Some recent results dealing with *(freely decaying)* isotropic turbulence dynamics

Pierre Sagaut *, Marcello Meldi **, Vincent Mons***
Claude Cambon****

*: M2P2 Laboratory, UMR CNRS 7340
Aix-Marseille Université, France

**: PPRIME Laboratory, Poitiers, France
***: D'Alembert Institute, Paris, France
****: LMFA, Lyon, France

NCTR IV School, Les Houches, March 23rd 2016

Motivations

Most isotropic turbulence decay theories:

- rely on a **self-similarity/self-preservation hypothesis** (single lengthscale, usually Taylor scale)
- are not fully assessed by experiments and DNS

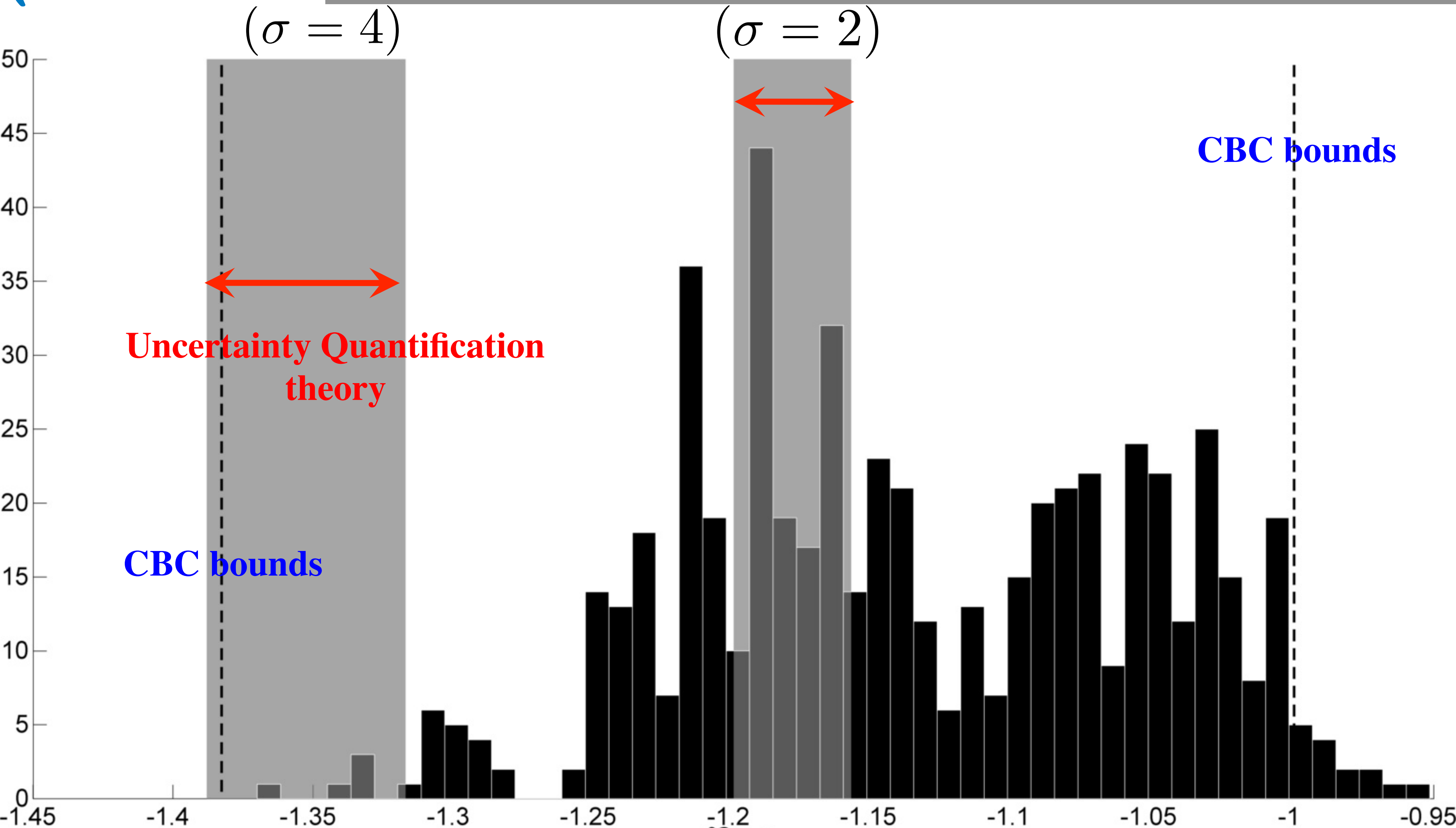
Examples:

- dimensional analysis (Lin or KH equation): George, Gonzalez, Antonia, Djenidi, Danaila …
- DA + simplified spectrum: Comte-Bellot/ Corrsin, Lesieur …
- fixed point of 2 eqs. model: KH, Batchelor, Panchev, Speziale & Bernard, Ristorcelli, …

Typical results:

- **decay exponent tied to** $E(k \to 0) \propto k^{\sigma}$ $f(r \to +\infty) \propto r^{-m}$

Do theories agree with experimental data ?



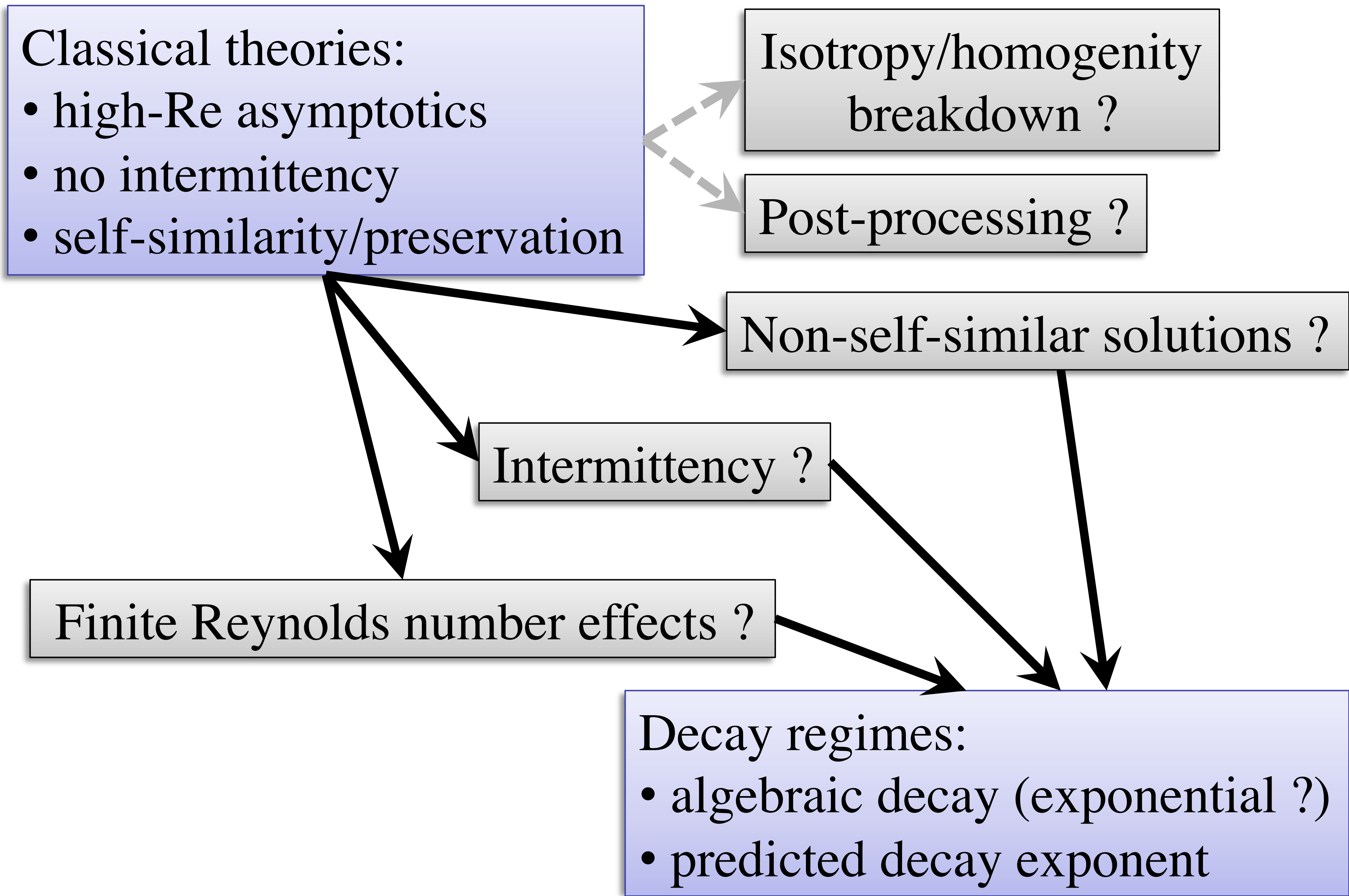
n_{u^2}

Decay exponent of kinetic energy

[Meldi et al., J. Fluid Mech. 668, 2011]

[Meldi & Sagaut, J. Fluid Mech. 711, 2012]

Motivations



Looking for an investigation tool

DNS:

- low- to medium Reynolds (far from high-Re asymptotics)
- large-scale resolution issue, ergodicity ?

Experiments:

- not strictly isotropic, restricted evolution time
- low-to medium Reynolds in practice (for grid turbulence)
- no direct measurement of very large-scales
- no control of initial solution & large scales

Orszag's EDQNM (1970):

- realizable closure of the 3rd order moment equation in Fourier space
- fully general: initial solution, Reynolds number, …
- no intermittency effects
- minimal complexity: *[Rubinstein & Clark, Phys. Fluids 17, 2005]*
  [Woodruff & Rubinstein, J. Fluid Mech. 565, 2006]

Aix*Marseille université

Outline

1. Tools & models:
   - EDQNM: a brief reminder
   - A glance at Comte-Bellot—Corrsin theory
2. Do self-similar solution exist for freely decaying HIT ?
3. Which scales govern the decay rate ?
4. Non-self-similar solutions over arbitrary long time ?
5. Existence of a universal attractor with -1 decay exponent?
6. Conclusions

Tools and Models

1. EDQNM: a brief reminder

Navier-Stokes equations in Fourier space

$$\frac{\partial}{\partial t}\hat{u}_i(\vec{k},t) + \nu k^2 \hat{u}_i(\vec{k},t) = T_i(\vec{k},t)$$

2nd-order moment equation

$$\left(\frac{\partial}{\partial t} + \nu(k^2+p^2)\right)\langle \hat{u}(k)\hat{u}(p)\rangle = \langle \hat{u}\hat{u}\hat{u}\rangle$$

Lin equation (= KH)

$$\frac{\partial}{\partial t}E(k,t) = T(k,t) + 2\nu k^2 E(k,t)$$

3rd-order moment equation

$$\left(\frac{\partial}{\partial t} + \nu(k^2+p^2+q^2)\right)\langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \langle \hat{u}\hat{u}\hat{u}\hat{u}\rangle$$

1. EDQNM: a brief reminder

$$\left(\frac{\partial}{\partial t} + \nu(k^2 + p^2 + q^2)\right) \langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \langle \hat{u}\hat{u}\hat{u}\hat{u}\rangle$$

Quasi-Normal Approximation
[Millionshikov, 1941]

$$\left(\frac{\partial}{\partial t} + \nu(k^2 + p^2 + q^2)\right) \langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle$$

Eddy-Damped QN
[Orszag, 1970]

$$\left(\frac{\partial}{\partial t} + \nu(k^2 + p^2 + q^2)\right) \langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle - \nu_t(k,p,q) \langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle$$

$$\left(\frac{\partial}{\partial t} + \theta_{kpq}^{-1}\right) \langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle$$

1. EDQNM: a brief reminder

$$\left(\frac{\partial}{\partial t} + \theta_{kpq}^{-1}\right)\langle \hat{u}(k)\hat{u}(p)\hat{u}(q)\rangle = \sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle$$

EDQN exact solution

$$\langle \hat{u}\hat{u}\hat{u}\rangle (t) = e^{-\theta_{kpq}^{-1} t}\langle \hat{u}\hat{u}\hat{u}\rangle (0) + \int_0^t e^{-\theta_{kpq}^{-1}(t-t')}\sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle (t')dt'$$

Markovianisation step

$$\langle \hat{u}\hat{u}\hat{u}\rangle (t) = e^{-\theta_{kpq}^{-1} t}\langle \hat{u}\hat{u}\hat{u}\rangle (0) + \sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle (t)\int_0^t e^{-\theta_{kpq}^{-1}(t-t')}dt'$$

EDQNM solution

$$\langle \hat{u}\hat{u}\hat{u}\rangle (t) = \theta_{kpq}\left(1 - e^{-\theta_{kpq}^{-1} t}\right)\sum \langle \hat{u}\hat{u}\rangle \langle \hat{u}\hat{u}\rangle (t)$$

1. EDQNM: a brief reminder

A few comments:

- Closure of the evolution equation of the 3rd-order Moments (≈ equation for the transfer rate)
- Dissipation is an output, not a prescribed quantity
- No self-similarity/self-preservation assumption
- No hypothesis about the Reynolds number
- No hypothesis about the spectrum shape
- No hypothesis about the initial condition
- No intermittency effect included
- Can be extended to many cases (HAT)

Rosette's stone

Karman-Howarth equation for velocity correlation function

$$\frac{\partial}{\partial t}(u'^2 f) = \left(\frac{\partial}{\partial r} + \frac{4}{r}\right)\left(u'^3 h(r,t) + 2\nu \frac{\partial}{\partial r}(u'^2 f)\right)$$

$$u'^2 f(r,t) = 2\int_0^{+\infty} E(k,t)\left(\frac{\sin(kr)}{k^3 r^3} - \frac{\cos(kr)}{k^2 r^2}\right) dk$$

$$E(k,t) = \frac{u'^2}{\pi}\int_0^{+\infty} kr(\sin kr - kr\cos kr) f(r,t) dr$$

$$u'^3 h(r) = \int_0^{+\infty}\left[\frac{(k^2r^2 - 3)\sin(kr)}{k^4 r^4} + \frac{3\cos(kr)}{k^3 r^3}\right]\frac{T(k)}{k} dk$$

$$T(k,t) = -2\frac{u'^3}{\pi}\int_0^{+\infty}\left[(k^2r^2 - 3)kr\sin(kr) + 3k^2r^2\cos(kr)\right]\frac{h(r)}{r} dr$$

Rosette's stone

Karman-Howarth equation for the velocity structure function

$$\frac{2}{3}\frac{\partial \mathcal{K}}{\partial t} = \frac{1}{2}\frac{\partial S_2}{\partial t} + \frac{1}{6r^4}\frac{\partial}{\partial r}(r^4 S_3) - \frac{\nu}{r^4}\frac{\partial}{\partial r}\left(r^4 \frac{\partial S_2}{\partial r}\right)$$

$$S_2(r) = \overline{[u(r) - u(0)]^2} = 2u'^2\left[1 - f(r)\right] = 4\int_0^{+\infty} E(k)a(kr)dk$$

$$S_3(r) = \overline{[u(r) - u(0)]^3} = -12u'^3 h(r) = 4\int_0^{+\infty} \frac{T(k)}{k^2}\frac{\partial a(kr)}{\partial r}dk$$

$$a(x) = \frac{1}{3} - \frac{\sin x - x\sin x}{x^3}$$

Aix*Marseille université

E(k) vs. f(r) : tricky maths

$$f(r \to +\infty) \propto r^{-m} \not\Longrightarrow E(k \to 0) \propto k^{\sigma}$$

[Davidson, 2011]
[Llorr & Soulard, 2013]

$$f(r) = \sum_{m=m_0}^{+\infty} c_m \left(\frac{r}{L}\right)^{-m}$$

$$\frac{\pi}{2u'^2} E(k) = \sum_{n=4}^{+\infty} (\phi_n - c_{n+1} \ln(kL)) a_n L^{n+1} k^n + \sum_{m=m_0}^{+\infty} c_m \alpha_m L^m k^{m-1}$$

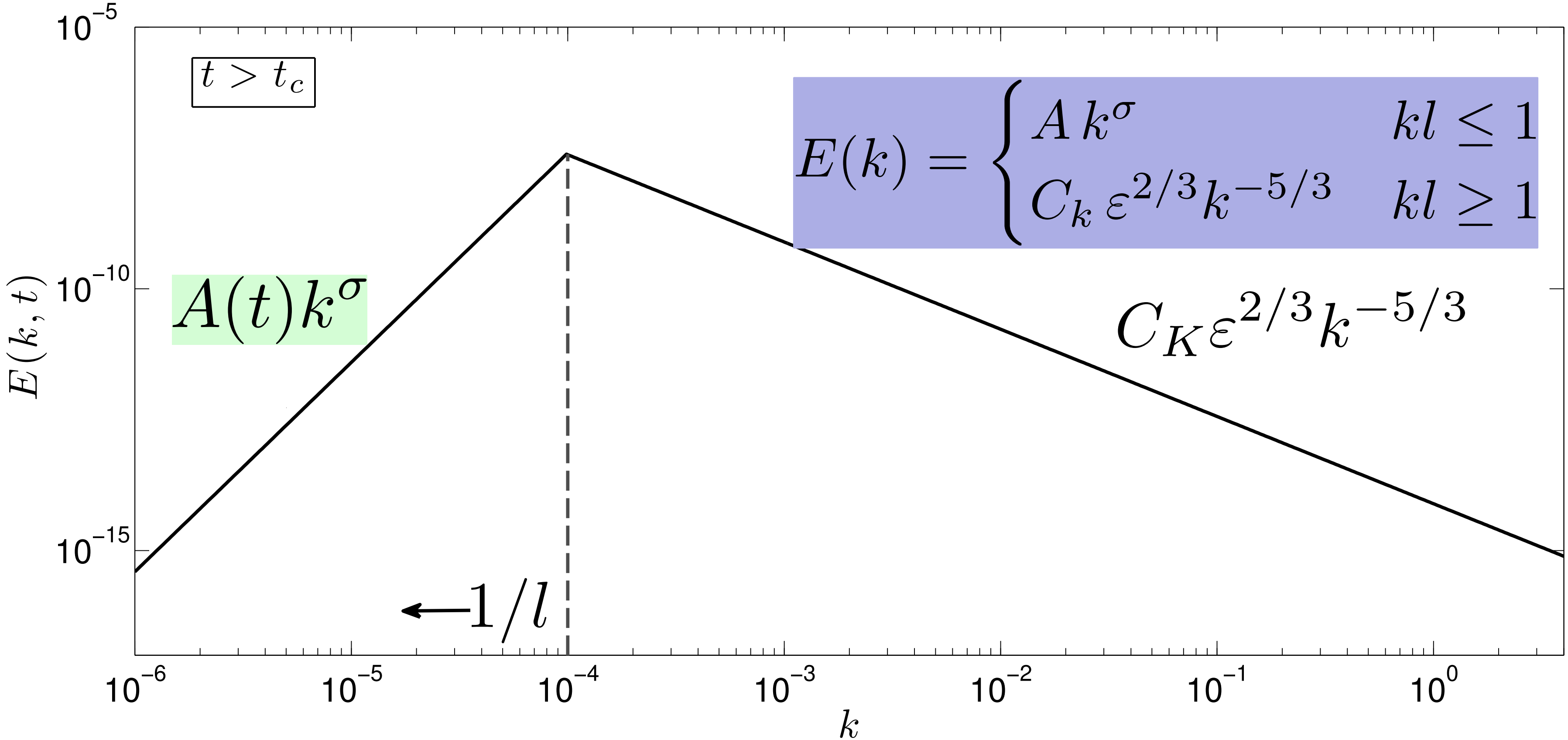
$$\phi_n = \lim_{R \to +\infty} \left(\int_0^R \left[f(r) - \sum_{m=m_0}^{<n+1} c_m \left(\frac{r}{L}\right)^{-m} \right] \frac{r^n}{L^{n+1}} dr - c_{n+1} \ln(R/L) \right)$$

$$\alpha_m = \lim_{\epsilon \to 0} \left(\int_{\epsilon}^{+\infty} \xi^{-m} \left[\xi(\sin \xi - \xi \cos \xi) - \sum_{n=4}^{<m-1} a_n \xi^n \right] d\xi \right)$$

2. A look at Comte-Bellot—Corrsin theory

[Comte-Bellot – Corrsin, 1966] [Lesieur, 1978] [Skbrek & Stalp, 2000] …

- simplified, 2-range initial energy spectrum E(k)
- single lengthscale = integral scale (~ spectrum peak)
- pure dimensional analysis
- decay regime exponents as functions of spectrum slope at large scales



CBC theory: decay exponents

	$\mathcal{K}(t)$	ε	L	λ	η	Re_L	Re_λ
High Re_λ	$-2\frac{\sigma-p+1}{\sigma-p+3}$	$-\frac{3(\sigma-p)+5}{\sigma-p+3}$	$\frac{2}{\sigma-p+3}$	$\frac{1}{2}$	$\frac{3(\sigma-p)+5}{4(\sigma-p+3)}$	$\frac{1-(\sigma-p)}{(\sigma-p+3)}$	$\frac{1}{2}\frac{1-\sigma+p}{\sigma-p+3}$
Saturated high Re_λ	-2	-3	0	$\frac{1}{2}$	$\frac{3}{4}$	-1	$-\frac{1}{2}$
Low Re_λ	$-\frac{\sigma+1}{2}$	$-\frac{\sigma+3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sigma+3}{8}$	$\frac{1-\sigma}{4}$	$\frac{1-\sigma}{4}$

Breakdown of PLE $\sigma \to (\sigma - p)$ $\sigma > 3,\ p(\sigma = 4) \sim 0.55$

Some important predictions:

- Taylor microscale evolution independent of the initial slope and Re
- Singular behavior for σ=1
  - Constant Re decay
  - High- and Low-Re regimes identical
  - All length scales collapse with growth exponent 1/2

[Meldi & Sagaut, 2013]

CBC theory: decay exponents

Invariant quantities ?

$$I_L(\sigma) \equiv \mathcal{K}^{\alpha} L \qquad I_\eta(\sigma) = \mathcal{K}^{\beta}\eta$$

$$\alpha = \begin{cases} \frac{1}{\sigma - p + 1} & Re_\lambda \geq 200 - 300 \\ \frac{1}{\sigma+1} & Re_\lambda \leq 0.1 - 0.01 \end{cases}$$

$$\beta = \begin{cases} \frac{1}{8}\frac{3(\sigma-p)+5}{\sigma-p+1} & Re_\lambda \geq 200 - 300 \\ \frac{1}{4}\frac{\sigma+3}{\sigma+1} & Re_\lambda \leq 0.1 - 0.01 \end{cases}$$

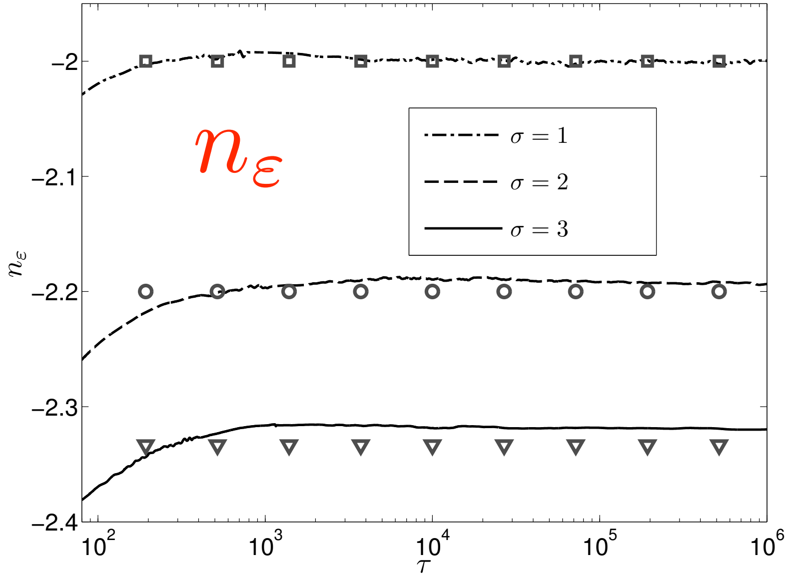
- No invariant over arbitrary long times (except singular regime σ=1)
- ***Infinite number of invariants*** for high/low regime (finite time)

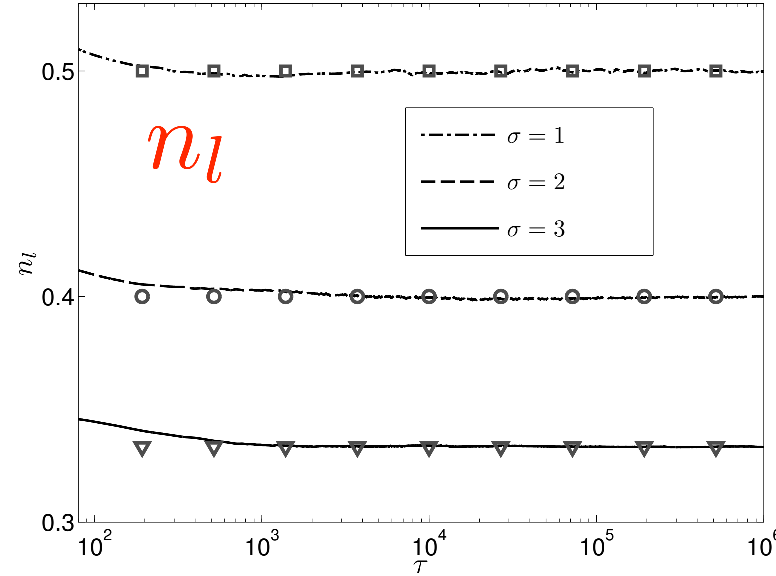
$$H(I_L, I_\eta)$$

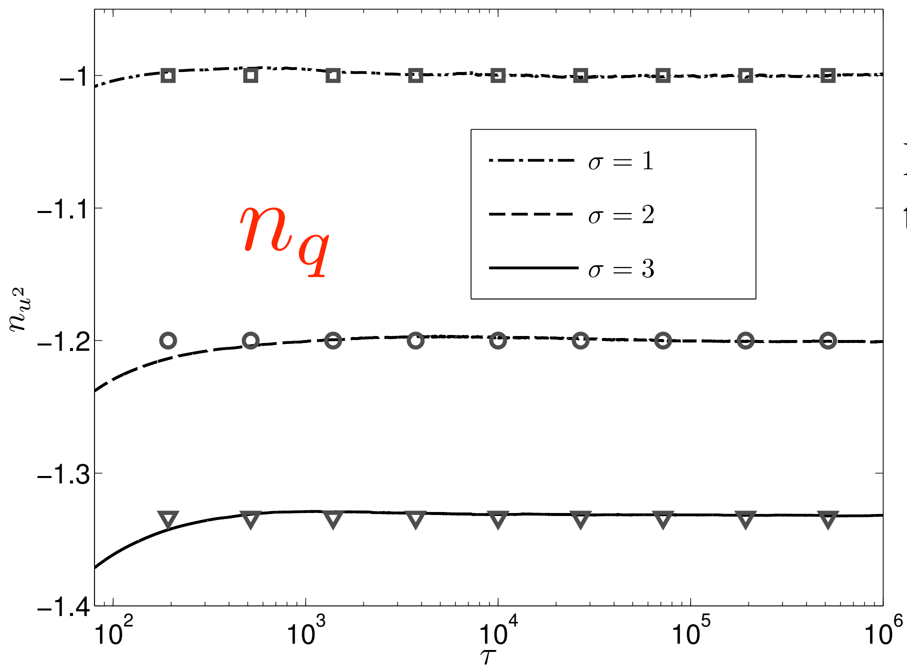
[Antonia & al., Vassilicos …]

Aix*Marseille université

CBC vs. EDQNM: high Re asymptotics





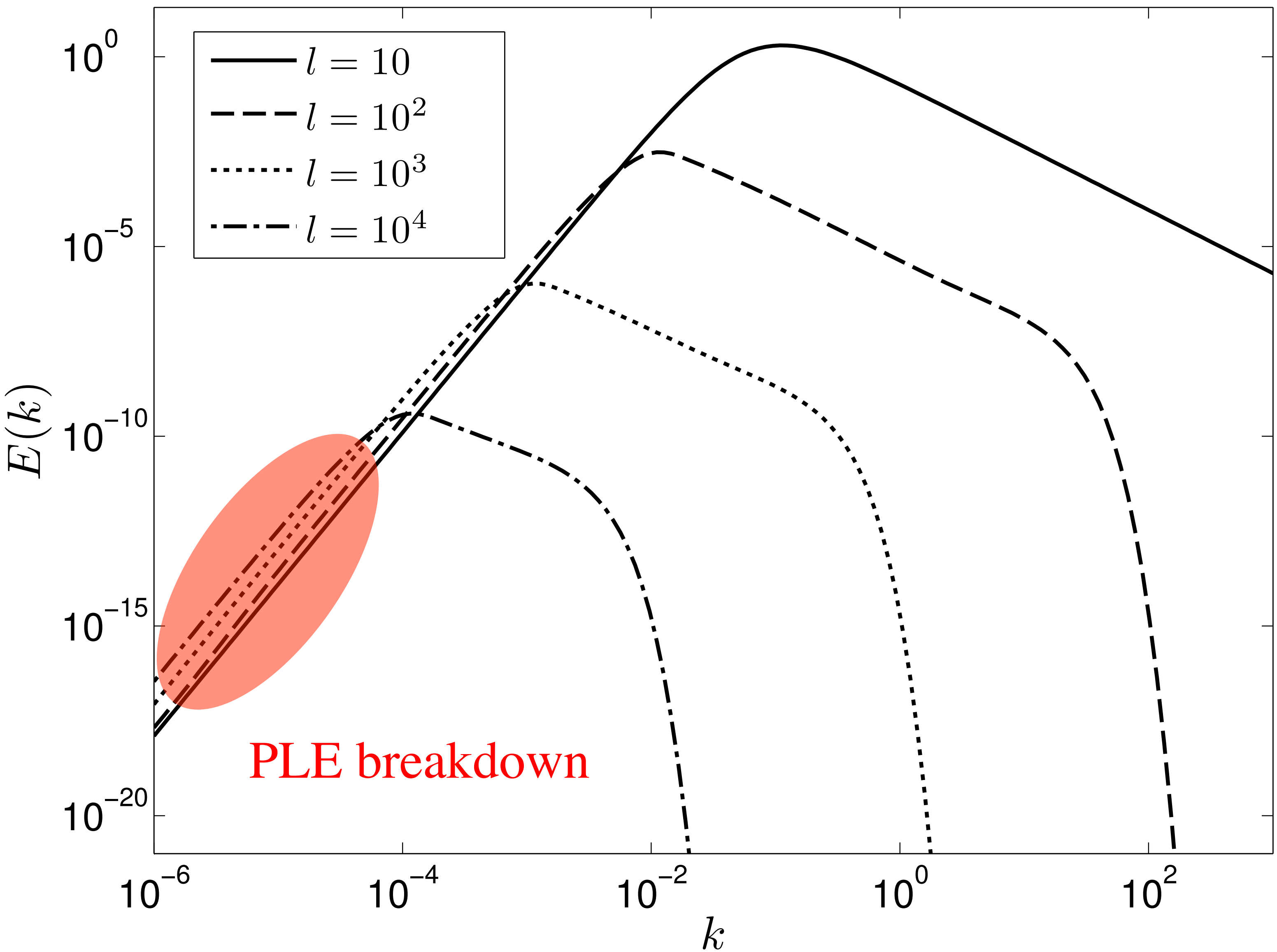


Perfect agreement with Comte-Bellot– Corrsin theory in all cases for two-range initial spectrum

<1% error

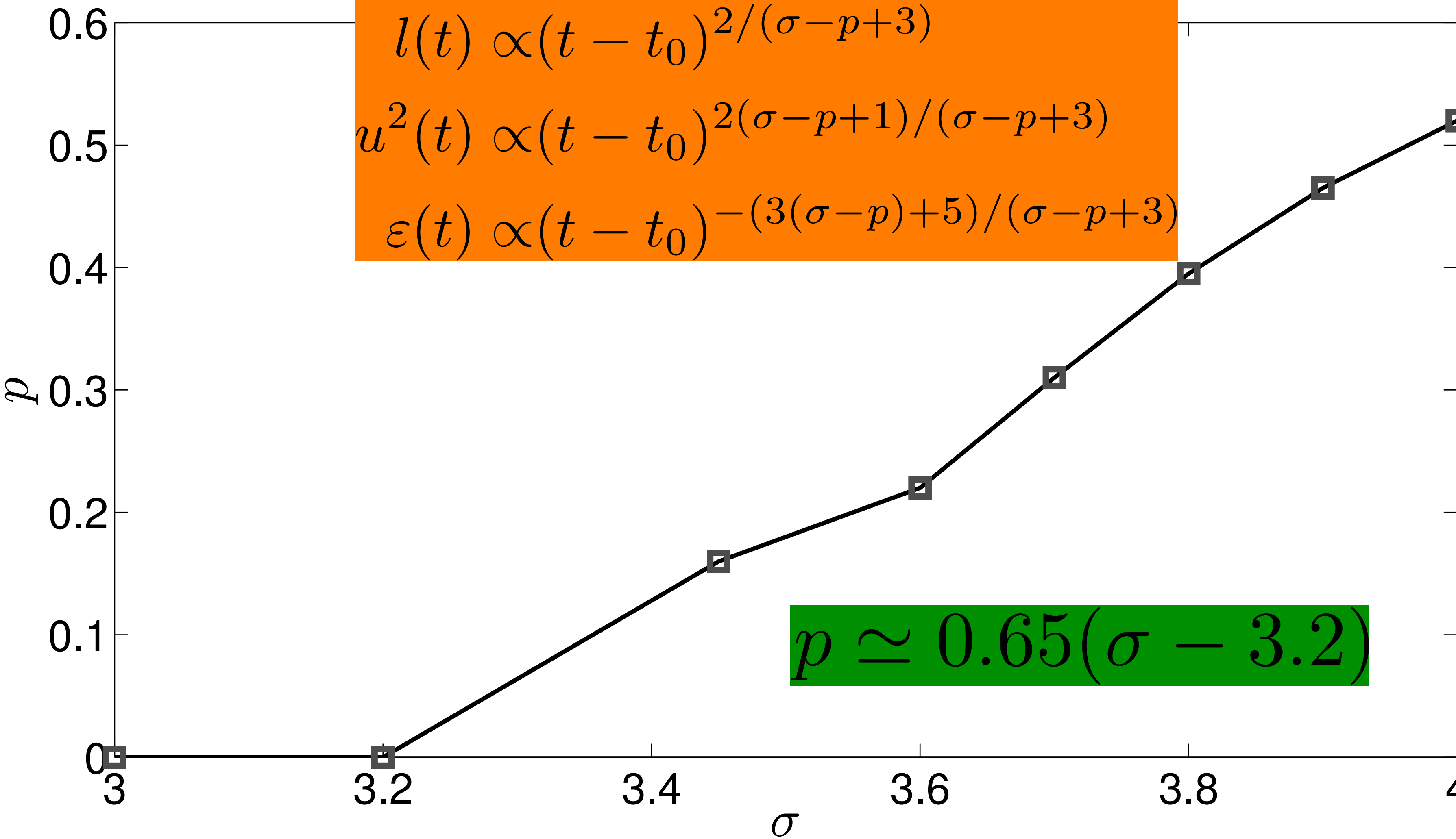
[Meldi et al., J. Fluid Mech. 668, 2011]
[Meldi et al., J. Fluid Mech., 711, 2012]

Batchelor turbulence: σ=4



Breakdown of PLE

[Meldi et al., J. Fluid Mech. 668, 2011]
[Meldi et al., J. Fluid Mech., 711, 2012]



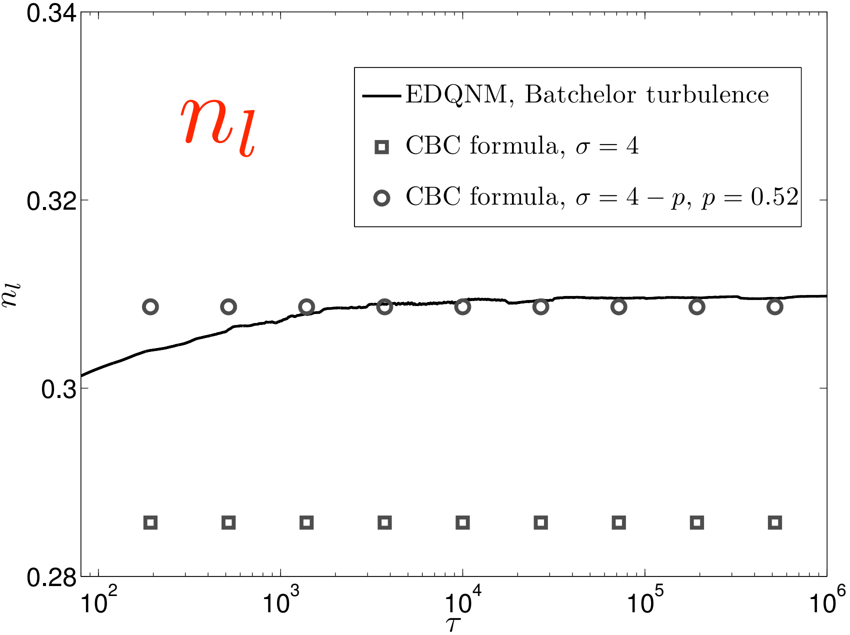
$$l(t) \propto (t - t_0)^{2/(\sigma - p + 3)}$$

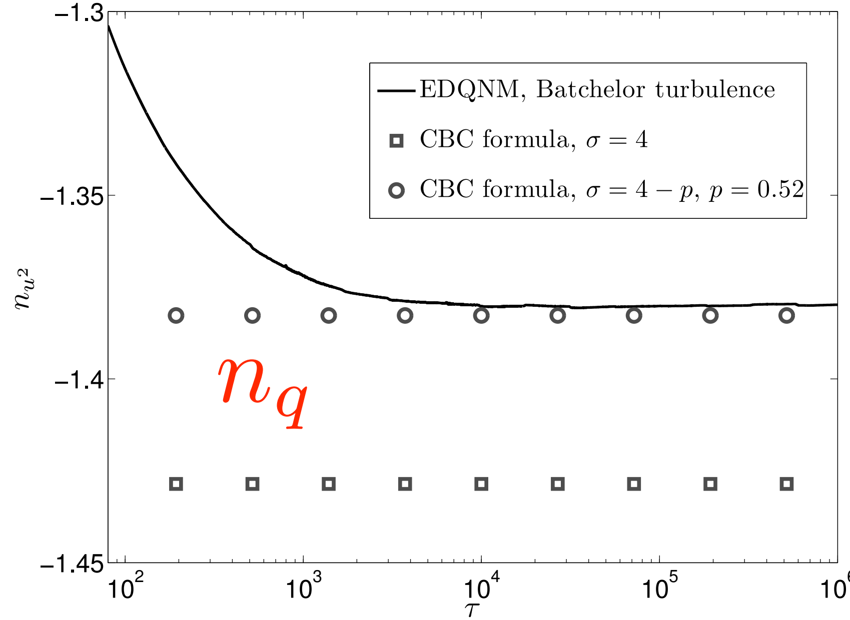
$$u^2(t) \propto (t - t_0)^{2(\sigma - p + 1)/(\sigma - p + 3)}$$

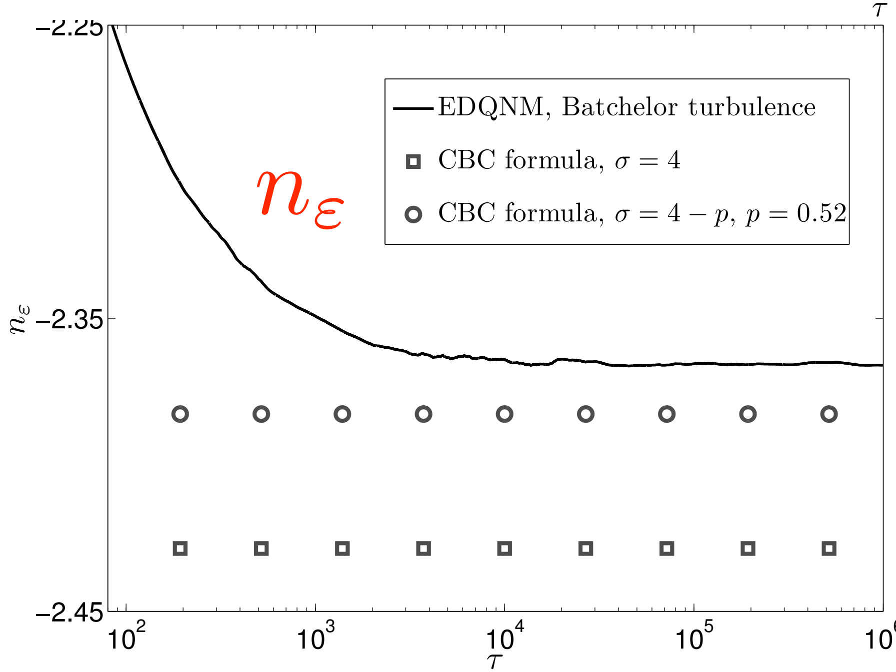
$$\varepsilon(t) \propto (t - t_0)^{-(3(\sigma - p) + 5)/(\sigma - p + 3)}$$

$$p \simeq 0.65(\sigma - 3.2)$$

Batchelor turbulence: σ=4







Do self-similar solutions exist ?

A few definitions

Self-preservation : there exist

- a unique length scale and
- a unique velocity scale
- A time independent shape function

$$E(k,t) = v^2(t)l(t)F(kl)$$

Partial Self-preservation : restricted to a range of scales

Self-similar solutions: solutions of equations (Lin, KH)

The assumption of similarity of shape of the statistical functions during decay in the earlier work was principally a mathematical device, used to enable definite results to be obtained ... To find such solution has been one task; to determine the conditions under which they can and do provide a correct description of turbulence is another. In this latter task which has engaged much attention in the last five years, but even so most of the established results are negative, and our positive results still rest insecurely on vague intuitive arguments (vague for most of us - clear and precise for the inspired few !)

Batchelor, 1953 (p.148)

A look at George theory (1992, 2009)

$$\begin{cases} E(k,t) = E_s(t,*) f(\eta,*) & \eta = k\ell \\ T(k,t) = T_s(k;*) g(\eta,*) & \ell = \ell(t,*) \end{cases}$$

$$\Rightarrow \quad \frac{\partial E}{\partial t} = \left[\frac{dE_s}{dt}\right] f(\eta,*) + \left[\frac{E_s}{\ell}\cdot\frac{d\ell}{dt}\right] \eta f'(\eta,*)$$

Lin equation

$$\left[\frac{\ell^2}{\nu E_s}\cdot\frac{dE_s}{dt}\right] f + \left[\frac{\ell}{\nu}\cdot\frac{d\ell}{dt}\right] \eta f' = \left[\frac{T_s \ell^2}{\nu E_s}\right] g - [1]\, 2\eta^2 f$$

George 's **Extended Self-Similarity** hypothesis

$$\left[\frac{\ell^2}{\nu E_s}\cdot\frac{dE_s}{dt}\right] = \left[\frac{\ell}{\nu}\cdot\frac{d\ell}{dt}\right] = \left[\frac{T_s \ell^2}{\nu E_s}\right] = [1]$$

George's theory: predicted decay regimes

$$\begin{cases} \ell^2 = 2A\nu(t - t_0) = \lambda^2 \\ E_s \propto t^p \\ T_s \propto \dfrac{E_s}{t} \propto t^{p-1} \end{cases}$$

⇒ Taylor Self-Similarity ?

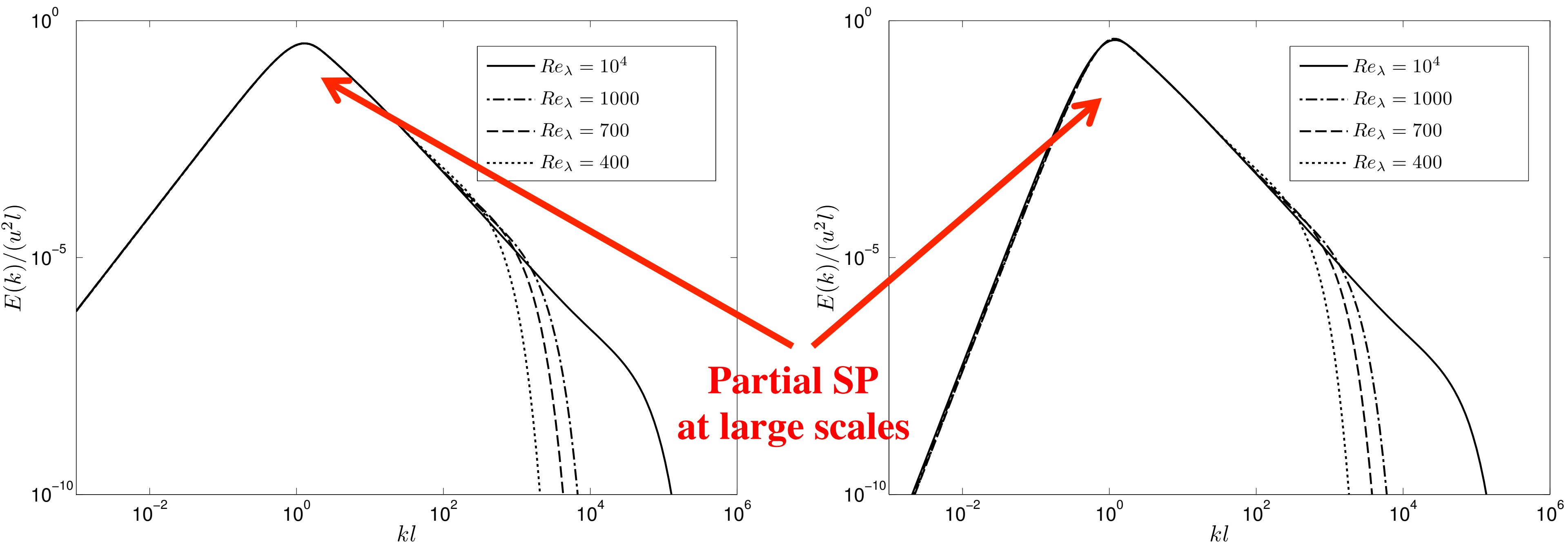
⇒ $u^2 \propto t^{p-1/2}$

Universal equilibrium	$dE_s/dt = d\ell dt = 0$	
Inviscid power law decay	$\nu = 0$	
visous power law decay	All terms retained	$u^2 \sim t^n,\ \lambda \sim t^{1/2},\ Re_\lambda \sim t^{\frac{n+1}{2}}$
Linearized "final period"	$T_s = 0$	
Viscous, constant length scale	$d\ell/dt = 0$	$u^2 = u_0^2 \exp\left[-10\nu(t-t_0)^2/\lambda_0^2\right]$, $\lambda = \lambda_0 \propto L_{11}$
Inviscid, constant length scale	$d\ell/dt = \nu = 0$	$u^2 = u_0^2(1 + Cu_0\left[t - t_0\right]/\ell_0)^{-2}$

Is E(k) self-similar/preserving ?

[Meldi & Sagaut, JoT, 2013]

$(\sigma = 2)$ $(\sigma = 4)$

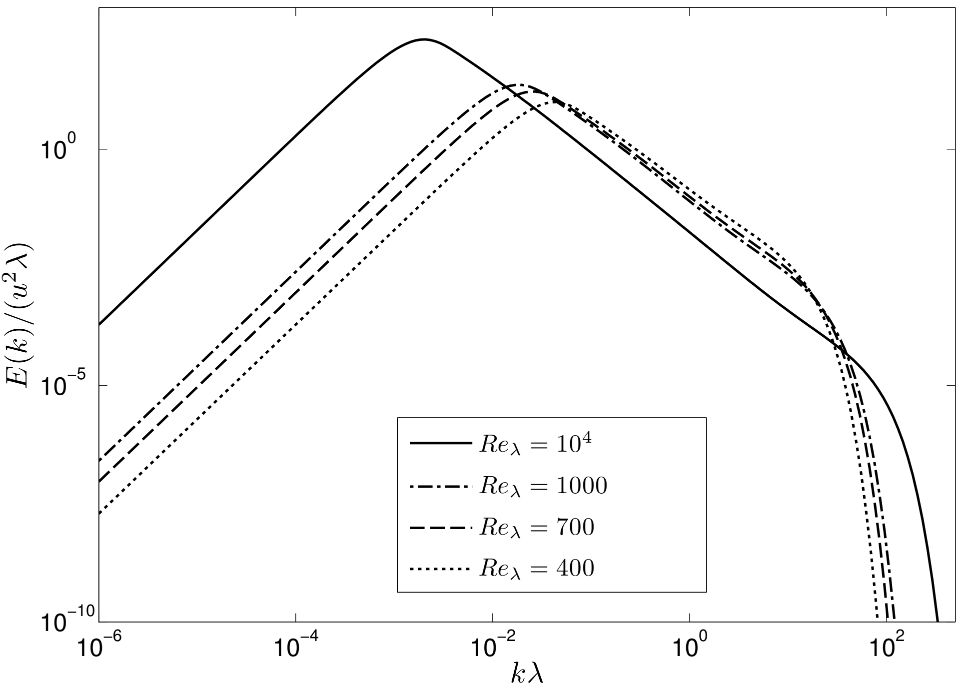


Normalization based on integral scale & kinetic energy

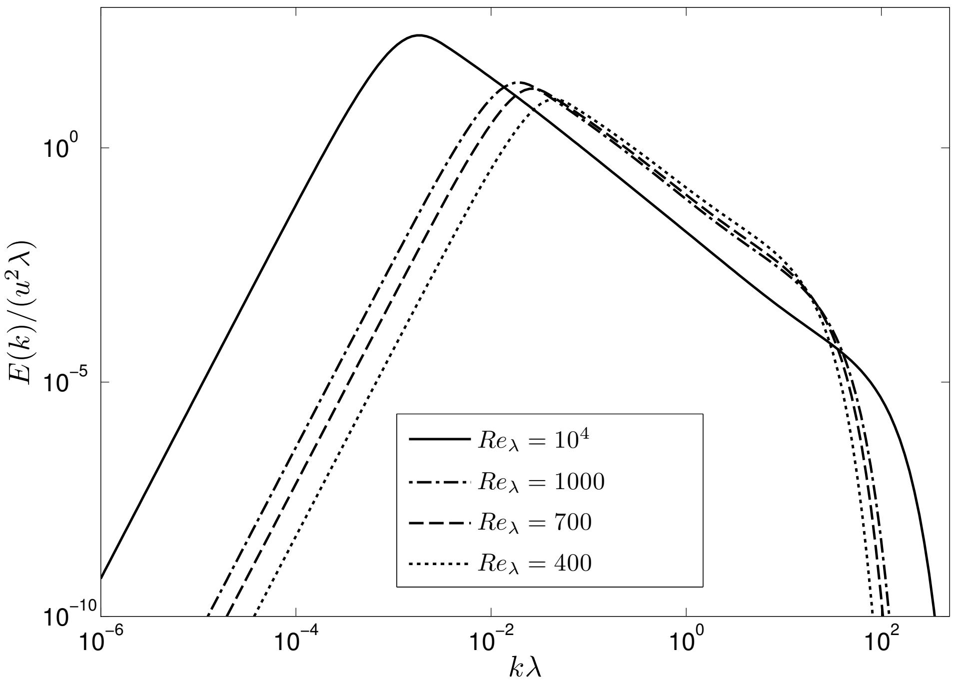
e.g. *[Clark & Zemach, 1998]*

Is E(k) self-similar/preserving ?

$(\sigma = 2)$



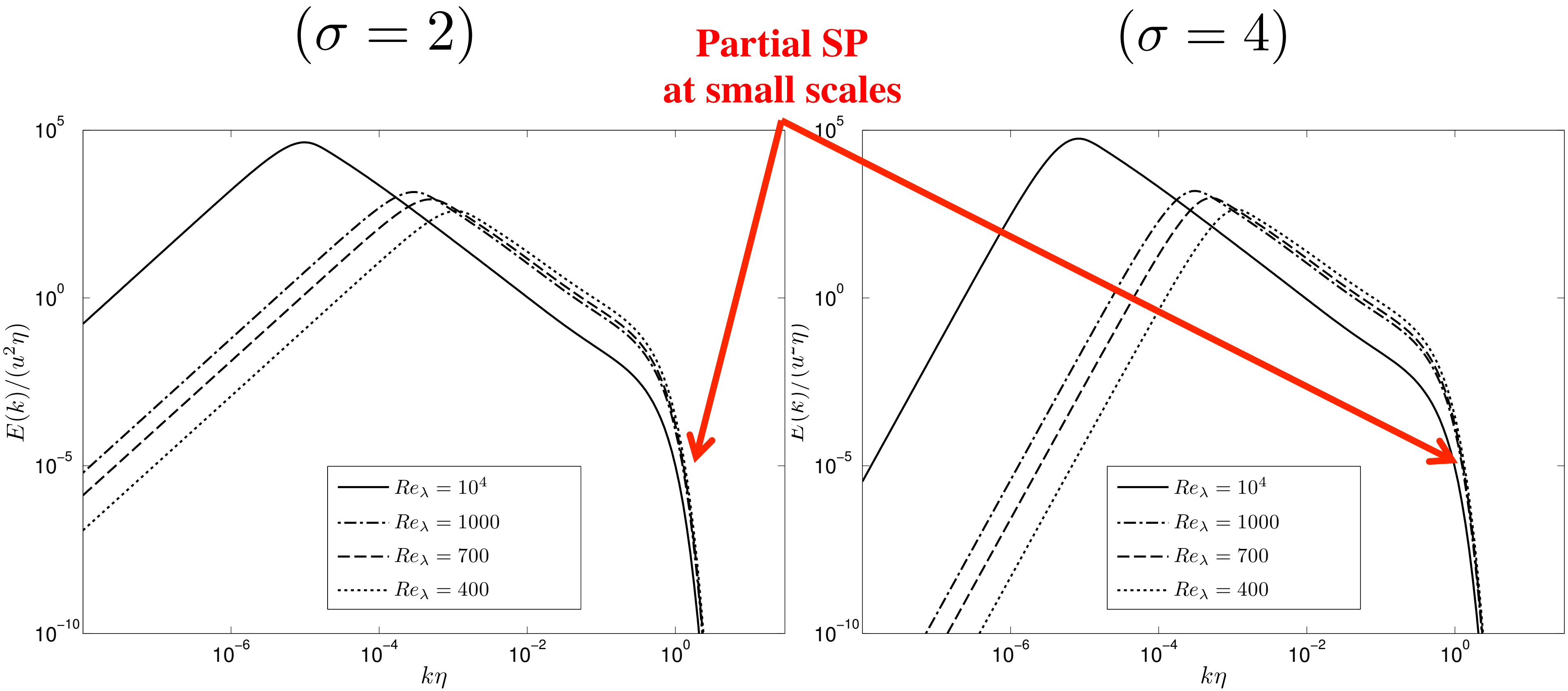
$(\sigma = 4)$



Normalization based on Taylor scale & kinetic energy (TSS)

e.g. *[George, 2002, 2009] [Speziale & Bernard, 1992] [Mohammed & Larue, 1990]*

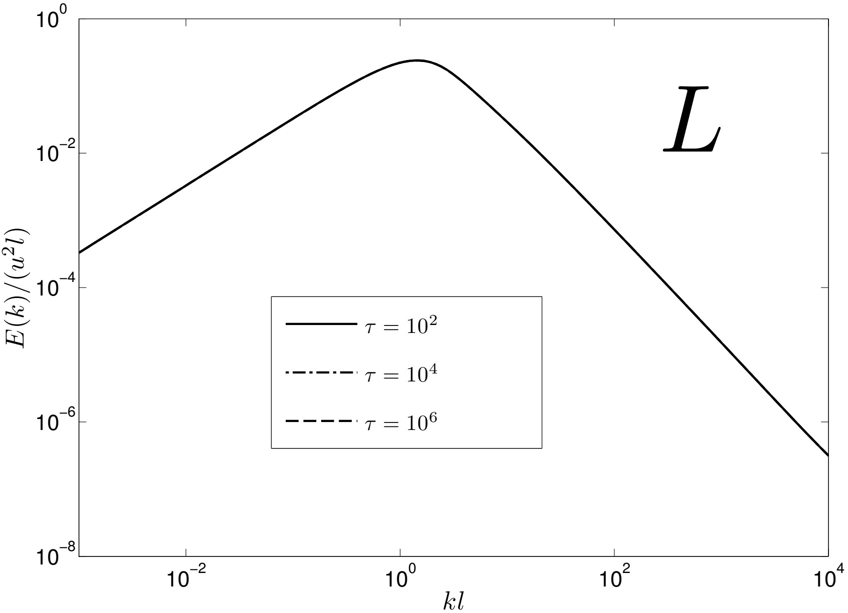
Is E(k) self-similar/preserving ?

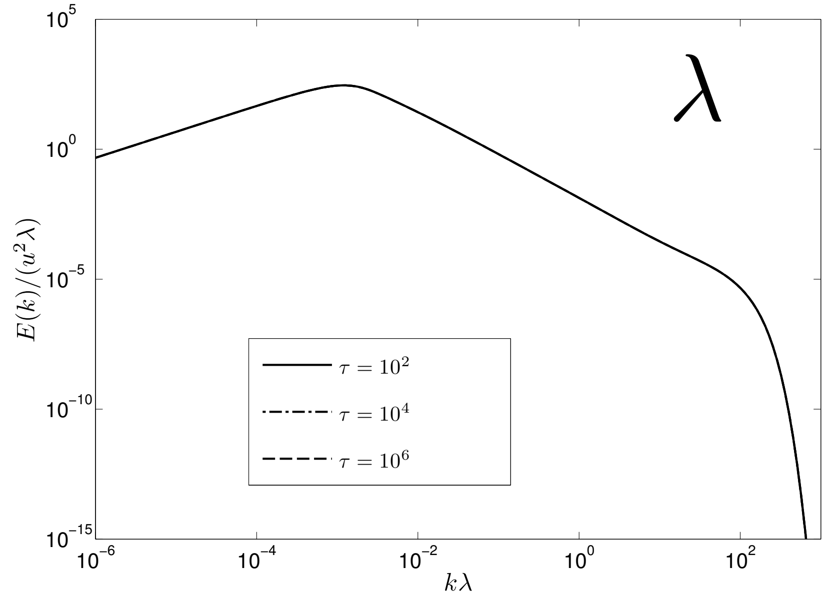


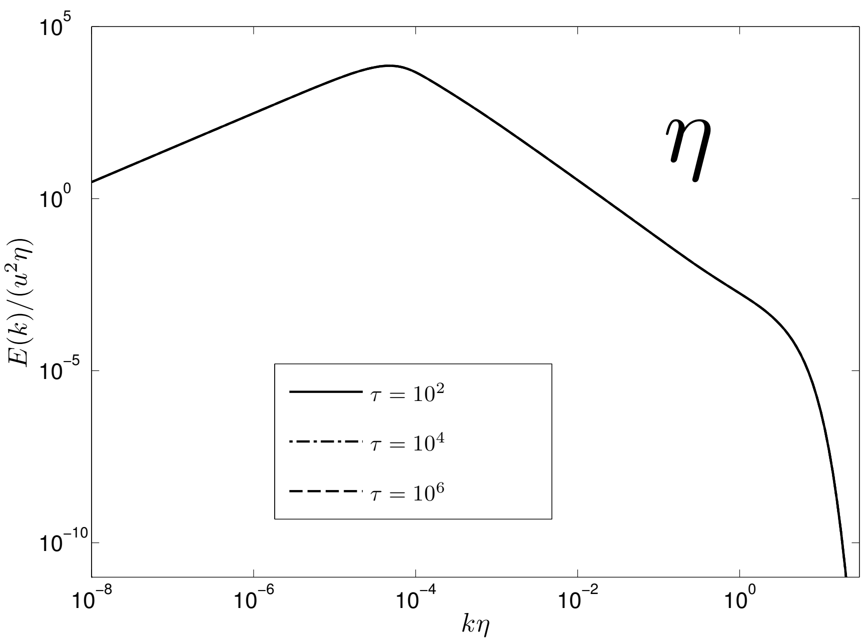
Normalization based on Kolmogorov scale & kinetic energy (KSS)

e.g. *[Antonia et al., 2002] [Ristorcelli et al., 2004, 2006]*

A singular case: σ=1



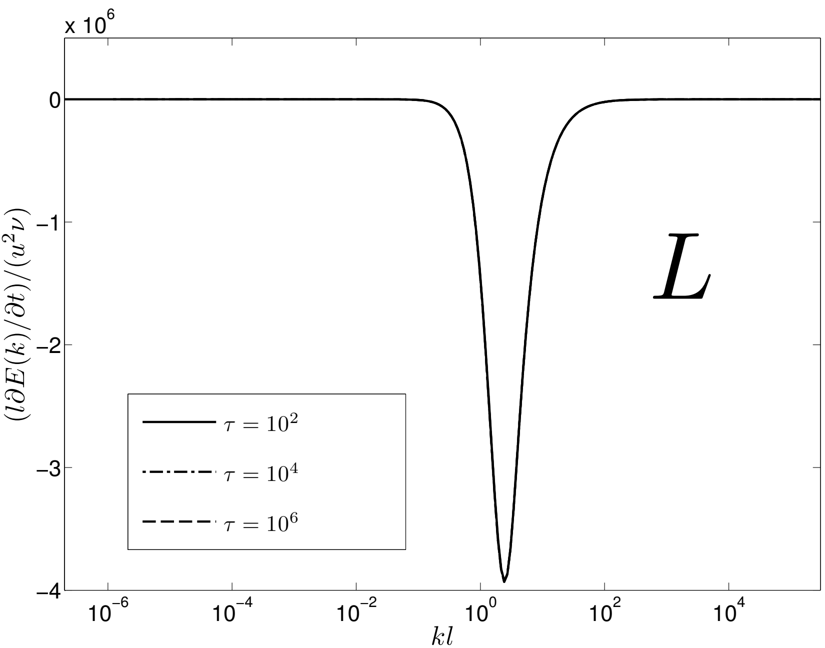


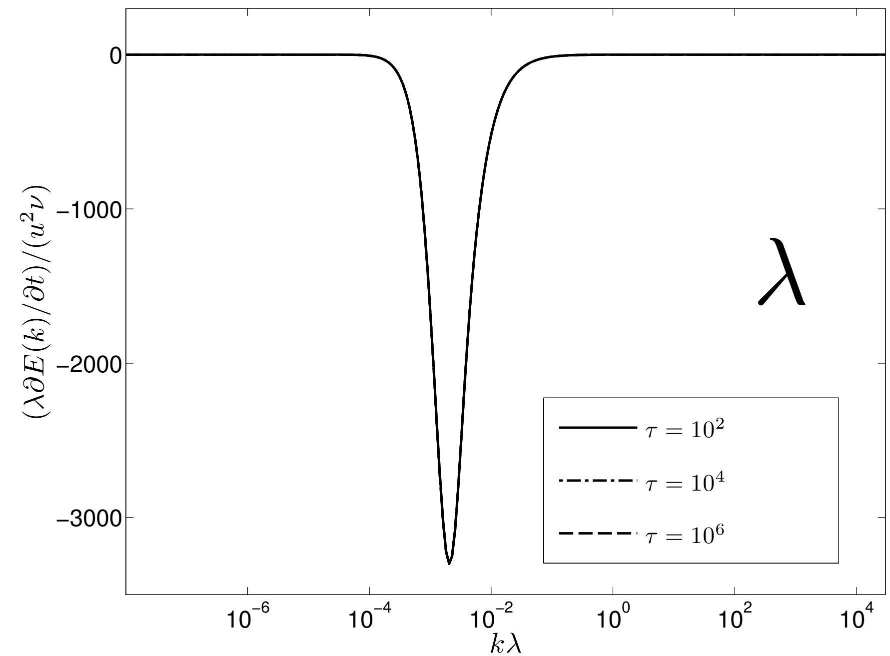


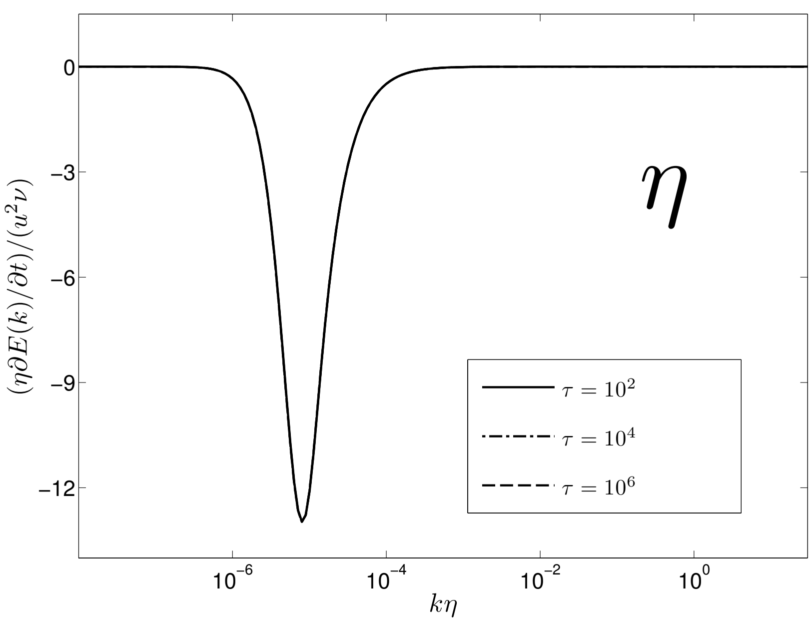
→ the unique self-similar solution

All theories collapse in this case !

A singular case: σ=1







George's theory also valid in this case

Which scales govern the decay rate?

3. Which scales govern HIT decay ?

Idea: use Data Assimilation techniques to identify governing scales

- Which scales ?
- Which features associated to these scales ?
- Dependency on observations ?

Data Assimilation: reminder

- Developed in weather forecast field
- Optimal recosntruction/identification of selected parameters (initial/boundary conditions, free parameters in physical/numerical models …)
- Two main families:
  - ***Variational methods based on optimal control theory***
  - Kalman filter type methods

3. Which scales govern HIT decay ?

[Mons et al. Phys. Fluids, 2014]

Problem :

Given an arbitrary decay law/realization, reconstruct the optimal initial condition $E(k,0)$

- Several possible observations levels:
  - full spectrum at sampling times
  - Integral quantities (***kinetic energy***, dissipation, lenghtscales, …)
  - Decay exponent (assuming algebraic decay)

Idea: starting from an arbitrary/random initial guess spectrum, DA optimization will reconstruct the optimal initial condition focusing on scales that govern the decay regime

DA: mathematical formulation

Find E(k,0) that minimizes the Lagrangian cost function

$$\mathcal{L}(E,\tilde{E}) = \frac{1}{2}\|y - H(E)\|^2_{\mathcal{O}} + \left\langle \tilde{E}, \frac{\partial E}{\partial t} + 2\nu k^2 E - T(E) \right\rangle_{\mathcal{M}}$$

Observations (available data)

Observation operator (applied to *E(k,t)*)

Optimality condition

Direct problem (Lin equation)

$$\frac{\partial E}{\partial t}(k,t) + 2\nu k^2 E(k,t) - T(E,k,t) = 0$$

Adjoint problem (adjoint Lin equation)

$$-\frac{\partial \tilde{E}}{\partial t}(k,t) + 2\nu k^2 \tilde{E}(k,t) - \tilde{T}|_E(\tilde{E},k,t) = \tilde{H}|_E\big(y - H(E),k,t\big)$$

$$\tilde{E}(k,T_f) = 0$$

DA iterative procedure

Arbitrary guess value for initial condition

$$E^0(k) = E(k,0)$$

Solve direct problem for *E(k,t)* from *t=0* to $t=T_f$

Evaluate cost function

Is it smaller than fixed convergence threshold ?

Yes → End

No

Solve adjoint problem from $t=T_f$ to t=0

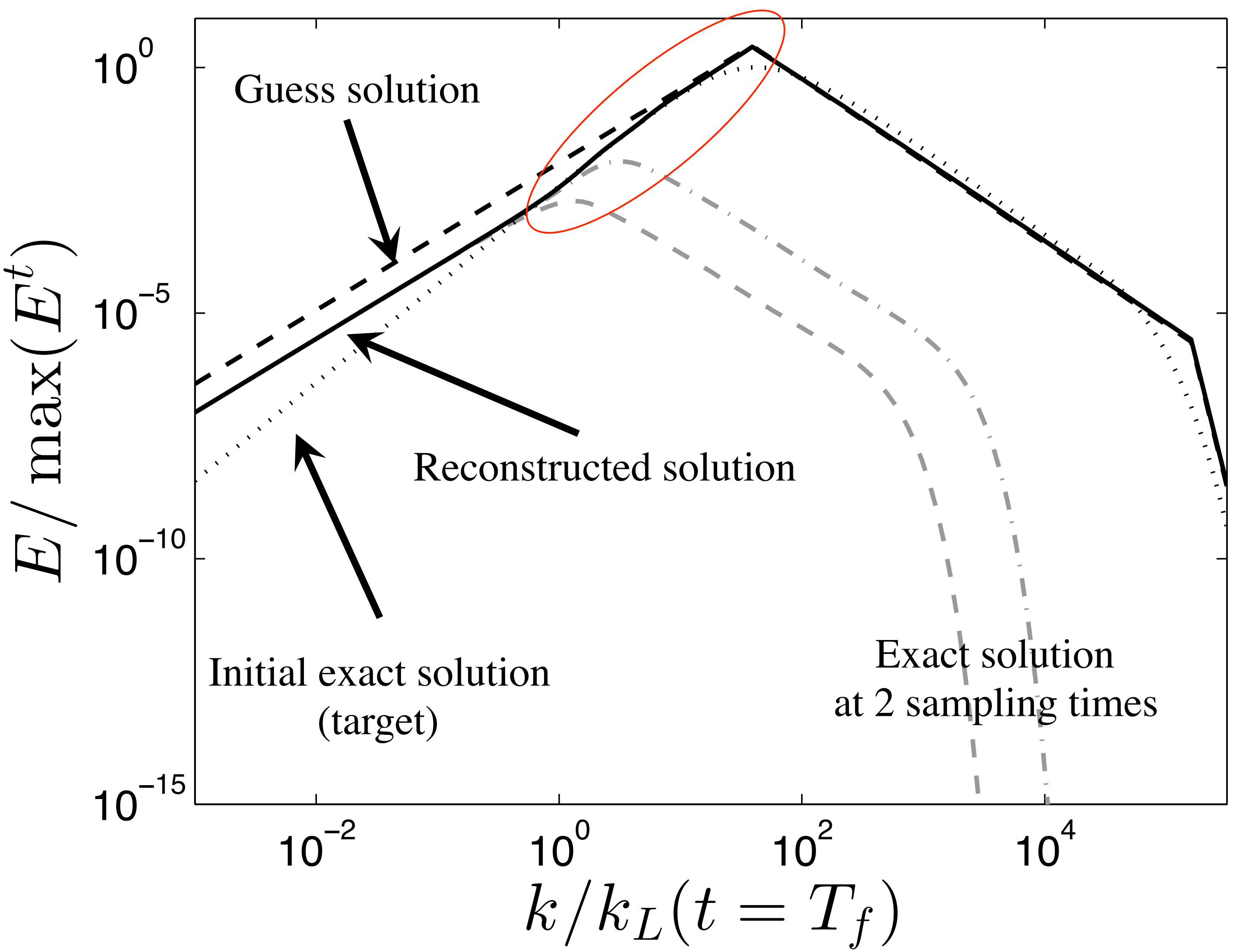
Compute gradient of cost function with respect to $E^0(k)$

$$\frac{\partial \mathcal{L}}{\partial E^0}(E, \tilde{E}, k) = -\tilde{E}(k,0)$$

Modify $E^0(k)$ with gradient-type method

DA results

Initial guess with single wrong slope at large scales

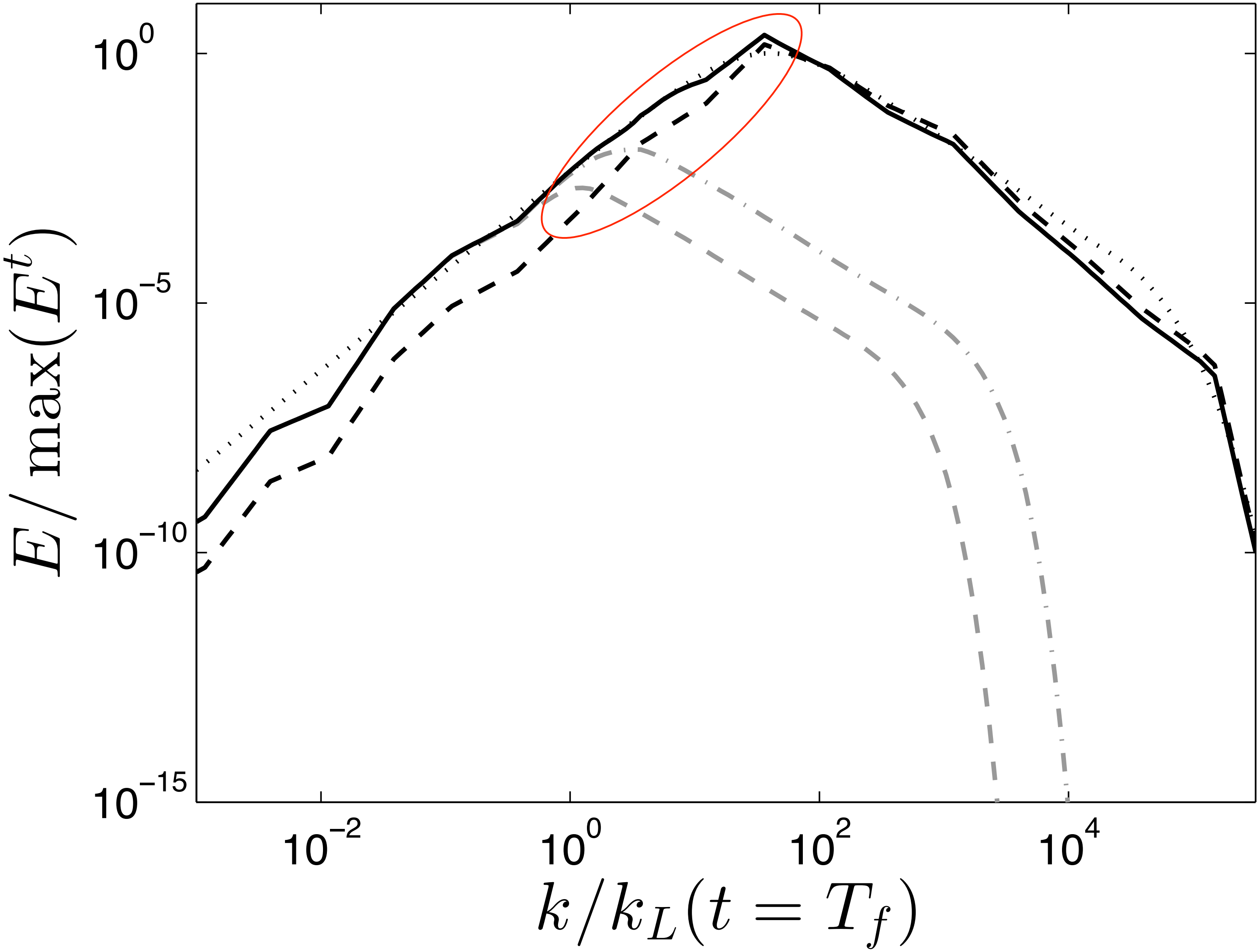


$T_f = 10^4\tau_0$

10 observations

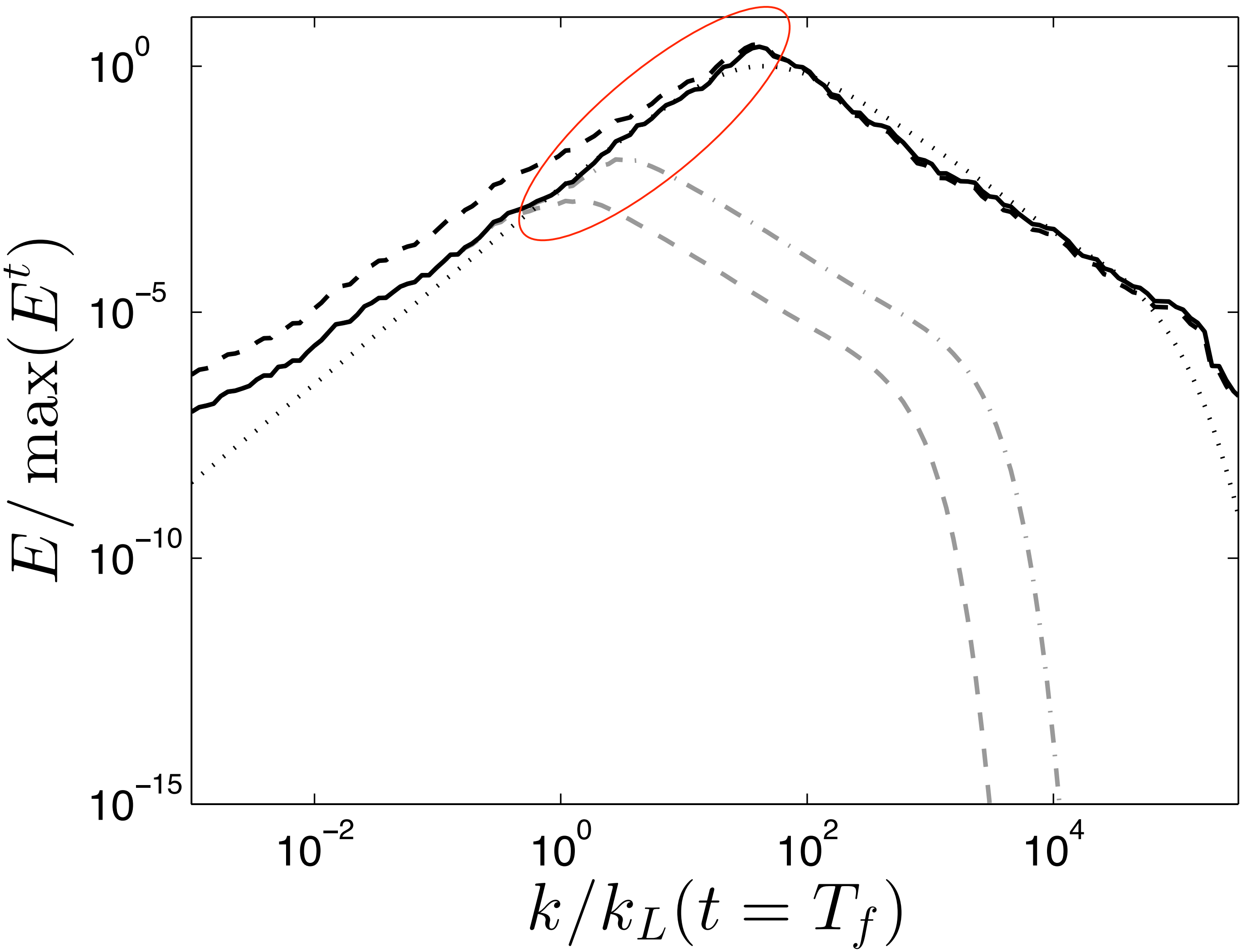
DA results

Initial guess with random local slope at large scales (by half decade)



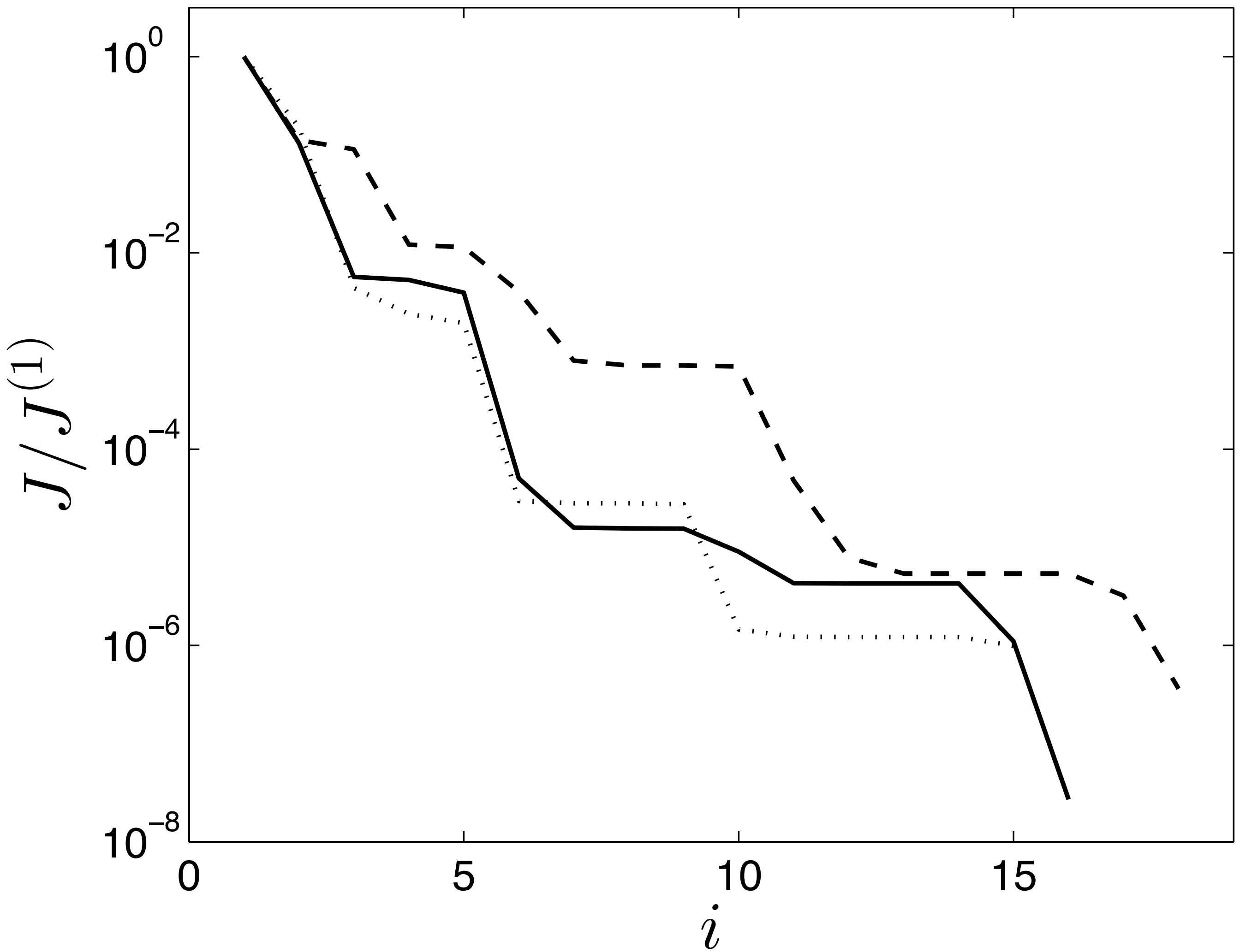
DA results

Initial guess with random local slope at large scales (each *k*)



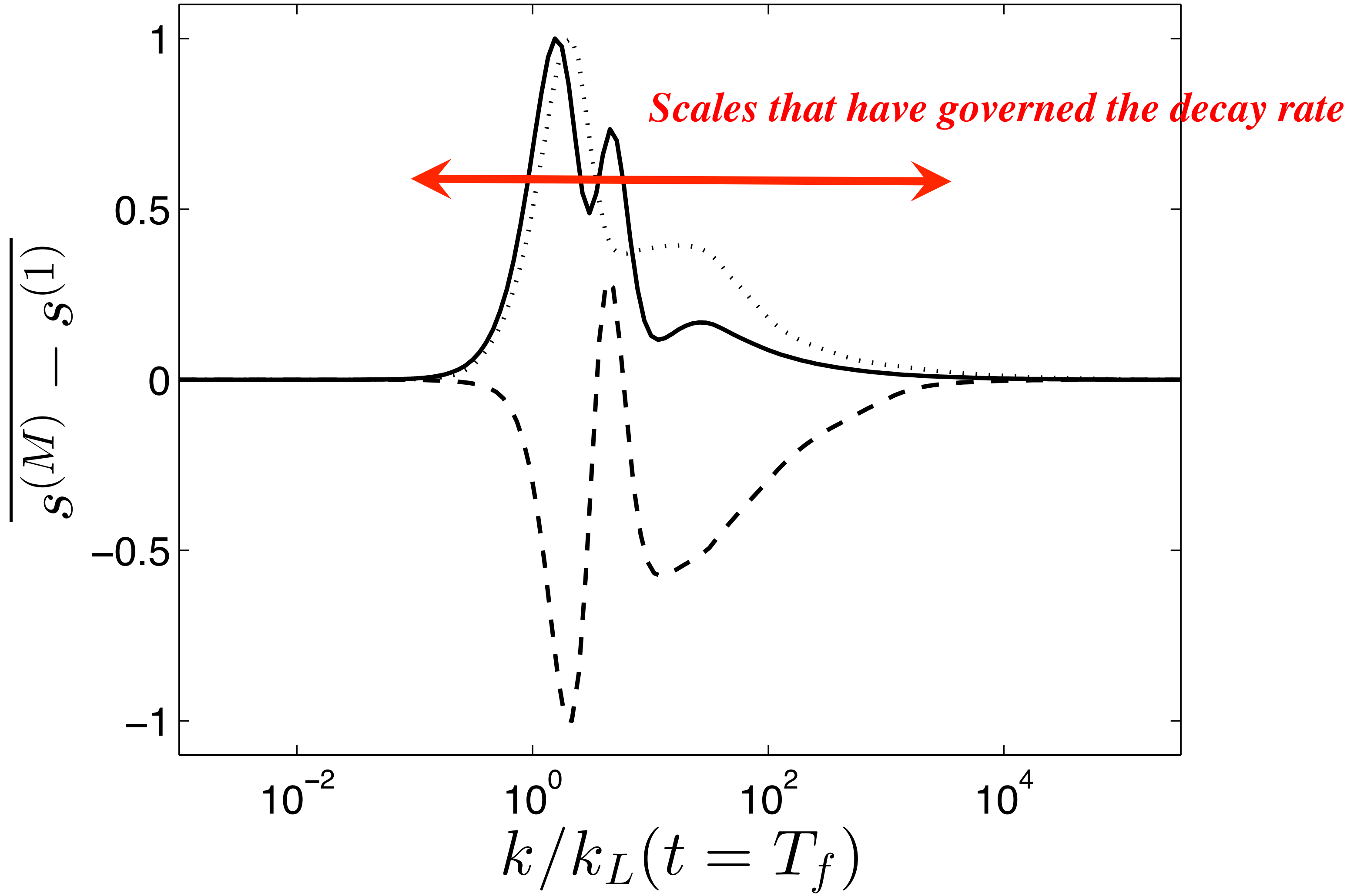
DA results

Convergence history



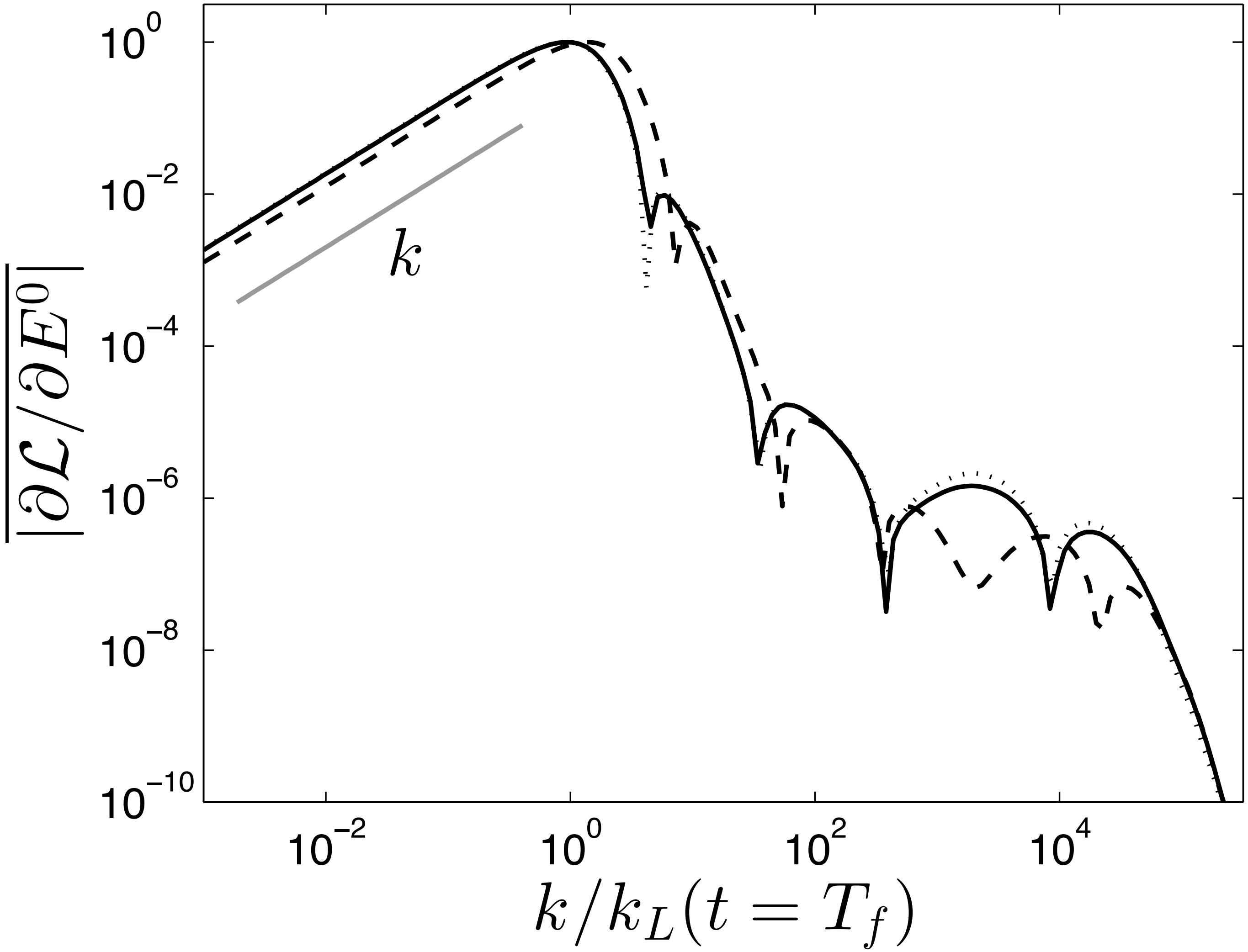
DA results

Change in the local slope (begining/end of DA procedure)



DA results

Gradient of the cost function versus initial spectrum



DA results

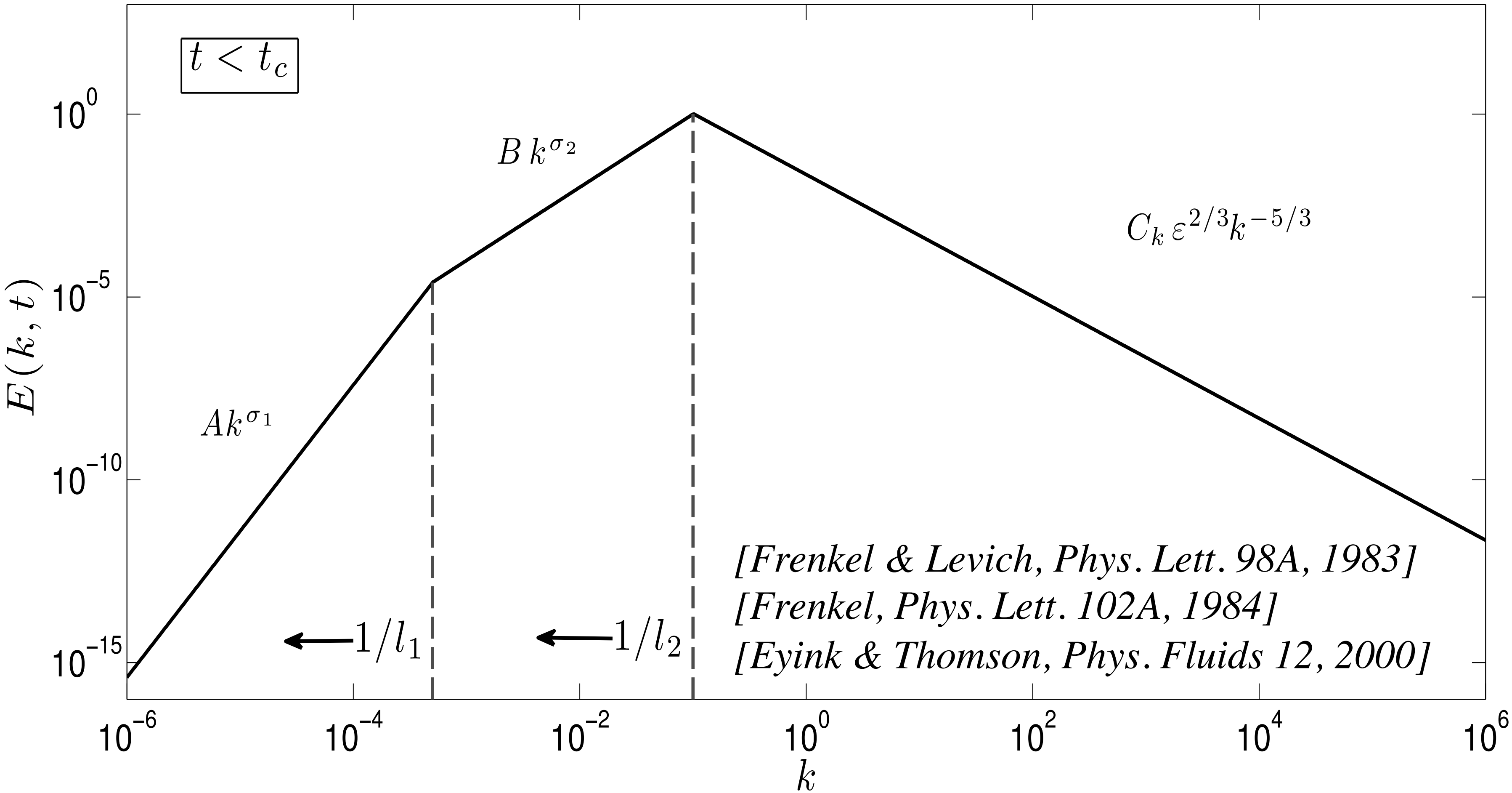
Conclusions:

- Initial conditions do matter !
- Non-classical (fast) decay have also been obtained
- Finite time decay regime can be controled manipulating the initial conditions
- Within HIT framework:
  - « model » of the turbulence generation process
  - Adding a forcing term (forced HIT with time-varying source term)

Non-self-preserving solutions over arbitrary long time ?

Initial Three-Range Spectrum

$$E(k) = \begin{cases} A\,k^{\sigma_1} & kl_1 \leq 1 \\ B\,k^{\sigma_2} & kl_1 \geq 1,\ kl_2 \leq 1 \\ C_k \varepsilon^{2/3} k^{-5/3} & kl_2 \geq 1 \end{cases}$$



Initial Three-Range Spectrum

Time evolution:

- Initial non-self-similar regime (three-range spectrum)
- Recovery of a classical self-similar regime (two-range spectrum)

Lengthscale evolution (CBC-like dimensional analysis)

$$l_2(t) \propto t^{2/(\sigma_2+3-p_2)} \qquad l_1(t) \propto t^{\frac{2p_2}{(\sigma_2-\sigma_1+p_1)(\sigma_2+1)}}$$

$$\Rightarrow \quad t_c = t_c(\sigma_1, \sigma_2, l_1(0)/l_2(0))$$

Caution: $t_c = +\infty$

Initial Three-Range Spectrum

Kinetic energy evolution law during first phase

$$u^2(t) = \frac{1}{2}\int_0^{+\infty} E(k,t)dk$$

$$u_e^2 = u^2 - \frac{\sigma_2 - \sigma_1}{(\sigma_1+1)(\sigma_2+1)} B^{2/3} \left(\frac{A}{B}\right)^{\frac{\sigma_2+1}{\sigma_2-\sigma_1}}$$

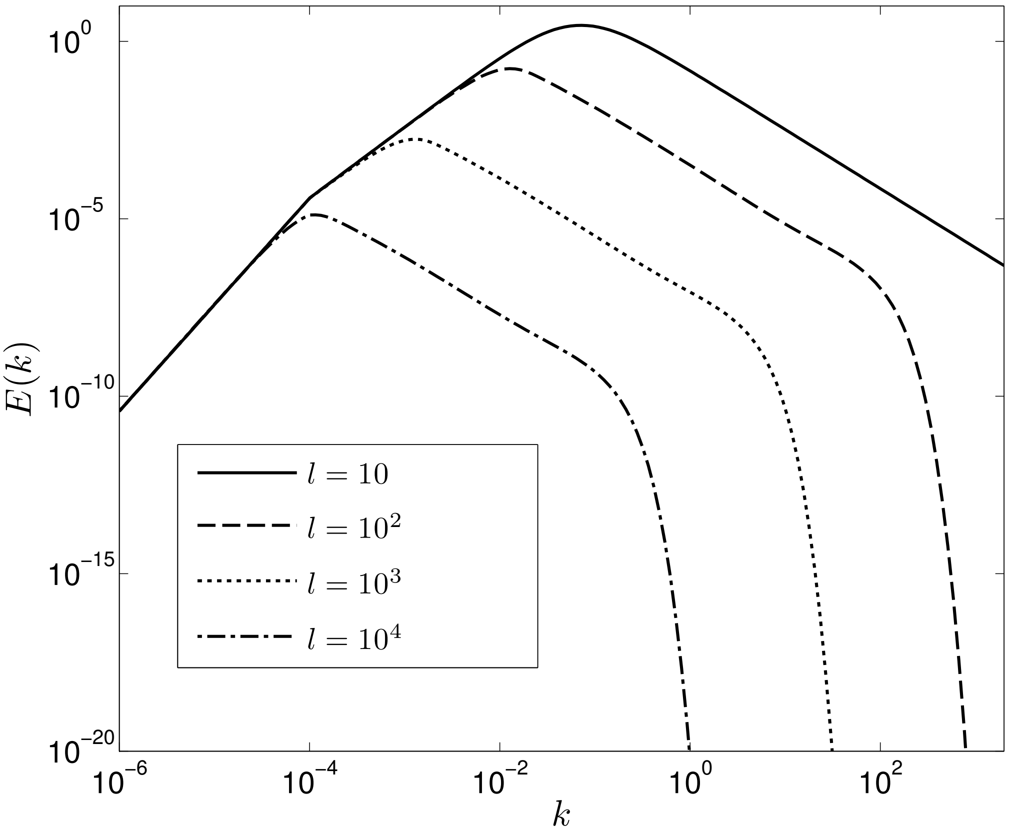
Time independent if PLE holds

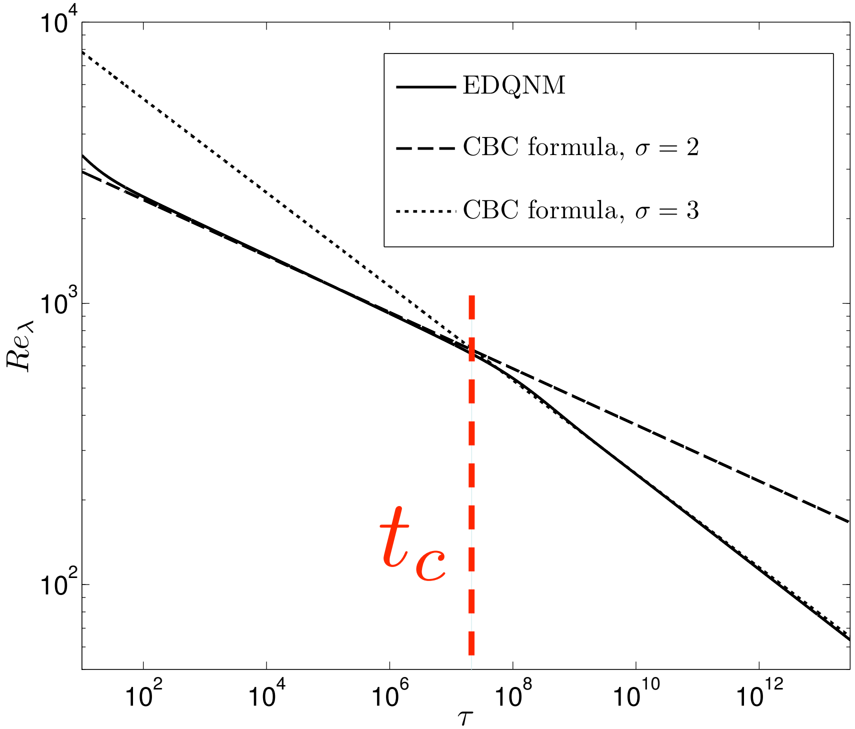
$$u_e^2(t) = E\left(t - t_1\right)^{-\frac{2(\sigma_2+1)}{\sigma_2+3}}$$

Pseudo self-similar law !

Initial Three-Range Spectrum

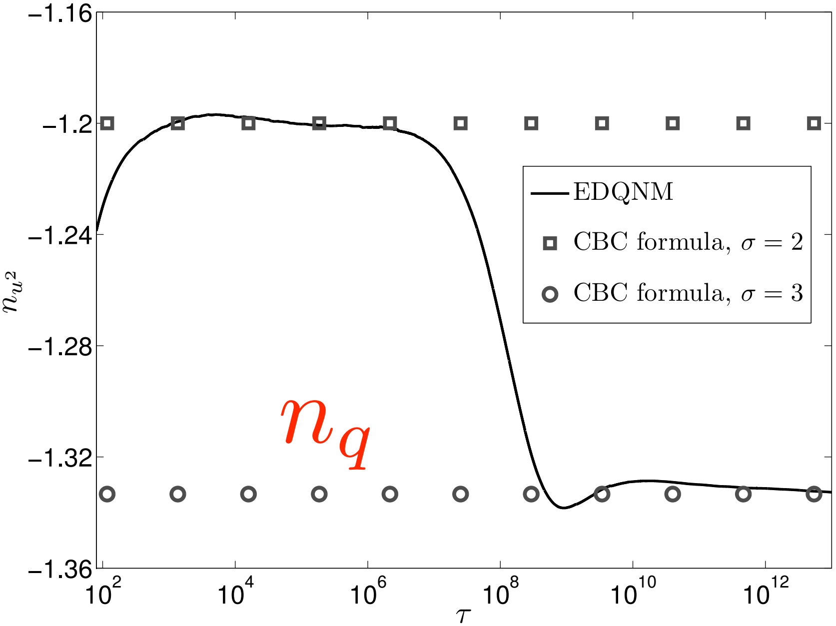
$$(\sigma_1 = 3, \sigma_2 = 2)$$

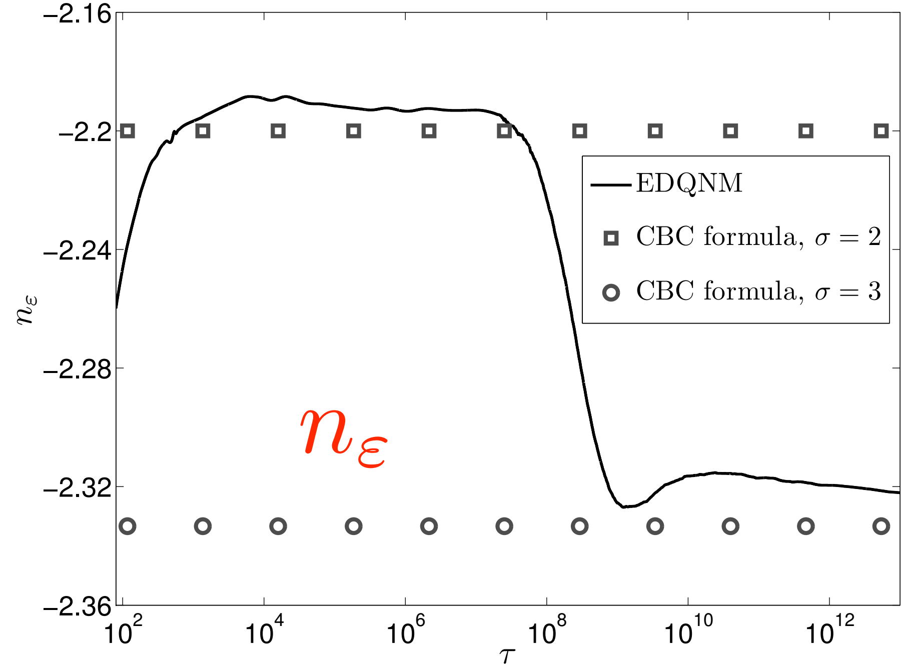


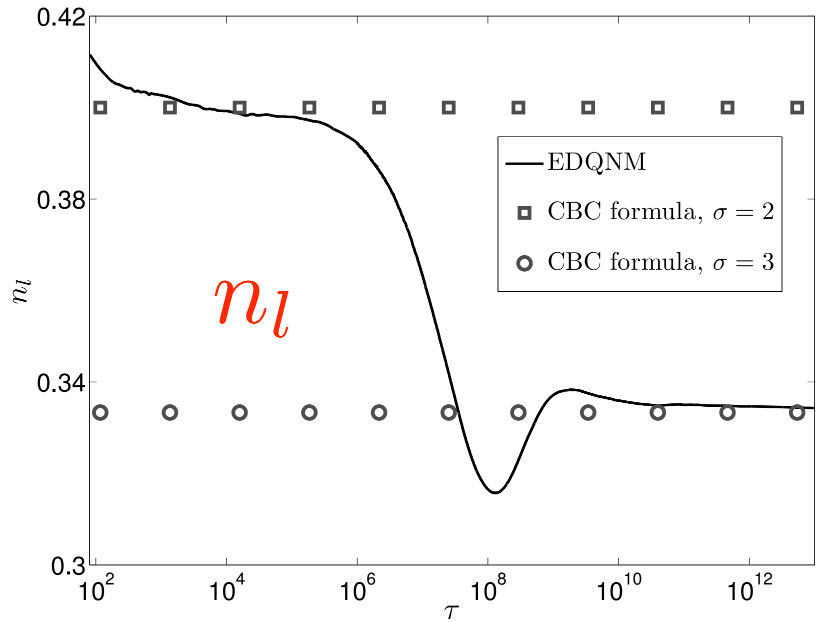


[Meldi & Sagaut, J. Fluid Mech., 711, 2012]

Initial Three-Range Spectrum (with PLE)



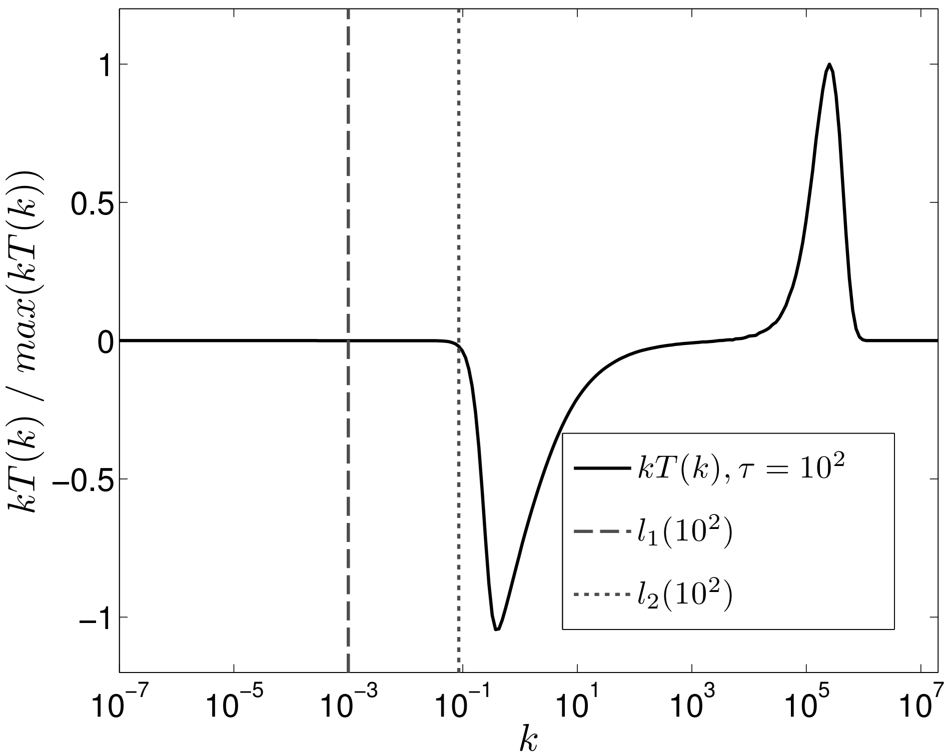




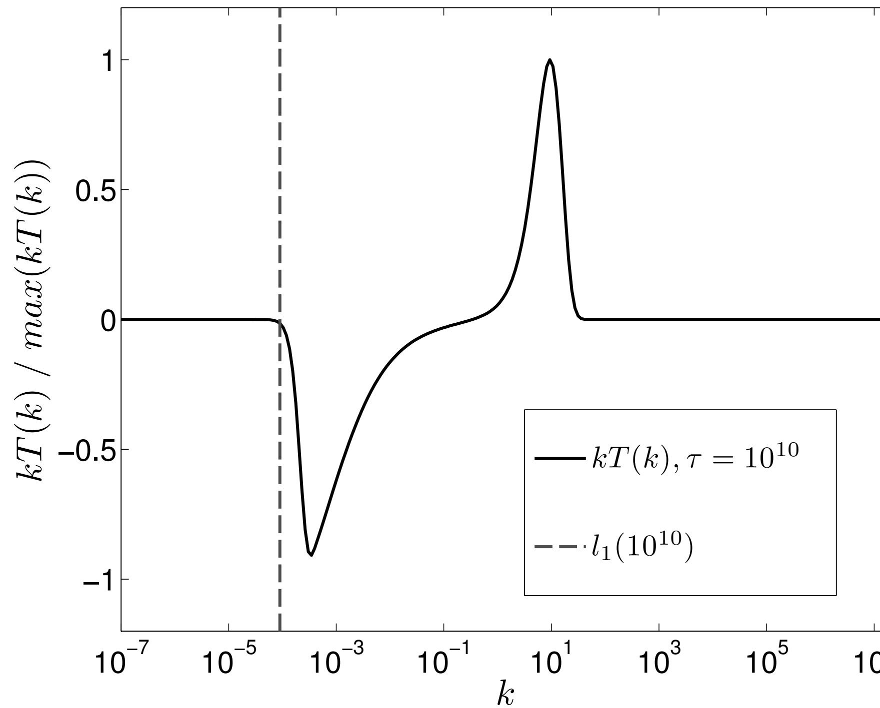
$$(\sigma_1 = 3, \sigma_2 = 2)$$

Pre-multiplied non-linear transfer T(k)

$$(\sigma_1 = 2,\ \sigma_2 = 3)$$

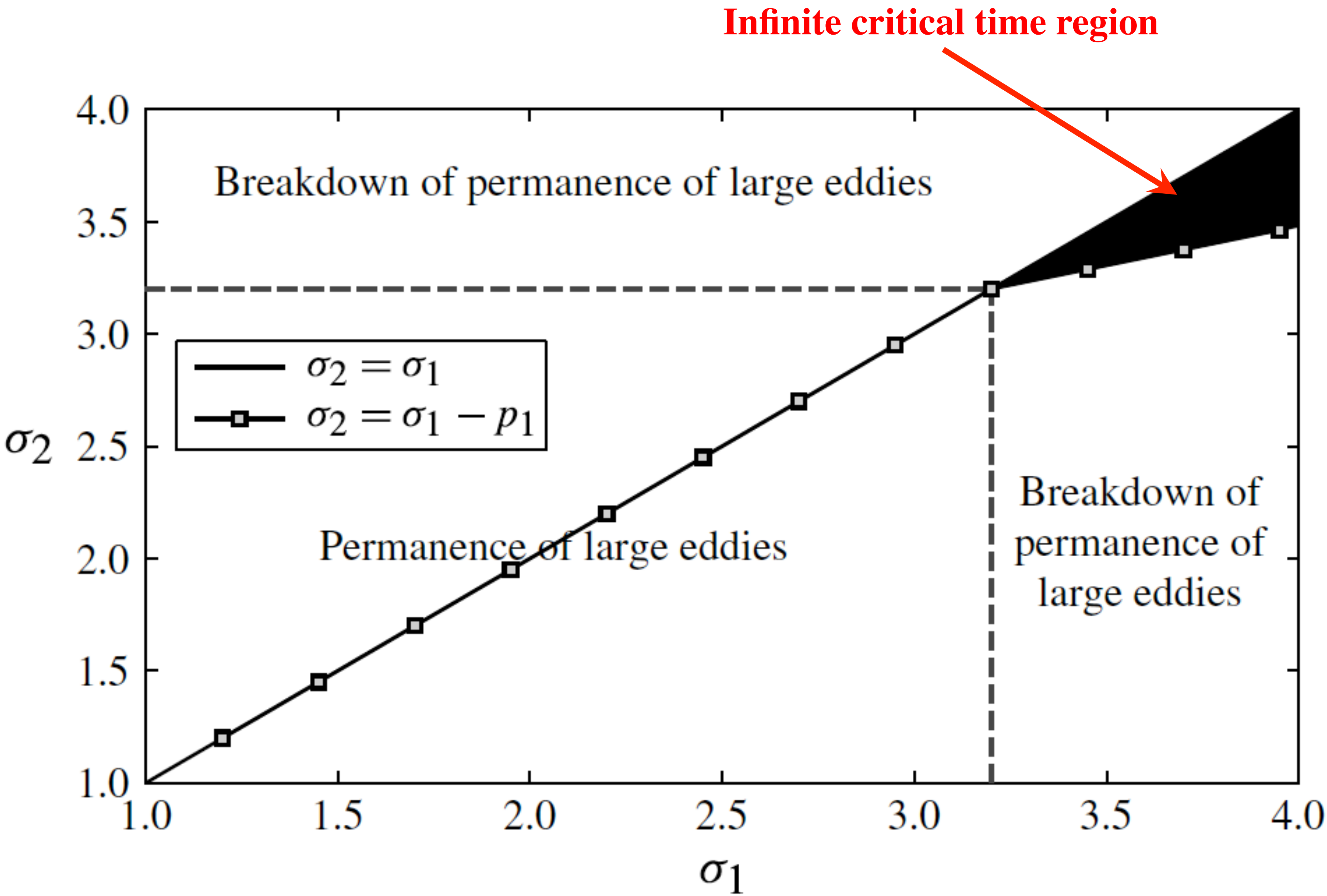


$$t < t_c$$



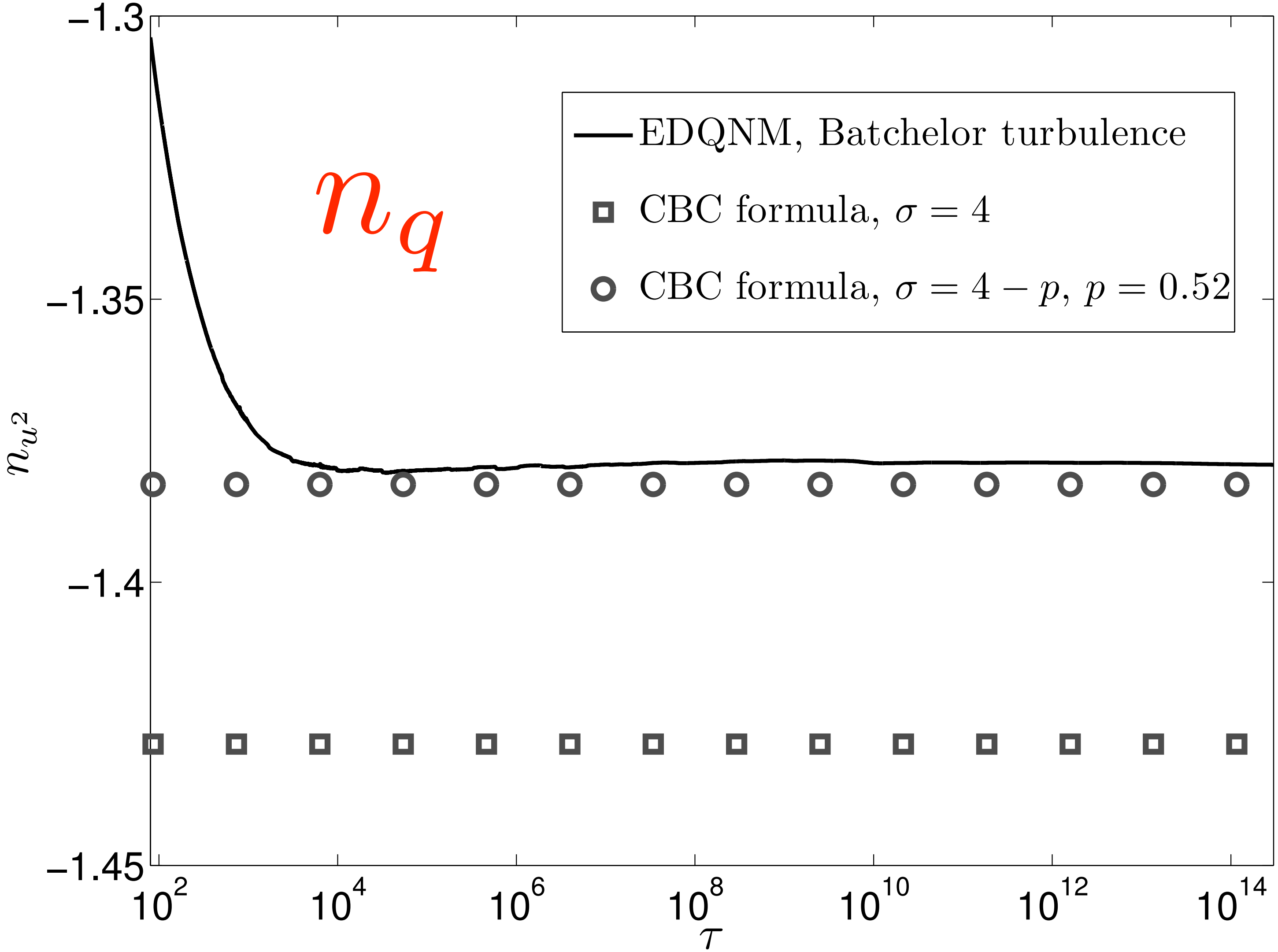
$$t > t_c$$

Initial Three-Range Spectrum



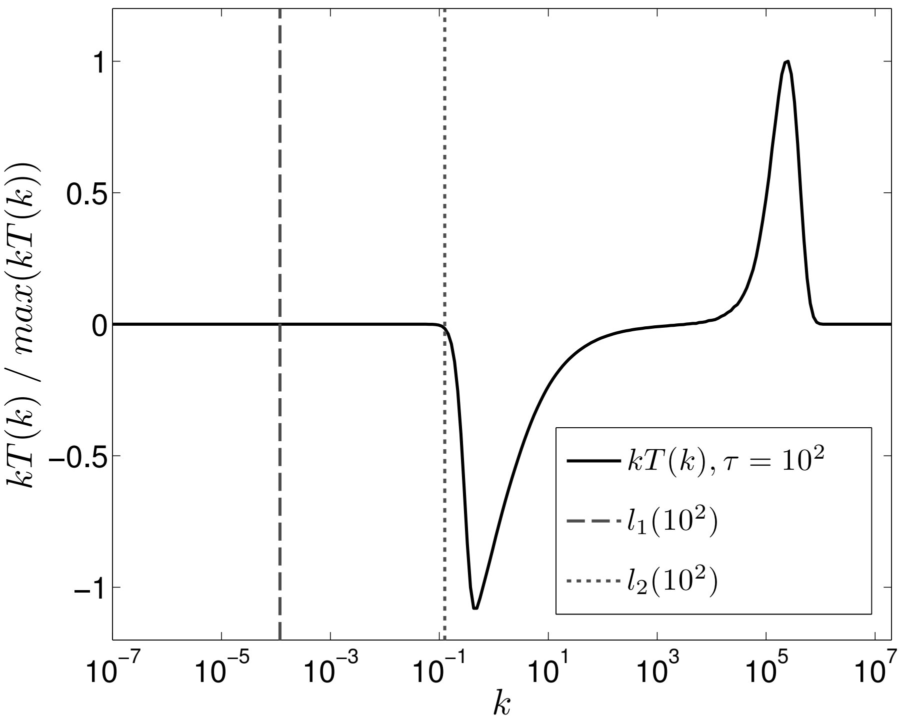
Solution with infinite critical time

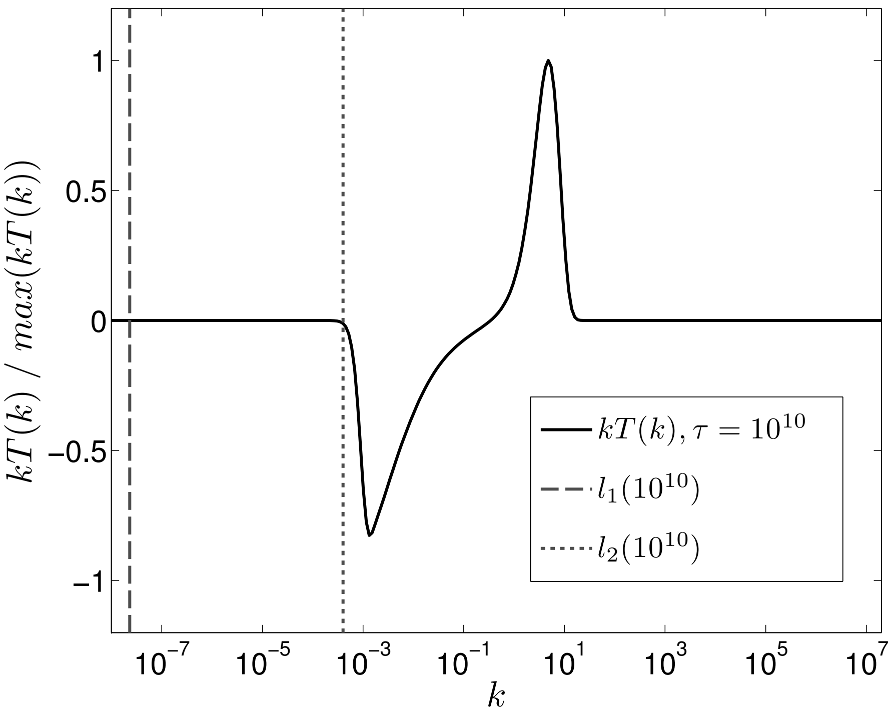
$(\sigma_1 = 3.7, \sigma_2 = 4)$ $t_c = +\infty$



Pre-multiplied non-linear transfer T(k)

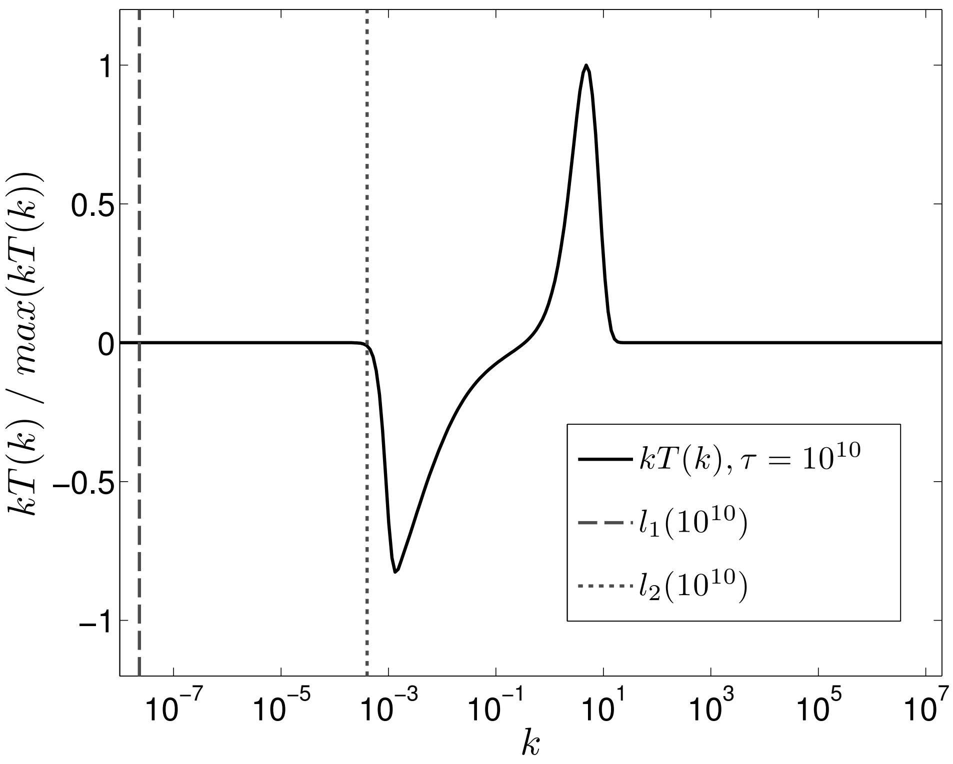
$$(\sigma_1 = 3.7, \sigma_2 = 4)$$

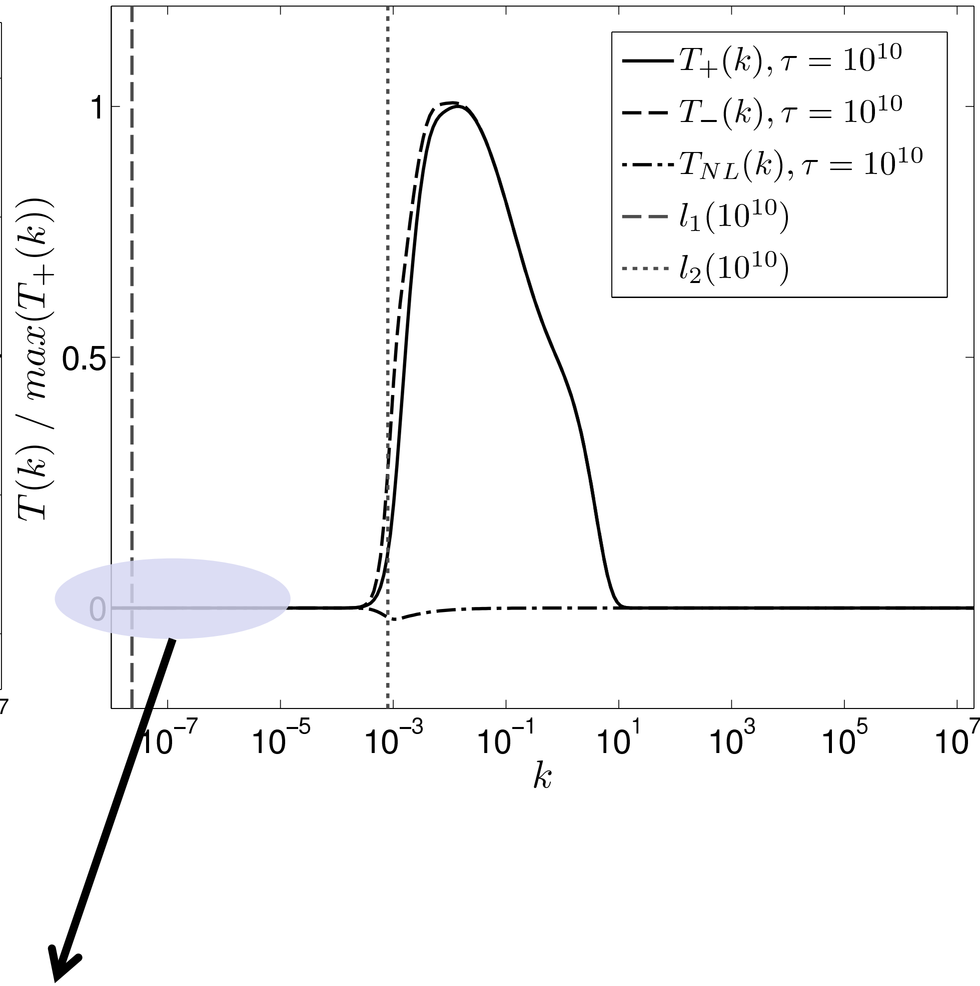




Pre-multiplied non-linear transfer T(k)

$$(\sigma_1 = 3.7, \sigma_2 = 4)$$





« inactive » scales → permanence

Existence of a universal attractor ?

Aix*Marseille université

A universal decay regime ?

$$\frac{d\varepsilon}{dt} = \int_0^\infty 2\nu k^2 T(k)dk - \int_0^\infty (2\nu k^2)^2 E(k)dk$$

$$= -\frac{7}{15}\left(\frac{1}{2}S(t)Re_\lambda(t) + G(t)\right)\frac{\varepsilon^2(t)}{\mathcal{K}(t)}$$

$$= \underbrace{\frac{7}{3\sqrt{15}}S(t)\sqrt{Re_L(t)}\frac{\varepsilon^2(t)}{\mathcal{K}(t)}}_{\text{Generation of dissipation}} - \underbrace{\frac{7}{15}G(t)\frac{\varepsilon^2(t)}{\mathcal{K}(t)}}_{\text{Destruction of dissipation}}$$

$\Rightarrow$

$$\frac{\dot{k}}{k} = -\frac{\varepsilon}{k}$$

$$\frac{\dot{\varepsilon}}{\varepsilon} = -\left(\frac{7}{3\sqrt{15}}SR_t^{1/2} + \frac{7}{15}G\right)\frac{\varepsilon}{k}$$

Karman & Howarth (1938)
Panchev (1971)
Speziale & Bernard (1992)
Antonia et al. (2001)
Ristorcelli (2003,2004,2006)

A universal decay regime ?

Skewness coefficient

$$S = -\frac{\overline{(\partial u'/\partial x)^3}}{\overline{(\partial u'/\partial x)^2}^{3/2}} = -\frac{3\sqrt{30}}{14}\frac{\int_0^{+\infty} k^2 T(k,t)dk}{\left[\int_0^{+\infty} k^2 E(k,t)dk\right]^{3/2}} = \frac{h'''(0)}{f''(0)}^{3/2}$$

Palinstrophy coefficient

$$G = \frac{30}{7}\frac{\nu\mathcal{K}}{\varepsilon}\frac{\overline{\frac{\partial\omega'_i}{\partial x_j}\frac{\partial\omega'_i}{\partial x_j}}}{\overline{\omega'_k\omega'_k}} = \frac{120}{7}\frac{\nu\mathcal{K}}{\varepsilon}\frac{\int_0^{+\infty} k^4 E(k,t)dk}{\int_0^{+\infty} k^2 E(k,t)dk} = \lambda^4 f^{(IV)}(0)$$

$$\mathcal{K}(t) \propto t^n$$

$$C_{\varepsilon_2} = \frac{n}{n+1} = \frac{7}{15}\left(\frac{1}{2}S(t)Re_\lambda(t) + G(t)\right)$$

A universal decay regime ?

Von Karman-Howarth hypothesis (1938) :

$$S(t) = S_\infty \qquad G(t) = G_\infty$$

$$\Rightarrow \quad \frac{\dot{Re}_L}{Re_L} = \left(\frac{7}{3\sqrt{15}} S_\infty Re_L^{1/2} + \frac{7}{15} G_\infty \right) - 2$$

Fixed point analysis + Taylor-scale-based Self-Similarity :

$$\Rightarrow \quad Re_L(t) \rightarrow Re_L(\infty) \neq 0$$

$$\mathcal{K}(t) \propto t^{-1}$$

[Speziale & Bernard, J. Fluid Mech, 1992]

A universal decay regime ?

[Antonia et al. , JoT, 2002]: Kolmogorov Self-Similarity

$$S(t) = S_\infty$$

$$G(t) = G_\infty + \left(\frac{dG}{dRe_L^{1/2}}\right)(Re_L^{1/2}(t) - Re_L^{1/2}(\infty)) = G_0 + G_0' Re_L^{1/2}(t)$$

$$\frac{\dot{Re}_L}{Re_L} = \left[\left(\frac{7}{3\sqrt{15}}S_\infty + \frac{7}{15}G_0'\right)Re_L^{1/2} + \frac{7}{15}G_0\right] - 2$$

Fixed point analysis + Kolmogorov-scale-based self-similarity :

$$Re_L(t) \rightarrow Re_L(\infty) \neq 0$$

$$\mathcal{K}(t) \propto t^{-1}$$

[Ristocelli et al., Phys. Fluids, 2003, 2004 ,2006]

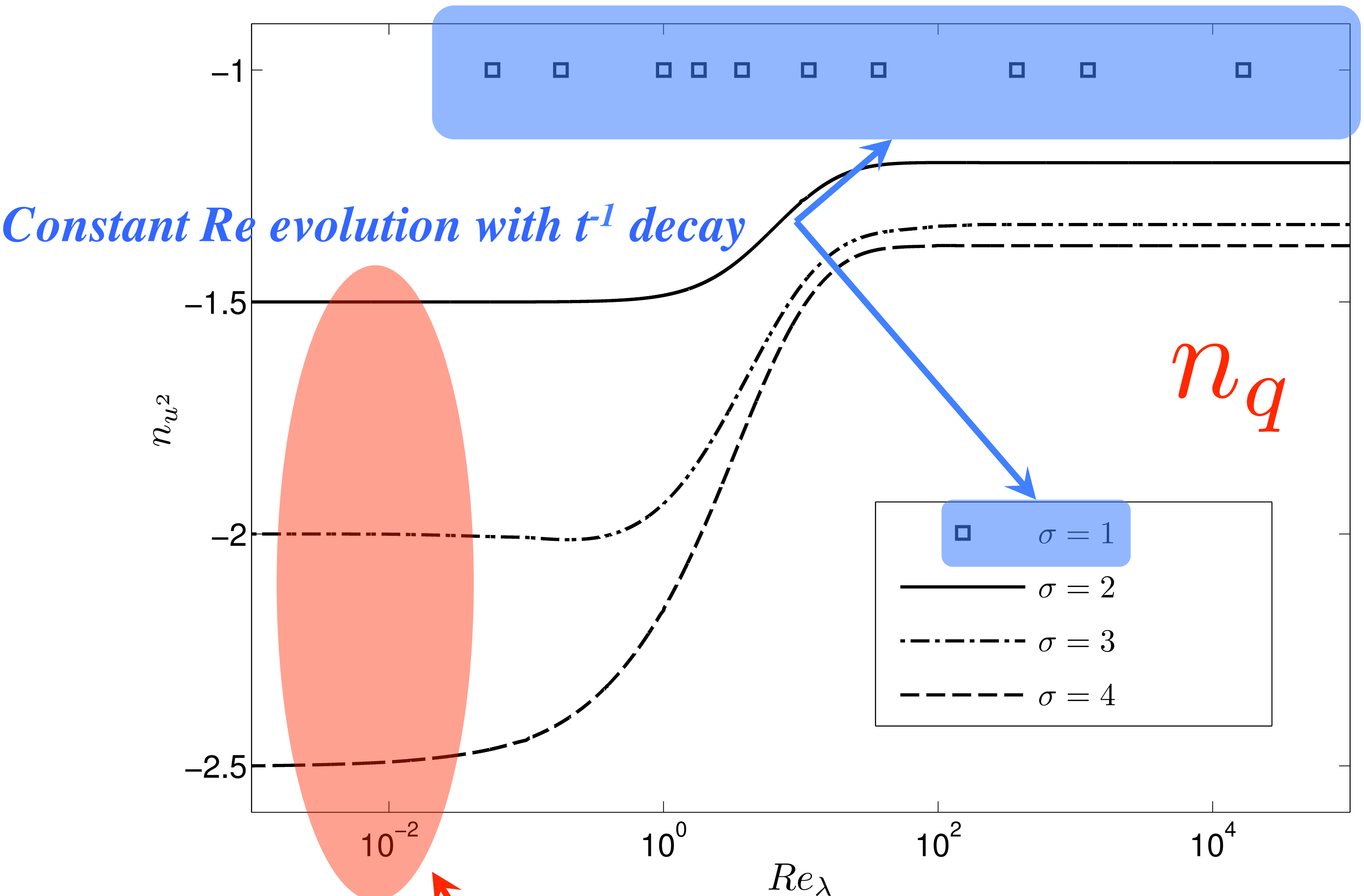
Aix*Marseille université

Observability of asymptotic decay regimes

The R_t fixed point and $k(t) \sim t^{-1}$: The reader is reminded that the presumed behavior of $S \sim$ const, and $G \sim$ const (or $G \sim R_\lambda$) implies, rigorously and self-consistently, that $R_t \to R_{t\infty} > 0$, and $k(t) \sim t^{-1}$. Such $k(t) \sim t^{-1}$ behavior has not been seen experimentally. The speculation has been that the approach to the t^{-1} regime is too slow to be seen experimentally.[26,27,31] However, the DNS, for example,[31,43] do all seem to exhibit this behavior when especial care has been taken to adjust for virtual origin effects. Using DNS to investigate fixed-point behavior requires long time computations and is a nontrivial problem; the two point correlation begins to approach box size and the most energetic modes are at the lowest wavenumber and the largest scales are represented by very few points.

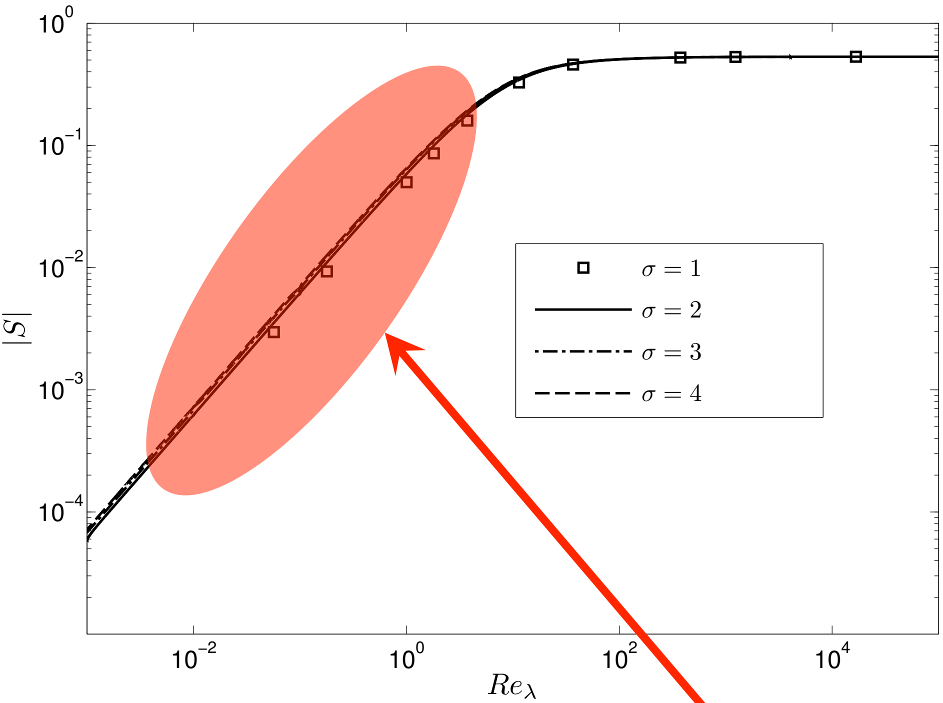
[Ristorcelli, Phys. Fluids 18, 2006]

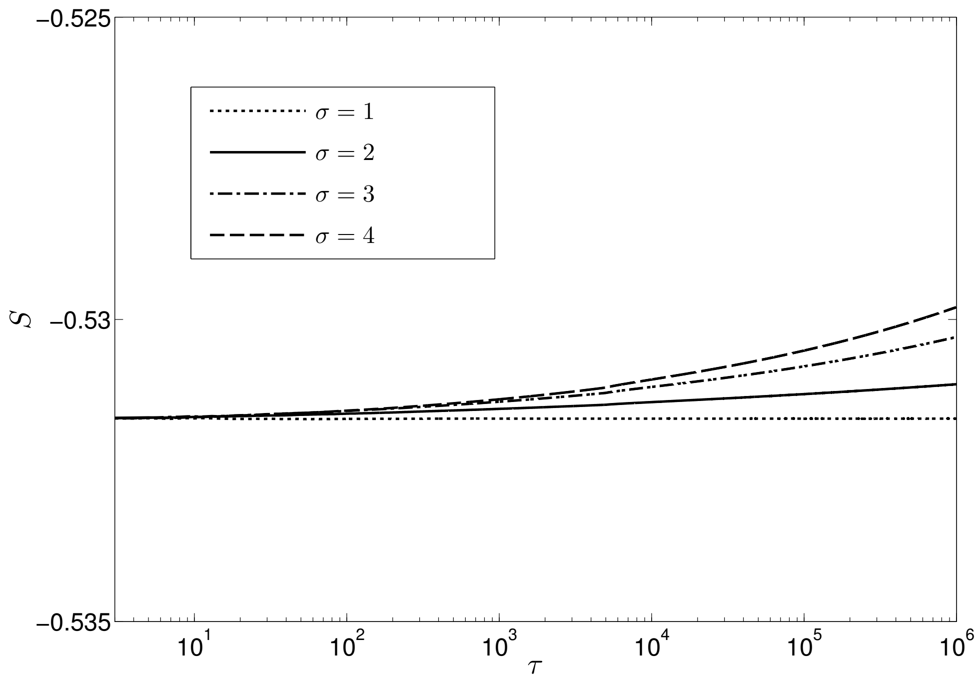
EDQNM results



no t^{-1} universal attractor

Skewness evolution

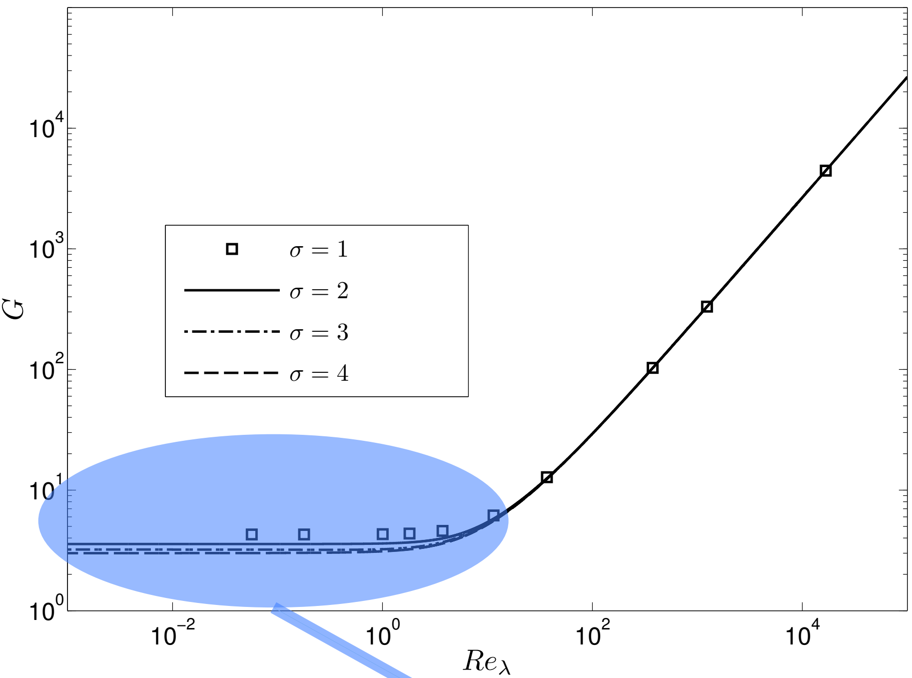


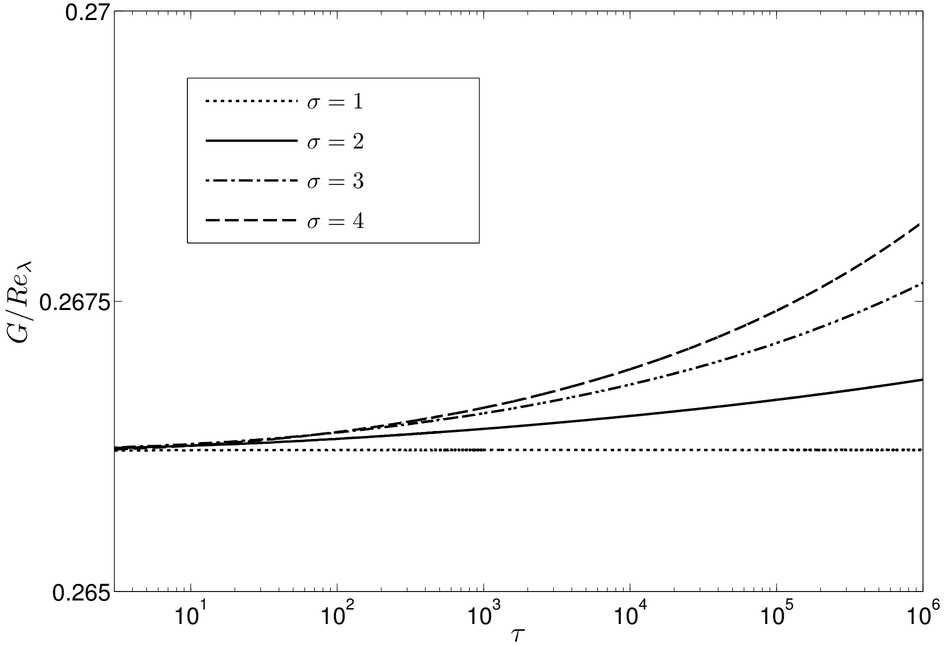


$$S(t) \rightarrow \begin{cases} 0 & \sigma \neq 1 \\ S(0) & \sigma = 1 \end{cases}$$

$$S(t) \propto \begin{cases} Re_\lambda^0 & Re_\lambda > 200 - 300 \\ Re_\lambda & Re_\lambda < 0.1 - 1 \end{cases}$$

Palinstrophy evolution





$$G(t) \propto \begin{cases} Re_\lambda & Re_\lambda > 200 - 300 \\ Re_\lambda^0 & Re_\lambda < 0.1 - 1 \end{cases}$$

$$G(t) \to G_\infty = \begin{cases} G(0) & \sigma = 1 \\ \frac{15}{7}\left(\frac{n_q - 1}{n_q}\right) & \sigma \neq 1 \end{cases}$$

Conclusions

• Decay is governed by initial conditions (the turbulence generation mechanisms in practice)

•Finite time evolution is governed by large scales ranging from initial to final spectrum peak

• No full self-similarity/exact self-preservation in general case

• No sensitivity to asymptotic behavior of the spectrum at large scales

→ Behavior at *k*→*0* escapes experiments/simulations and should not be a parameter in theories for decay (***real turbulence lives in a finite world !***)

→ Idem: analyticity of the spectrum, integral invariants depending on asymptotic behavior …

• Non-self similar solutions possible over arbitrary long times

Thank you!

Aix*Marseille université

TKE spectrum with fractal forcing

[Meldi, Lejemble, Sagaut, JFM, 2014]

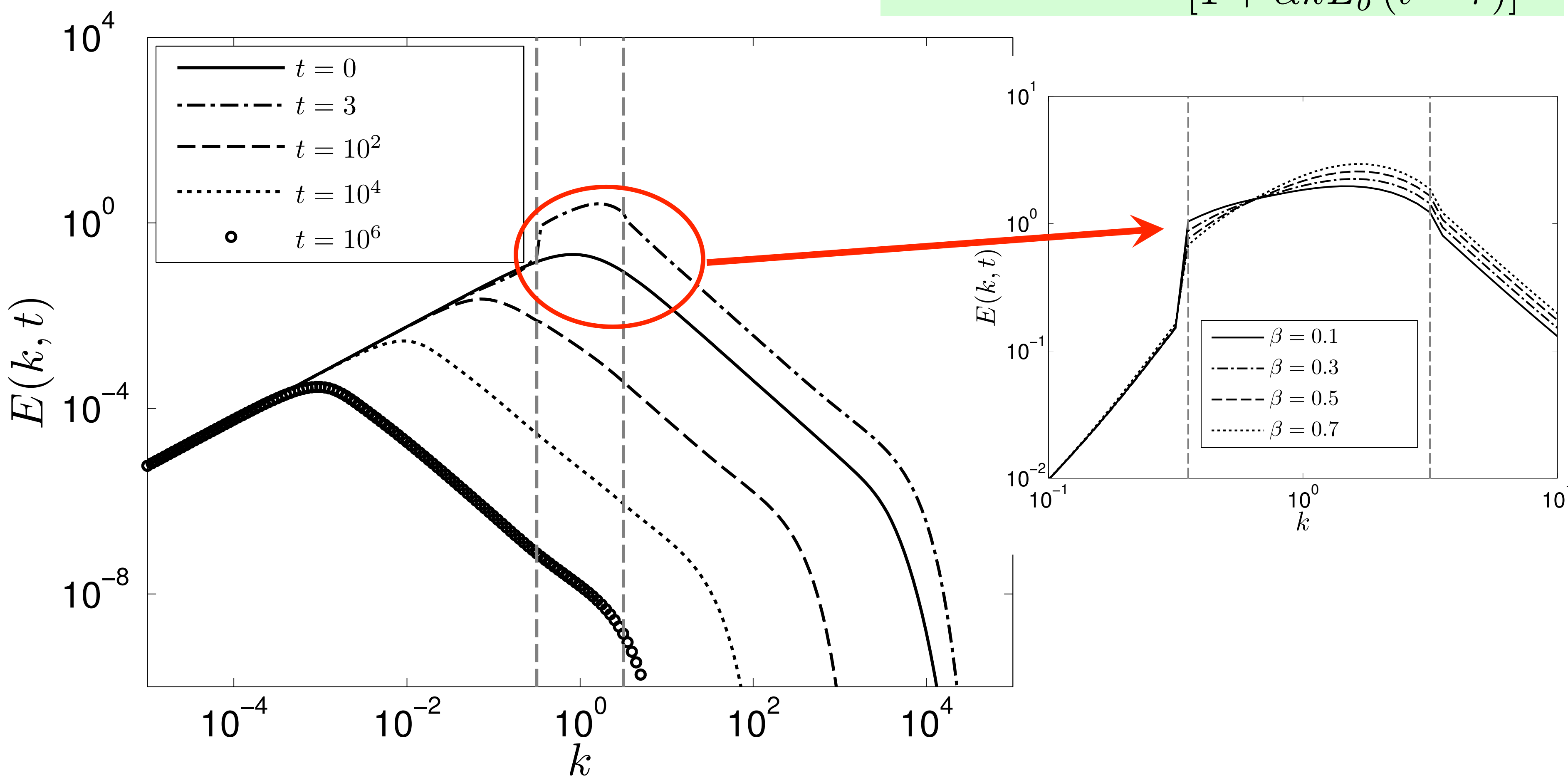
Fractal forcing

→ broadband energy production

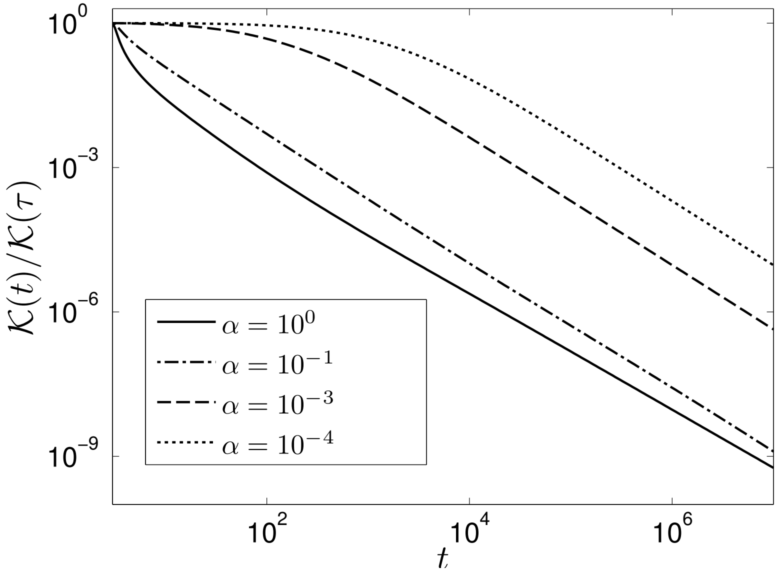
→ flatened spectrum peak

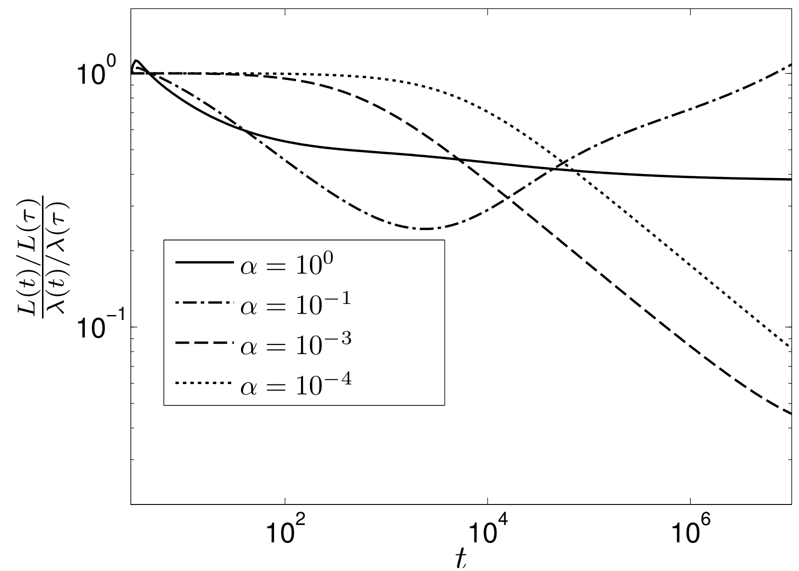
$$\frac{\partial E(k,t)}{\partial t} + 2\,\nu\,k^2\,E(k,t) = T(k,t) + \langle\,\mathcal{F}(k,t)\,\rangle$$

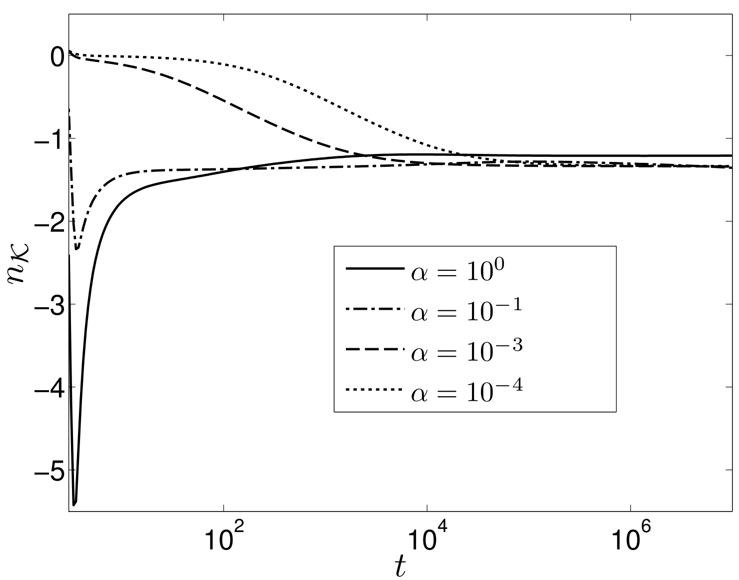
$$\langle\,\mathcal{F}(k,t)\,\rangle = a_k\,\frac{(k\,L_b)^{\beta}\sqrt{8\pi k^2 E(k,t)}}{\left[1+\alpha k L_b\,(t-\tau)\right]^2}$$

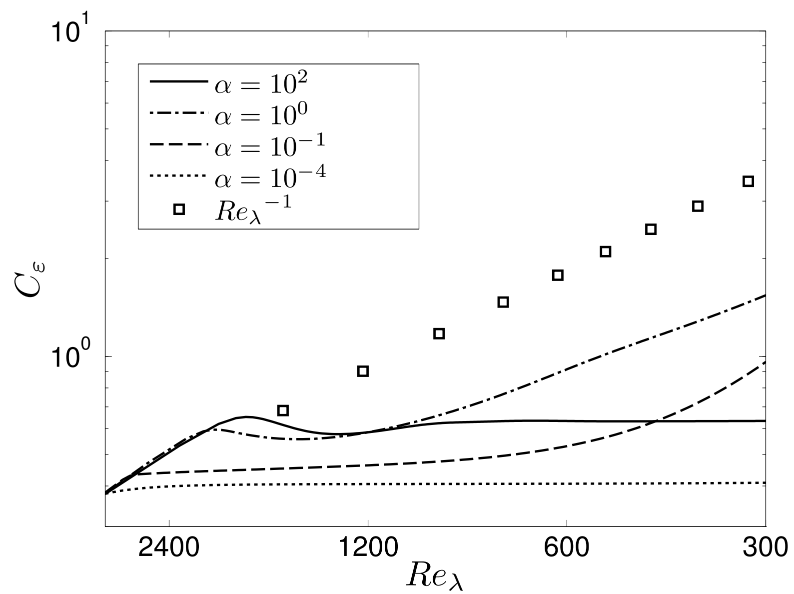


TKE spectrum with fractal forcing







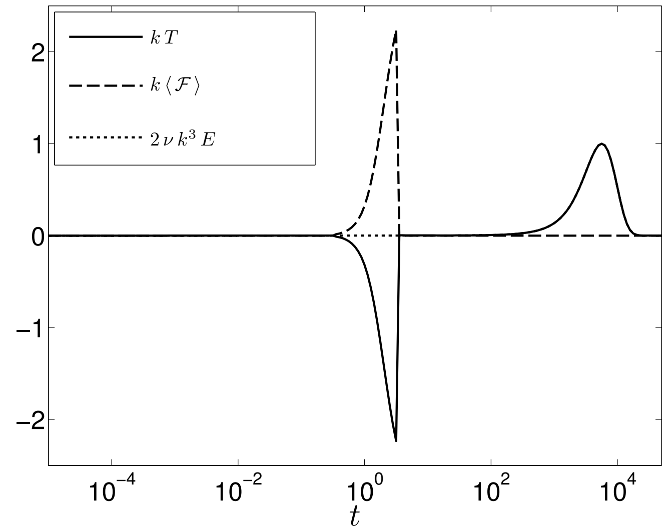


Aix*Marseille université

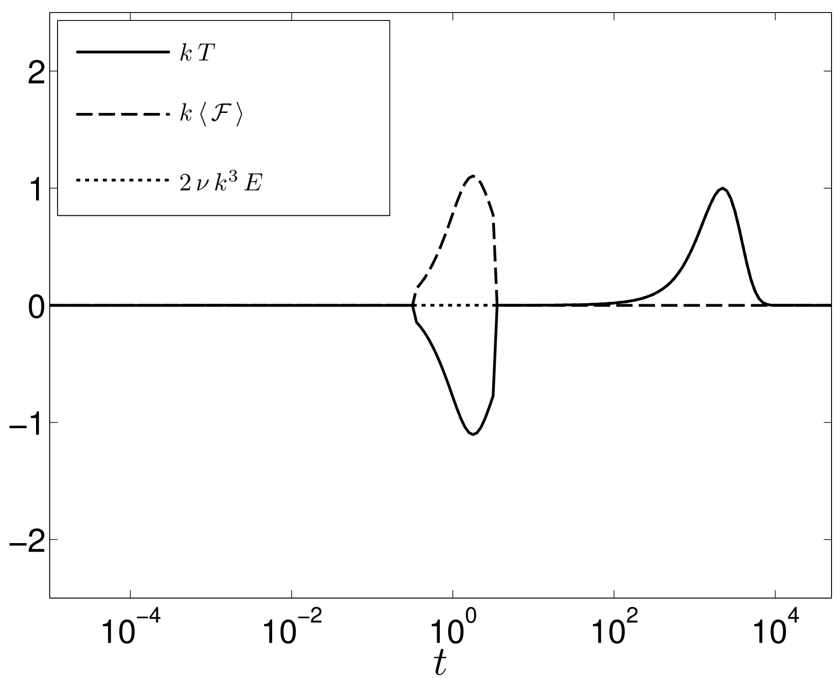
TKE spectrum with fractal forcing

$$\alpha = 10^{-4}$$

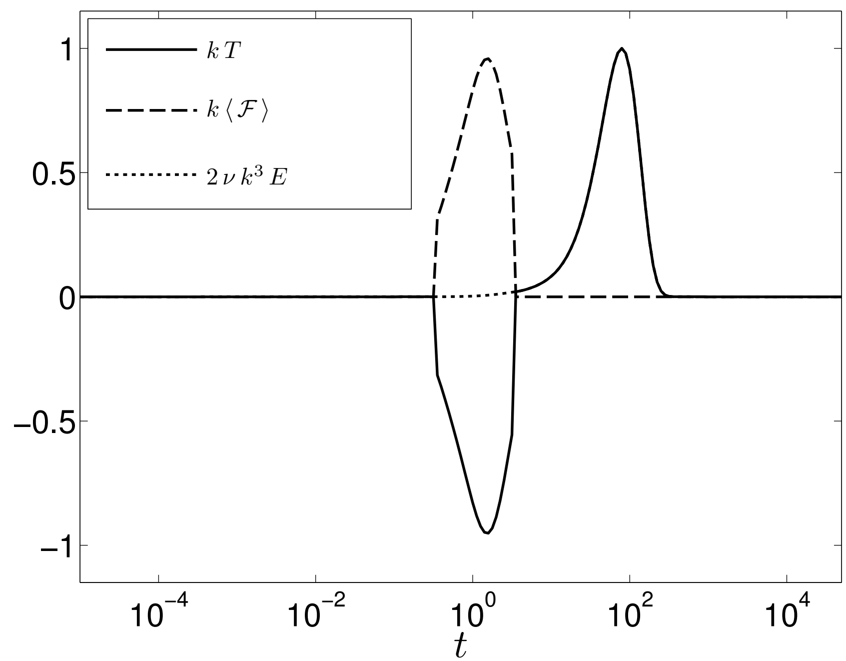
$t = \tau$



$t = \tau + 1/\alpha$



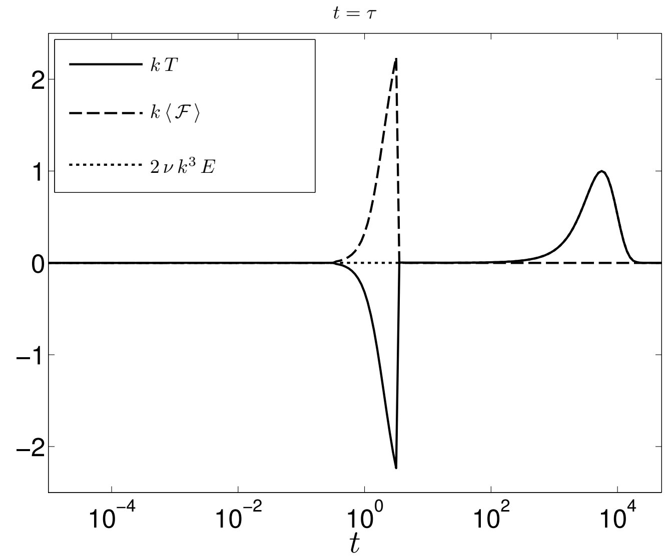
$t = \tau + 10^3/\alpha$

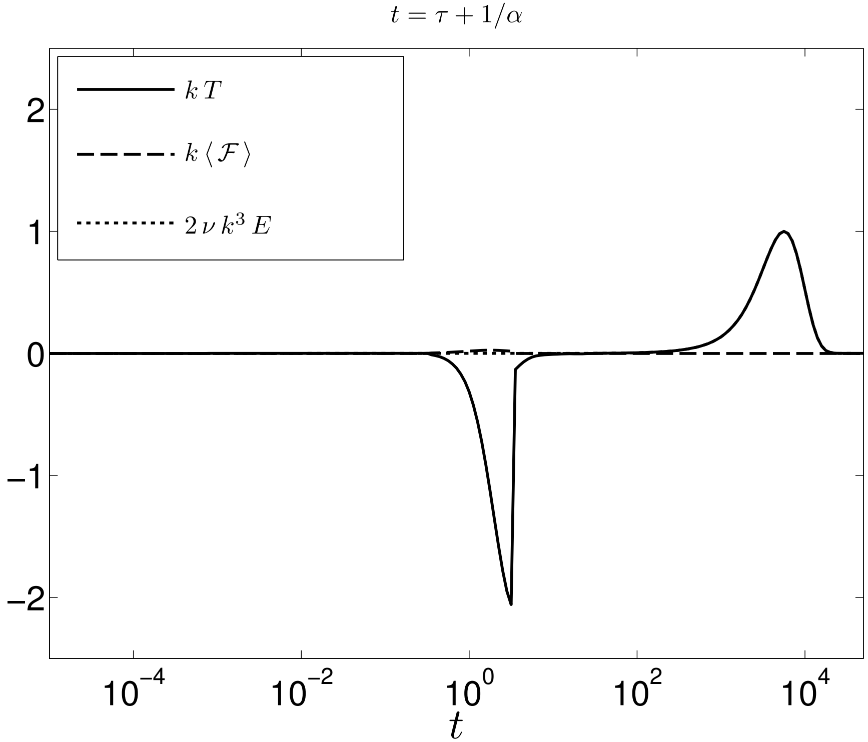


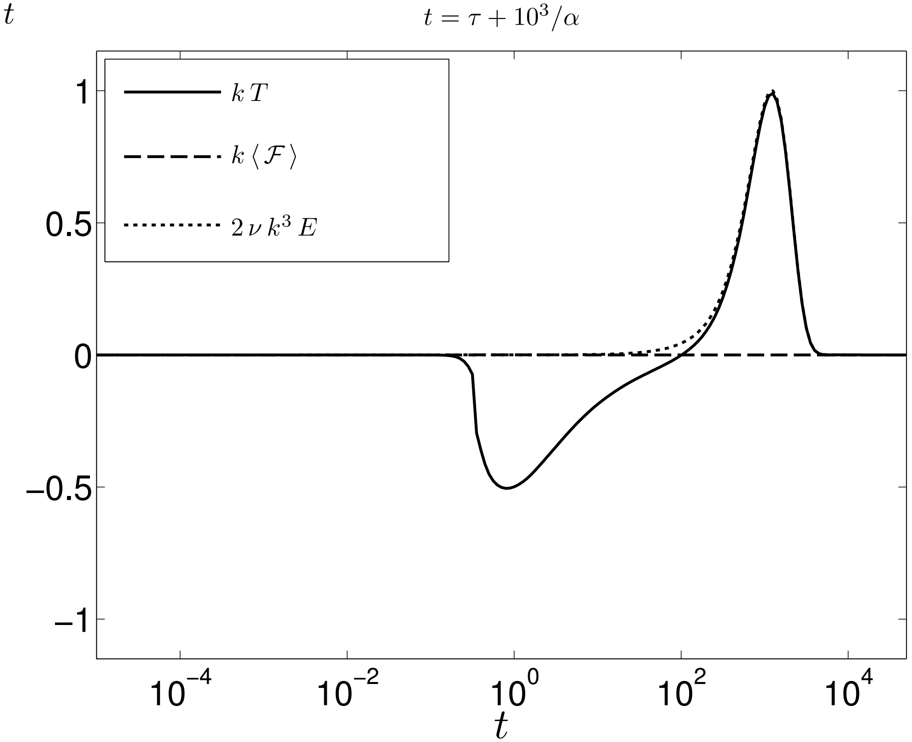
Aix*Marseille université

TKE spectrum with fractal forcing

$\alpha = 100$

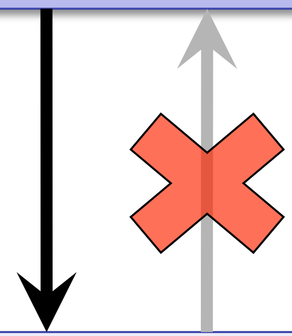






Motivations

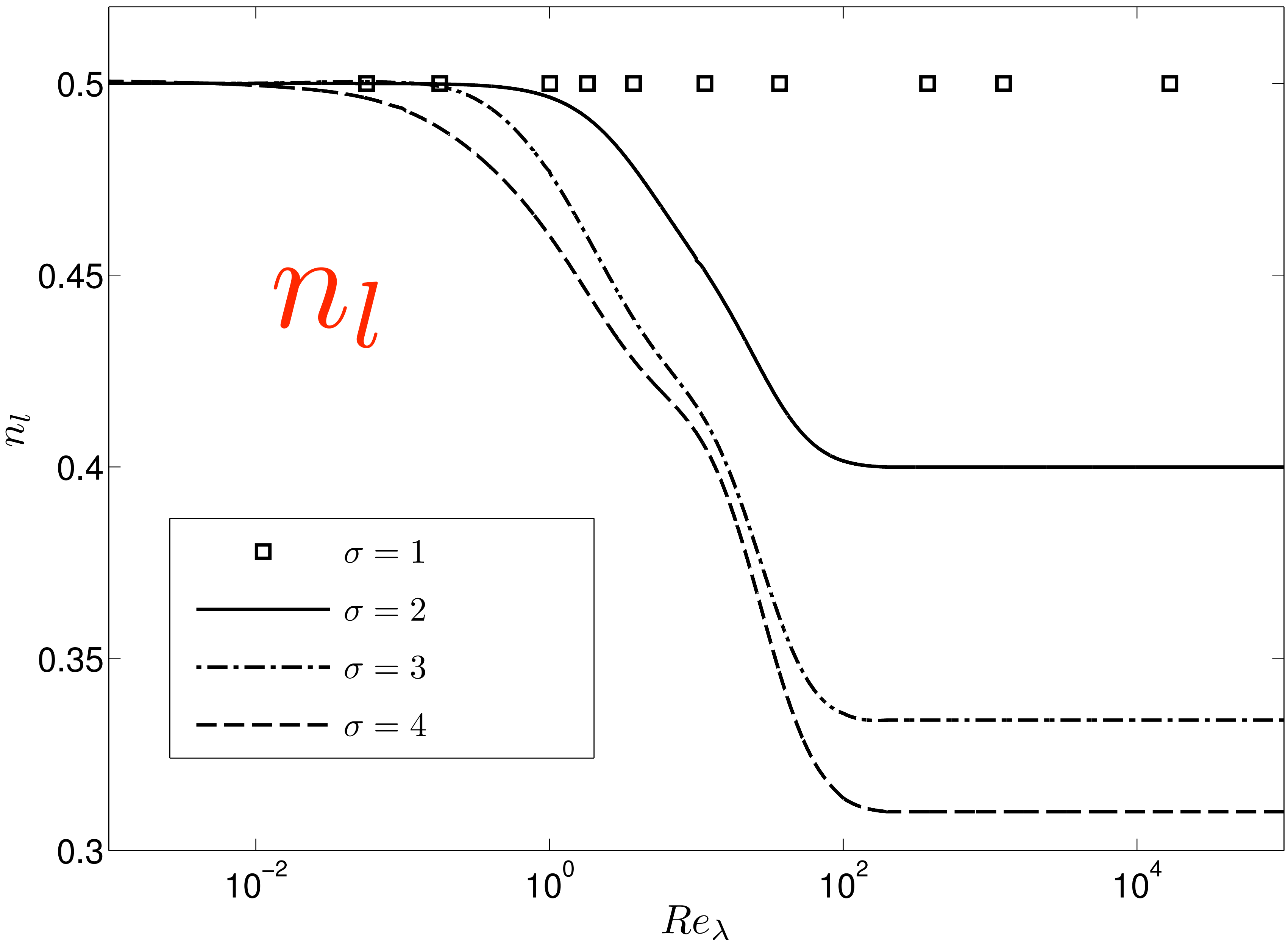
Classical theories:
- high-Re asymptotics
- no intermittency
- self-similarity/self-preservation



Decay regimes:
- algebraic decay (*others ?*)
- predicted decay exponent

Do theories agree with experimental data ?

EDQNM results



Observability of asymptotic decay regimes