Cold, quantum, turbulent: disorder near absolute zero

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Outline

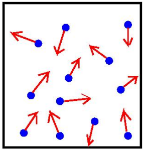
Three strands:

- **Quantum**
- **Cold**
- **Turbulent**

which come together in the study of **quantum turbulence**

Guillaume Amontons (1663-1705)

Daniel Bernoulli (1700-1782)



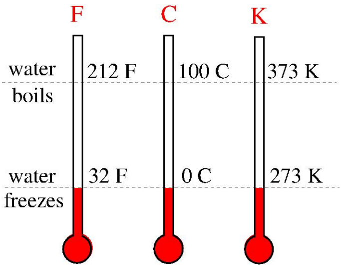
- Amontons measured ΔP induced by ΔT in a gas. P cannot become negative - he argued - hence there exists a minimum T or **absolute zero** estimated at -240 C

- Daniel Bernoulli had a similar idea based on the existence of atoms (absolute zero = no motion)

The idea was put aside when **Lavoisier's caloric theory** was popular, but came back in the mid XIX century.

Cold

- Temperature scales: Daniel Fahrenheit (1686-1736)
  Anders Celsius (1701-1744)
  Lord Kelvin (1824-1907)



Warming: solid $\rightarrow$ liquid $\rightarrow$ gas
Cooling: gas $\rightarrow$ liquid $\rightarrow$ solid

- Mid-late XIX century: race to liquifiy gases and approach absolute zero (0 K)

- The lower the temperature, the less the thermal disorder, the more apparent the fundamental properties of matter

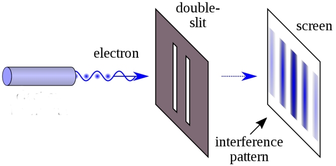
Cold

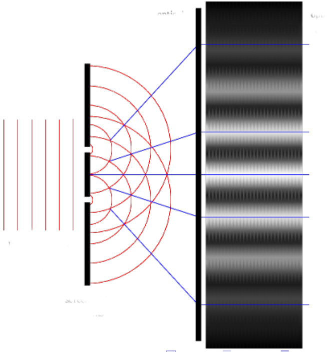
273.15 K	Water freezes (0 C)
184 K	Antarctica lowest (-89 C)
77 K	Nitrogen becomes liquid
63 K	Nitrogen becomes solid
20 K	Hydrogen becomes liquid
14 K	Hydrogen becomes solid
4.2 K	Helium becomes liquid
4.1 K	Mercury becomes **superconductor**
2.725 K	Cosmic microwave background radiation
2.1768 K	Liquid helium (^{4}He) becomes **superfluid**
10^{-3} K	Rare isotope liquid ^{3}He becomes **superfluid**
10^{-6} K	**Bose-Einstein condensation** in atomic gases
...	
0 K	Absolute zero (-273.15 C)

Quantum

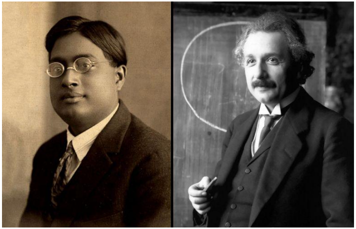
- Quantum mechanics deals with **microscopic** objects (atoms, electrons, quarks ...)

- **Particle-wave duality:**
  Particle of mass m, velocity v is a wave of wavelength $\lambda = \frac{h}{mv}$
  ($h =$ Planck's constant)

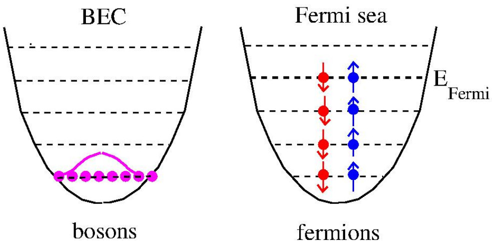




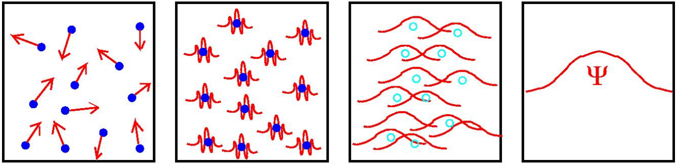
- **Quantum mechanics:** Atom with mass m and velocity v has de Broglie wavelength $\lambda = h/(mv)$
- **Thermodynamics:** Average kinetic energy is $mv^2/2 \approx k_B T$ hence $\lambda \sim T^{-1/2}$
- What happens if $T \to 0$? This is particularly relevant for **bosons** (integer spin) which, unlike **fermions** (half-integer spin), are not prevented by Pauli's principle from occupying the same state



Satyendra Nath Bose and Albert Einstein, 1924

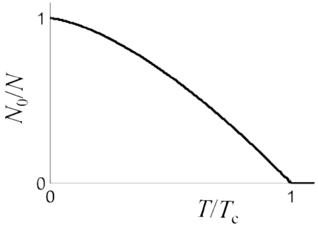


Compare λ (de Broglie wavelength) and d (average distance between atoms in the gas): as T decreases, λ increases

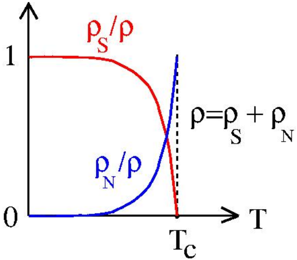


When $\lambda \approx d$ a **Bose-Einstein Condensate (BEC)** arises, governed by a **macroscopic** wavefunction Ψ

- **Superconductivity**: current flows without resistance (1911)
- **Superfluidity**: fluid flows without viscosity
  liquid helium ^{4}He (1938) and ^{3}He (1972),
  cold atomic gases (1995), neutron stars, etc



ideal gas:
(Bose & Einstein 1924)
$N_0/N = 1 - (T/T_c)^{3/2}$



Liquid helium:
superfluid and normal fluid fractions

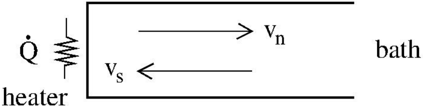
At nonzero T:

Atomic BECs: dilute ballistic **thermal gas**
Helium: thermal exitations make up the **normal fluid**
(**two-fluid model** of Landau & Tisza)

Component	Density	Velocity	Entropy	Viscosity
Normal fluid	ρ_n	$\mathbf{v}_n$	S	η
Superfluid	ρ_s	$\mathbf{v}_s$	0	0

where $\rho = \rho_n + \rho_s$

- second sound
- thermal counterflow



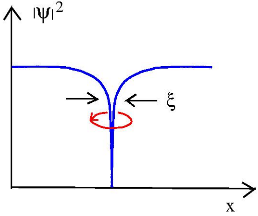
Cold + Quantum

Let $\Psi(\mathbf{r},t)=\sqrt{n(\mathbf{r},t)}e^{i\phi(\mathbf{r},t)}$

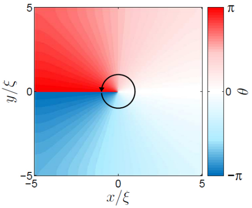
- Number density $n=|\Psi|^2$, Velocity $\mathbf{v}=(\hbar/m)\nabla\phi$
- Circulation

$$\oint_C \mathbf{v}\cdot\mathbf{dr}=n\frac{h}{m}=n\kappa \quad (\text{n} > 1 \text{ unstable})$$

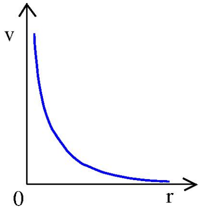
- A **quantum vortex** is a hole ($n\to 0$ as $r\to 0$) around which the phase changes by 2π, hence $v=\kappa/(2\pi r)$



Density profile

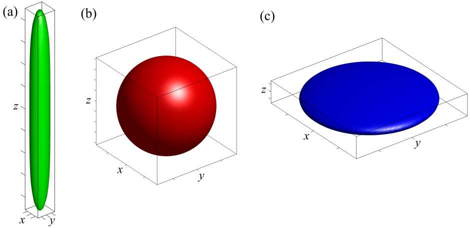


Phase



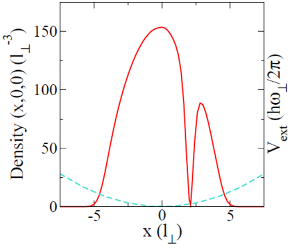
Velocity

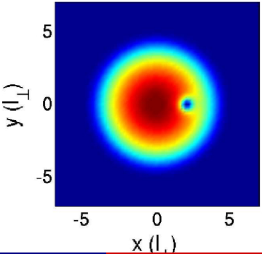
Cold + Quantum

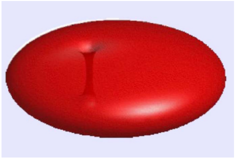


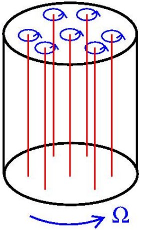
Atomic condensates are small clouds of gases trapped into arbitrary shape of non-uniform density.

BECs without (above) and with (below) a quantum vortex



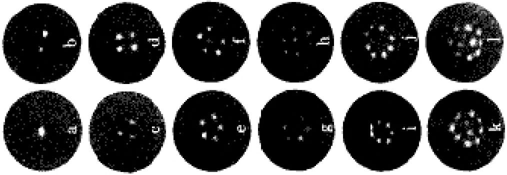




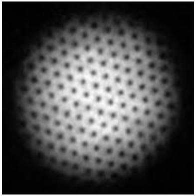


Vortex lattice

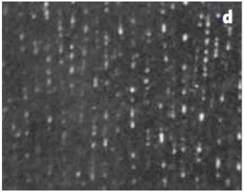
$n = 2\Omega/\kappa$



Yarmchuck *& al* 1979



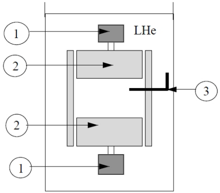
Abo Shaeer *& al* 2001



Bewley *& al* 2006

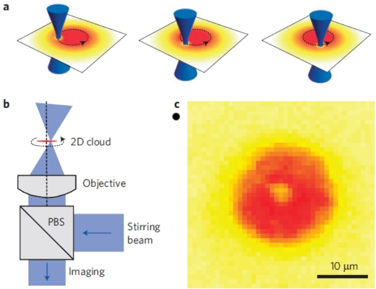
Quantum turbulence is easily excited:

- in helium, by thermal or mechanical stirring (propellers, grids, forks, wires), injecting vortex rings, etc
- in atomic condensates, by laser stirring, shaking the trap, vortex phase imprinting, thermal quench, etc



Mauerer & Tabeling 1998

(1) motor; (2) propellers;
(3) probe

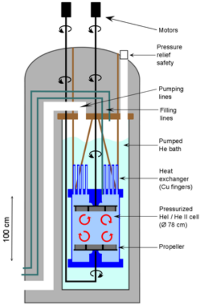


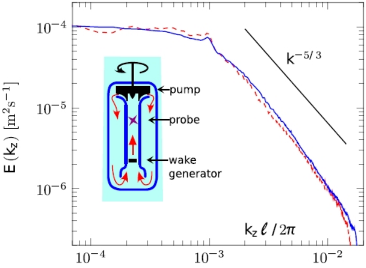
Desbuquois *& al* 2012

Quantum turbulence: experiments

Energy spectrum $E(k)$: distribution of kinetic energy over the length scales $2\pi/k$

$$E = \frac{1}{V}\int_V \frac{\mathbf{v}^2}{2}dV = \int_0^\infty E(k)dk, \qquad E(k) \sim k^{-5/3}$$





as in classical turbulence
Salort & al 2012

SHREK (Grenoble)

The Gross-Pitaevskii equation

The GPE is:

- a quantitative model of atomic BECs at low temperatures,
- a qualitative model of helium

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + g|\Psi|^2\Psi + V(\boldsymbol{r}, t)\Psi$$

$$\int |\Psi(\mathbf{r}, t)|^2 d^3\mathbf{r} = N$$

$\Psi(\boldsymbol{r}, t) =$ complex wavefunction
$V(\boldsymbol{r}, t) =$ trapping potential (for atomic BECs)
$g =$ strength of interactions
$N =$ number of atoms
$m =$ atomic mass

Fluid dynamics interpretation of the GPE

Let $\psi(\mathbf{r},t)=\sqrt{n(\mathbf{r},t)}e^{i\phi(\mathbf{r},t)}$

velocity $\mathbf{v}(\boldsymbol{r},t)=(\hbar/m)\nabla\phi(\boldsymbol{r},t)$, density $\rho(\boldsymbol{r},t)=mn(\boldsymbol{r},t)$

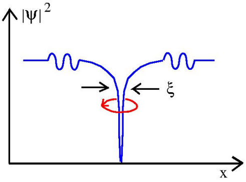
$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0 \qquad \text{(continuity eq.)}$$

$$\rho\left(\frac{\partial v_j}{\partial t}+v_k\frac{\partial v_j}{\partial x_k}\right)=-\frac{\partial p}{\partial x_j}+\frac{\partial Q_{jk}}{\partial x_k} \qquad \mathrm{j}=1,2,3\ (\mathrm{quasi-Euler\ eq.})$$

$$\text{Pressure } p=\frac{V_0}{2m^2}\rho^2, \qquad \text{Quantum stress } Q_{jk}=\left(\frac{\hbar}{2m}\right)^2\rho\frac{\partial^2\ln\rho}{\partial x_j\partial x_k}$$

- At scales larger than $\xi=\hbar/\sqrt{m\mu}$ ($\approx$ vortex core size) the quantum stress is negligible and the GPE reduces to

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{1}{\rho}\nabla p \quad \text{(Euler equation)}$$

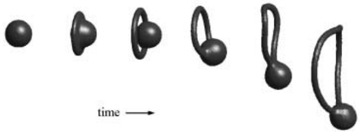


The GPE describes:

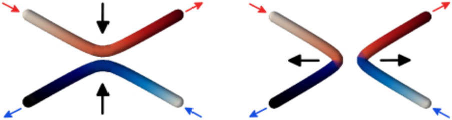
- waves
- vortex lines

and (via the quantum stress) effects which go beyond Euler:

- vortex nucleation
- vortex reconnections



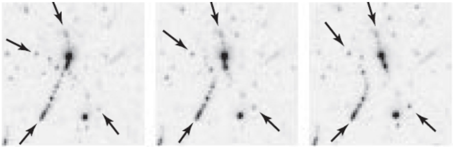
Winiecki & Adams 2000

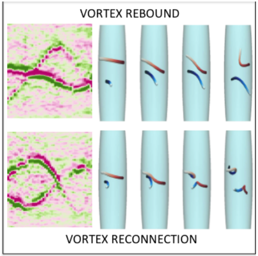


Galantucci & al 2018

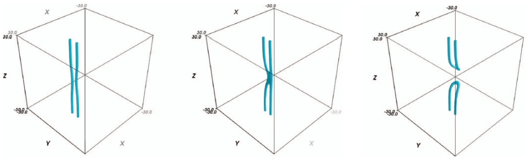
Vortex reconnections

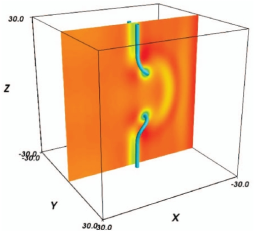
Reconnections have been observed in liquid helium (Paoletti & al, PNAS 2008) and in atomic BECs (Serafini & al, PRX 2017)





Reconnection ⇒ acoustic kin. energy losses (Leadbeater & al 2001, Zuccher & al 2012)





The vortex filament model (VFM)

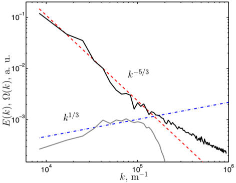
- At length scales $\gg \xi$, a vortex line is a 3D space curve $\boldsymbol{s}(\zeta, t)$ which obeys

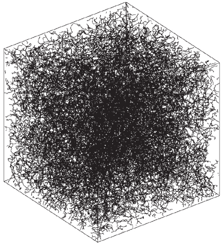
$$\frac{d\boldsymbol{s}}{dt} = \boldsymbol{v}(\boldsymbol{s}) + \alpha \boldsymbol{s}' \times [\boldsymbol{v}_n(\boldsymbol{s}) - \boldsymbol{v}(\boldsymbol{s})]$$

- $\zeta =$ arc length
- $\boldsymbol{s}' = d\boldsymbol{s}/d\zeta$ (unit tangent)
- $\alpha =$ temp. dep. friction coeff
- $\boldsymbol{v}_n =$ normal fluid velocity

$$\mathrm{Biot-Savart\ law:}\ \ \boldsymbol{v}(\boldsymbol{s}) = -\frac{\kappa}{4\pi} \oint_{\mathcal{T}} \frac{(\boldsymbol{s} - \boldsymbol{r})}{|\boldsymbol{s} - \boldsymbol{r}|^3} \times \boldsymbol{dr}$$

- In the VFM, vortex reconnections are done algorithmically



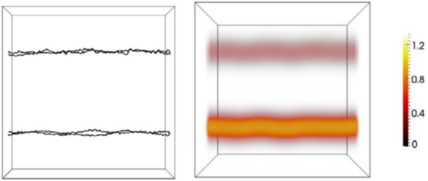


Where does the $E_k \sim k^{-5/3}$ spectrum come from ?

What happens inside a tangle of vortex lines ?

Coarse-grain vortex lines over the intervortex distance ℓ:

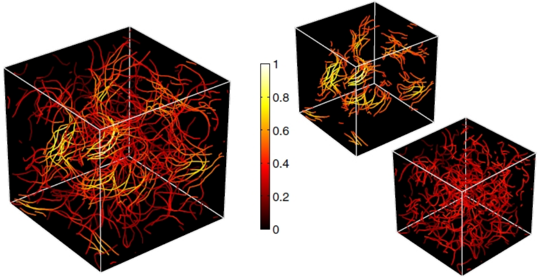
$$\omega(\boldsymbol{r}, t) = \kappa \sum_{j=1}^{N} \frac{\boldsymbol{s}'_j \, \Delta\zeta}{(2\pi\sigma^2)^{3/2}} e^{-|\boldsymbol{s}_j - \boldsymbol{r}|^2/2\sigma^2}$$

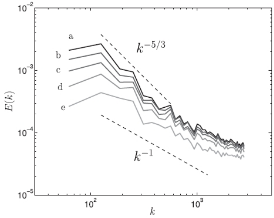


Baggaley, CFB, Shukurov, Sergeev, EPL 2012

Quantum turbulence: numerics

Kolmogorov energy spectrum arises from polarized vortex bundles

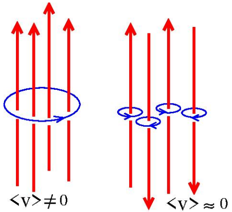




All and Pol. $E_k \sim k^{-5/3}$

Unpol. $E_k \sim k^{-1}$

Left: All vortex lines
Right: Polarized lines (yellow),
Unpolarized lines (red)

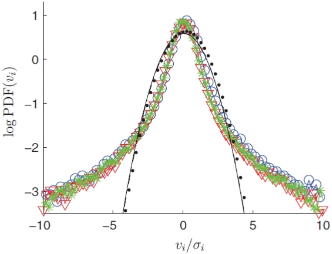


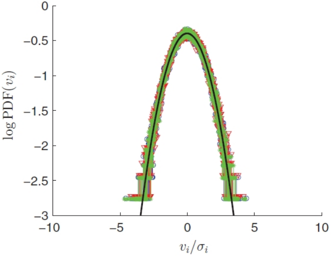
- Classical at scales $\gg \ell$

Baggaley, Laurie & CFB (PRL 2012)

- Classical turbulence: Gaussian (Vincent & Meneguzzi 1991)
- Quantum turbulence: power-laws
  (exp. by Paoletti & al PRL 2008, sim. by White & al PRL 2010)
- Cross-over (sim. by Baggaley & CFB, PRE 2011):
  ($\Delta =$ measurement region, $\ell =$ average inter-vortex distance)

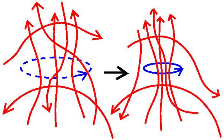
For $\Delta < \ell$ (left): power-laws
For $\Delta > \ell$ (right): Gaussians



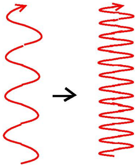


Cross-over confirmed by exp. of La Mantia & Skrbek, EPL 2014

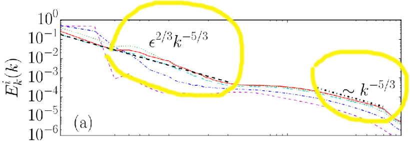
Turbulence in the $T \to 0$ limit



Kolmogorov cascade



Kelvin wave cascade



Leoni & al PRA 2017

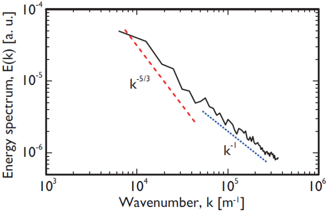
- E injected at large scale
- Kolmogorov cascade of eddies
- bottleneck
- Kelvin cascade of waves on individual vortices
- E becomes sound (phonons)

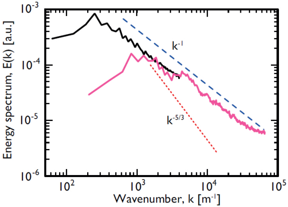
- Kelvin wave cascade: spectrum controversy solved by Krstulovic (PRE 2012)

A different quantum turbulence: Vinen turbulence

Another form of turbulence, called **Vinen turbulence**, has also been observed in:

- ^{4}He experiment (Walmsley & Golov, PRL 2008)
- ^{3}He experiments (Bradley & al, PRL 2006)
- simulations of Walmsley & Golov's experiment (Baggaley, CFB & Sergeeev, PRB 2012)
- simulations of counterflow (T1) turbulence in ^{4}He (Baggaley, Sherwin, CFB & Sergeev, PRB 2012)
- simulations of thermal quench of a Bose gas (Stagg, Parker & CFB, PRA 2016)
- simulations of turbulence in atomic gases (Cidrim, CFB & Bagnato, PRA 2017)
- simulations of dark matter (Mocz & al, MNRAS 2017)



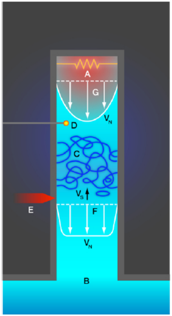


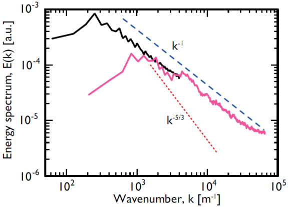
- **Kolmogorov:** $E(k)$ peaks at low k, $E(k) \sim k^{-5/3}$ for $k < 2\pi/\ell$, coherent structures, $L \sim t^{-3/2}$, $E_{kin} \sim t^{-2}$
- **Vinen:** $E(k)$ peaks at intermediate k, $E(k) \sim k^{-1}$ at larger k, $L \sim t^{-1}$, $E_{kin} \sim t^{-1}$, velocity correlation decays rapidly with r

Interpretation of Vinen turbulence:
turbulence without a cascade, random-like flow
(CFB, Sergeeev & Baggaley, Sci Rep 2016)

Vinen turbulence in thermal counterflow

Normal fluid and superfluid driven in opposite direction by a small applied heat flux (T1 state)



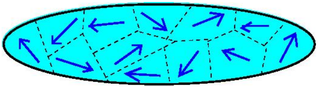


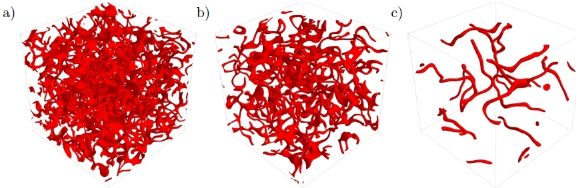
If normal fluid is uniform
$E(k)$ peaks at intermediate k
Vinen turbulence
(Baggaley & al PRB 2012)

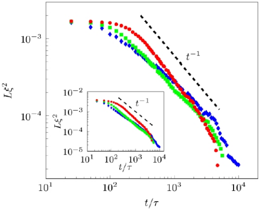
- At larger heat flux (T2 state) **both normal fluid and superfluid become turbulent** (Babuin & al PRB 2016)
- Visualization: La Mantia, Skrbek & al (PRB 2016); Guo, Vinen & al (PRB 2017)

Vinen turbulence in a thermal quench (Kibble-Zurek)

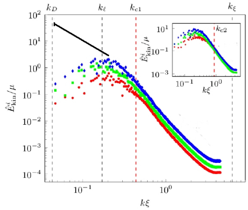
Stagg, Parker & CFB, PRA 2016



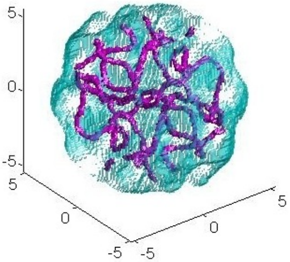




Decay



Energy spectrum

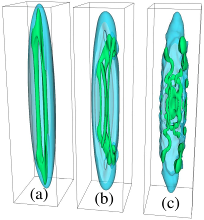


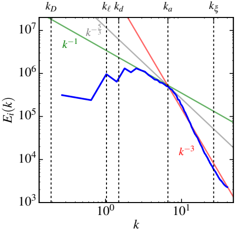
(White & al, PRL 2010)

D = system size
ℓ = average intervortex distance
ξ = vortex core radius

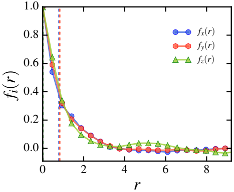
- In superfluid helium $D/\xi \approx 10^{10}$ and $D/\ell \approx 10^5$
- In atomic BECS $D/\xi \approx 100$ and $D/\ell \approx 10$
  (not much k-space available)

Cidrim, Allen, White, Bagnato & CFB, PRA 2017

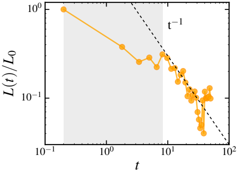




Energy spectrum

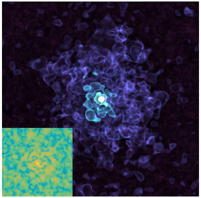


Correlation

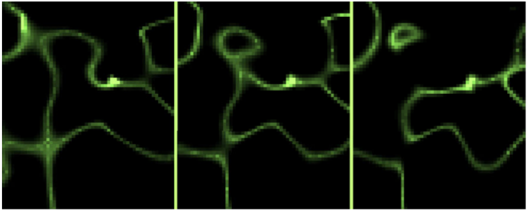


Decay

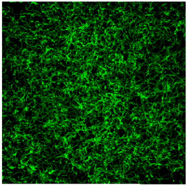
Dark matter BEC of axions in galactic halos modelled by the self-gravitating GPE (Mocz & al MNRAS 2017)



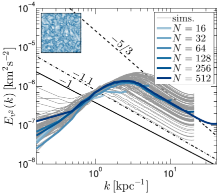
Density



Vortex reconnection

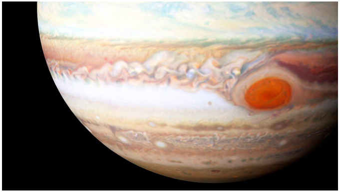


Vortex tangle

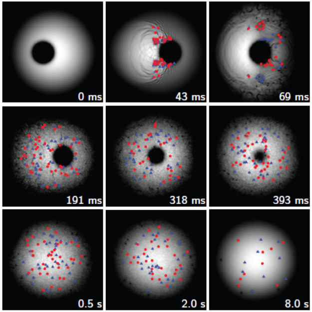


Energy spectrum

Two-dimensional turbulence: **negative absolute temperature**
(Onsager vortex gas, reverse energy cascade, Red Spot effect...)



Jupiter's Red Spot (NASA)



2D vortices
created by a laser spoon

Kwon et al (Phys Rev A 2014)
Stagg et al (J. Phys Rev A 2015)

Conclusions

There seem to be **three kinds** of quantum turbulence:

- Kolmogorov-like turbulence
  - $T \neq 0$: normal fluid and superfluid are stirred together
  - $T = 0$: pure superfluid tangle (skeleton of classical turbulence) Kolmogorov+Kelvin (K & K) cascades
- Vinen turbulence = turbulence without a cascade
- Double turbulence of coupled normal fluid and superfluid in thermal counterflow at high heat flux (like turbulent velocity and magnetic fields of VKS dynamo)

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