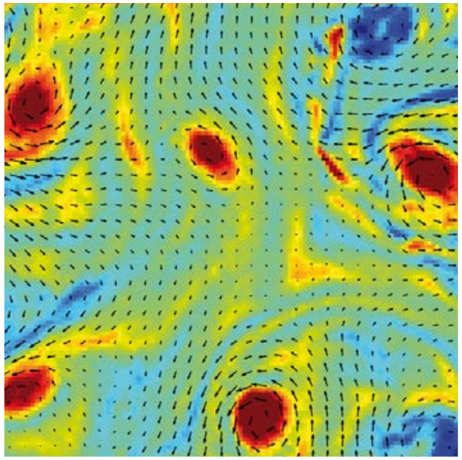
Rate of energy dissipation in a rotating turbulence



Pierre-Philippe Cortet

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with Antoine Campagne
Basile Gallet
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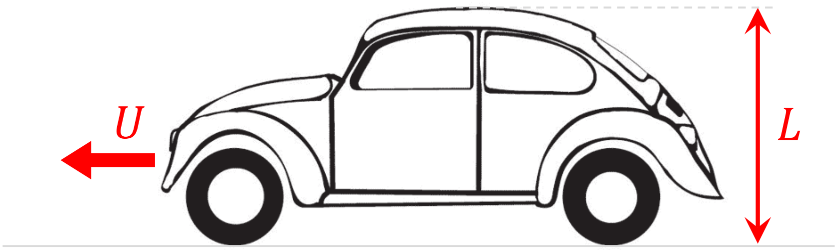
Hydrodynamic turbulence

Navier-Stokes equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

$$Re = \frac{|(\vec{u} \cdot \vec{\nabla})\vec{u}|}{|\nu \Delta \vec{u}|} \sim \frac{UL}{\nu} \gg 1$$



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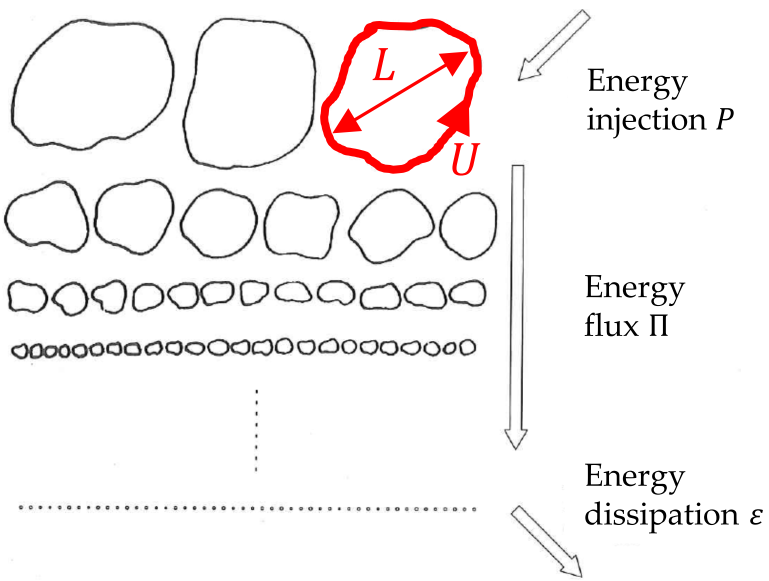
Energy cascade
from large to small scales



In stationnary regime,
energy flux conservation

$$P = \Pi = \varepsilon \equiv \nu \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2}$$

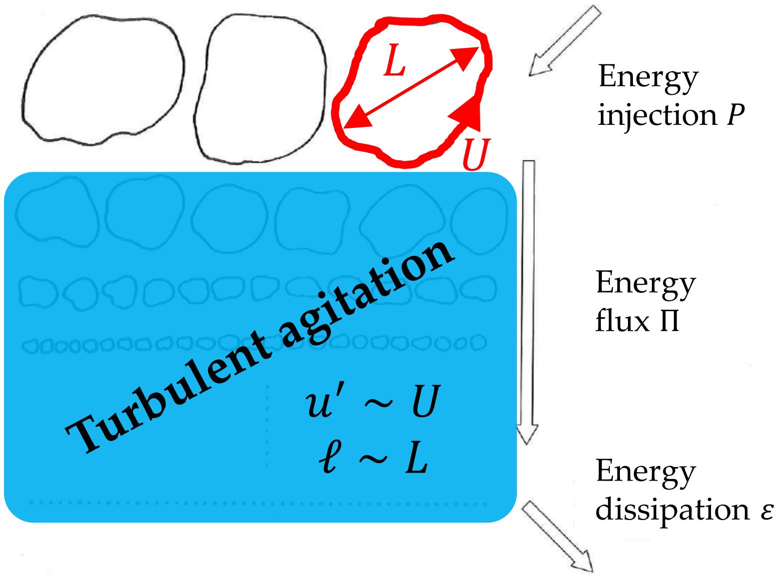
Hydrodynamic turbulence

Energy cascade
from large to small scales

Mean rate of energy dissipation

$$\varepsilon = \Pi \sim \frac{U^3}{L}$$

→ Dissipation anomaly



Turbulent viscosity

$$\varepsilon \equiv \nu_T \left(\frac{U}{L}\right)^2 \quad \rightarrow \quad \nu_T \sim u'\ell \sim UL$$

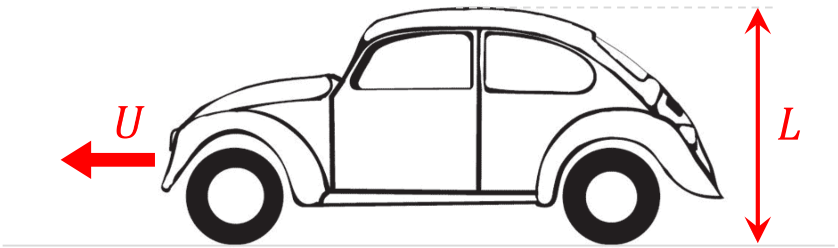
Transport velocity (u') — Mixing length (ℓ)

Dissipated power and drag coefficient

Navier-Stokes equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

$$Re = \frac{\left|(\vec{u} \cdot \vec{\nabla})\vec{u}\right|}{|\nu \Delta \vec{u}|} \sim \frac{UL}{\nu} \gg 1$$



Dissipated power D

and drag coefficient C_x

$$\varepsilon \sim \frac{U^3}{L} \quad \Rightarrow \quad D = \int \rho \varepsilon d\vec{x}^3 = C_x \, \rho L^2 U^3$$

Turbulence under rotation

Navier-Stokes equation in a rotating frame

$\vec{\Omega}$ 

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega} \times \vec{u} + \nu \Delta \vec{u}$$

Coriolis force

Reynolds number $Re = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|\nu\Delta\vec{u}|} \sim \frac{UL_\perp}{\nu} \gg 1$

Rossby number $Ro = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{U}{2\Omega L_\perp} \leq 1$

Non-dimensionnal frequency $\sigma^* = \frac{|\partial\vec{u}/\partial t|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{\sigma}{2\Omega}$

Rotation drives turbulence towards a 2D state

$$\sigma^* \frac{\partial \vec{u}}{\partial t} + Ro(\vec{u} \cdot \vec{\nabla})\vec{u} = -\vec{\nabla}p - \vec{e_z} \times \vec{u} + Ro\ Re^{-1} \Delta\vec{u}$$

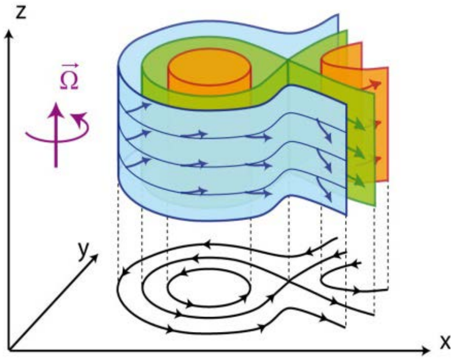
In the limit $Ro = Ro_t = 0$, NS becomes

$$\frac{1}{\rho}\vec{\nabla}p = -2\vec{\Omega} \times \vec{u}$$

Taking its curl gives $(\vec{\Omega} \cdot \vec{\nabla})\vec{u} = \vec{0}$

Taylor-Proudman Theorem = Geostrophic equilibrium

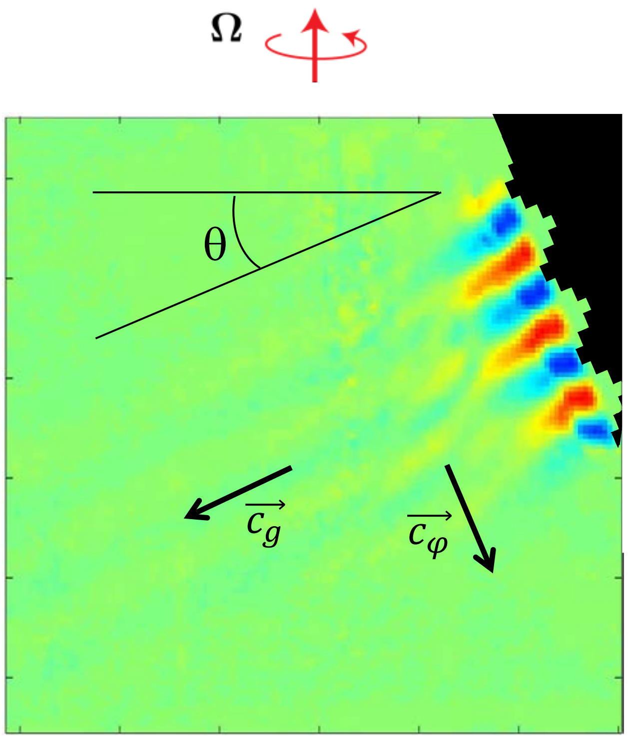
$Ro = 0$ → 2D 3C flow, but no turbulence



2D-3C flow

Inertial waves in fluids under rotation

Bordes, Moisy, Dauxois & Cortet, PoF (2012)

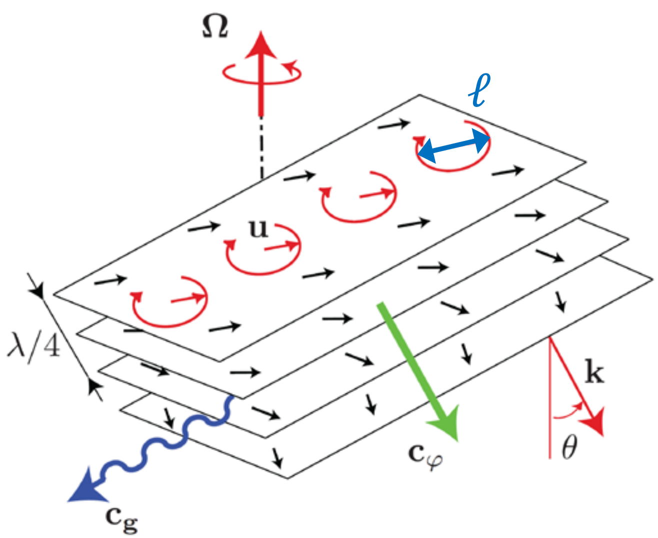


Dispersion relation

$$\frac{\sigma}{2\Omega} = \cos(\theta)$$

NS equation in a rotating frame at $Ro \ll 1$

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho}\vec{\nabla} p - 2\vec{\Omega} \times \vec{u}$$



$$\ell \sim \frac{U}{\Omega}$$

i.e. typical radius of gyration of fluid particles in the wave

Rate of energy dissipation of rotating turbulence at $Ro \to 0$

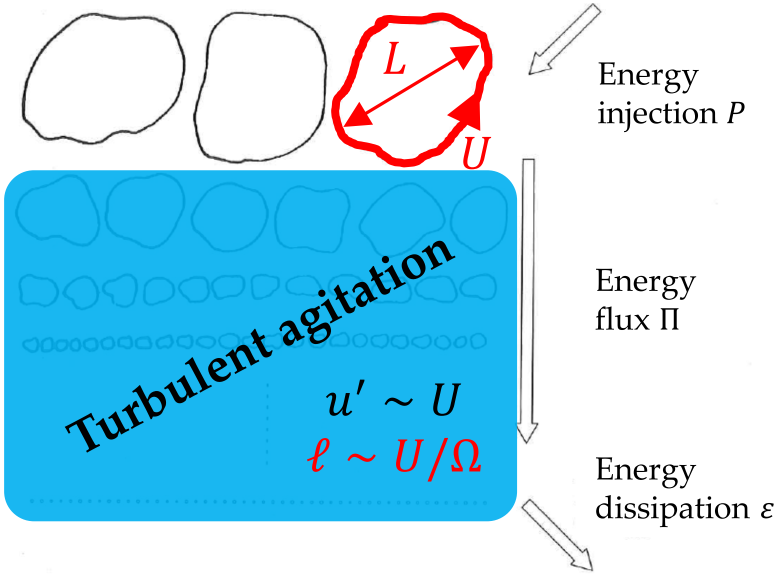
Turbulent viscosity

$\nu_T \sim U\,\ell$

Mixing length

~~$\ell \sim L$~~

$\ell \sim U/\Omega$



$$\varepsilon = \frac{U^4}{\Omega L^2} = Ro\,\frac{U^3}{L} = Ro\;\varepsilon_\infty$$

Jacquin, Leuchter, Cambon & Mathieu, JFM (1990)
Zhou, PoF (1995)
Smith, Chasnov & Waleffe, PRL (1996)

Velocity field decomposition on the helical mode basis

Craya (1958)
Herring, PoF (1974)
Cambon & Jacquin, JFM (1989)
Waleffe, PoF (1992)

$$\mathbf{u}(\mathbf{x},t) = \sum_{\substack{\mathbf{k} \\ s_{\mathbf{k}}=\pm 1}} A_{s_{\mathbf{k}}}(\mathbf{k},t)\, \mathbf{h}_{s_{\mathbf{k}}}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

avec $$\mathbf{h}_{s_{\mathbf{k}}}(\mathbf{k}) = \frac{\mathbf{k}}{|\mathbf{k}|} \times \frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|} + i s_{\mathbf{k}} \frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|}$$

Navier-Stokes equation becomes

$$\left(\frac{\partial}{\partial t} + \nu \mathbf{k}^2\right) A_{s_{\mathbf{k}}}(\mathbf{k},t) = \frac{1}{2} \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} C_{\mathbf{kpq}}^{s_{\mathbf{k}} s_{\mathbf{p}} s_{\mathbf{q}}} A_{s_{\mathbf{p}}}^{*} A_{s_{\mathbf{q}}}^{*}$$

avec $$C_{\mathbf{kpq}}^{s_{\mathbf{k}} s_{\mathbf{p}} s_{\mathbf{q}}} = \frac{1}{2}\left[s_{\mathbf{q}}\kappa_{\mathbf{q}} - s_{\mathbf{p}}\kappa_{\mathbf{p}}\right]\left(\mathbf{h}_{s_{\mathbf{p}}}^{*}(\mathbf{p}) \times \mathbf{h}_{s_{\mathbf{q}}}^{*}(\mathbf{q})\right)\cdot \mathbf{h}_{s_{\mathbf{k}}}^{*}(\mathbf{k})$$

Without rotation, the timescale of energy transfers is

$$\tau_{tr} \sim \frac{1}{C_{\mathbf{kpq}}\, A} \sim \frac{1}{kU} \sim \tau_{nl} \quad \rightarrow \quad \Pi \sim \frac{U^2}{\tau_{tr}} \sim \frac{U^3}{L}$$

Weakly non-linear rotating turbulence, *Ro*→0

$$A_{s_\mathbf{k}}(\mathbf{k},t) = B_{s_\mathbf{k}}(\mathbf{k},t)\, e^{-i\sigma_\mathbf{k} t}$$

If $(\sigma_\mathbf{k}, \mathbf{k})$ vérifies the wave dispersion relation,

Helical mode ≡ Plane inertial wave

$$\left(\frac{\partial}{\partial t} + \nu \mathbf{k}^2\right) B_{s_\mathbf{k}}(\mathbf{k},t) = \frac{1}{2} \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=\mathbf{0}} C^{s_\mathbf{k} s_\mathbf{p} s_\mathbf{q}}_{\mathbf{kpq}}\, B^*_{s_\mathbf{p}} B^*_{s_\mathbf{q}}\, e^{i(\sigma_\mathbf{k}+\sigma_\mathbf{p}+\sigma_\mathbf{q})t}$$

Under rotation ($Ro \to 0$), there is a scrambling effect at times longer than

$$\tau_\Omega = \frac{1}{\sigma_\mathbf{k} + \sigma_\mathbf{p} + \sigma_\mathbf{q}} \sim \frac{1}{\Omega}$$

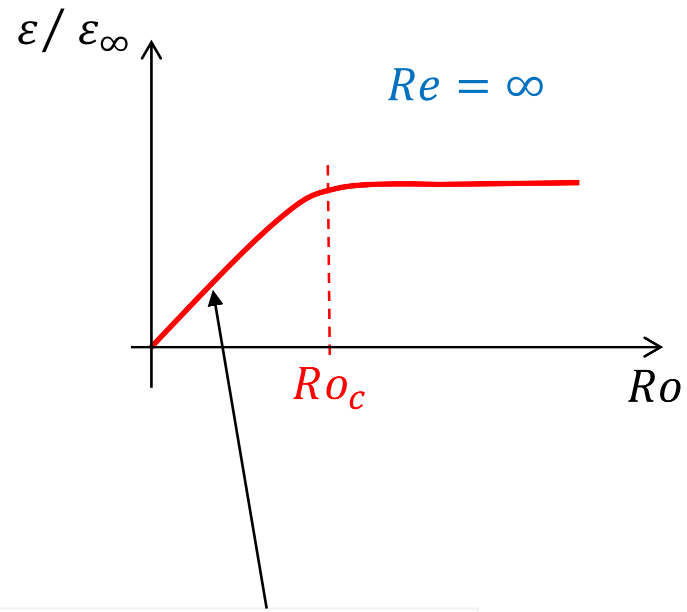
Interactions are efficient during a relative time $\frac{\tau_\Omega}{\tau_{nl}} \sim Ro$

→ $\varepsilon = \Pi \sim Ro \left(\frac{U^2}{\tau_{tr}}\right) \sim Ro\, \frac{U^3}{L_\perp}$

Iroshnikov (1964)
Kraichnan (1965)

Rate of energy dissipation of homogeneous turbulence

$$\varepsilon = \begin{cases} \varepsilon_\infty & \text{for} \quad Ro \gg Ro_c \\ Ro\, \varepsilon_\infty & \text{for} \quad Ro \ll Ro_c \end{cases}$$

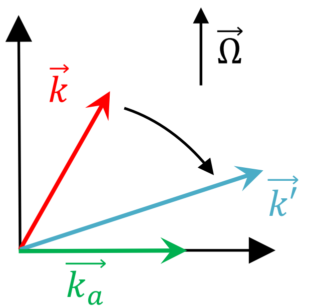


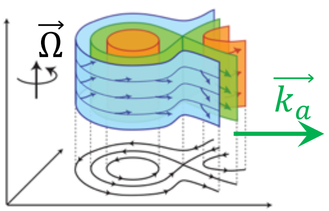
Does this « wave turbulence » scaling exist

in real rotating turbulence ?

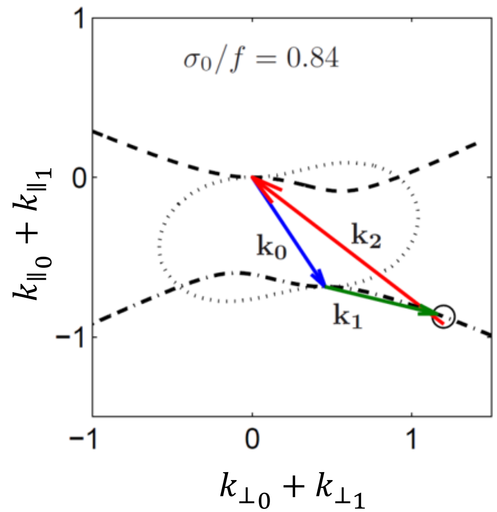
Inertial wave turbulence, $Ro \to 0$

$\frac{k_\parallel}{k_\perp} \searrow$ Anisotropic and direct energy cascade





Mode 2D



Anisotropic energy spectrum

$$E(k_\perp, k_\parallel) \sim k_\perp^{-5/2} k_\parallel^{-1/2}$$

Galtier, PRE (2003)
Cambon, Rubinstein, Godeferd, NJP (2004)

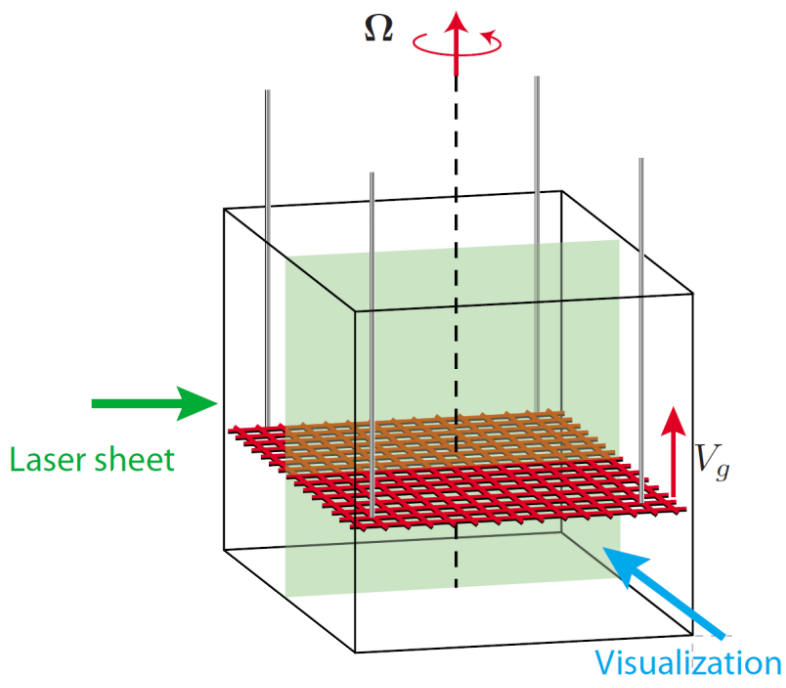
Rate of energy transfer

$$\Pi \sim Ro\ \varepsilon_\infty = Ro \frac{U^3}{L}$$

Kraichnan, PoF (1965)

Decaying grid turbulence under rotation

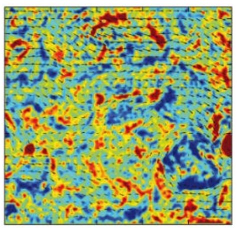
Lamriben, Cortet & Moisy, PRL (2011)



Turbulence decay, $Ro(t)$

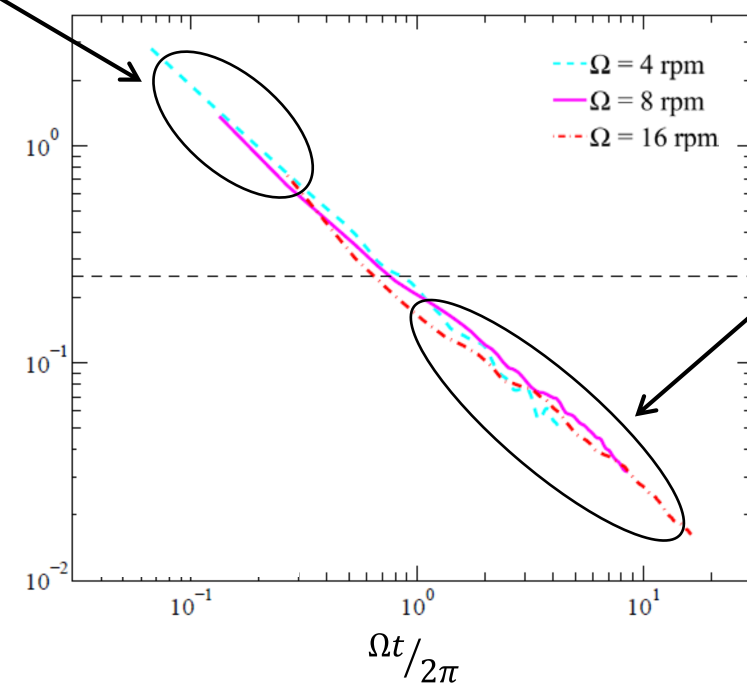
$Ro \gtrsim 1$

Negligible rotation effects



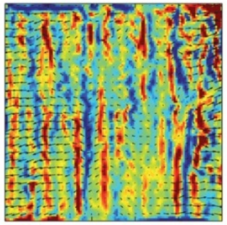
Isotropic

$$Ro(t) = \frac{u_{\mathrm{rms}}(t)}{2\Omega\, L_{\perp}(t)}$$



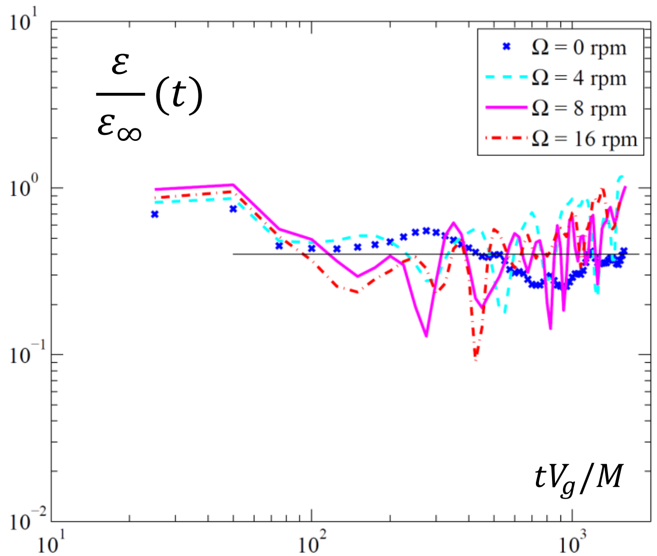
$Ro \lesssim 1$

Rotation at play



Anisotropic

Rate of energy dissipation

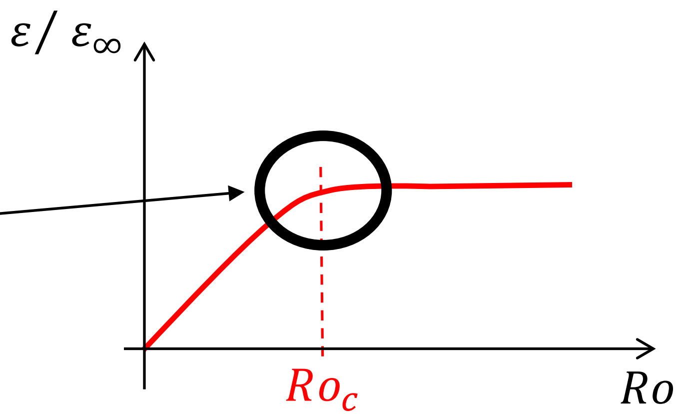


$$\varepsilon \sim \frac{U^3}{L_\perp} \sim \varepsilon_\infty$$

ε is not affected by rotation

Too large Rossby number ?

Strongly non-linear rotating turbulence : $Ro \sim 1$



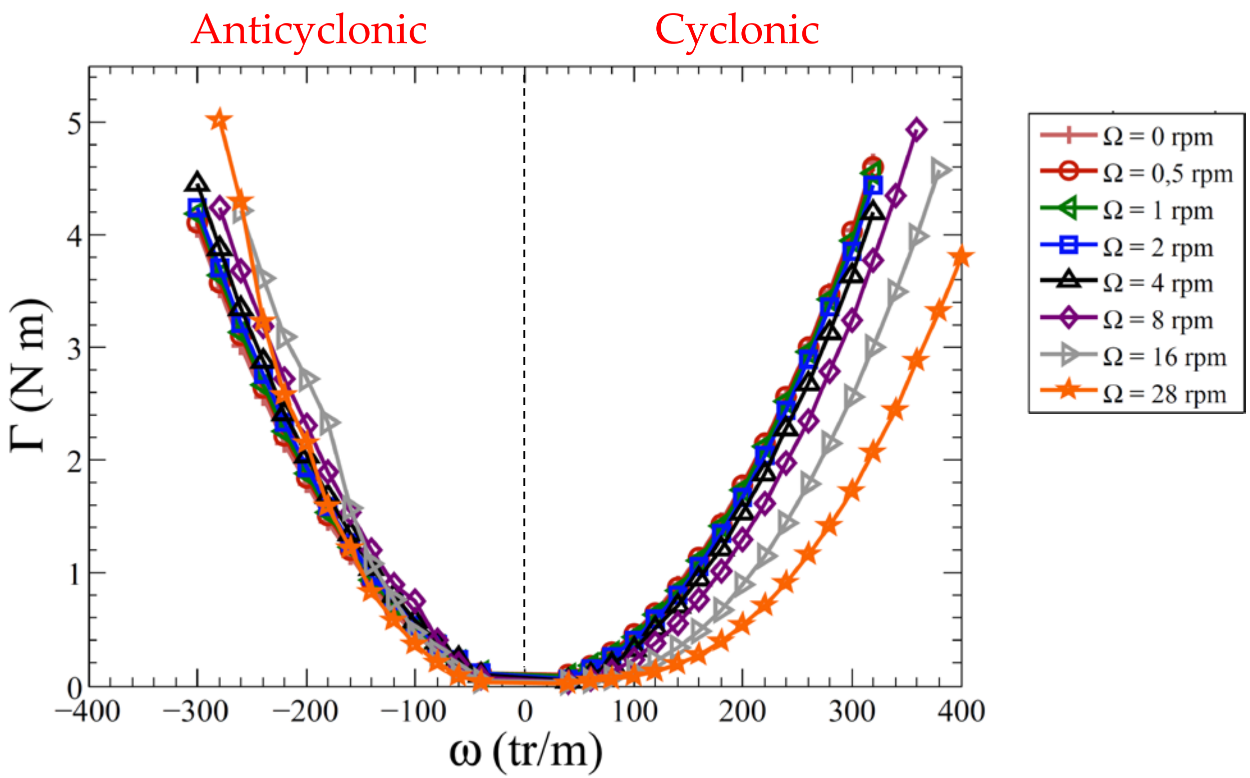
Impeller in rotation in a tank under rotation

Direct measurements of the injected power

$$P = \Gamma\,\omega$$

Torque — Rotation rate

Torque applied by the motor



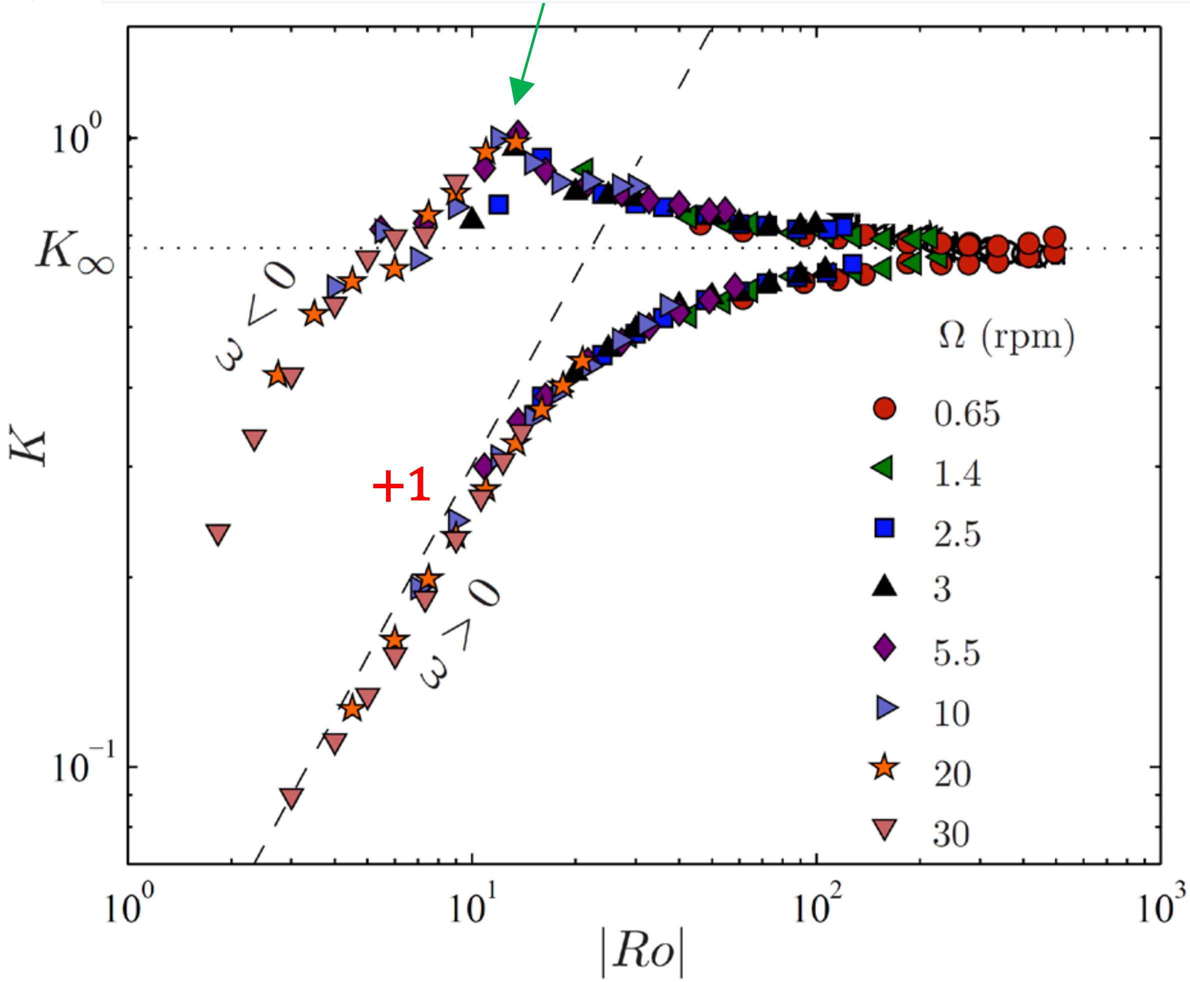
- Cyclonic-anticyclonic asymetry
- Fully turbulent scaling $\Gamma \sim \omega^2$?

Non-dimensionnal torque = Drag coefficient

$$K \equiv \frac{\Gamma}{\rho R^4 H \omega^2} = \frac{\varepsilon}{\varepsilon_\infty}$$

Maximum of dissipation:
Relation with the centrifugal instability maximum for $\vec{\omega} \sim -\vec{\Omega}$?

Kloosterziel, Van Heijst, JFM (1991)
Mutabazi, Normand, Wesfreid, PoF (1992)



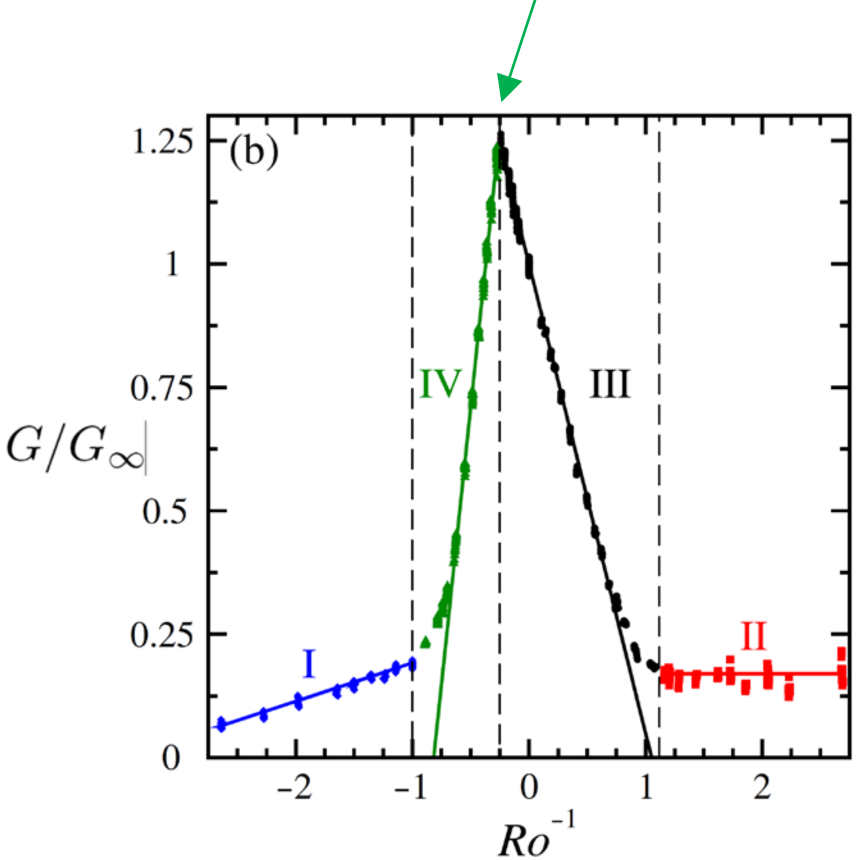
Dissipation peak in Taylor-Couette flow

Dubrulle et al. 2005
Van Gils et al. 2011
Paoletti & Lathrop 2011

PRL **106,** 024501 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

Angular Momentum Transport in Turbulent Flow between Independently Rotating Cylinders

M. S. Paoletti[1] and D. P. Lathrop[1,2,*]



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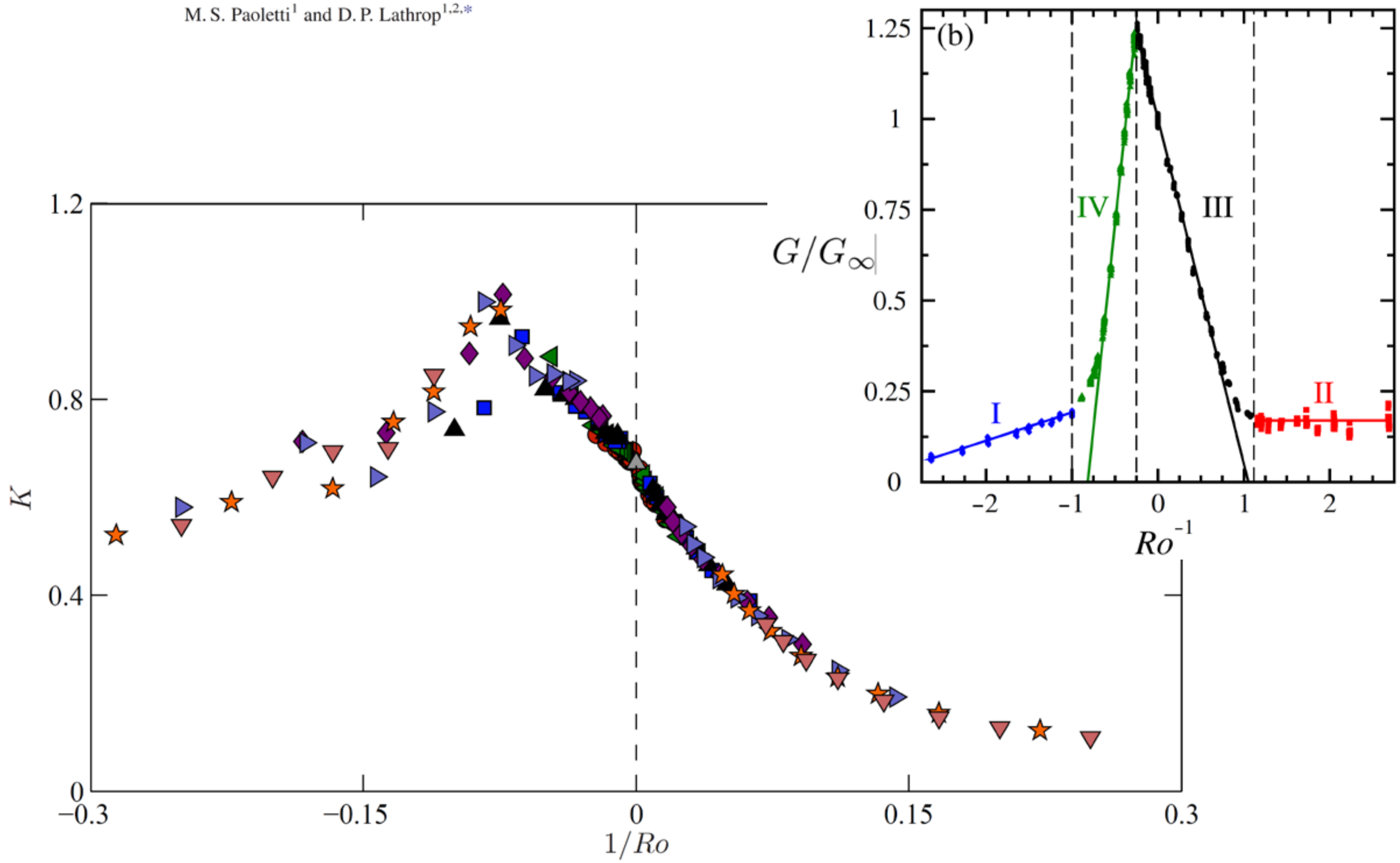
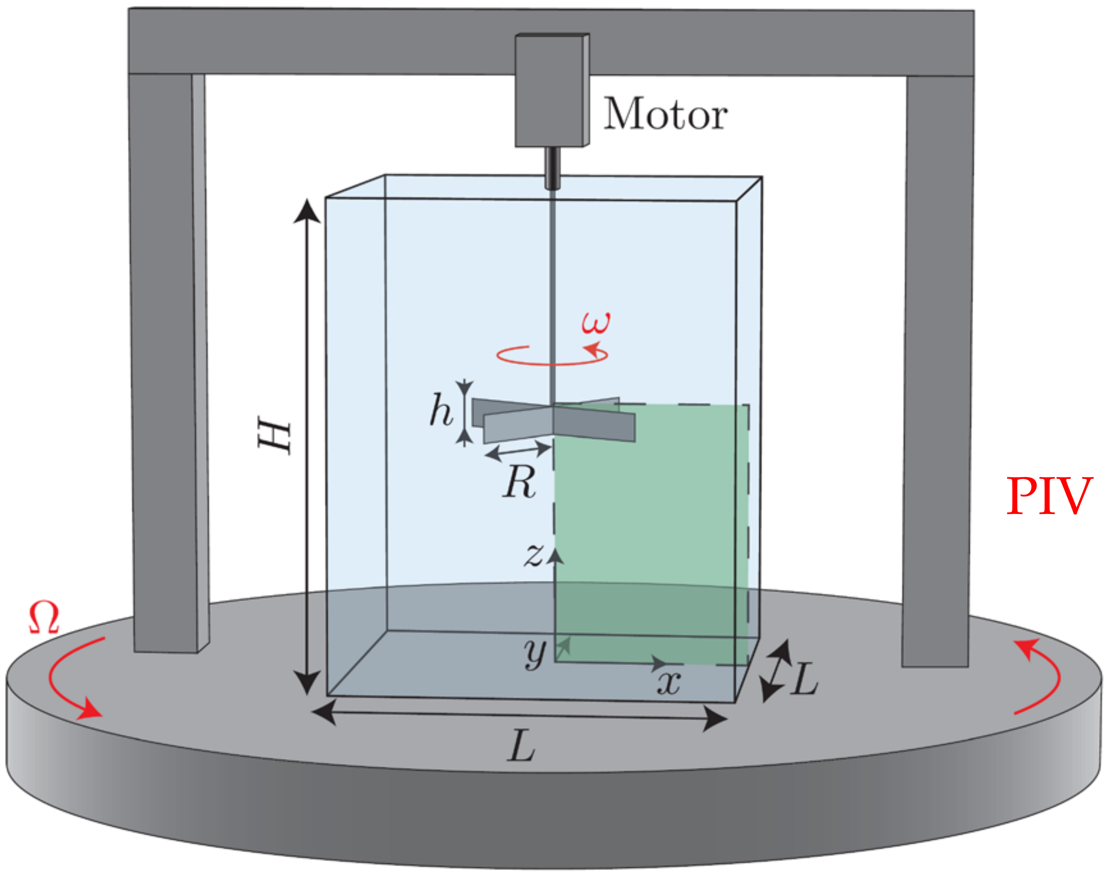
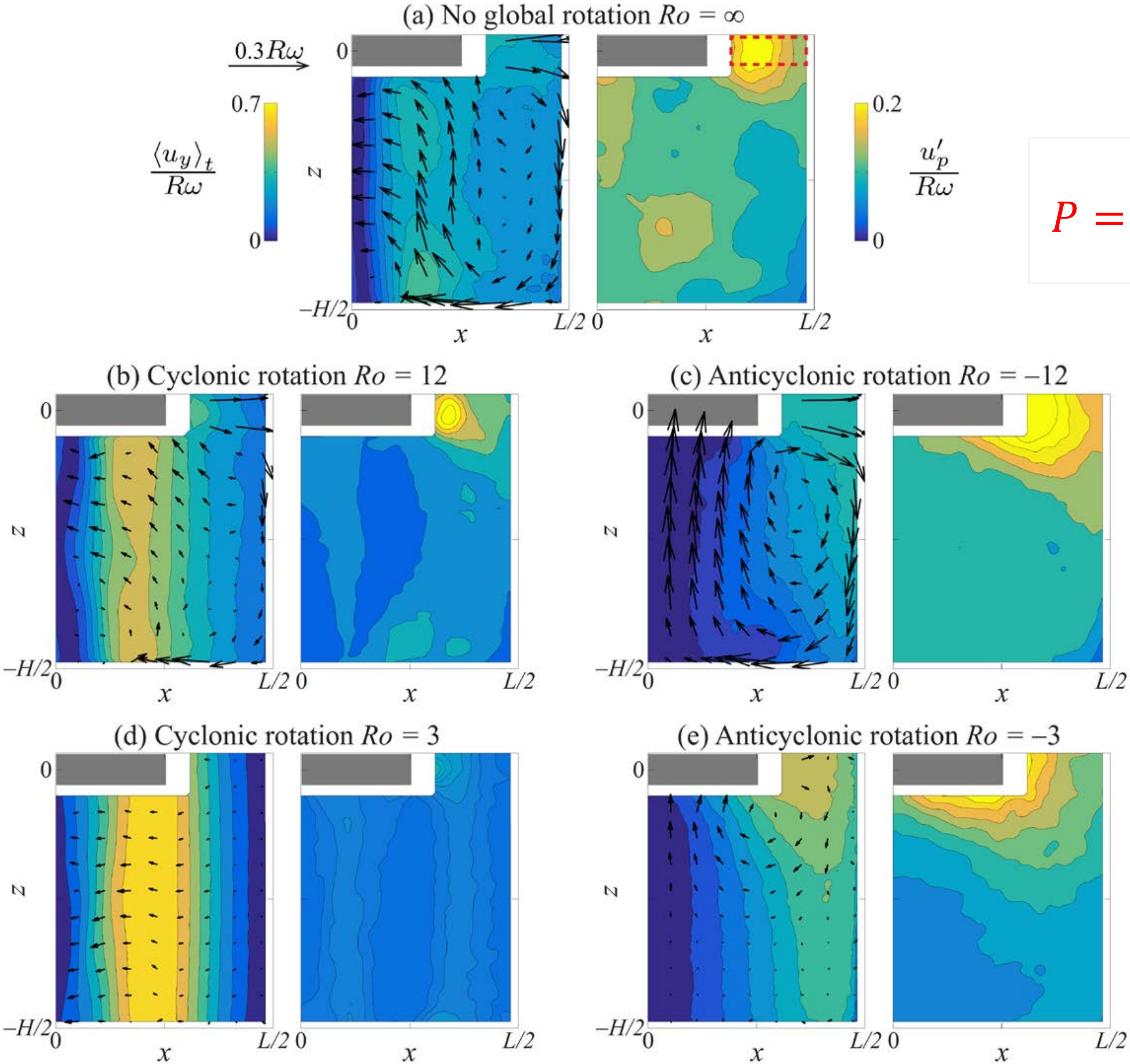


Rate of energy dissipation = Rate of energy transfers ?



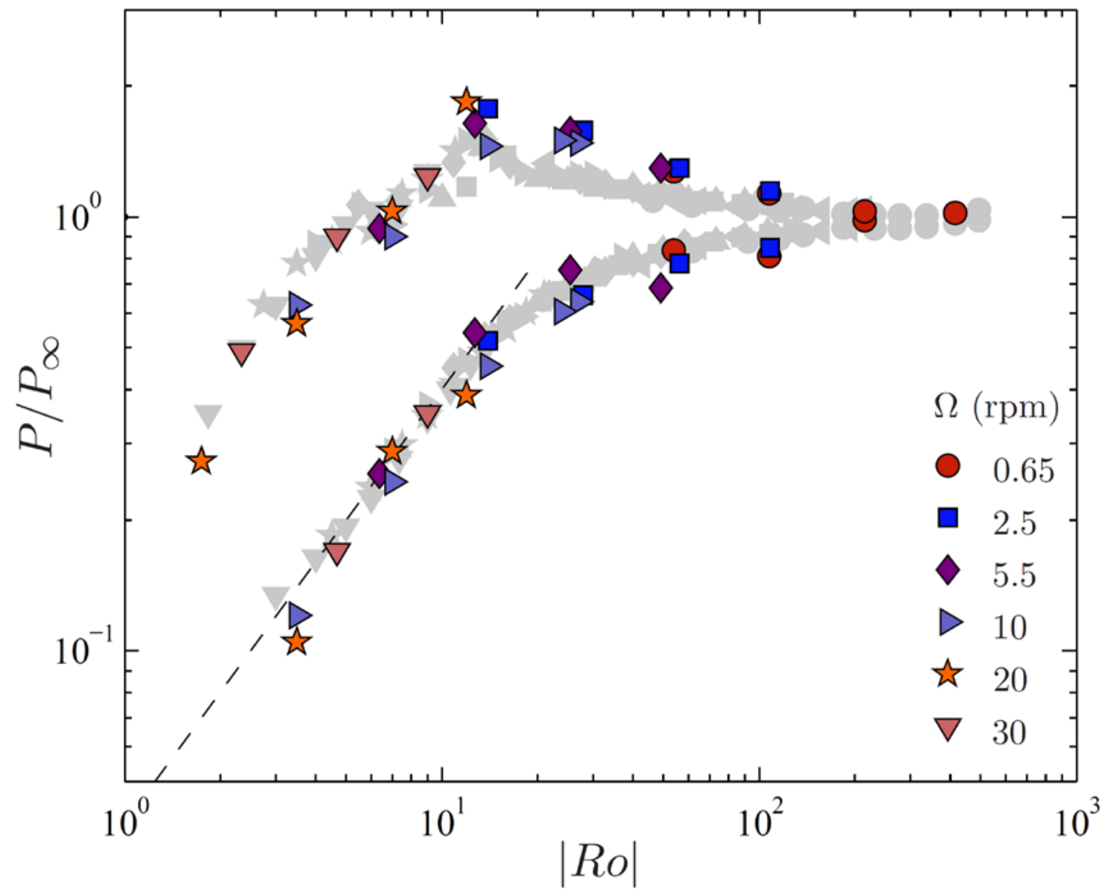
PIV measurements !



$$P = \int \rho \frac{{u'_p}^3}{h} d^3\vec{x}$$

Rate of energy dissipation and rate of energy transfers

$$P = \int \rho \frac{u_p'^{\,3}}{h} d^3\vec{x}$$



The rate of energy transfers follows the non-rotating turbulence scaling

$$\varepsilon = \frac{u'^3}{h}$$

Rate of energy dissipation and rate of energy transfers

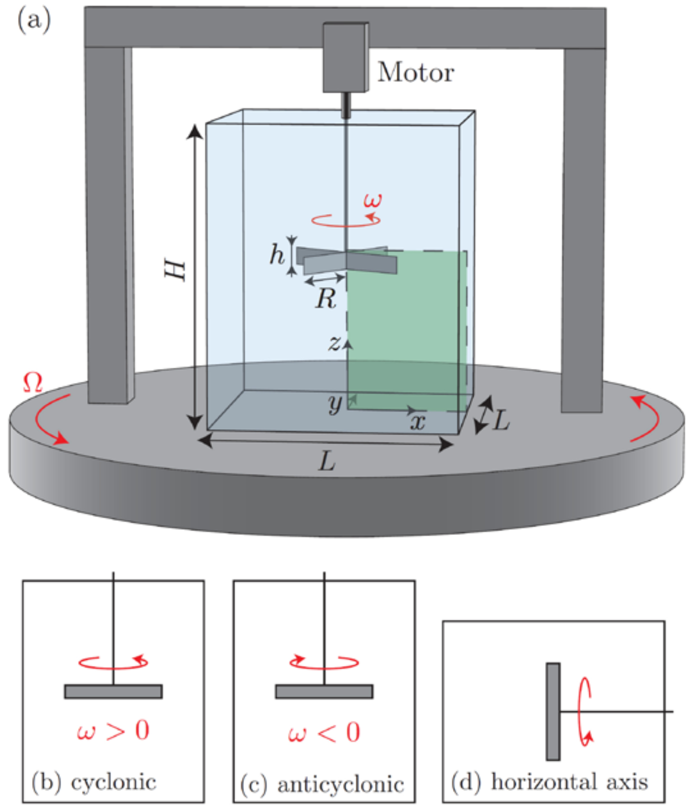
$$\mathrm{P} = \int \rho \frac{u'^3}{h} d^3\vec{x} = Ro\ \rho U^3 R h$$

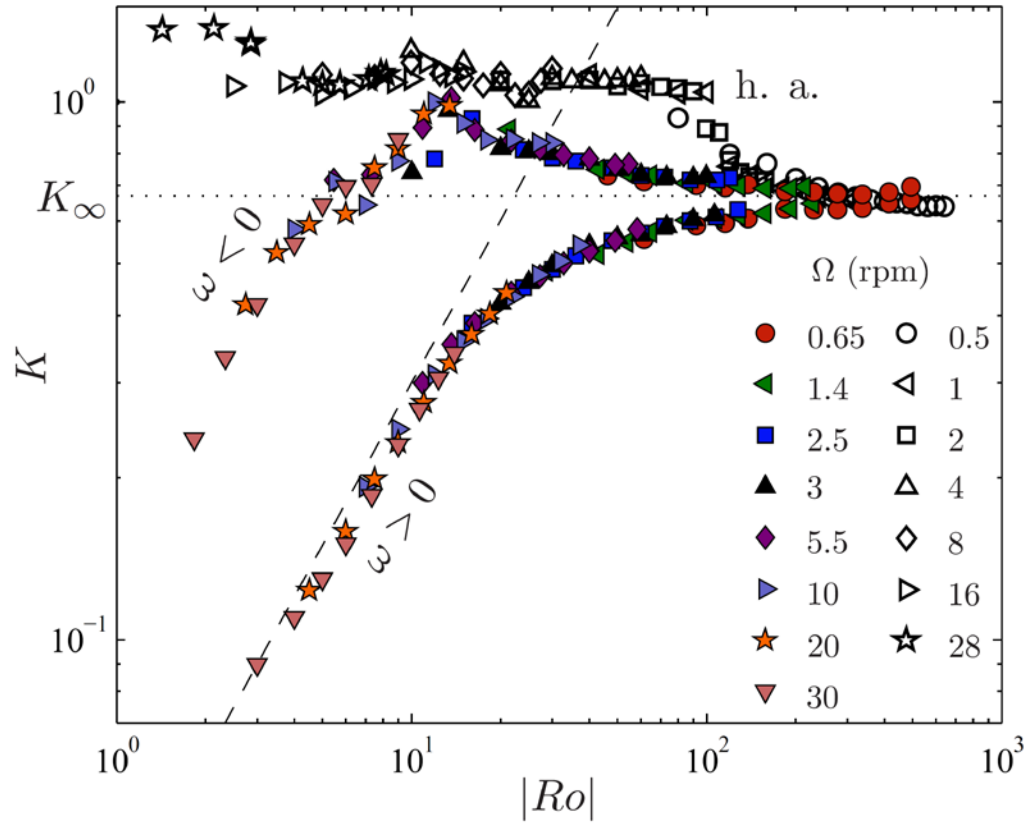
$$\frac{\int u'^3 d^3\vec{x}}{U^3 R h^2} = Ro$$

The efficiency of the forcing is weakened by rotation

The 2 dimensionnalization of the large scale flow by rotation kills the poloidal circulation and its fluctuations

Impeller rotating around a horizontal axis





Direct numerical simulations under rotation with 3D forcing

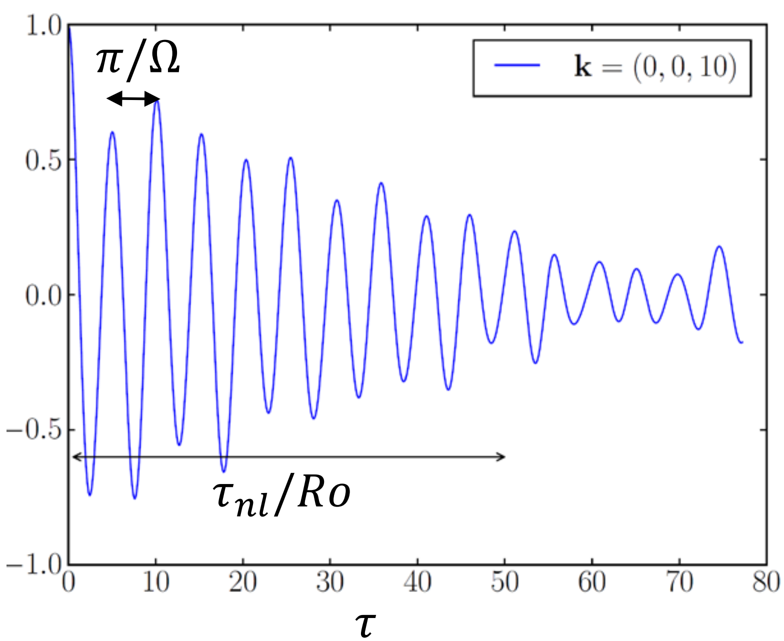
PHYSICS OF FLUIDS **26**, 035106 (2014)

Quantification of the strength of inertial waves in a rotating turbulent flow

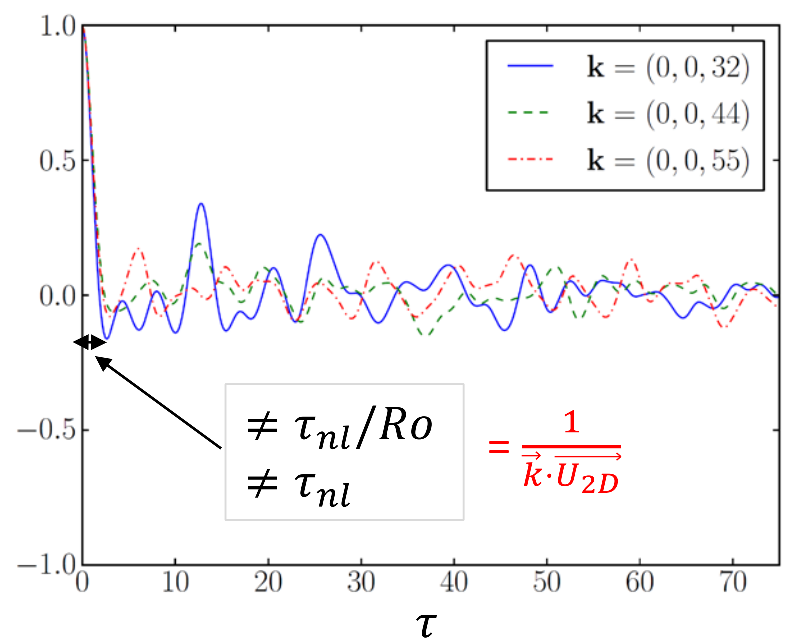
P. Clark di Leoni,[1] P. J. Cobelli,[1] P. D. Mininni,[1] P. Dmitruk,[1] and W. H. Matthaeus[2]
[1] *Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires and IFIBA, CONICET, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
[2] *Bartol Research Institute and Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*

Two-times correlation of spatial Fourier modes

$$\frac{\langle \tilde{\boldsymbol{u}}(\boldsymbol{k},t)\cdot\tilde{\boldsymbol{u}}^*(\boldsymbol{k},t+\tau)\rangle}{\langle \tilde{\boldsymbol{u}}(\boldsymbol{k},t)\cdot\tilde{\boldsymbol{u}}^*(\boldsymbol{k},t)\rangle}$$



$\frac{\tau_{nl}}{Ro}$ ≡ Energy transfer time-scale of inertial wave turbulence



Sweeping by the strong 2D vortex mode disrupts the « wave turbulence physics »