Turbulence of surface water waves

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WATU project

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Wave turbulence ?

Weak Turbulence Theory

Weak Turbulence formalism

dynamical non linear wave equations

statistical theory

main hypotheses of weak turbulence theory:

- **large system**
- **weak non linearity**: time scale separation $T_{linear} << T_{nonlinear}$

➥ statistical theory for the **slow time evolution of statistical quantities** (PDF, Fourier spectrum…)

see S. Nazarenko, *Wave Turbulence*, Springer 2011

example: thin elastic plate (flexion waves)

G. Düring, S. Rica, C. Josserand, PRL **97** (2006)

- **dynamical nonlinear wave equation** (Föppl - Von Karman eqs)

$$\rho \frac{\partial^2 \zeta}{\partial t^2} = -\frac{Eh^2}{12(1-\sigma^2)}\Delta^2 \zeta + \{\zeta, \chi\} \qquad \frac{1}{E}\Delta^2 \chi = -\frac{1}{2}\{\zeta, \zeta\}$$

non linearity

$\zeta(x, y, t)$ deformation from flat plate

$\chi(x, y, t)$ Airy stress function

$\{f, g\} = f_{xx}g_{yy} + f_{yy}g_{xx} - 2f_{xy}g_{xy}$

h plate thickness

E Young's modulus

σ Poisson's coefficient

ρ specific mass

- **weak turbulence theory** ➙ evolution equation for the wave action spectrum (kinetic equation)

$$\frac{dn_k^s}{dt} = 12\pi \int |J_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}|^2 \sum_{s_1 s_2 s_3} n_{\mathbf{k}_1} n_{\mathbf{k}_2} n_{\mathbf{k}_3} n_{\mathbf{k}} \left(\frac{1}{n_{\mathbf{k}}} + \frac{s_1}{n_{\mathbf{k}_1}} + \frac{s_2}{n_{\mathbf{k}_2}} + \frac{s_3}{n_{\mathbf{k}_3}}\right) \boxed{\delta(\omega_k + s_1\omega_{k_1} + s_2\omega_{k_2} + s_3\omega_{k_3})}$$

$$\times \boxed{\delta^{(2)}(\mathbf{k} + s_1\mathbf{k}_1 + s_2\mathbf{k}_2 + s_3\mathbf{k}_3)} d^2\mathbf{k}_{123}$$

energy exchanges through wave resonances

- **stationary solutions** ➙ Kolmogorov-Zakharov spectrum

$$E_\zeta(k) = C \frac{P^{1/3}}{[12(1-\sigma^2)]^{1/6}} \frac{\ln^{1/3}(k^\star/k)}{\sqrt{E/\rho}\,k^3}$$

P : energy dissipation rate

wave turbulence

very diverse phenomenology

- **weak** turbulence
- **strong** wave turbulence: structures
  solitons, singularities
- **multiple cascades in scales**
  (energy, action, momentum,…)
  Bose-Einstein condensate
- **coexistence** with Fluid Turbulence
- finite size effects

wide range of applications:

not restricted to fluids !

- **geophysics** - oceanic/atmospheric waves
  ➥ climate modeling, wave forecasting
- **plasmas** - confinement issues in ITER, solar winds
- **non linear optics** - semiconductor/fiber lasers
- **condensed matter** - formation of BEC,
  superfluid turbulence

➥ need for **experimental input** and for **deeper statistical analysis**

Probe the fundamental nonlinear coupling for water waves

Gravity-capillary surface water waves

Linear dispersion relation :

$$\omega = \sqrt{gk + \frac{\gamma}{\rho}k^3}$$

Gravity | 15Hz | Capillary → Frequency

our experiment:

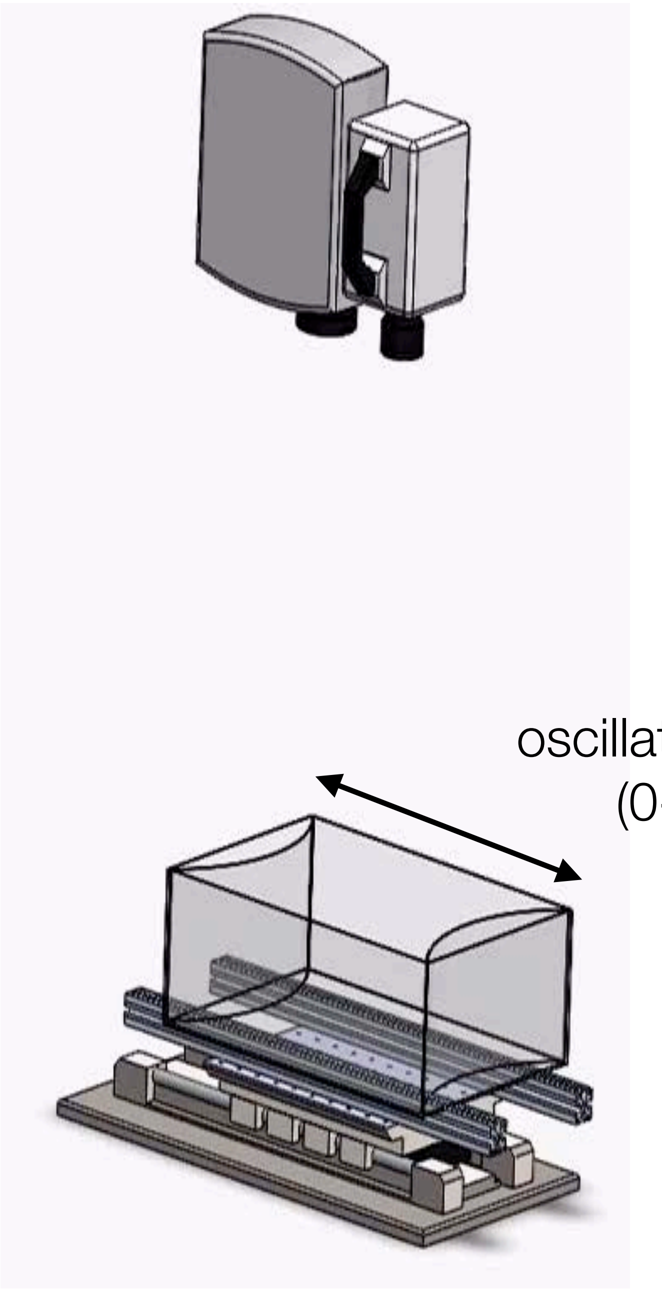
$\omega/2\pi \approx$ **1 to 60 Hz** $\quad \lambda = 2\pi/k \approx$ **0.01 to 1m**

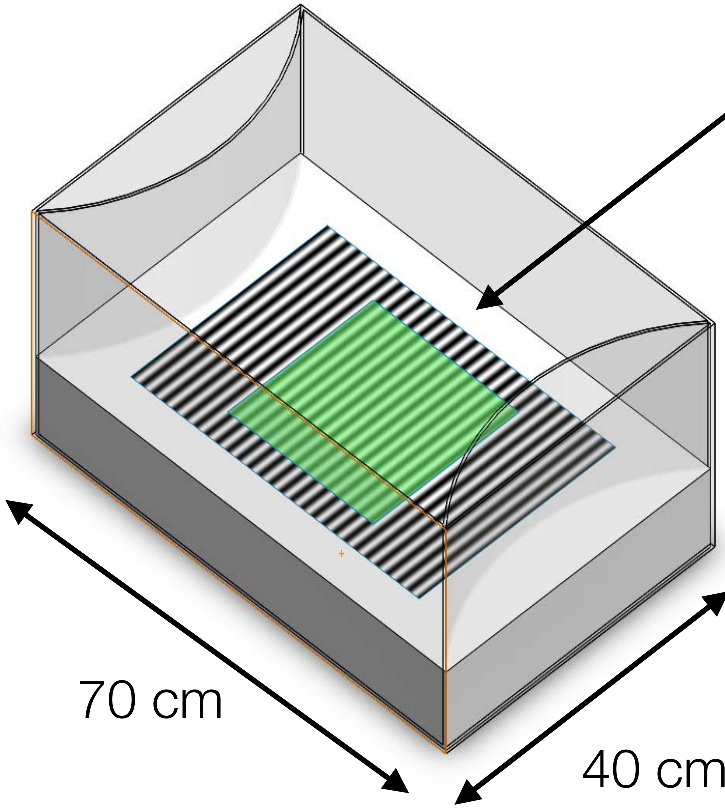
across the gravity/capillary crossover

high speed Fourier transform profilometry: space and time resolved measurement

water surface waves around the gravity/capillary crossover

$$\omega = \sqrt{gk + \frac{\gamma}{\rho}k^3}$$

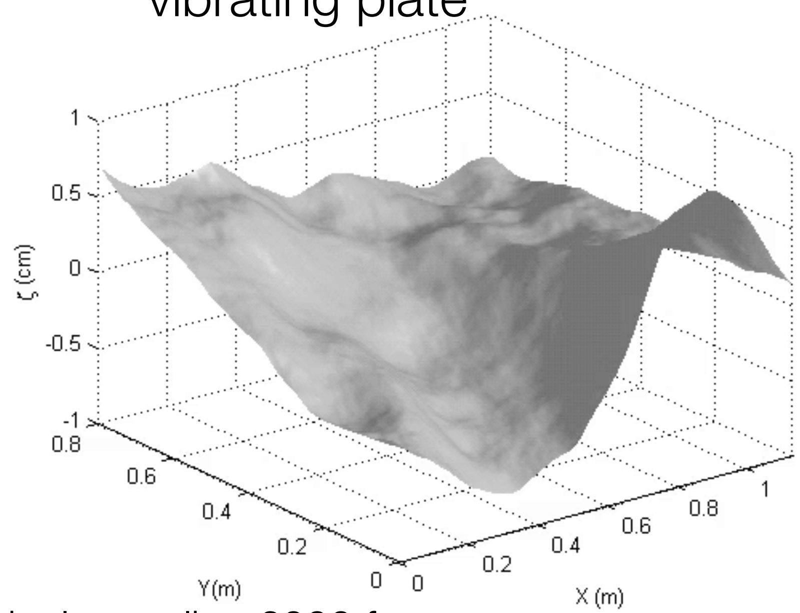




FTP profilometry: Maurel et al. *Applied optics* 2009

Experimental observation of Weak Turbulence:
space and time resolved measurement

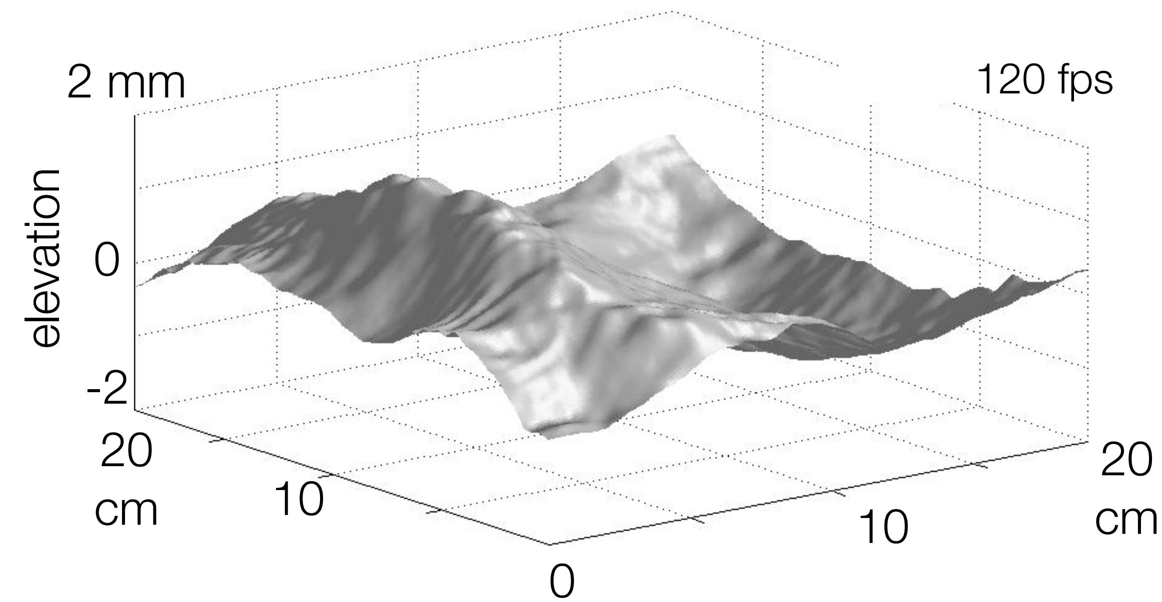
vibrating plate

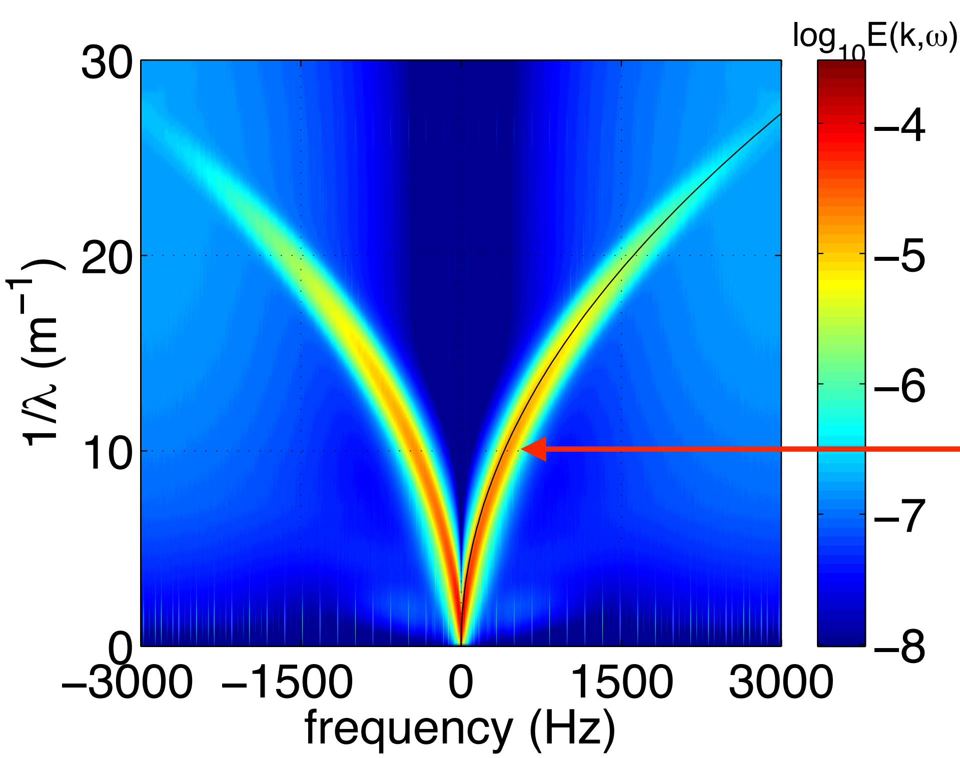


original sampling 6000 fps

time-resolved movies

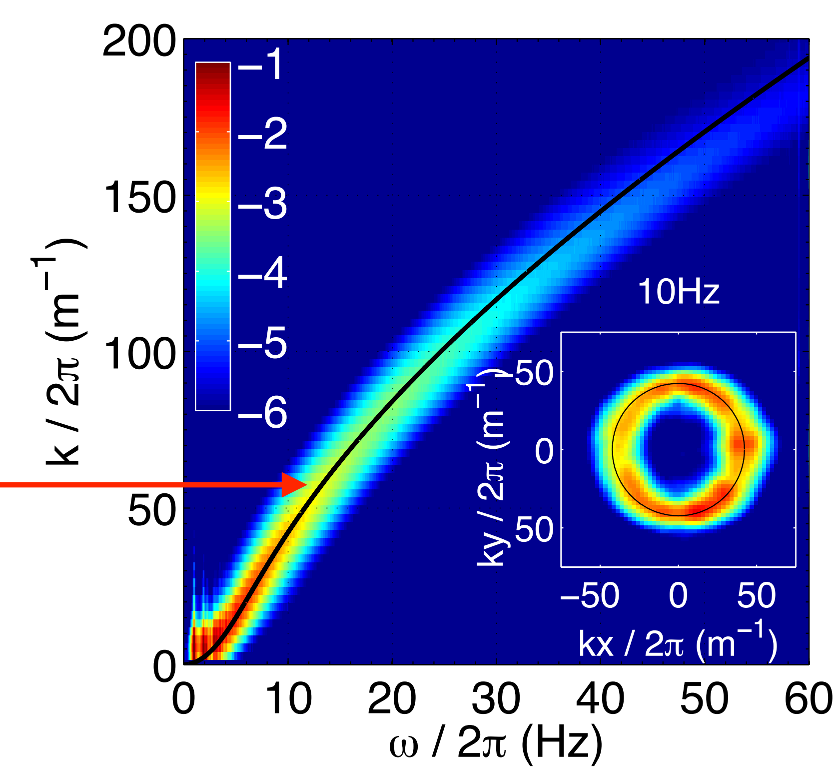
water waves





(k,ω) spectrum

linear dispersion relation
(black line)



→ weak turbulence

PRL'09, PRL'11,PRE '11, PRE'14

PRL'10, PRL'15

Spectral analysis

Velocity field

$$v(x,y,t) = \frac{\partial h(x,y,t)}{\partial t}$$

Fourier transform in space and time

$$v(k_x, k_y, \omega)$$

Space-time power spectrum

$$E^v(k_x,k_y,\omega) = \langle |v(k_x,k_y,\omega)|^2 \rangle$$

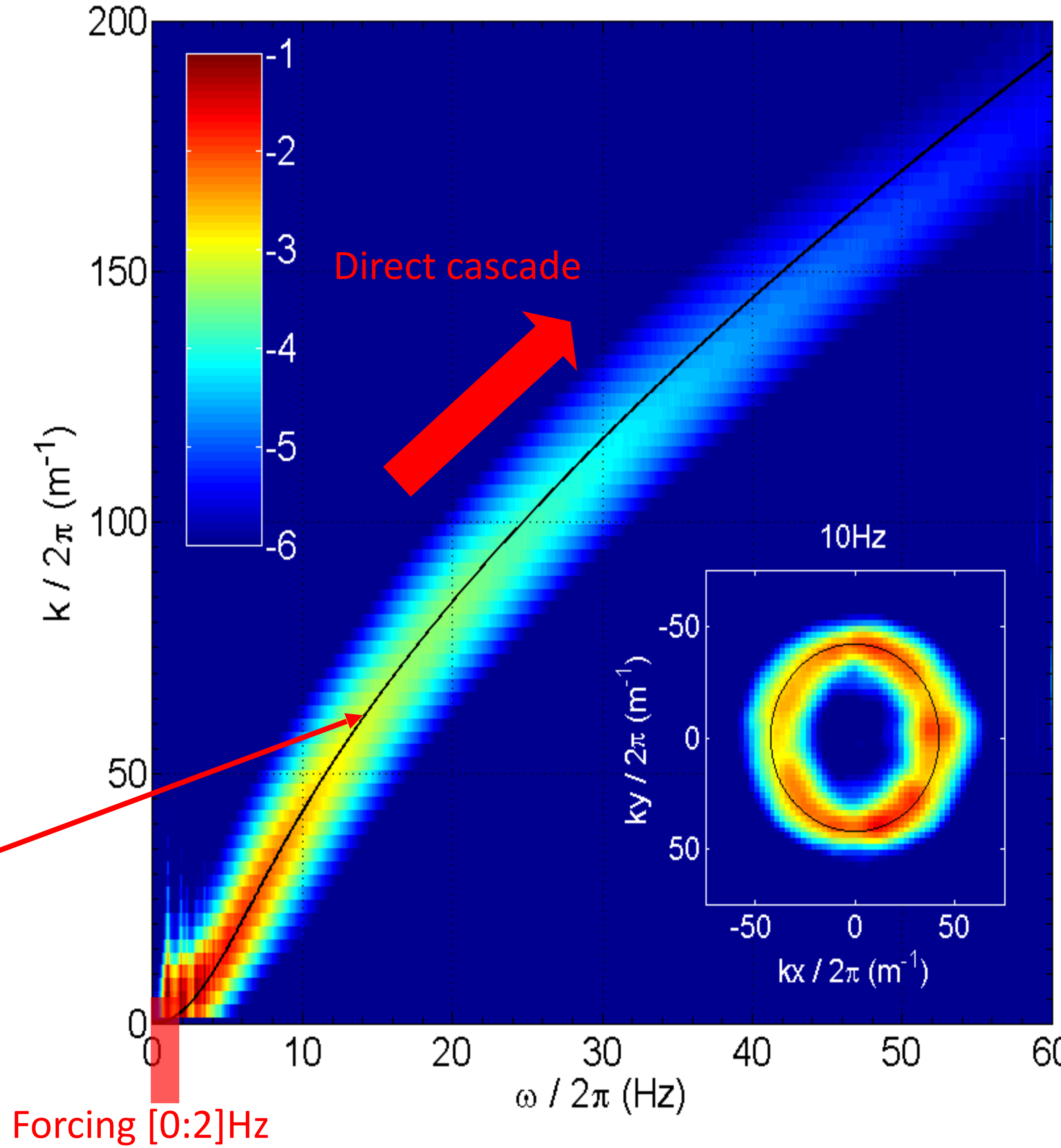
Angular integration

$$E^v(k_x,k_y,\omega) \Rightarrow E^v(k,\omega)$$

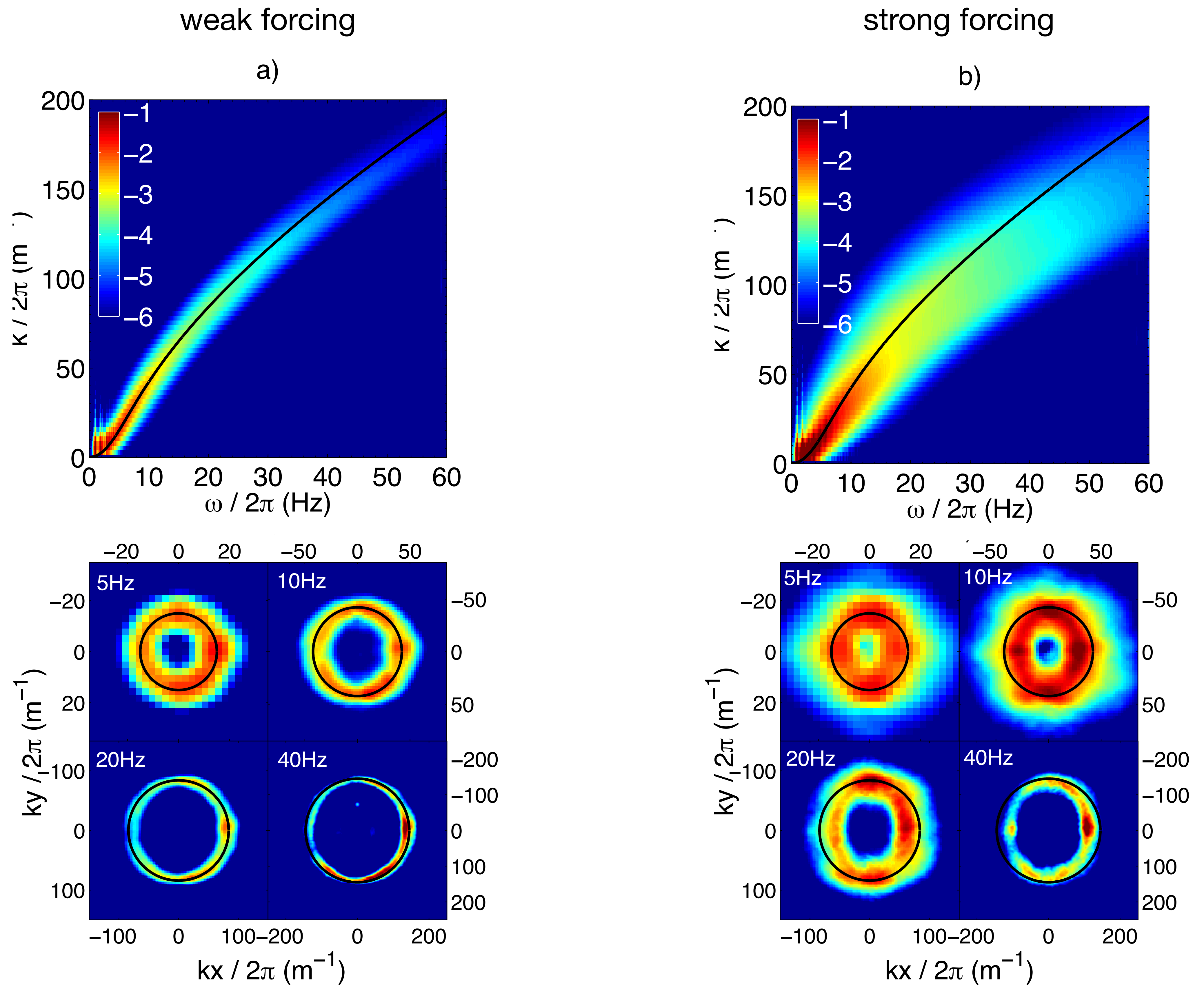
Linear dispersion relation

$$\omega = \sqrt{gk + \frac{\gamma}{\rho}k^3}$$

Power spectrum $E^v(k,\omega)$



influence of magnitude of forcing



wave resonances ?

Resonance investigation

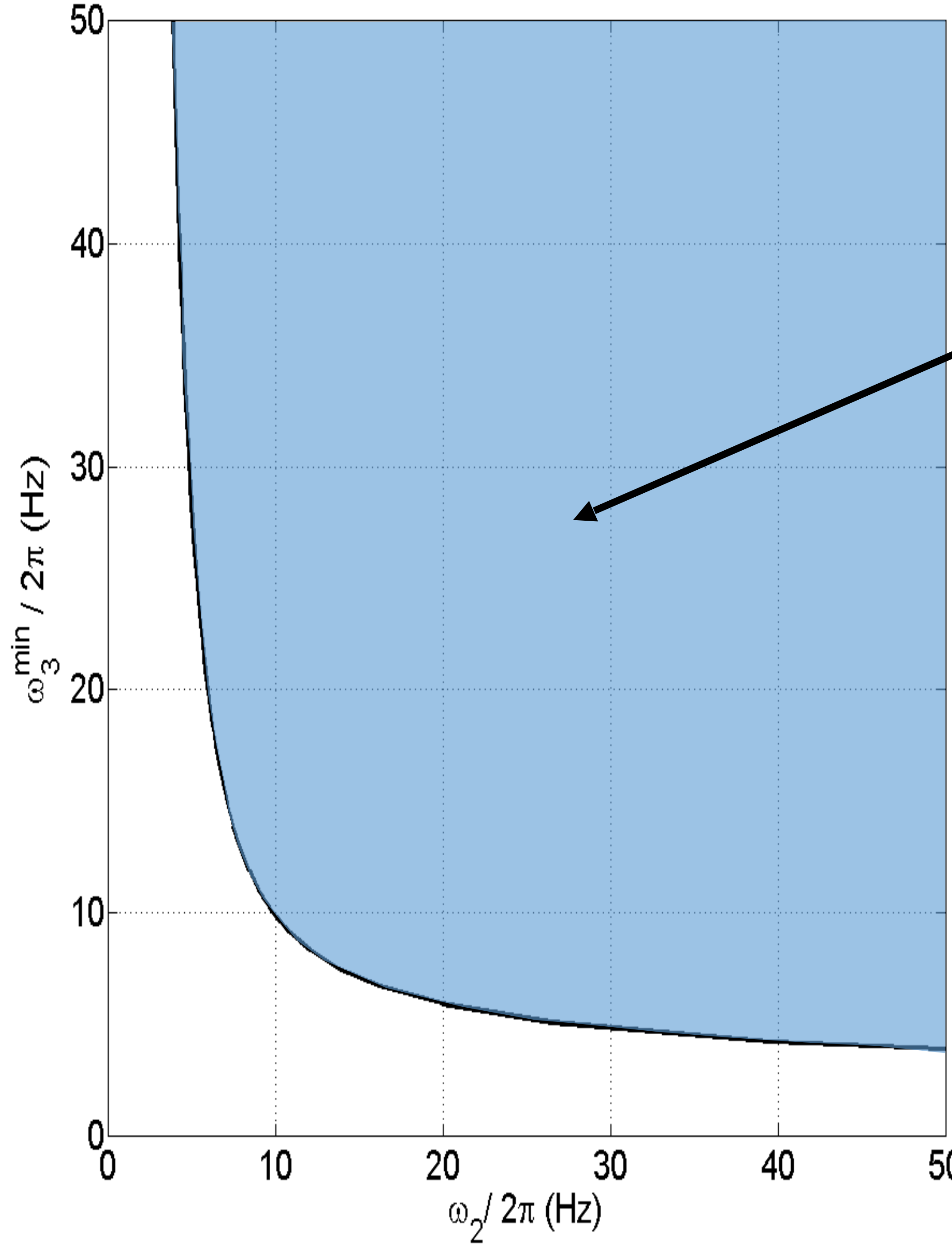
Theoretical exact solutions

3-wave resonant interaction

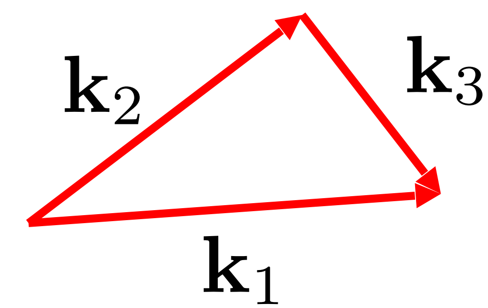
$$\omega_1 = \omega_2 + \omega_3$$
$$\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$$

Linear dispersion relation

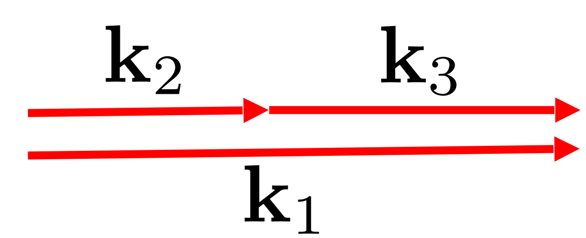
$$\omega = \sqrt{gk + \frac{\gamma}{\rho}k^3}$$



2D solutions



1D solutions



Resonance investigation

Velocity field

$$v(x, y, t)$$

Fourier transform in time

$$v(x, y, \omega)$$

3rd order correlation in frequency

$$C(\omega_1, \omega_2, \omega_3) = |\langle\langle v^*(\omega_1) v(\omega_2) v(\omega_3)\rangle\rangle|$$

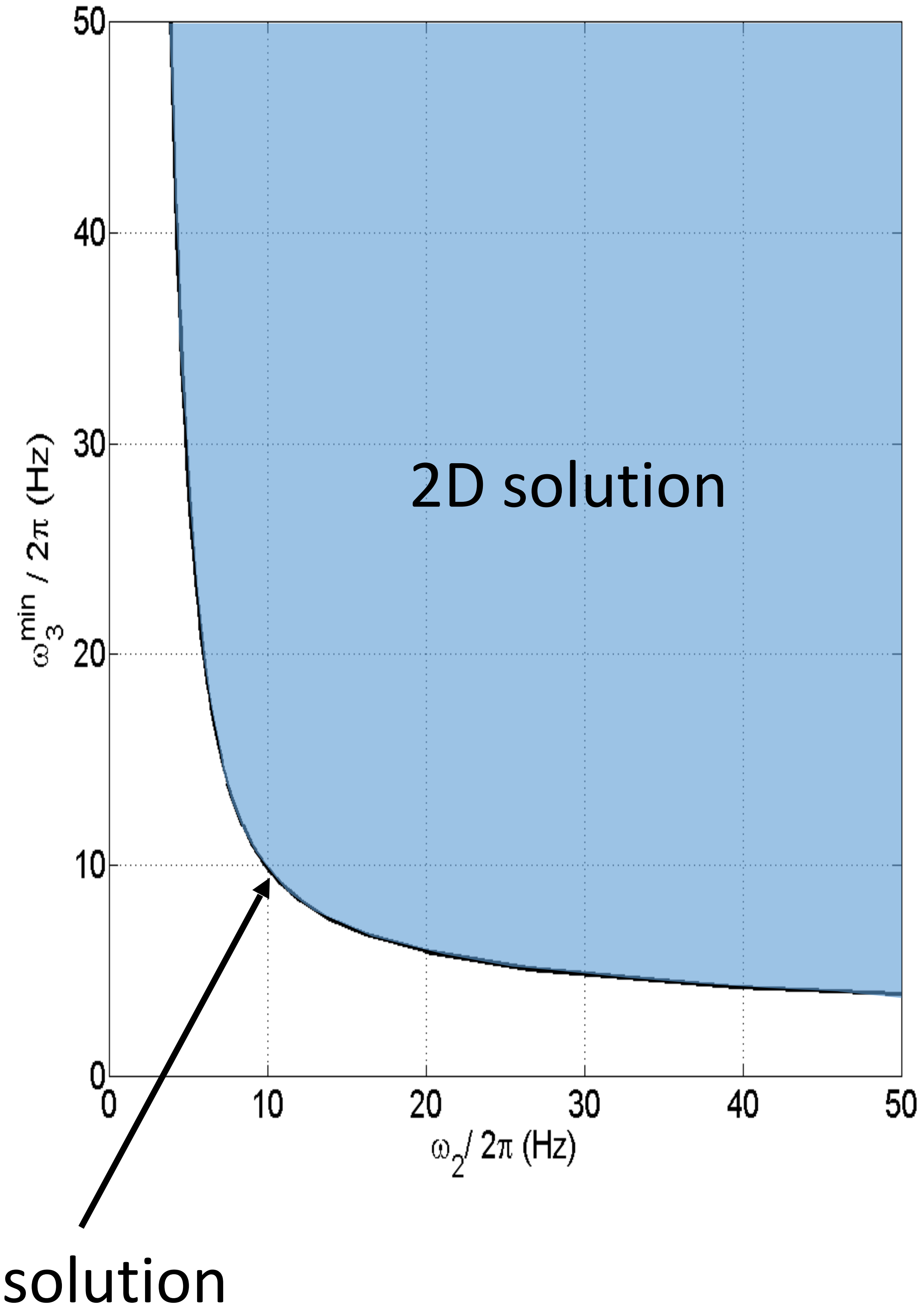
for resonant interactions

$$C(\omega_2, \omega_3) = |\langle\langle v^*(\omega_2 + \omega_3) v(\omega_2) v(\omega_3)\rangle\rangle|$$

Bi-coherence

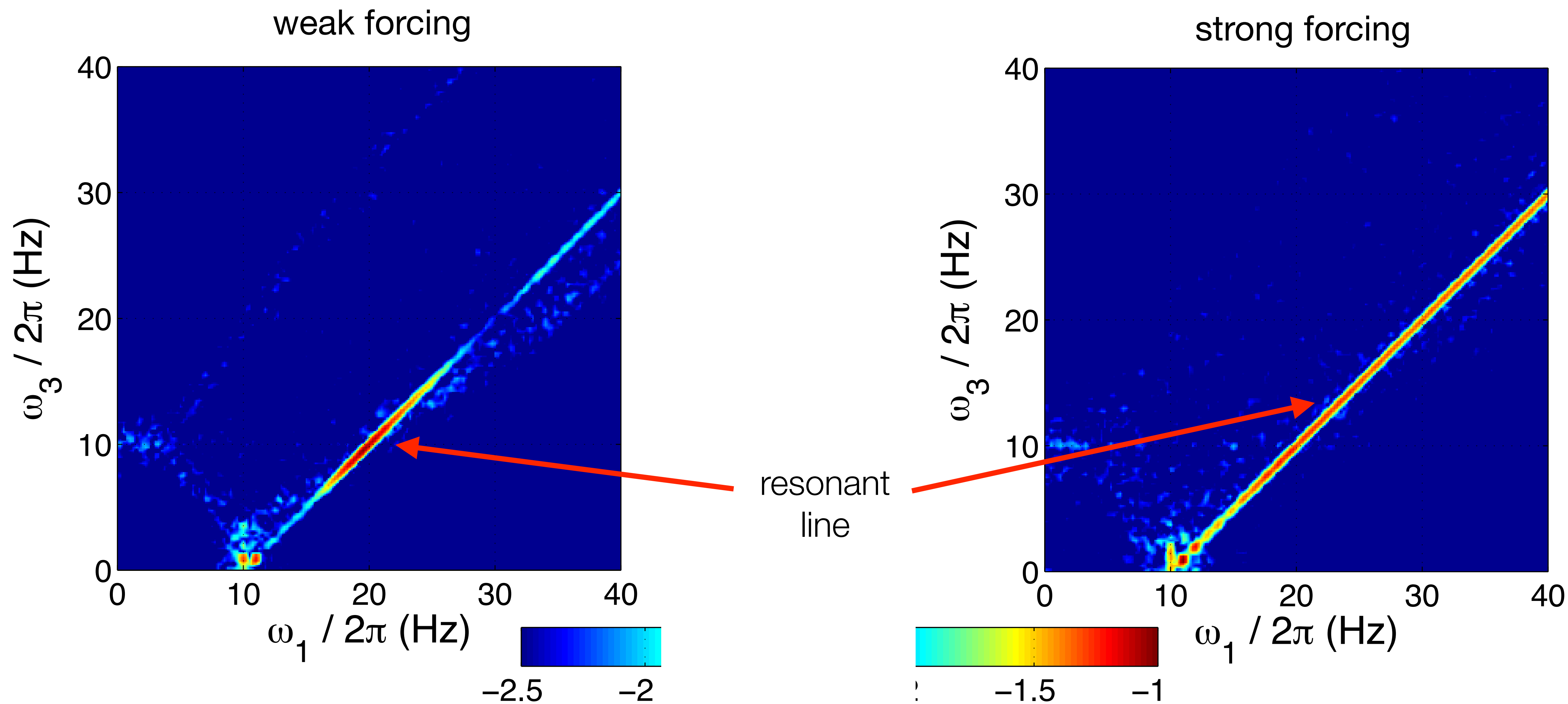
$$B(\omega_2, \omega_3) = \frac{|\langle\langle v^*(\omega_2 + \omega_3) v(\omega_2) v(\omega_3)\rangle\rangle|}{\sqrt{E^v(\omega_2 + \omega_3) E^v(\omega_2) E^v(\omega_3)}}$$

Theoretical exact solutions

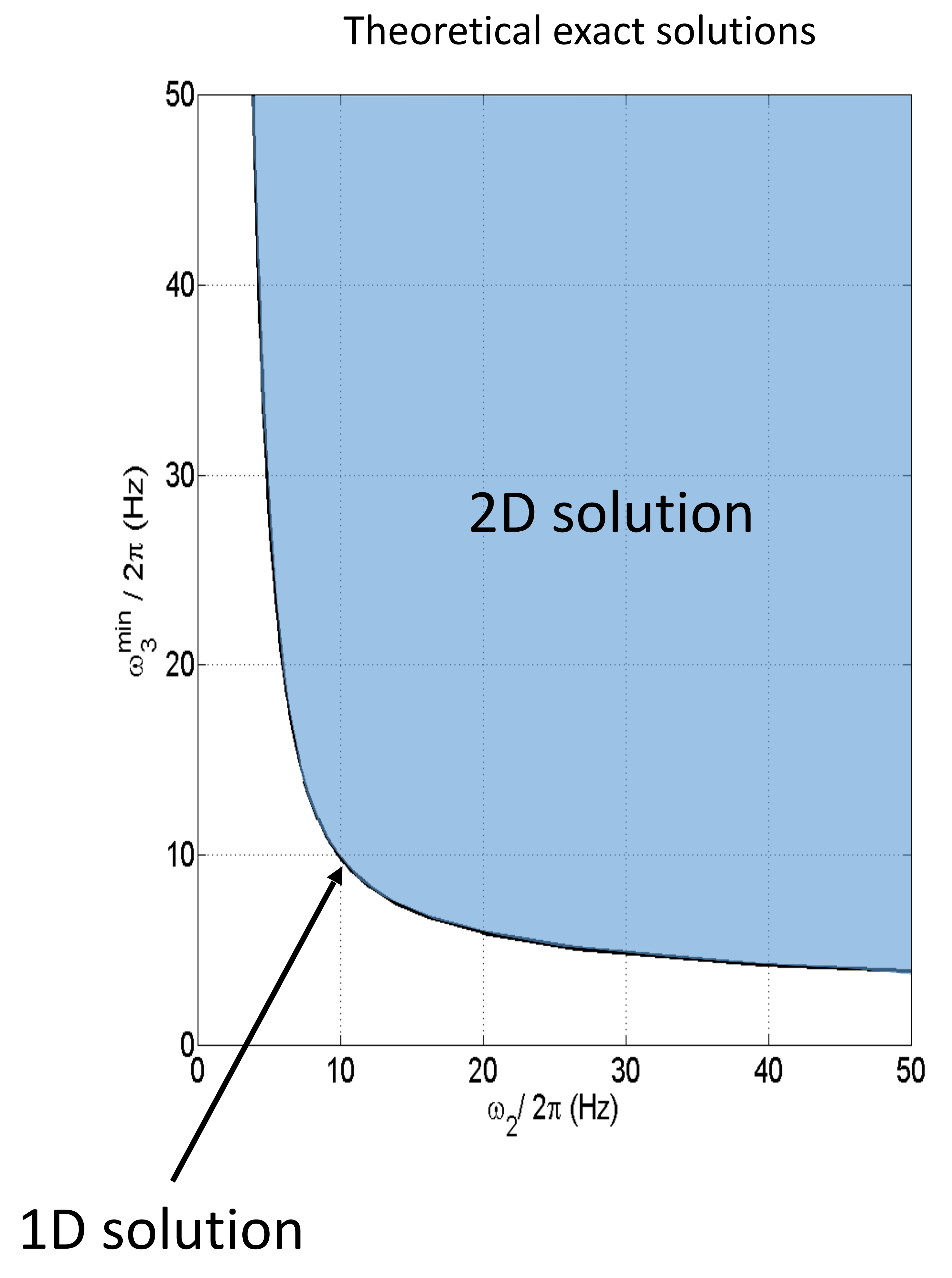


3 wave coherence

$$C(\omega_1, \omega_2, \omega_3) = |\langle\langle v^*(\omega_1) v(\omega_2) v(\omega_3) \rangle\rangle|$$



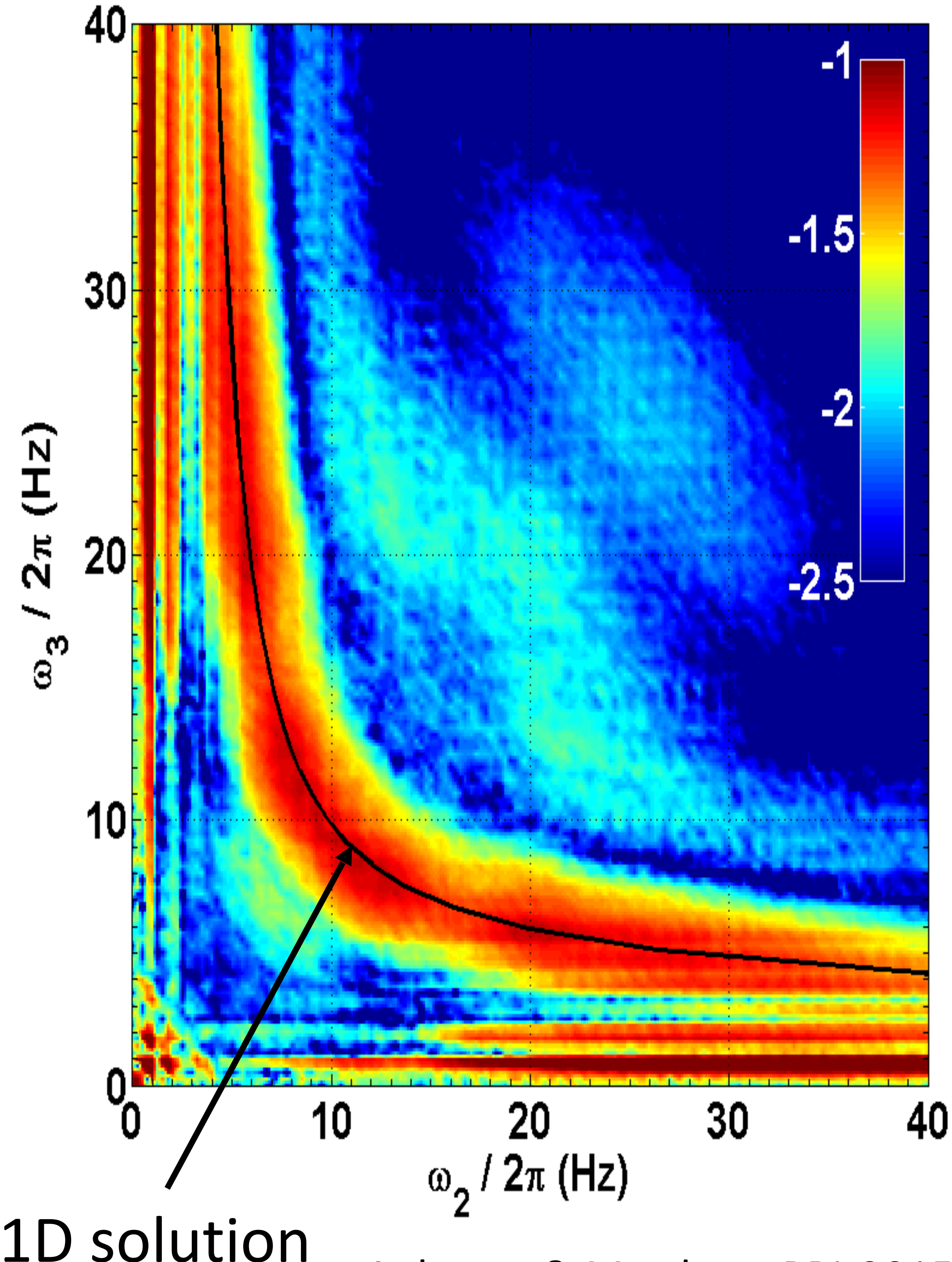
Resonance investigation



Resonance investigation (weak forcing)

Bi-coherence $B(\omega_2, \omega_3)$

- 3-waves processes are present
- "weak" correlations near 5%
- 1D coupling is dominant



Aubourg & Mordant, *PRL* 2015

Resonance investigation:

3-wave correlations in wavenumber space

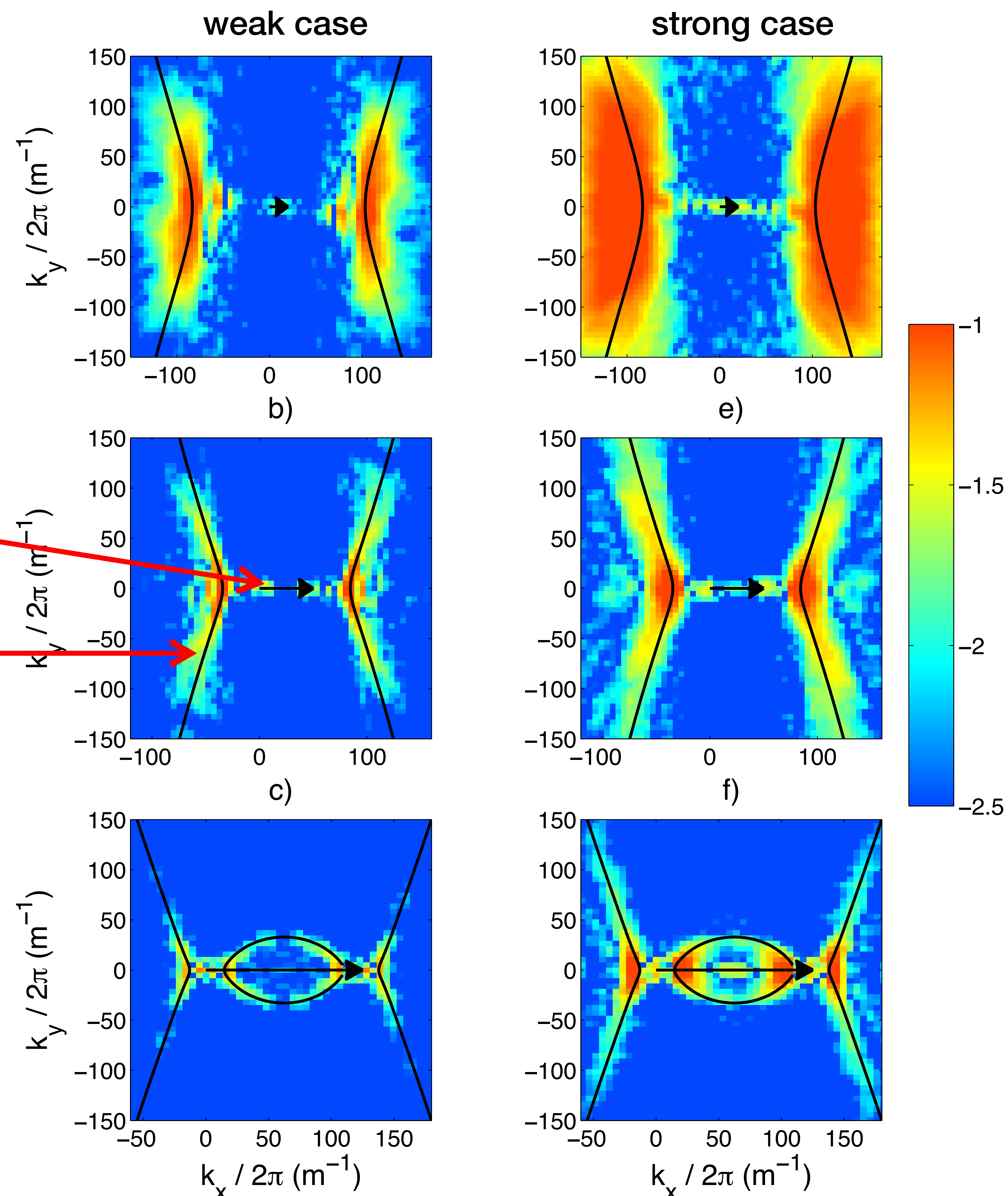
$$B(\mathbf{k}_2, \mathbf{k}_3) = \frac{|\langle\langle v^*(\mathbf{k}_2 + \mathbf{k}_3) v(\mathbf{k}_2) v(\mathbf{k}_3)\rangle\rangle|}{\sqrt{E^v(\mathbf{k}_2 + \mathbf{k}_3) E^v(\mathbf{k}_2) E^v(\mathbf{k}_3)}}$$

4 dimensions space…

$\mathbf{k}_2$ is given

exact resonances

➥ dominant correlations for almost aligned wavevectors

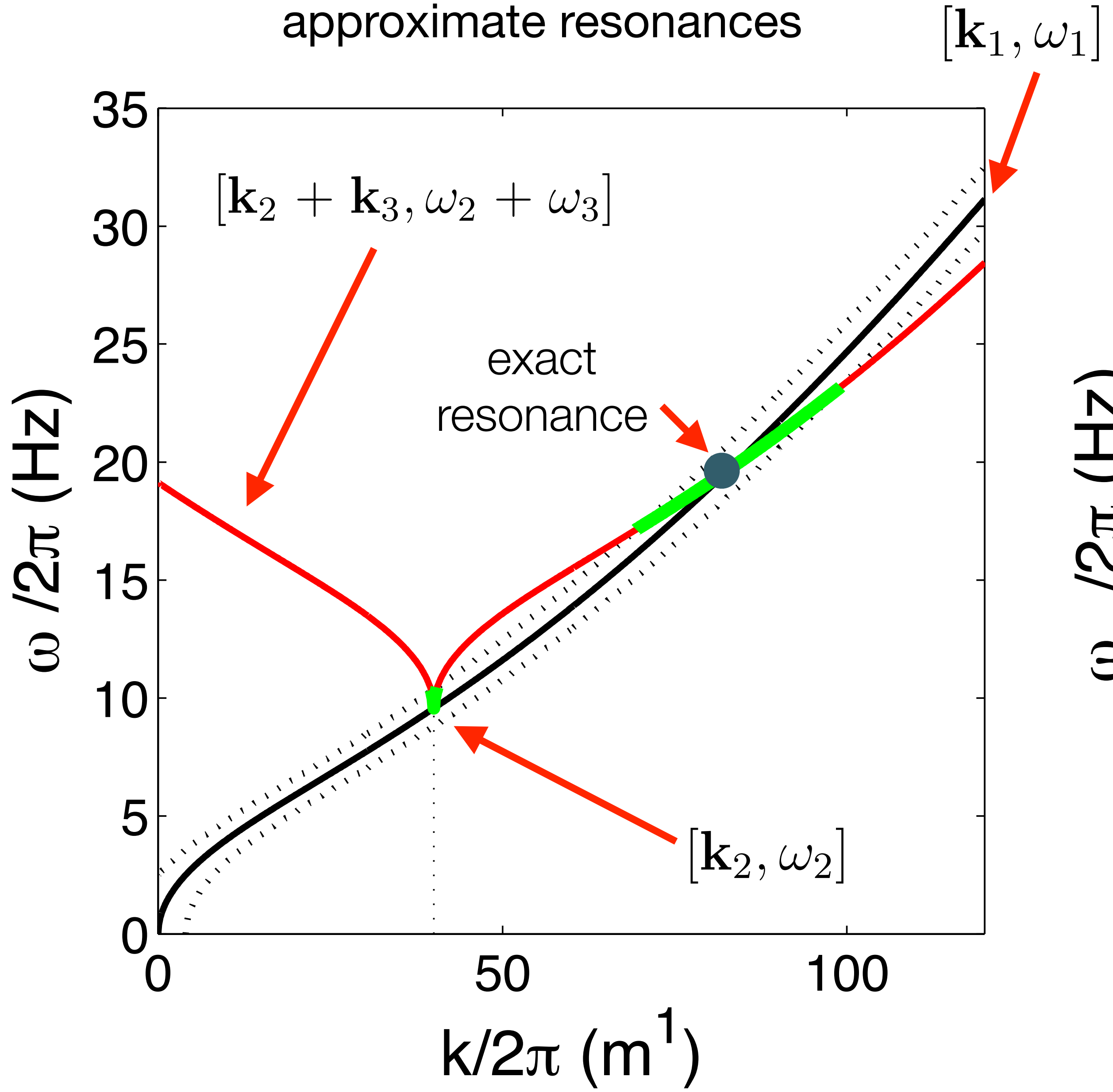


role of approximate resonances: case of 1D interactions

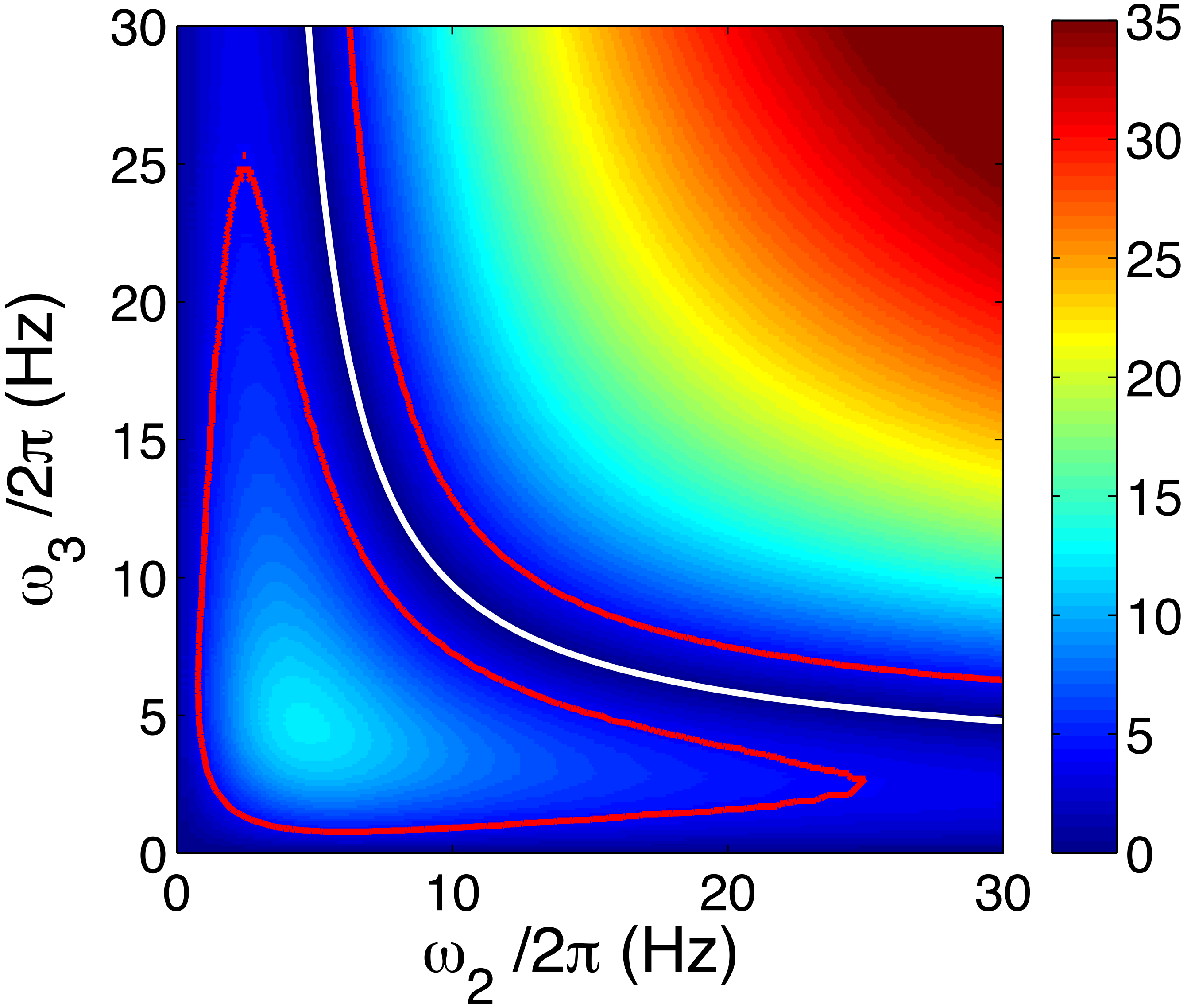
3-wave quasi resonant interactions (1D case):

$$k_1 \pm \delta k = k_2 + k_3$$
$$\omega_1 = \omega_2 + \omega_3$$

green region:
approximate resonances

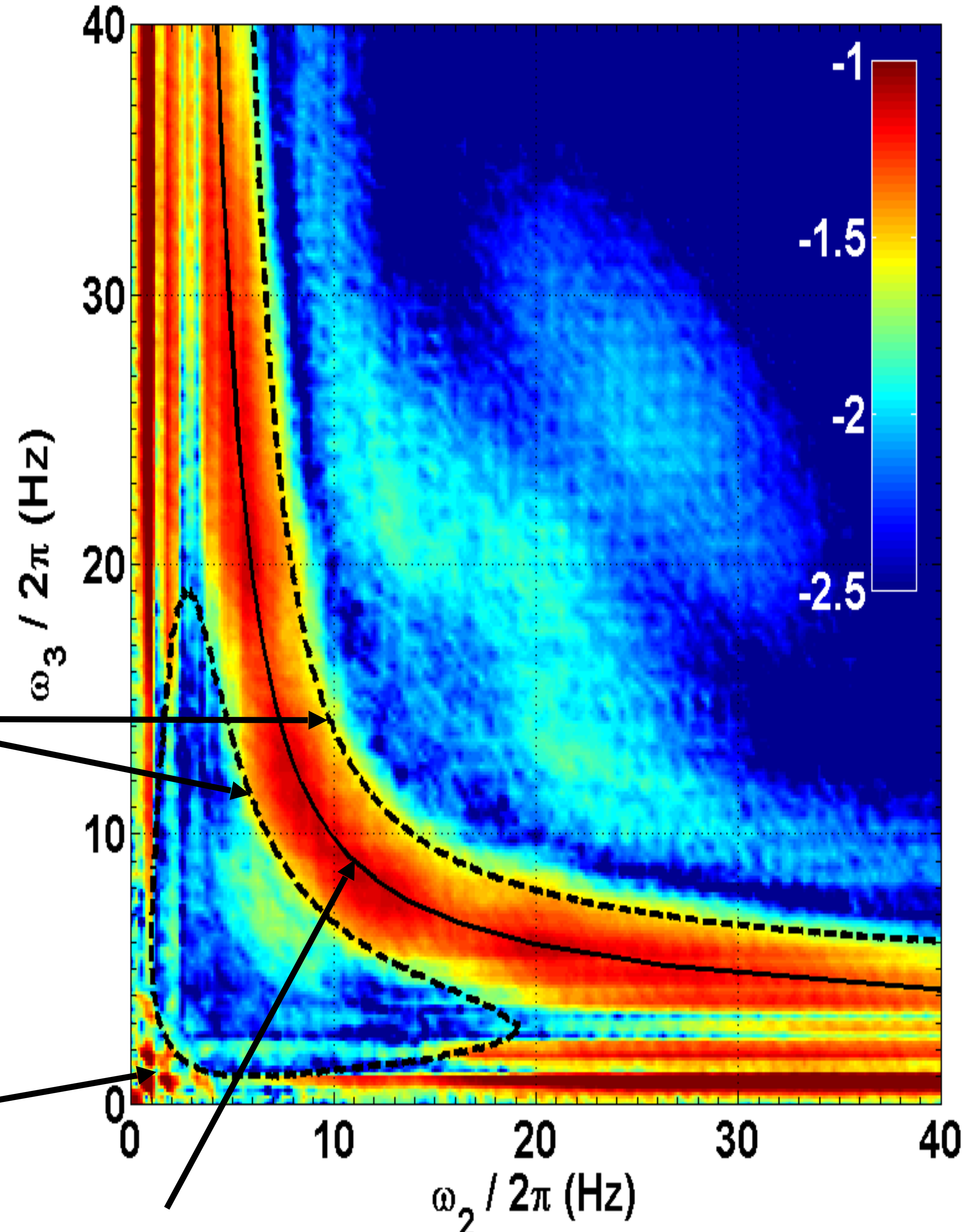


region of allowed
1D resonances



Resonance investigation (weak forcing)

Bi-coherence $B(\omega_2, \omega_3)$



- 3-waves processes are present
- "weak" correlations near 5%
- 1D coupling is dominant
  (checked with 3-wavevector correlations)
- importance of
  1D Approximate resonances

$$\omega_1 = \omega_2 + \omega_3$$
$$k_1 = k_2 + k_3 + \delta k$$

- 3 wave interaction of gravity waves ?
  (no exact 3-wave resonances)

1D solution

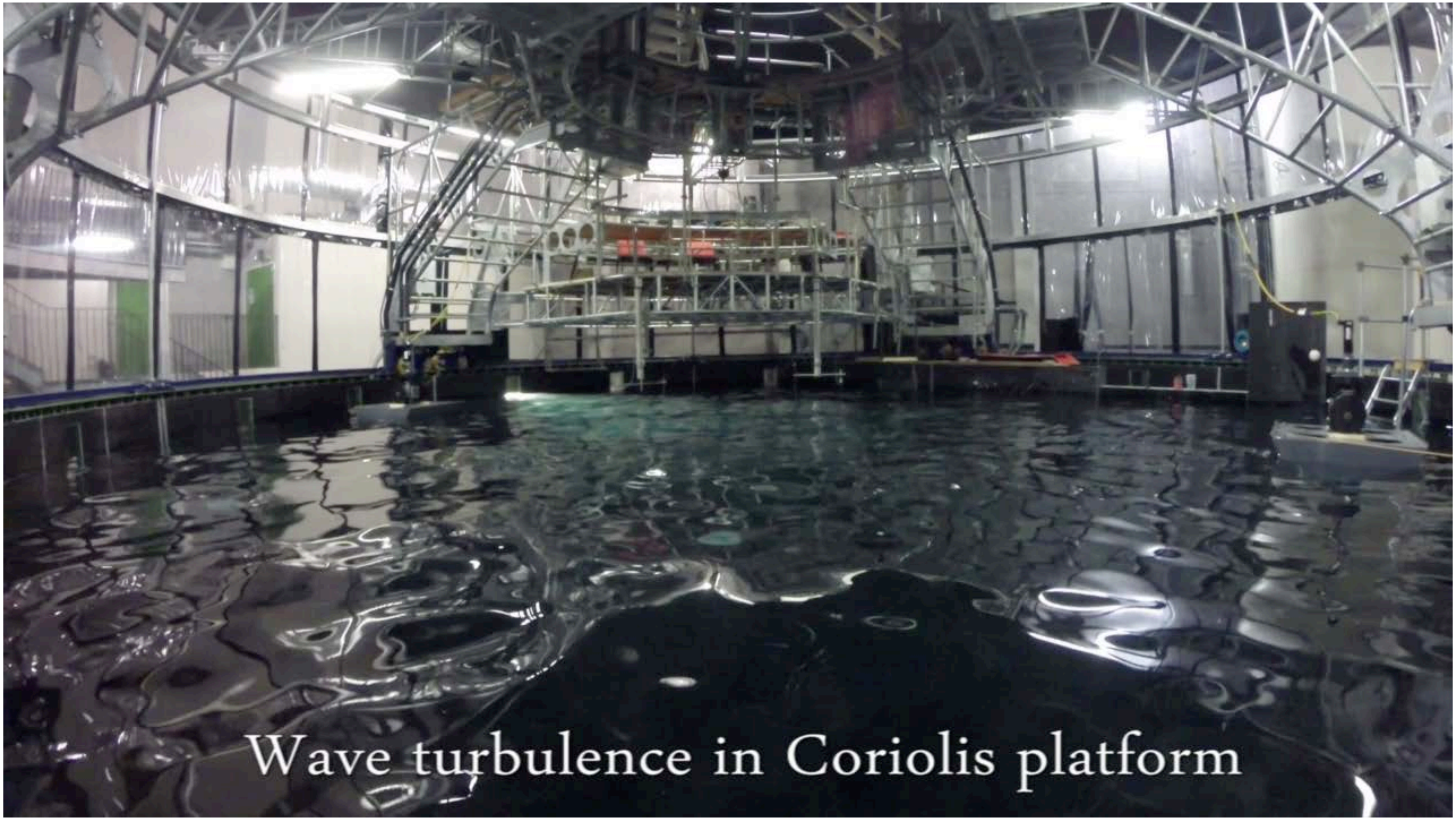
Aubourg & Mordant, *PRL* 2015

Perspectives

gravity surface waves: think bigger…

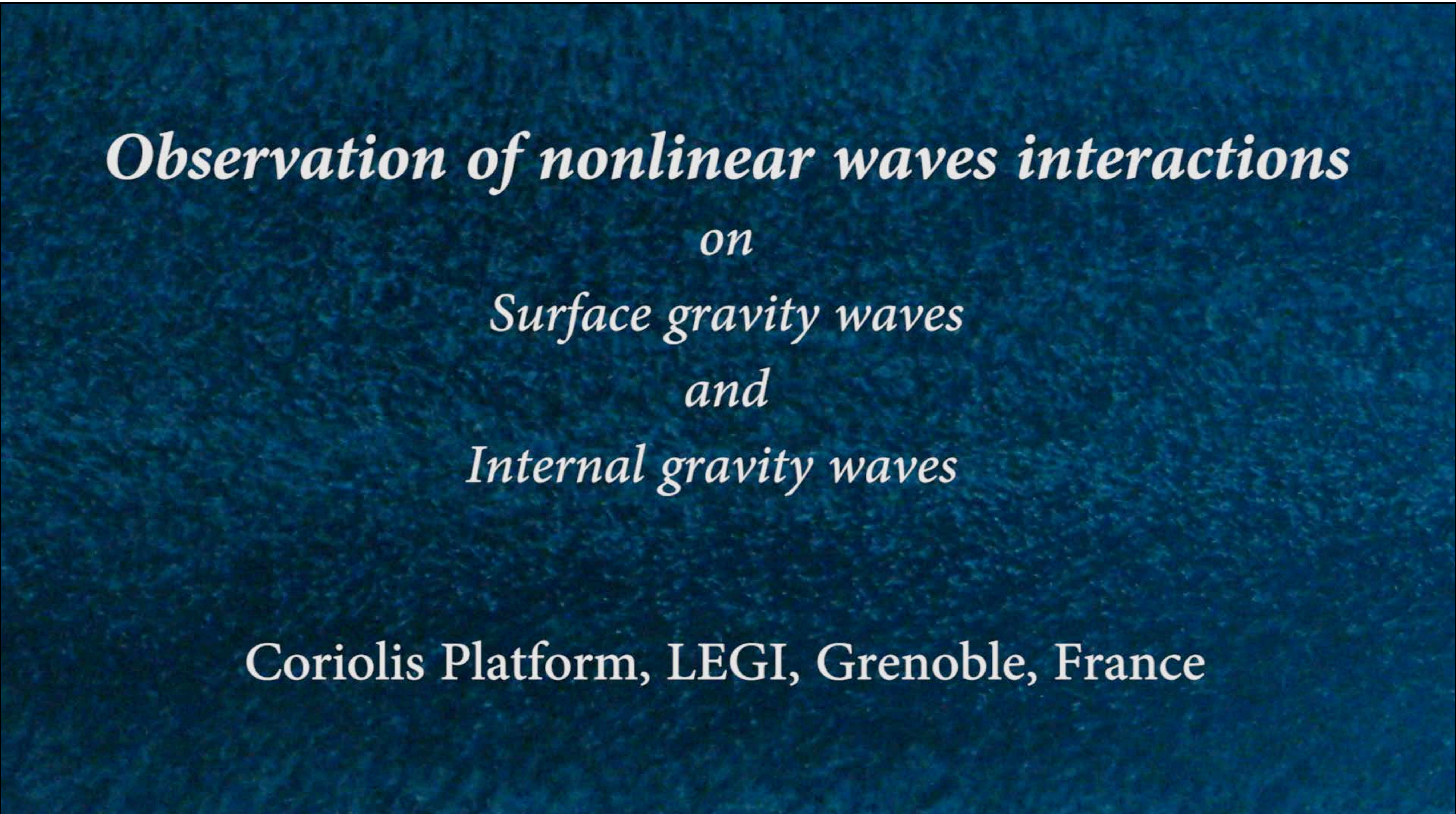
Coriolis facility: 13 m in diameter

surface waves or
bilayer case (clear/salted water, no capillarity)



gravity surface waves: think bigger…

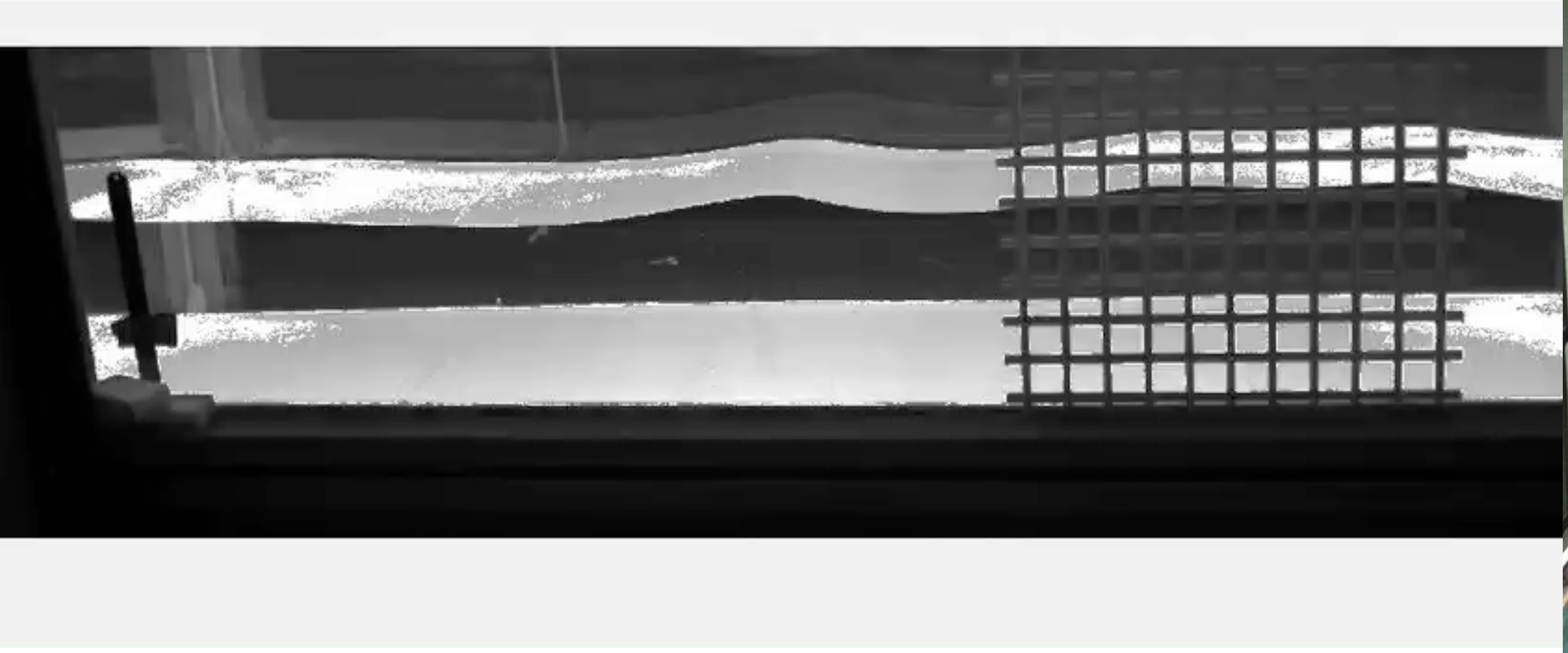
with Q. Aubourg, A. Campagne, J. Sommeria & S. Viboud



movie by Q. Aubourg

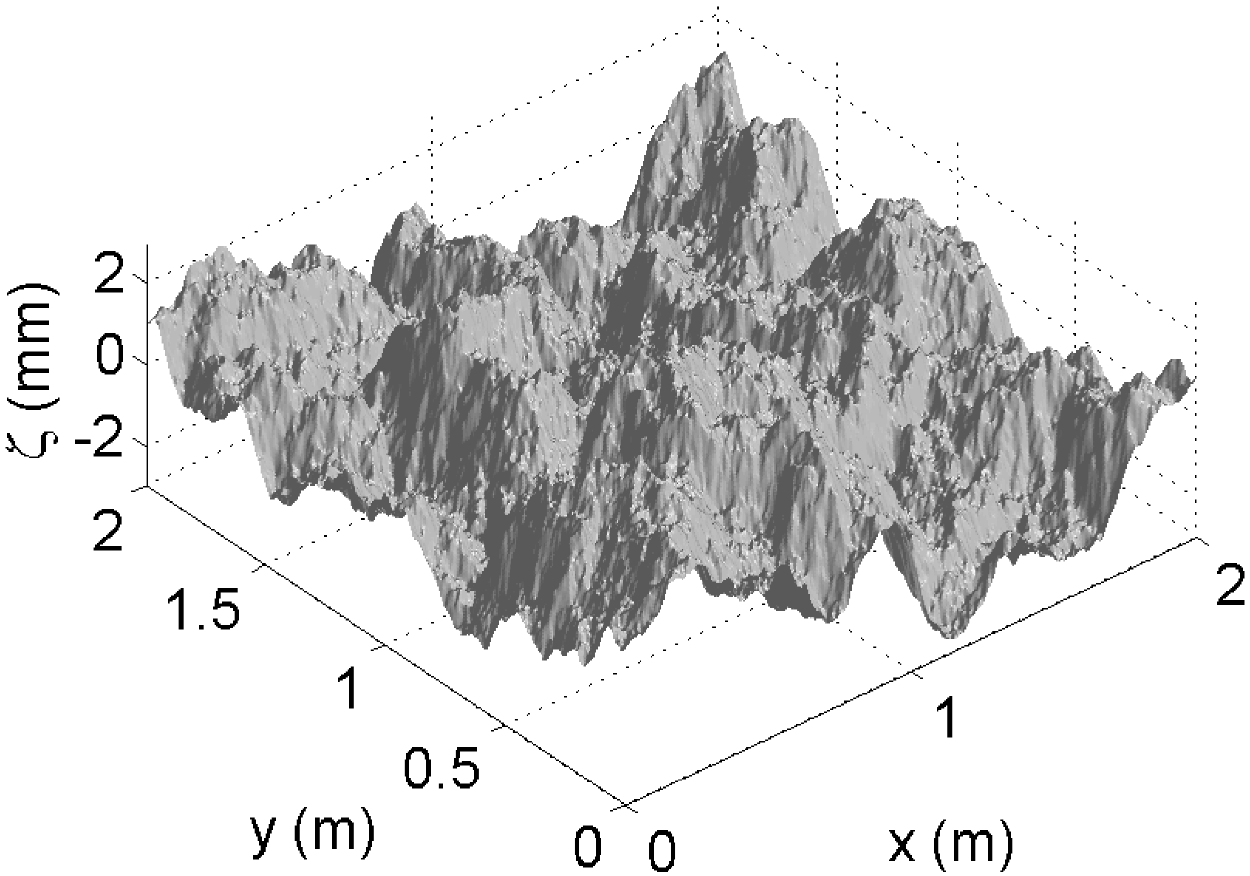
1D gravity surface waves (I. Redor)

1D case: 36 m linear wave flume



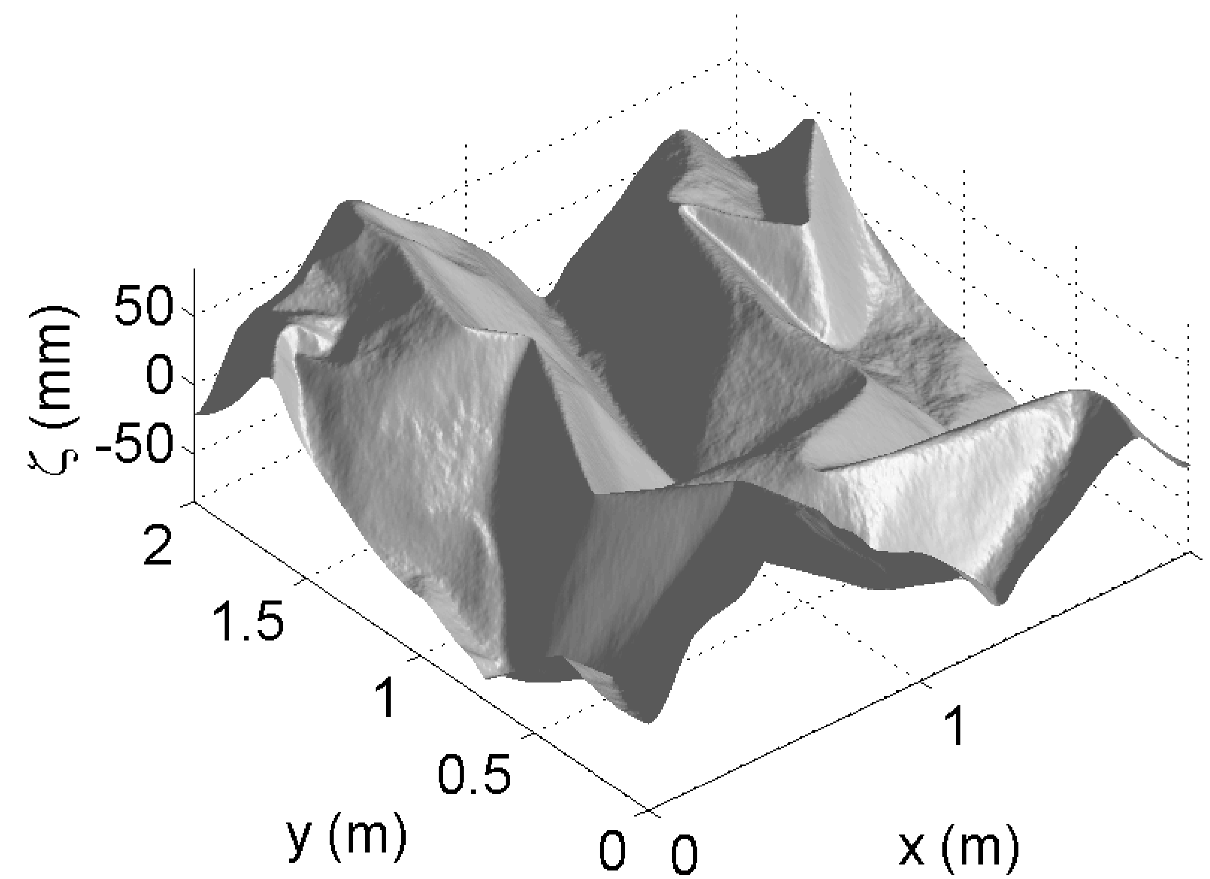
strongly non linear wave turbulence in a vibrated plate: emergence of singularities

weak forcing



(DNS)

strong forcing



emergence of **coherent structures** (singularities)
and **intermittency** coexisting with weak turbulence

influence of structures on energy transfer ?

the vibrating plate: a "simple" but rich model for wave turbulence !

Miquel, Alexakis, Josserand & Mordant, *Phys. Rev. Lett.* 2013

Wave Turbulence of Internal waves ?
(with P. Augier, A. Campagne, B. Voisin & J. Sommeria)

WATU

European Research Council
Established by the European Commission

- generate **turbulence in stratified fluid** or stratified+rotating fluid
  at both low Fr number et high Re number

compare:

- forcing with waves (oscillating bodies or topography)
- forcing with vortices (moving cylinders)

Coriolis facility in Grenoble
(fully reconstructed in 2014)
supervisor: J. Sommeria

13 m in diameter
1m deep
hydraulic scheme for salt/alcohol stratification
rotation down to 10 s period

3D, 3 components
time resolved PIV
+ Lagrangian particle tracking