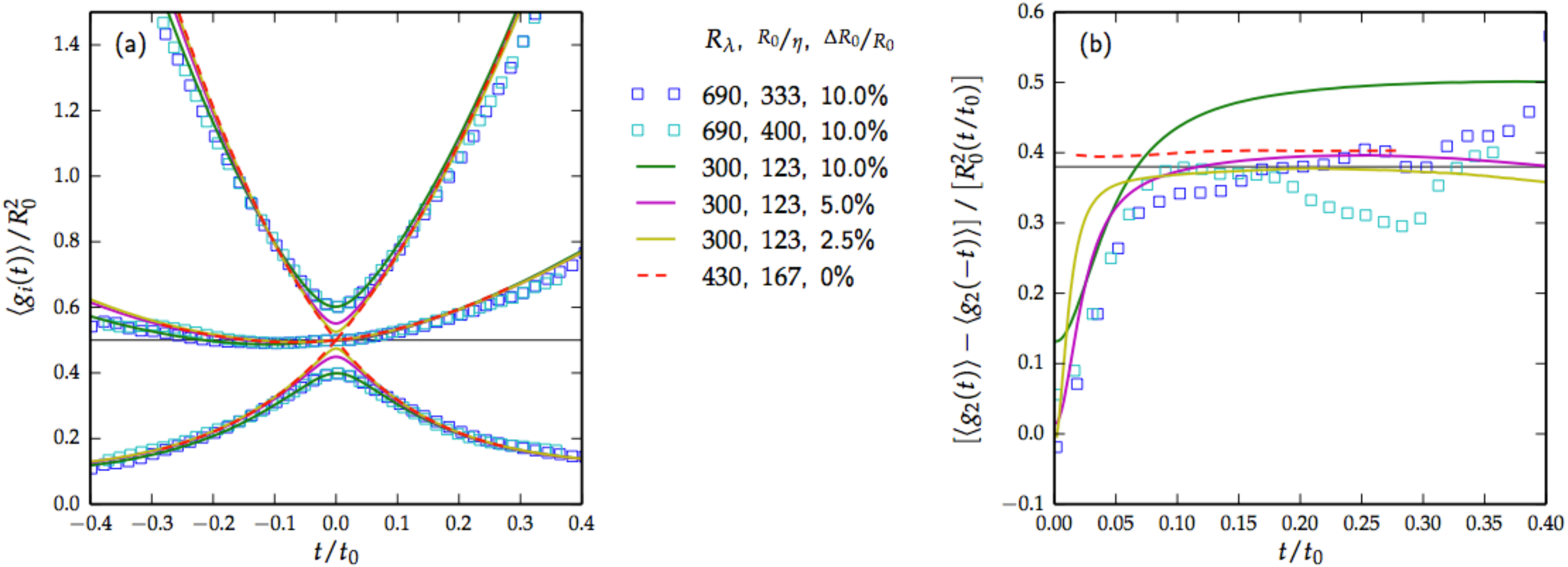


Jucha et al, 2014

Shape deformation and time asymmetry t -> -t

Consequence:

The intermediate eigenvalue $\langle g_2(t) \rangle$ is sensitive to the $t \to -t$ asymmetry, at ***first*** order in t !



Jucha et al, PRL 2014

Shape deformation and time asymmetry t -> -t

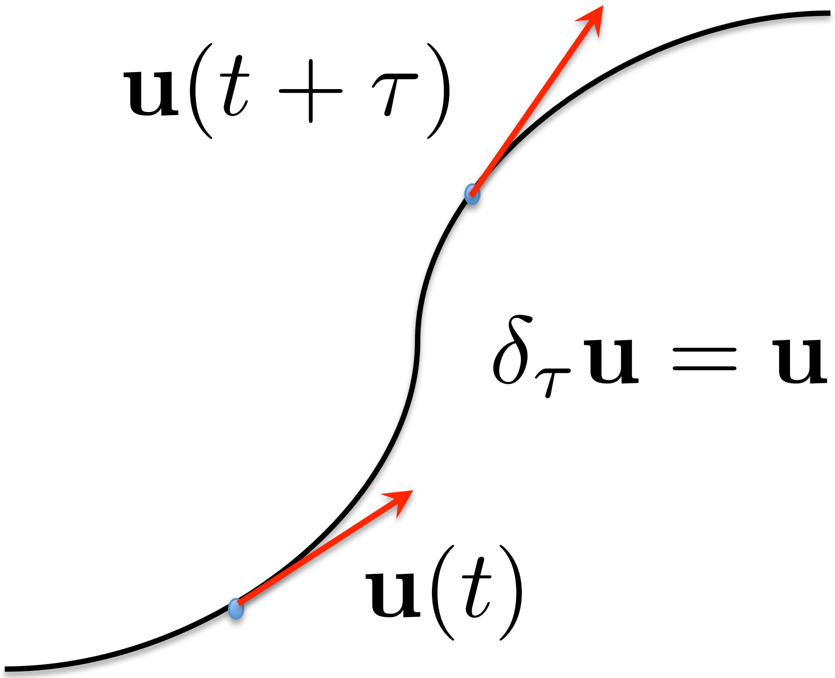
Physical meaning:

An extended, initially isotropic object, flattens differently depending on the arrow of time

~ consequence of the statistical asymmetry of the rate of strain tensor, which is itself a consequence of the small-scale generation in turbulence.

Can one "train the eye" by following one particle only ?

Lagrangian velocity increments



$$\delta_\tau \mathbf{u} = \mathbf{u}(t+\tau) - \mathbf{u}(t)$$

Lagrangian structure functions: $D_n(\tau) = <(\delta_\tau\ u)^n>$.

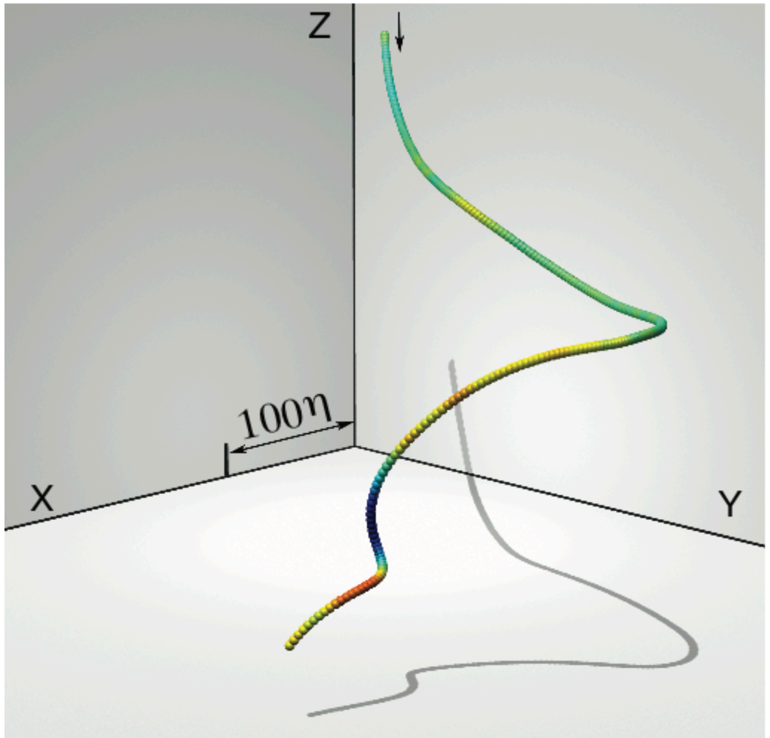
If one flips the direction of time: $t \to -t$,

$D_2(\tau) = \left\langle (\delta_\tau u)^2 \right\rangle$ is unchanged (Falkovich et al, 2012) !

See Leveque and Naso, EPL 2014, for a more precise analysis.

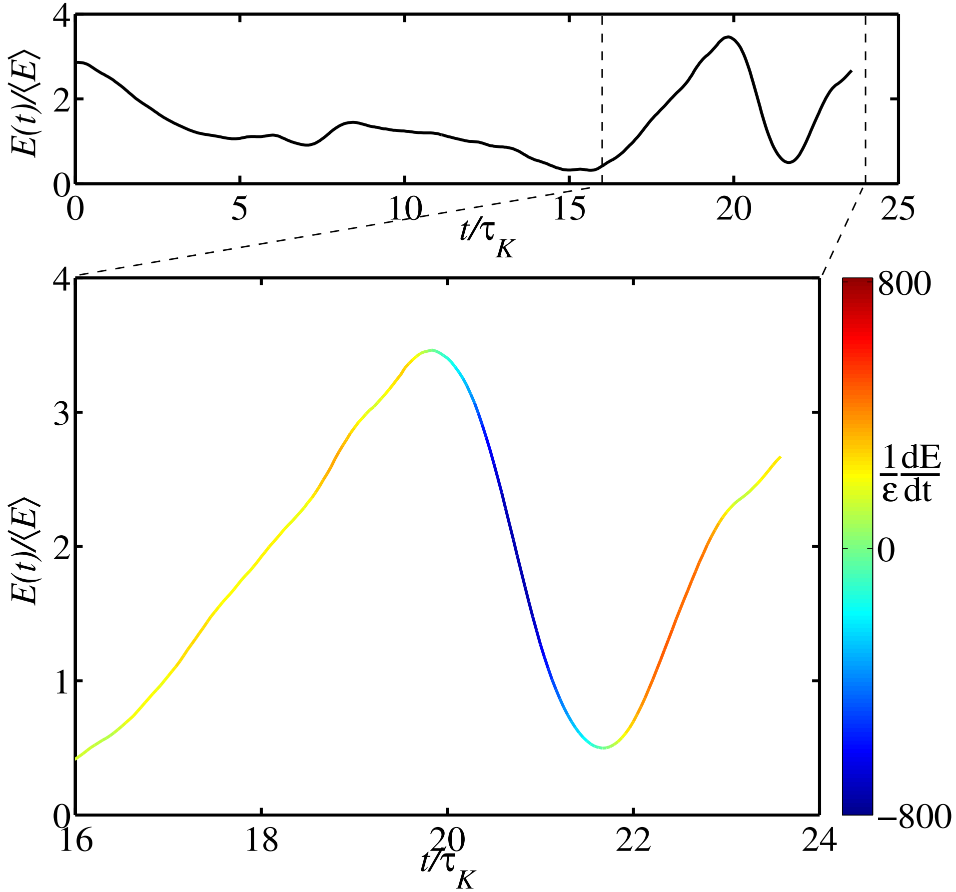
Can one detect irreversibility from one single particle trajectory ?

Observation: large velocity jumps of one given trajectory are associated with a stronger particle deceleration than acceleration.



Data from particle-tracking experiment.

$R_\lambda = 690,\ L/\eta \approx 2300$



Flight-and-crash event

Detecting time irreversibility from single-particle trajectory

Consider the *kinetic energy increments*:

$$W(\tau) = E(t+\tau) - E(t)$$

and their moments: $\langle W^n(\tau) \rangle$

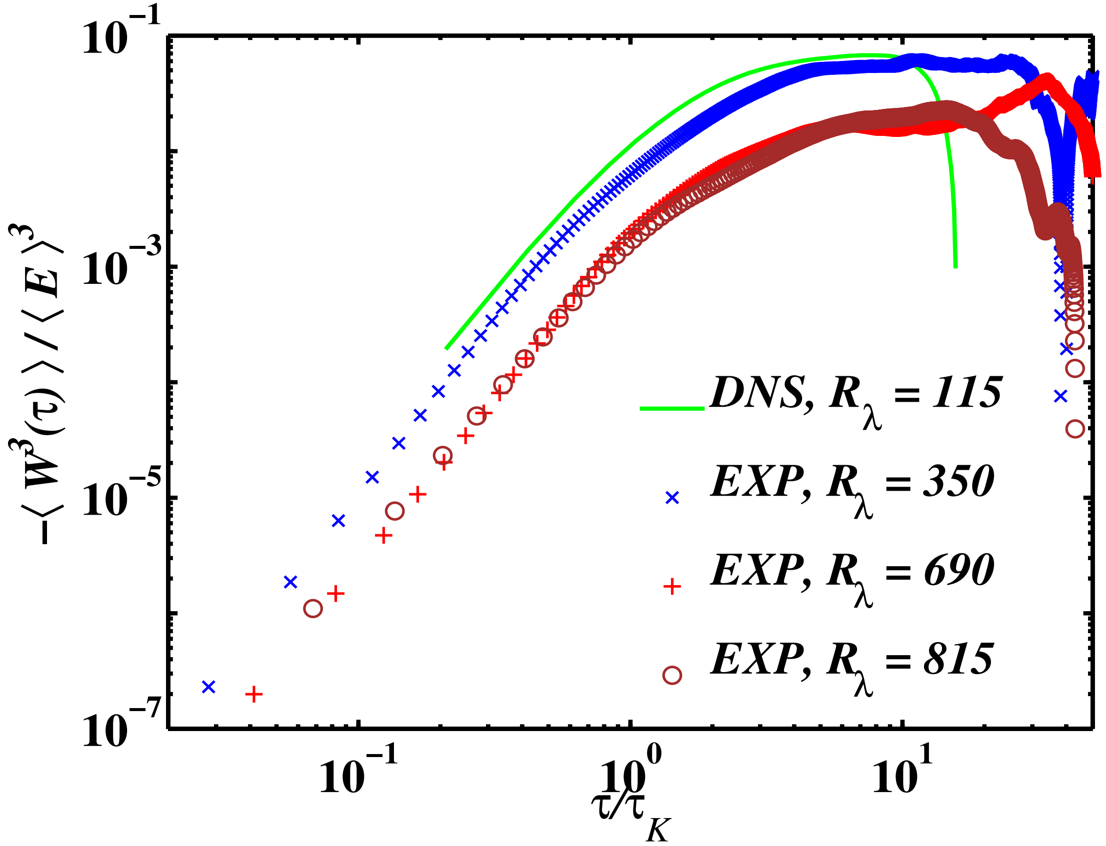
The odd moments are not invariant under $t \to -t$

They can pick up the lack of symmetry seen experimentally !

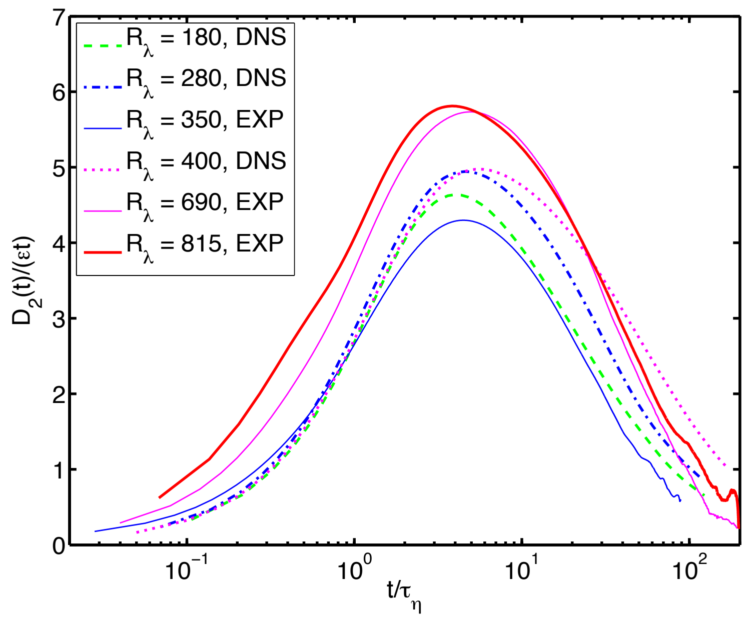
n.b.: the moments $\langle W^n(\tau) \rangle$ *cannot* be expressed in terms of velocity increments only

Detecting time irreversibility from single-particle trajectory

3D



The third moment of W(τ) is negative, and remains ~ constant when τ/τ_K is larger than ~2.



Note that plateau range is much more significant than that of the velocity structure functions.

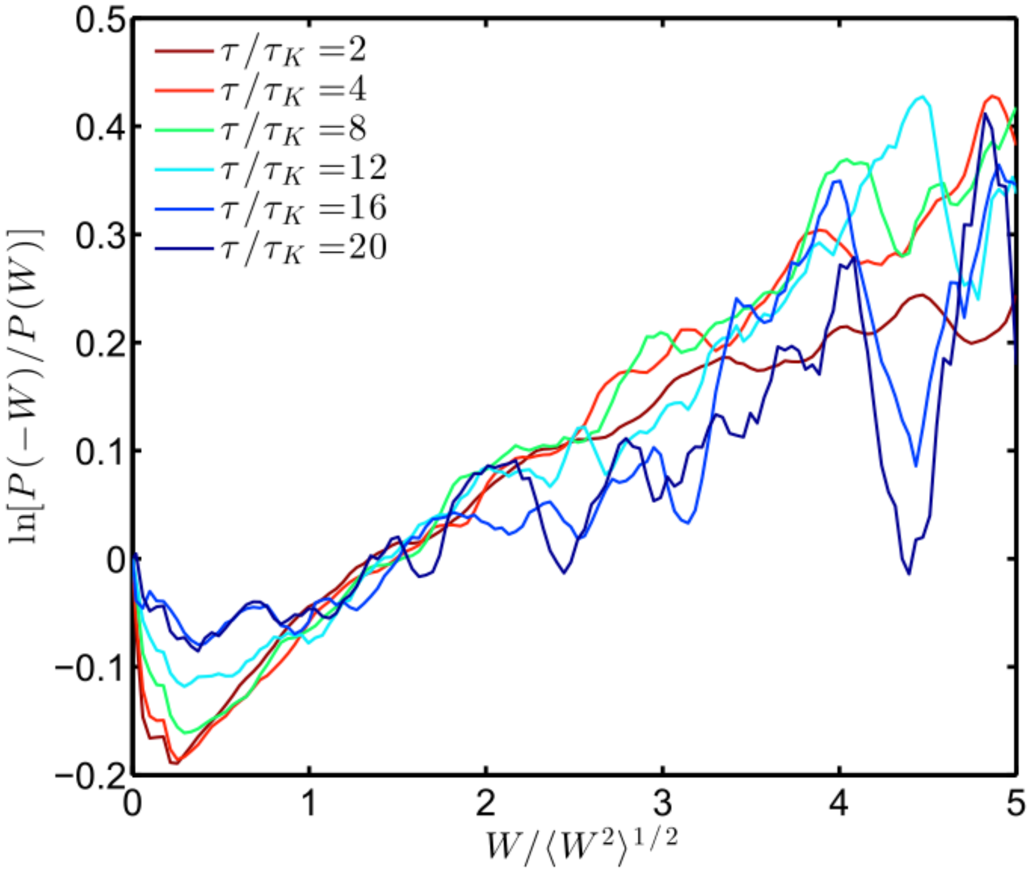
Asymmetry of the PDF of *W* and breaking of detailed balance

The observed lack of symmetry $W \longrightarrow -W$ from the PDFs implies that:

$$P(E \to E + \Delta E) \neq P(E + \Delta E \to E)$$

Detailed balance is broken !!

… no obvious connection with the existing results on statistical thermodynamics…



A quantitative measure of irreversibility

How to measure irreversibility in a turbulent flow ?

- When the Reynolds number increases, the range of excited scale becomes larger.

Can one quantify the irreversibility, and get a notion as how it depends on the Reynolds number ?

First problem: find a proper measure of irreversibility !

A quantity measuring Irreversibility (Ir)

A naïve suggestion:

Origin of irreversibility: the energy flux ε.

$$Ir = \varepsilon \text{ ??}$$

Problem: ε is dimensional – ie, it can be made arbitrarily large or small by changing the units.

At short times:

$$\langle W^3(\tau)\rangle = \left\langle \left(\frac{dE}{dt}\right)^3 \right\rangle \tau^3 + h.o.t.$$

$$= \langle(\mathbf{a}\cdot\mathbf{v})^3\rangle\tau^3 + h.o.t.$$

The observation of nonzero odd moments of W suggests that time-irreversibility should also be reflected in the statistics of the instantaneous power on a fluid particle

$$p \equiv \mathbf{a}\cdot\mathbf{v}$$

In particular, it implies that the PDF of power p is *negatively skewed*.

A quantity measuring Irreversibility (Ir)

A better suggestion:

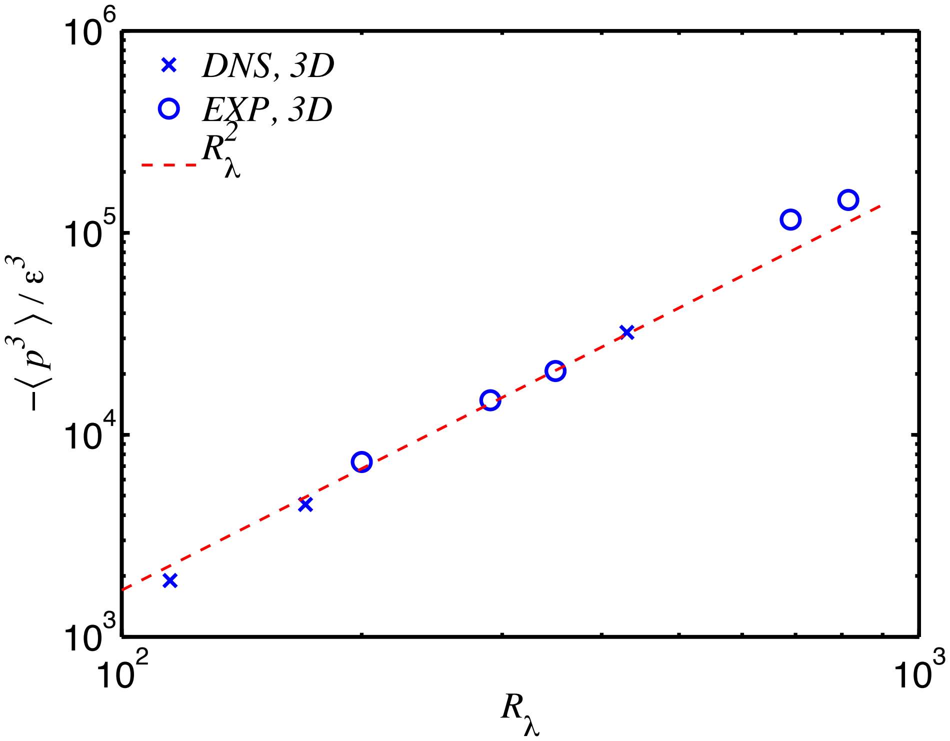
Use the simplest non-obvious moment of p, $\langle p^3 \rangle$, made properly dimensionless.

~notice that p and ε have the same dimension.

$$Ir = - \langle p^3 \rangle / \varepsilon^3$$

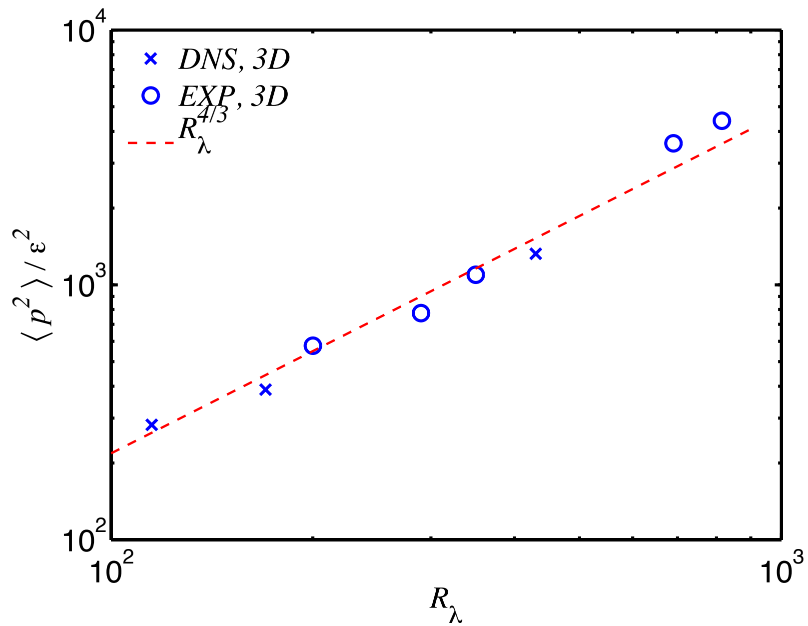
Scaling of *Ir*

3rd moment



$Ir \sim R_\lambda^{\,2}$

nb: 2nd moment ~ $R_\lambda^{4/3}$



Small scale generation in 3d turbulence and irreversibility

What is the content of $<p^3>$?

$$<p^3> \propto -\varepsilon^3 R_\lambda^2$$

Main result: in 3D flows:

$$<p^3> < 0 \quad \Leftrightarrow \quad <\omega.S.\omega> > 0$$

In other words: the manifestation of *irreversibility* observed from the motion of individual particles is a consequence of small-scale generation by turbulence.

Irreversibility "equals" small scale generation

Decomposition of acceleration

- Use the identify: $a = du/dt = \partial_t u + (u.\nabla)\ u$

=>

$$p = a.u = u.\partial u/\partial t + u.(u.\nabla u)$$

$$= p_L + p_C$$

where $p_L \equiv u.\ \partial u/\partial t$ and $p_C \equiv u.\ (u.\nabla)u$ *(see also Tsinober et al, 2001)*.

n.b.: p_C can be expressed as $p_C = u.S.u$, where S is the (symmetric) rate of strain tensor.

An identity

- Assume that the statistical properties of

<u>u and S are statistically independent</u>

(reasonable, since u is controlled by the largest scales of the flow, whereas S is controlled by the smallest scale quantity, which are expected to be only weakly coupled).

Then: $$\left\langle p_C^3 \right\rangle = \left\langle (u.\ S.\ u)^3 \right\rangle = \frac{8}{105} \left\langle u^6 \right\rangle \left\langle tr(S^3) \right\rangle$$

- Use the relation for homogeneous flows (Betchov 1956):

$$\left\langle tr(S^3) \right\rangle = -\frac{3}{4} \left\langle \omega.S.\omega \right\rangle$$

Consequence: $$\left\langle \omega.S.\omega \right\rangle > 0 \Rightarrow \left\langle p_C^3 \right\rangle < 0$$

Elementary scaling considerations

- The usual scaling arguments impy that:

$$\left\langle p_C^3 \right\rangle \propto \varepsilon^3 R_\lambda^3 \;\; ; \; \left\langle p_L^3 \right\rangle \propto \varepsilon^3 R_\lambda^3$$

$$\left\langle p_C^2 \right\rangle \propto \varepsilon^2 R_\lambda^2 \;\; ; \; \left\langle p_L^2 \right\rangle \propto \varepsilon^2 R_\lambda^2$$

- *How come does one find*

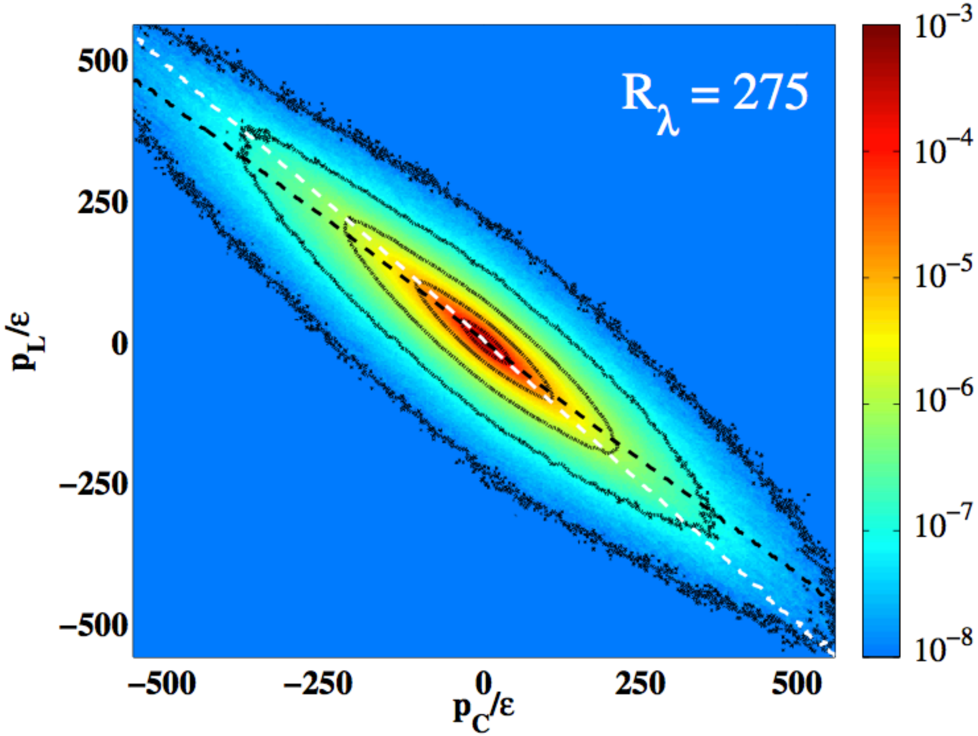
$$\left\langle p^2 \right\rangle \propto \varepsilon^2 R_\lambda^{4/3}; \; \left\langle p^3 \right\rangle \propto \varepsilon^3 R_\lambda^2$$

… *and what does it imply for the moments of* p_C, p_L ???

Partial cancellation between p_L and p_C

- Observation: to a large extent, p_C and p_L cancel each other!

$$|p_C + p_L| << |p_C|, |p_L|$$

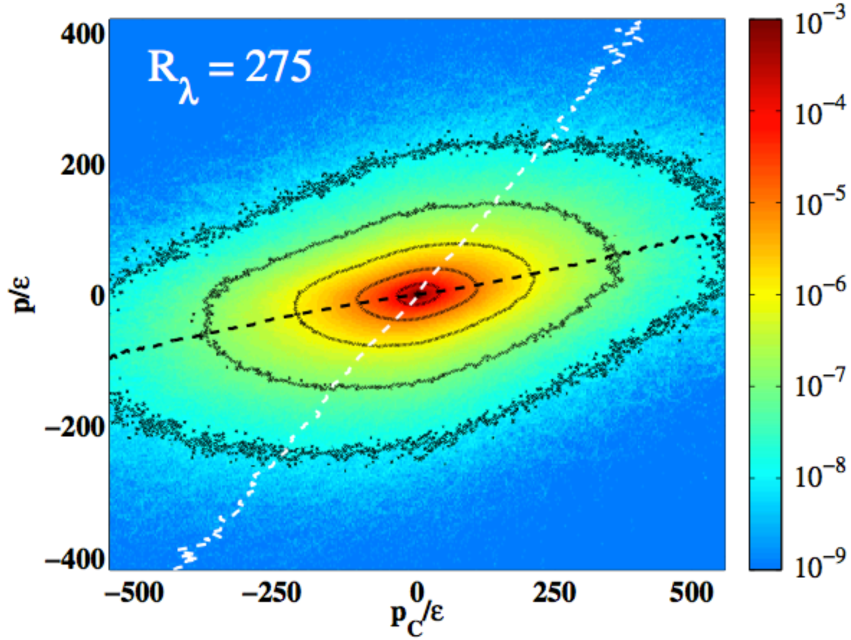


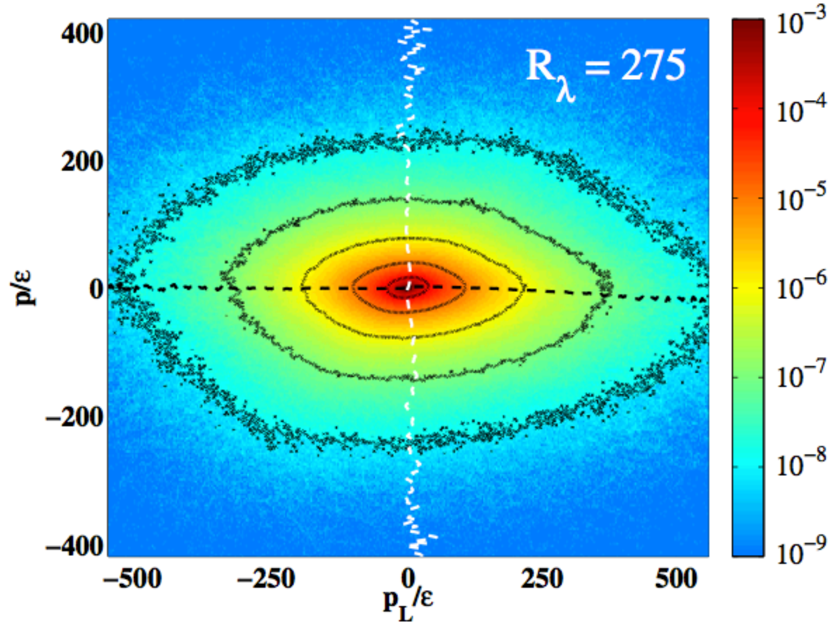
$$\langle p_L \,|\, p_C \rangle \approx -\beta\, p_C; \;\; \beta < 1$$

$$\langle p_C \,|\, p_L \rangle \approx -\, p_L$$

Partial cancellation between p_L and p_C

- Weak correlations between p_C, and p; almost no correlation between p_L and p.





$$\langle p \mid p_C \rangle \approx (1-\beta) p_C; \;\; \beta < 1$$

$$\langle p_C \mid p \rangle \approx p$$

$$\langle p \mid p_L \rangle \approx 0$$

$$\langle p_L \mid p \rangle \approx 0$$

Implication for the moments of p

- Taking into account the independence of p_L and p, obtain: $< p^2 | p_C > \approx (1\text{-}\beta)\, p_C^2$

so $<p^2> \approx (1\text{-}\beta) <p_C^2>$

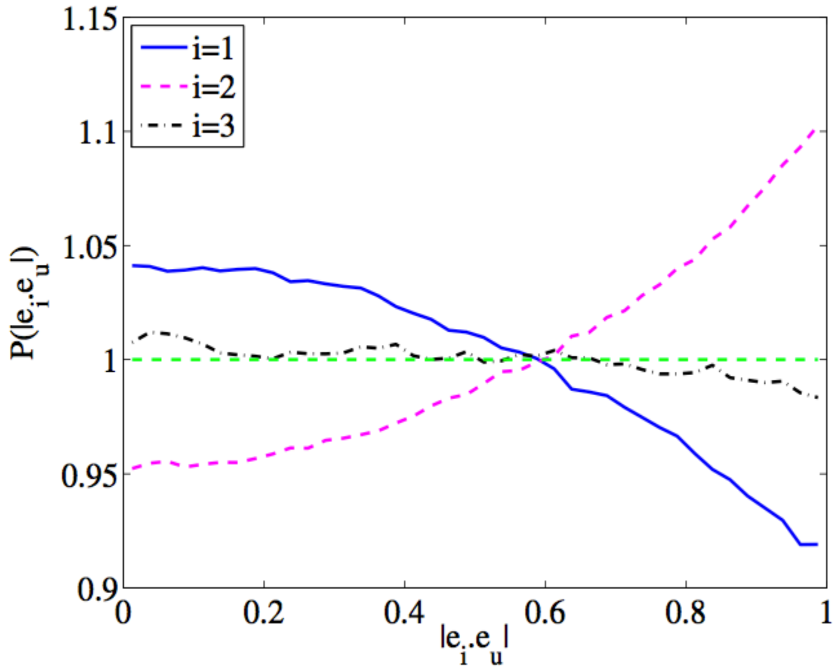
- For the third moment:

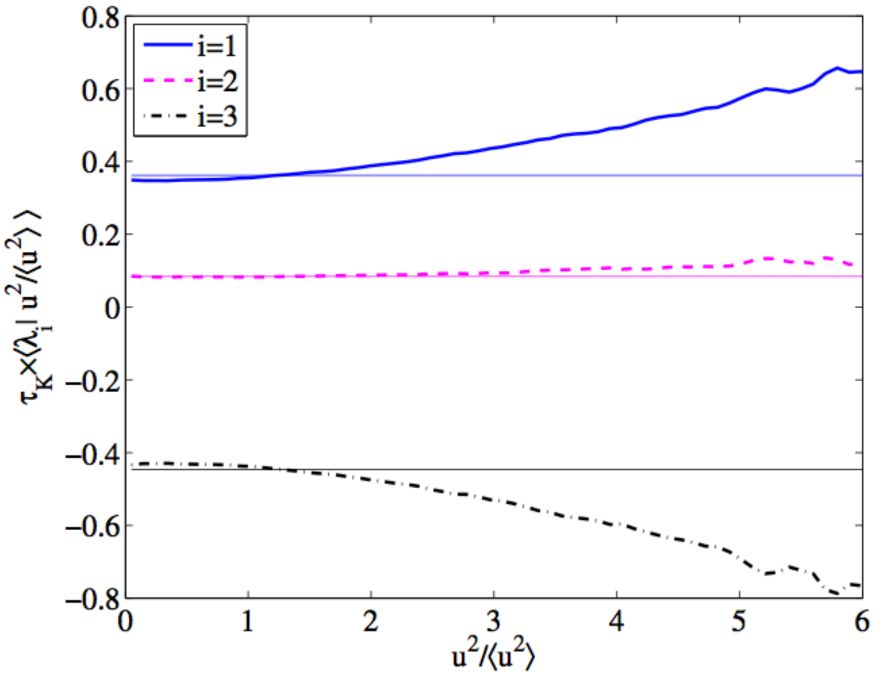
(…) $<p^3> \approx (1\text{-}\beta') <p_C^3>$

where $(1-\beta')$ is a number smaller than $(1-\beta)$, but positive.

<u>Consequence:</u> $(1\text{-}\beta) \sim R_\lambda^{-2/3}$ and $(1\text{-}\beta') \sim R_\lambda^{-1}$

Independence of u and S





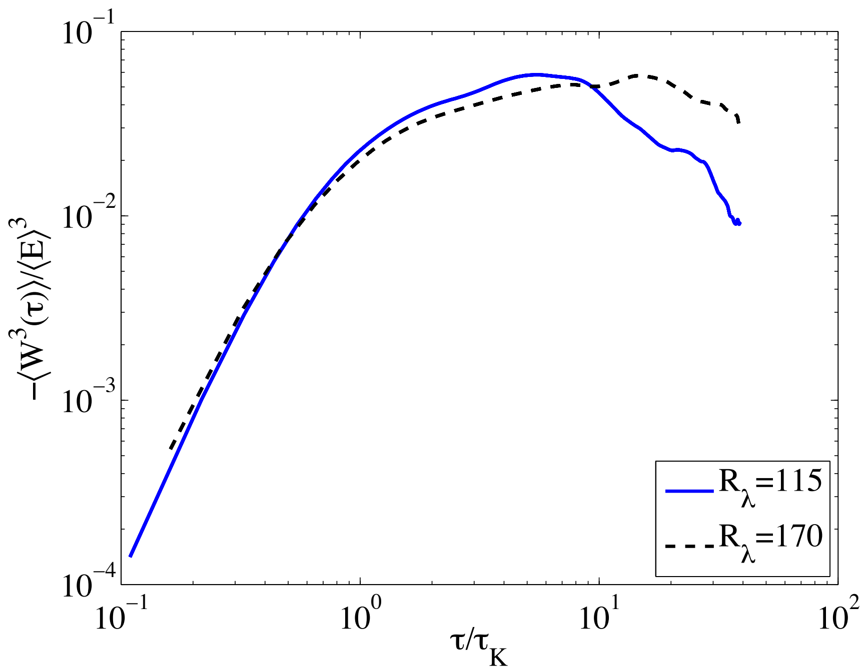
Velocity does not show any particular alignment with any of the eigenvectors of strain

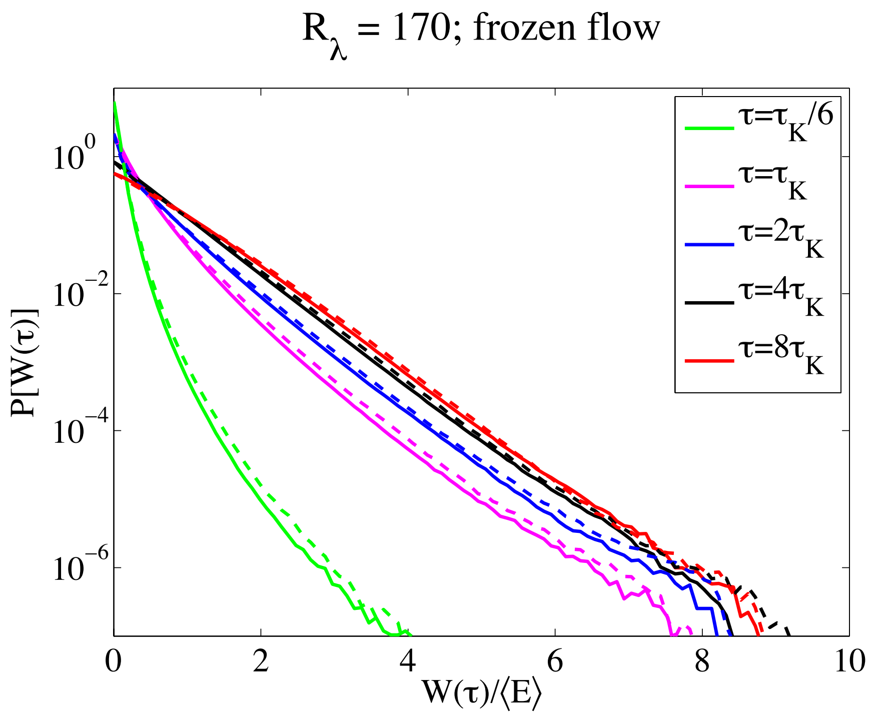
nb: here, $\lambda_1 > \lambda_2 > \lambda_3$

The strain eigenvalues conditioned on $\boldsymbol{u}^2$ depend weakly on the magnitude of velocity.

Implication of $\langle p_C^3 \rangle < 0$ for frozen flows

- In fact, the 3rd moment of $W(\tau)=1/2\ [\boldsymbol{u}^2(\tau) - \boldsymbol{u}^2(0)]$ are *negative*.
- Corresponding to negatively skewed distributions of $W(\tau)$.





Action of the different forces acting on tracer particles

A breakdown of the contributions to *p*

$$p = \mathbf{a} \cdot \mathbf{v} = -\mathbf{v} \cdot \nabla P + \mathbf{v} \cdot \mathbf{f} + \mathbf{v} \cdot \mathbf{D}$$

In 3D: $\mathbf{D} = \nu \nabla^2 \mathbf{v}$

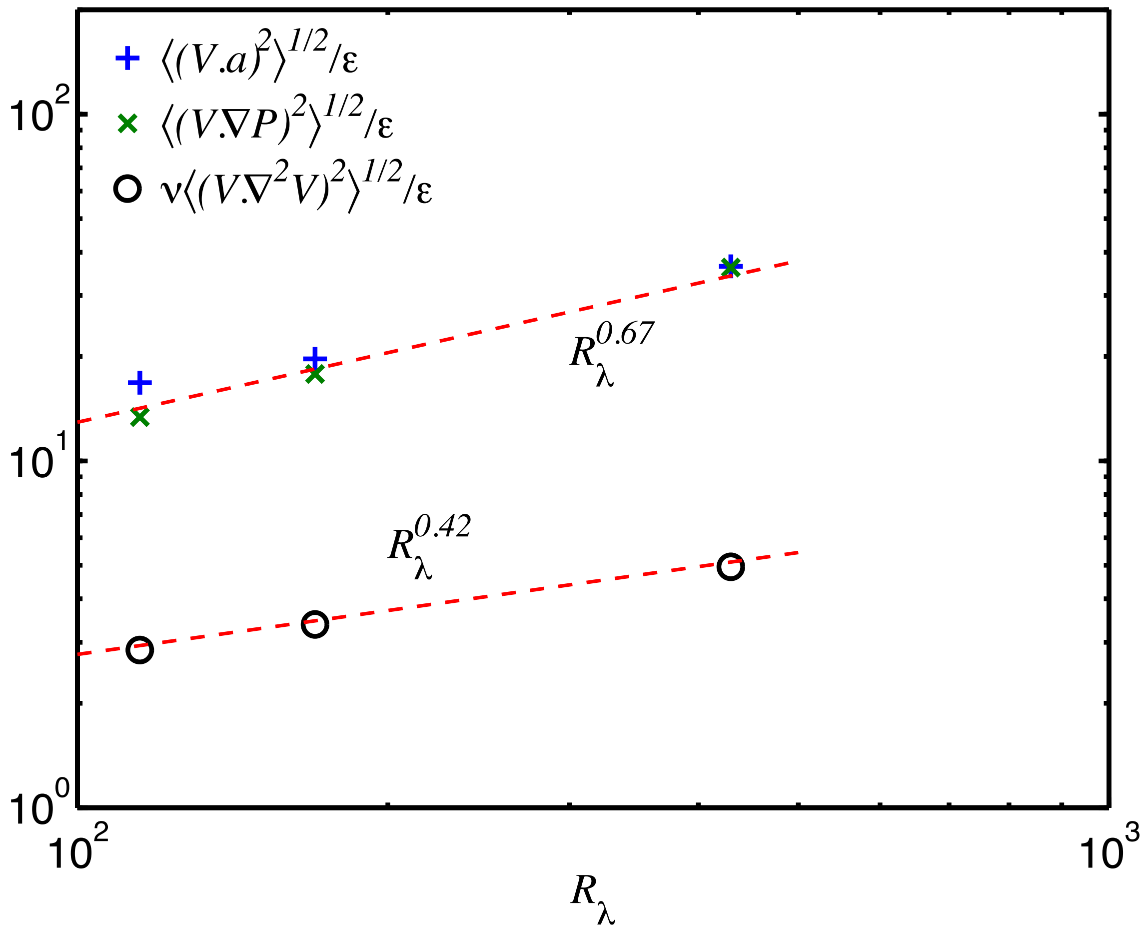
In 2D: $\mathbf{D} = \nu \nabla^2 \mathbf{v} - \alpha \mathbf{v}$

For stationary homogeneous turbulence:

$$\langle \mathbf{v} \cdot \nabla P \rangle = 0$$

$$\langle \mathbf{v} \cdot \mathbf{f} \rangle = -\langle \mathbf{v} \cdot \mathbf{D} \rangle = \epsilon$$

A breakdown of the contributions to *p*

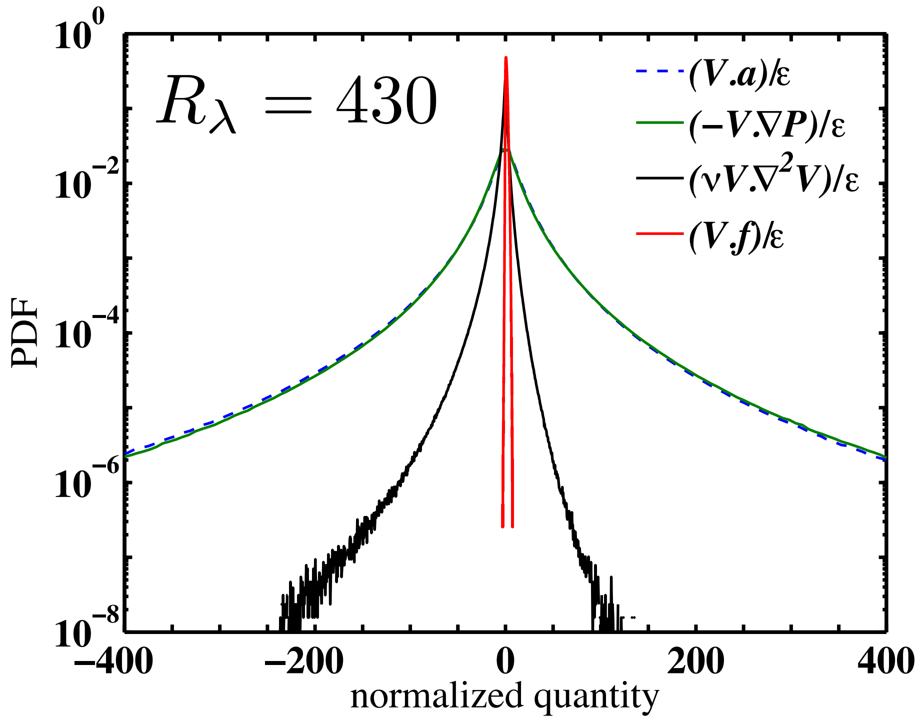


3D data.

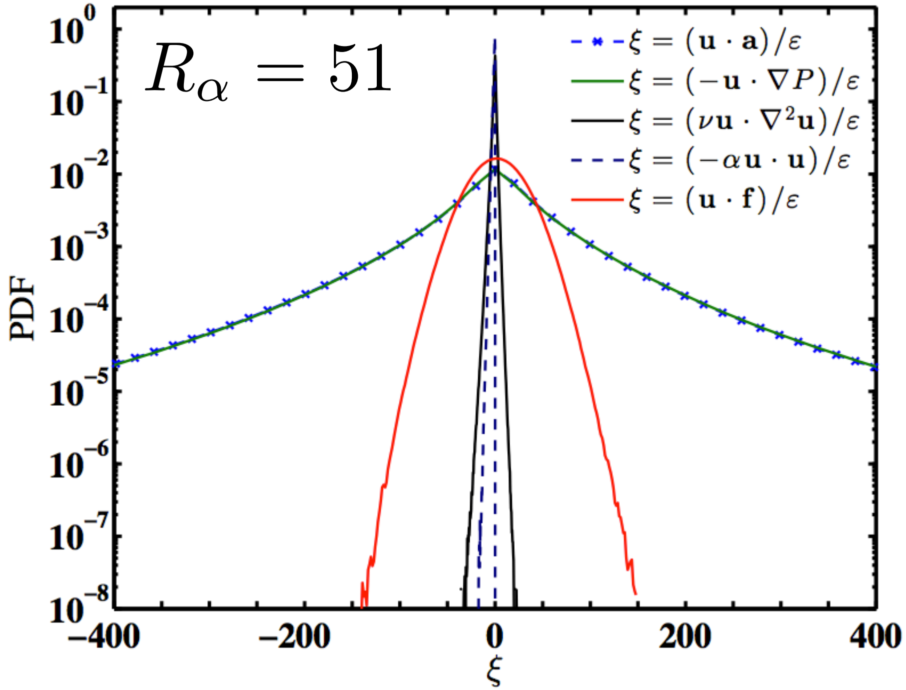
The magnitude of the pressure gradient term also grows faster than the dissipation term.

A breakdown of the contributions to *p*

3D



2D



The magnitude of the pressure gradient term overwhelms the others.

Main contributions to the third moment of the power

In ***2D***, $-\langle \mathbf{v} \cdot \nabla P \rangle$ contributes to 2/3 of the third moment of p.

➔ Pressure contributes to the large, negative accelerations.

$$\langle p^3 \rangle \approx \langle (-\mathbf{u} \cdot \nabla P)^3 \rangle + \langle 3(-\mathbf{u} \cdot \nabla P)^2 (-\alpha \mathbf{u}^2) \rangle$$

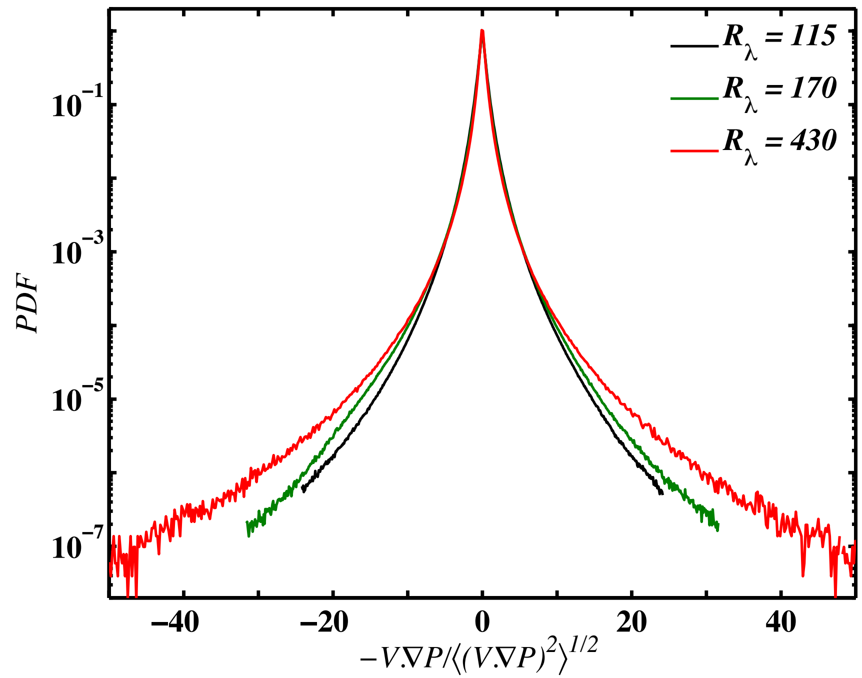
In ***3D***, the third moment of the pressure gradient term does not contribute; the third moment of p comes from cross-terms.

$$\langle p^3 \rangle \approx 3 \langle (\nu \mathbf{u} \cdot \nabla^2 \mathbf{u})(-\mathbf{u} \cdot \nabla P)^2 \rangle + 3 \langle (\mathbf{u} \cdot \mathbf{f})(-\mathbf{u} \cdot \nabla P)^2 \rangle$$

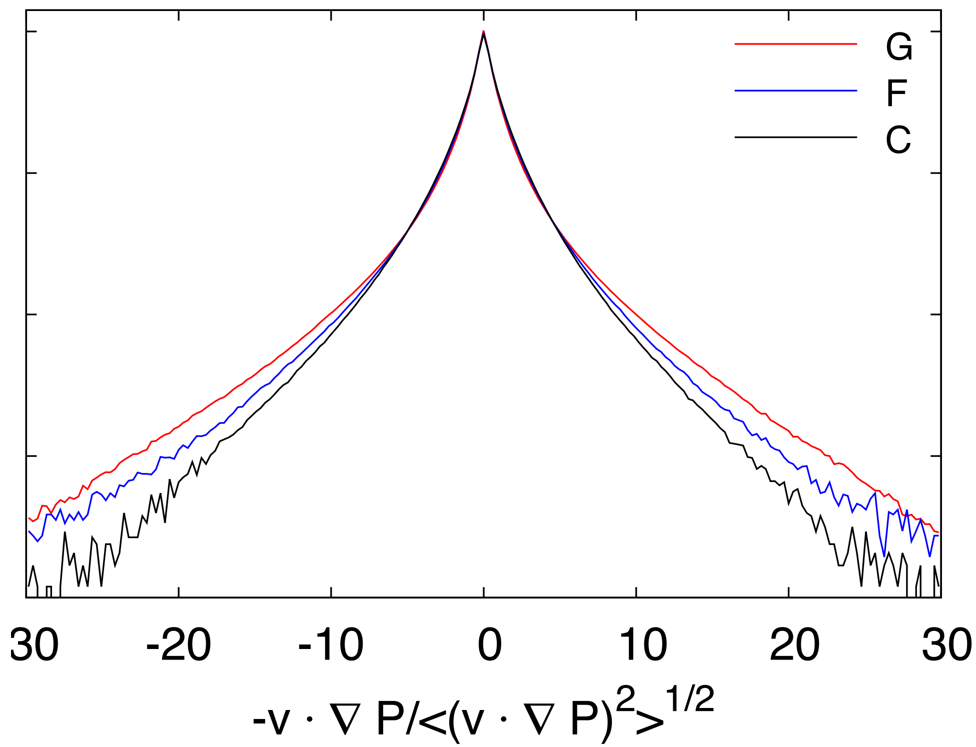
Pressure forces: in 3D, if anything, they contribute more to ***large energy increases*** than ***energy losses***…

Pressure gradient term is differently *skewed* in 2D and 3D!

3D



2D

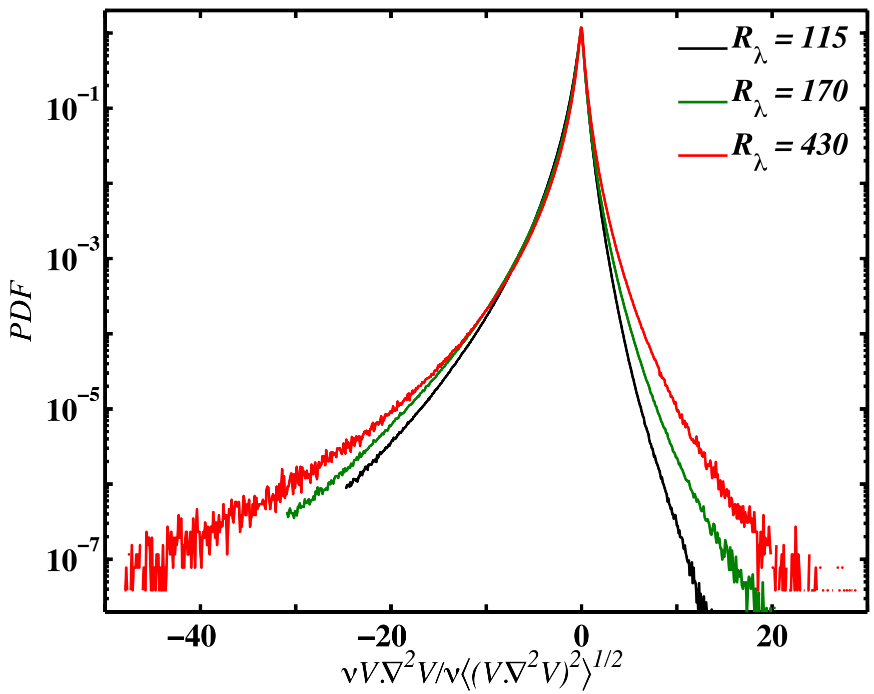


In 2D, $-\langle \mathbf{v} \cdot \nabla P \rangle$ contributes to 2/3 of the (negative) skewness of p.

In 3D, pressure gradient term is slightly positively skewed!

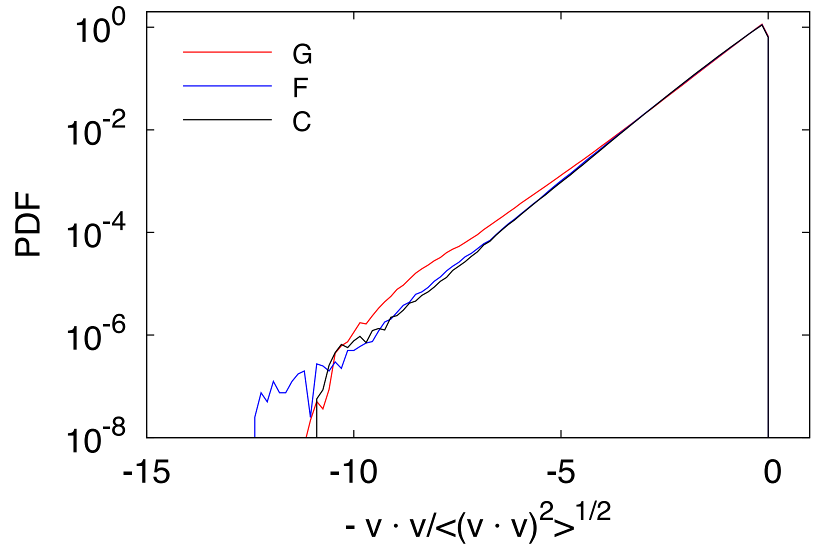
Where does the skewness of *p* come from?

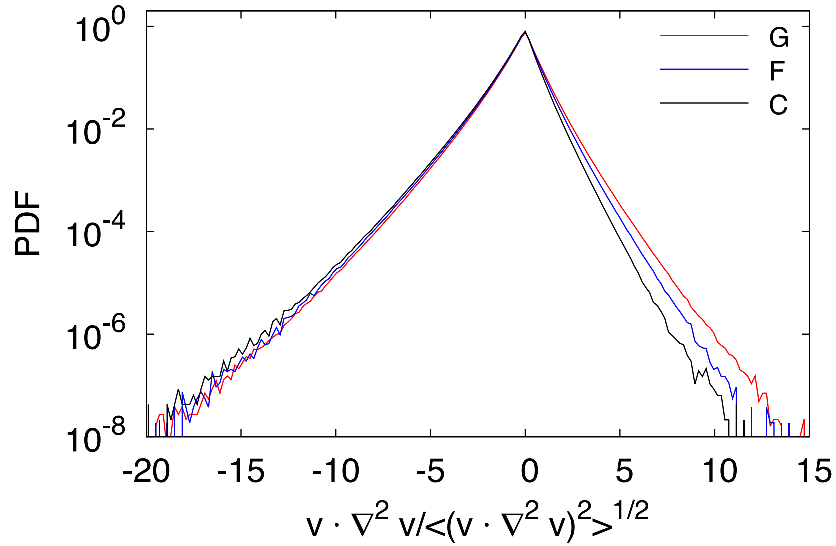
3D



Dissipation term is skewed, but its third moment is small and does not contribute much to the skewness of power.
However, cross terms do matter !

2D

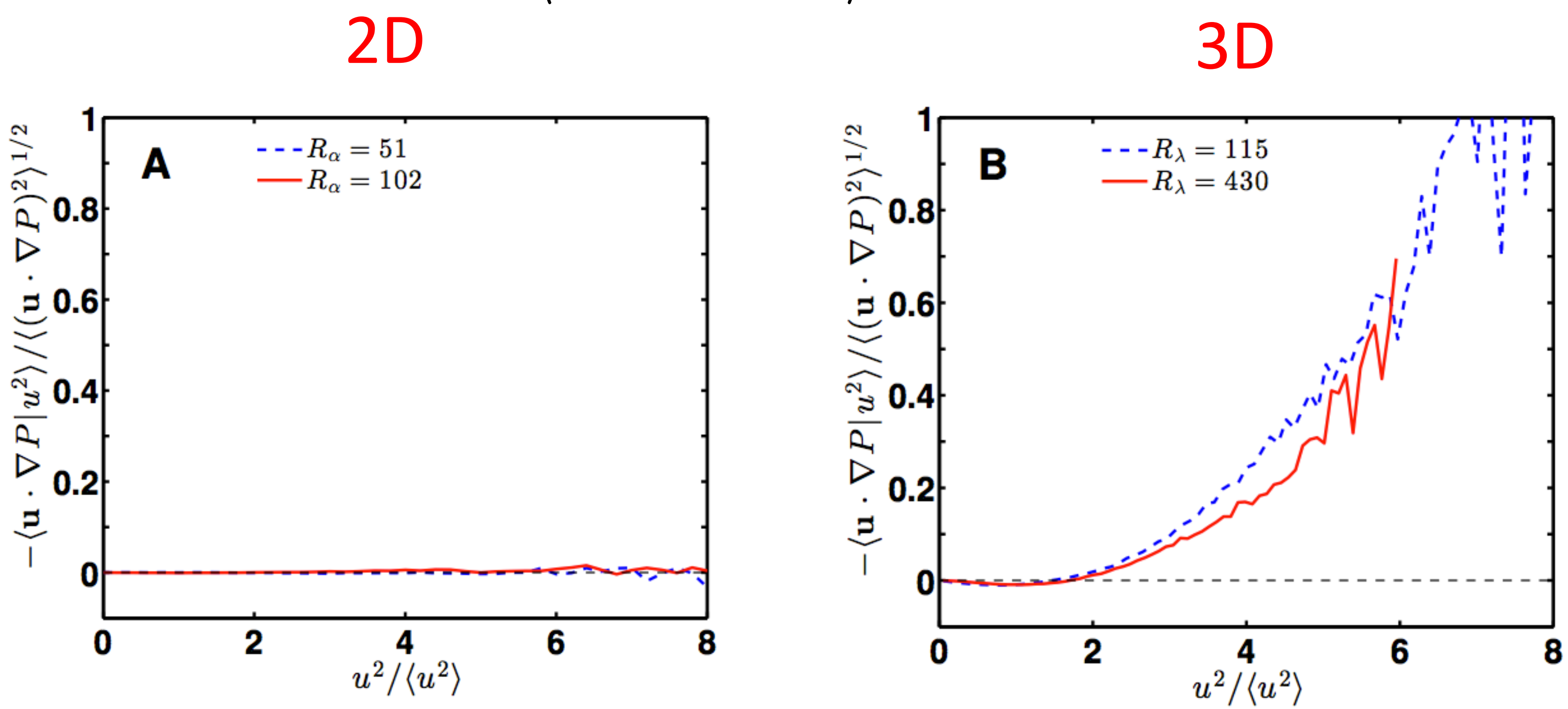




Peculiar behavior of pressure gradient

Pressure gradient term conditioned on the kinetic energy of the fluid particle:
Note that

$$\langle -\mathbf{v} \cdot \nabla P \rangle = 0$$



In 3D, pressure gradient tends to take kinetic energy away from *slow* particles and give it to *fast* particles!

A runaway mechanism related to the singularity problem of the NSE?

Summary

Can one detect time-irreversibility from the motion of particles ?

- With ***2 particles or more***: YES [Jucha et al, PRL, 2014; related to known-properties of turbulent flows]
- With ***only 1 particle*** ? YES [Xu et al, PNAS, 2014]!
- The irreversibility in the motion of single particles *IS* related to small-scale production [Pumir et al, PRL 2016].
- The normalized third moment of the instantaneous power on a fluid particle provides a good measure of how far the system is away from equilibrium.

Xu et al, PNAS, 2014 ; Jucha et al, PRL, 2014, Pumir et al, PRX 2014, Pumir et al, PRL 2016; .

Perspectives

Concerning the physics of turbulence:

much to be learned from studying the motion of particles in a turbulent flow !

From a broader perspective:

Turbulence: system very far from equilibrium [energy flux in 3D, several "fluxes" in other cases].

How does one quantify irreversibility ?

What does one relate possible manifestations of irreversibility to the existing fluxes in the problem ?

How does one generalize "statistical thermodynamics" to systems very far from equilibrium ?

What happens in other flow systems, of more geo/astro-relevance (with rotation and stratification, etc) ?

THANK YOU !

Questions ??

Acknowledgements

KITP, Santa Barbara, USA, “The Nature of Turbulence” program, 2011

KITPC, Beijing, China, “New Directions in Turbulence Theory” program, 2012

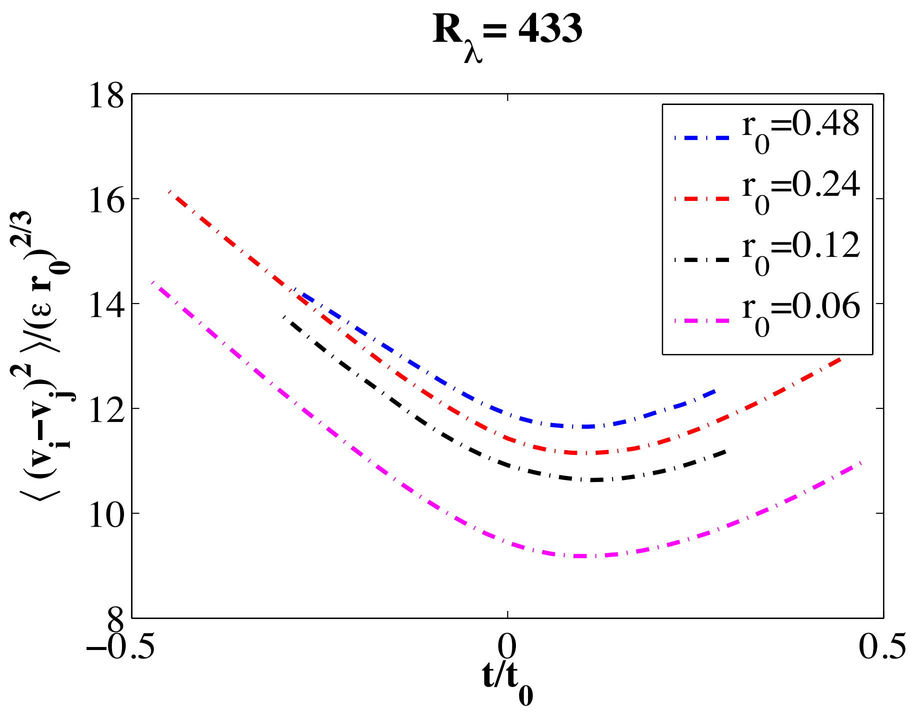
COST MP0806 “Particles in Turbulence”

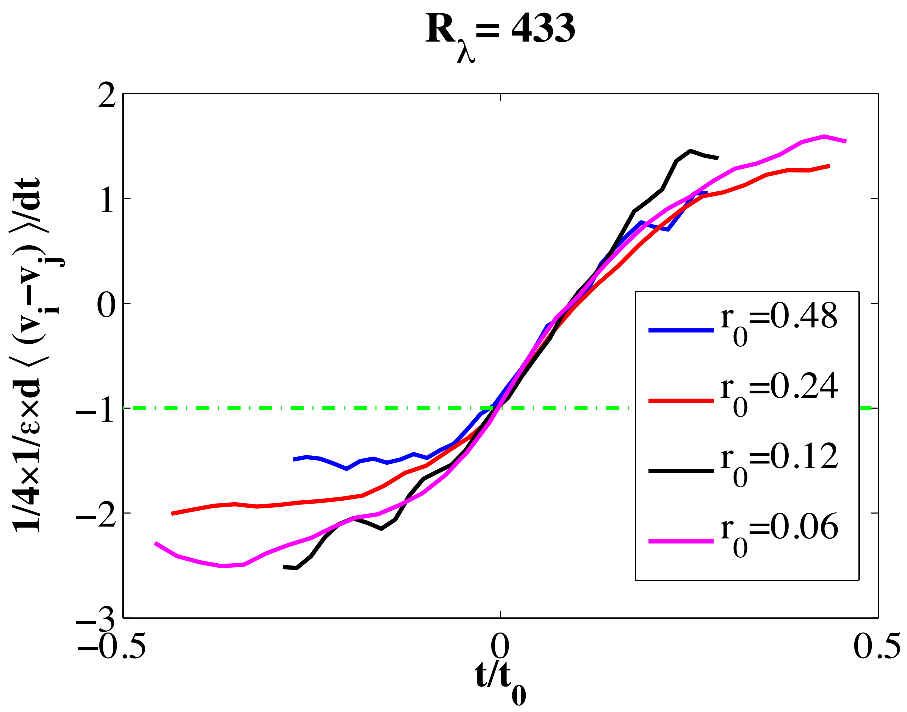
Max Planck Society

Agence Nationale pour la Recherche (ANR, France)

Deutsche Forschungsgemeinschaft (DFG, German Science Foundation)

Numerical results





Homogeneous turbulent flow from the Johns Hopkins database (η=0.003; L=1.38).

All pairs have the exactly same length at t = 0, well within the inertial range.