Dynamics of large scales in turbulent flows

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New Challenges in Turbulence Research VI
Les Houches 11/02/2021

Outline

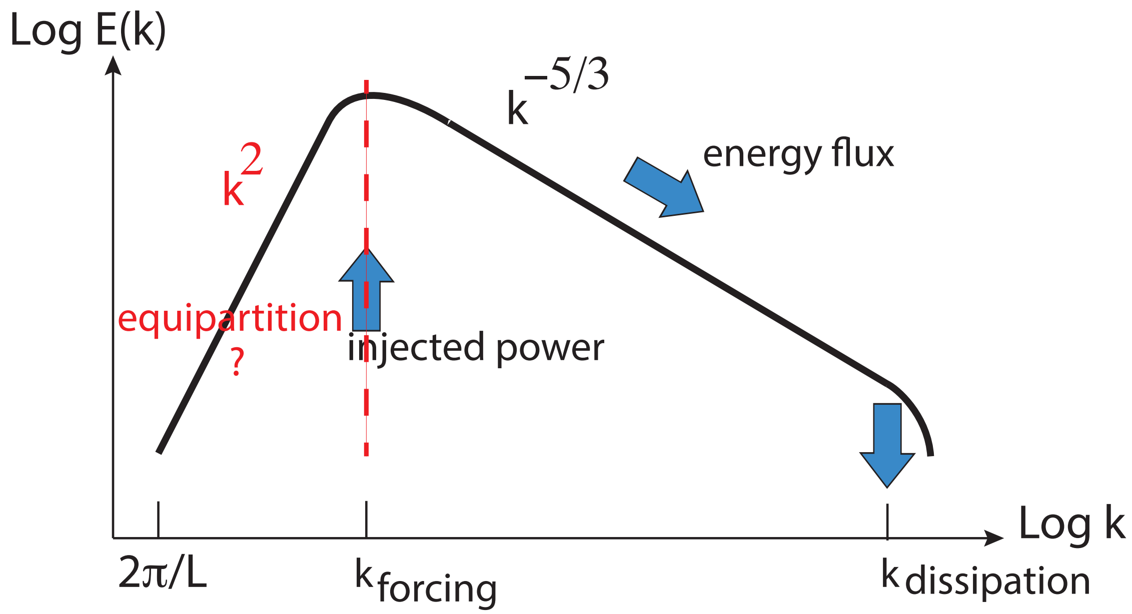
- Some turbulent flows display equipartition of energy at large scales
  - 3D hydrodynamic turbulence
  - Capillary wave turbulence

- An experimental study of transitions between different turbulent regimes in 2D spatially periodic forced flows
  - Homogeneous turbulence with Gaussian velocity PDF
  - Bimodal velocity PDF and reversals of large scale circulation
  - Large scale condensate involving a large part of the kinetic energy

  Bifurcations between turbulent flows that involve different large scale behaviors characterized by the velocity PDF described by equilibrium statistical mechanics?

- Low frequency behavior of turbulent flows : dynamics of coherent structures and 1/f noise

Are the large scales of a turbulent flow in statistical equilibrium?



$F(\mathbf{k})\, k^2\, dk \sim E(k)\, dk$

$E(k) \sim k^2$

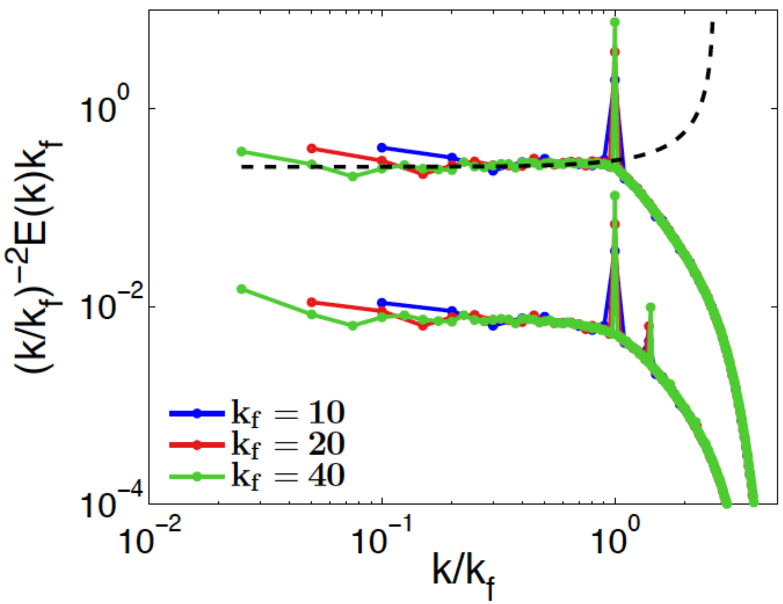
Rayleigh-Jeans spectrum at large scales

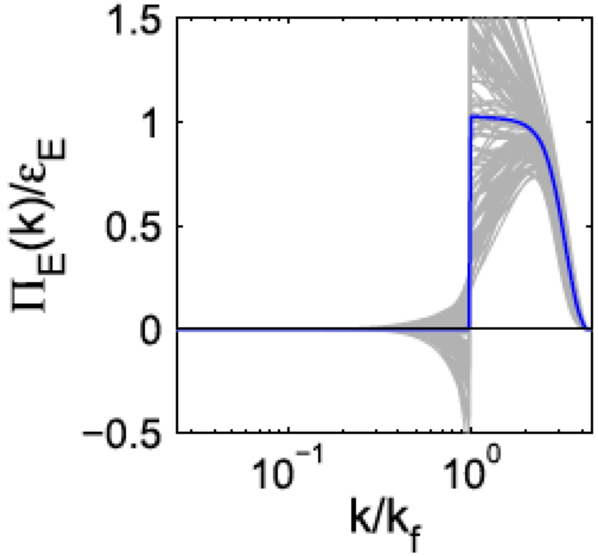
≠ Decaying turbulence: k^2 versus k^4 spectrum at small k (Batchelor, Saffmann)

Believed for a long time but experiments or direct simulations with scale separation between the forcing scale and the box size need to be performed

Large scale energy spectra for small scale periodically forced 3D turbulent flows

Dallas, Fauve, Alexakis, PRL 115, 204501 (2015)





Equipartition below the forcing scale k_f

$E(k) \sim v_f^2\ k_f^{-3}\, k^2$

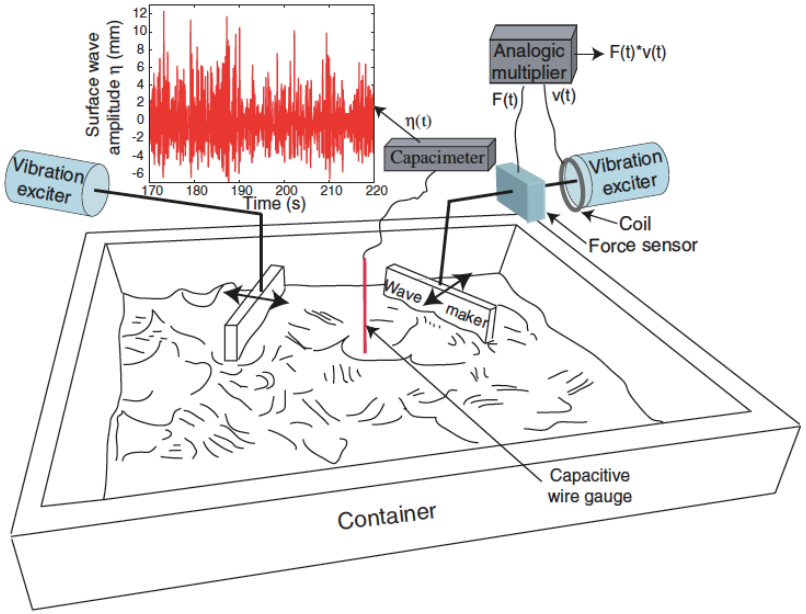
Fluctuations of the flux : $\sigma(k) \sim k^4$

$\sigma(k) / E(k) \sim E^{1/2}\, k^2$

2 large k modes beats and give energy to a small k mode which in turn is damped by turbulent viscosity (analogy with a Brownian particle)

Capillary wave turbulence forced at high frequency

G. Michel, F. Pétrélis, S. Fauve, PRL 118, 144502 (2017)

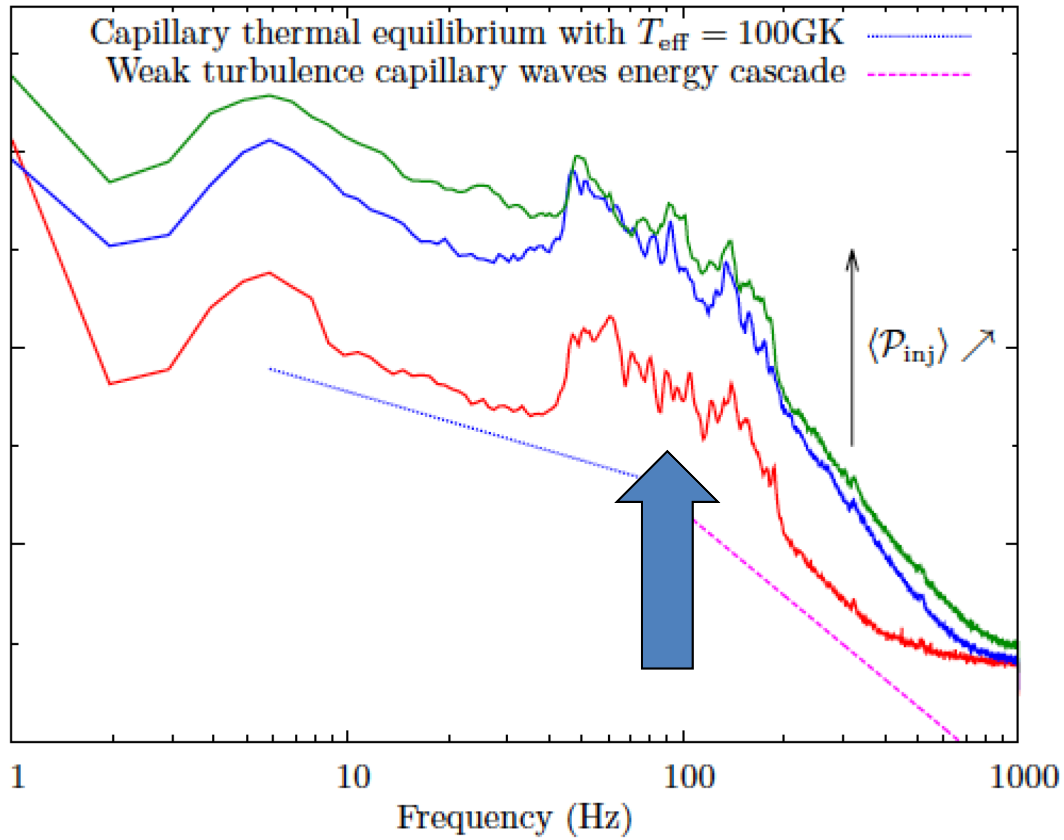


Weakly interacting surface waves forced by random motion of paddles

S(f) ~ 1/f ⟹ S(k) ~ 1/k

E(k) ~ k i.e equipartition

Modes at scales larger than the forcing scale are in equipartition



Large scales in 2D hydrodynamic turbulence

- Inverse cascade : depending on the scale at which large scale friction stops the inverse cascade, one observes either a turbulent regime or condensation of energy in the largest scale modes (Kraichnan 1967)
- The generation of large scales has been also explained using statistical mechanics of the Euler equation: Onsager (points vortices), Kraichnan, Montgomery, Salmon, Kuz'min, Robert, Sommeria, Miller, Chavanis, Bouchet, Venaille,..

- An experiment on a nearly 2D confined flow with a spatially periodic forcing: sequence of transitions within the turbulent regime when the large scale friction is decreased:

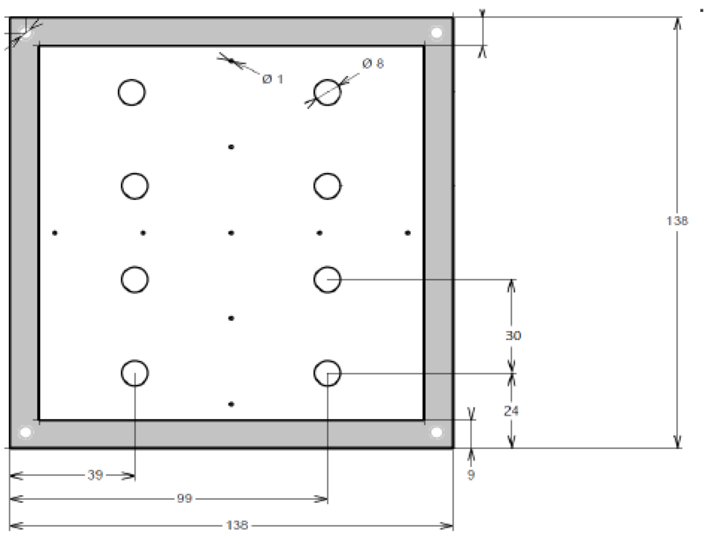
  - a turbulent flow with a gaussian velocity pdf

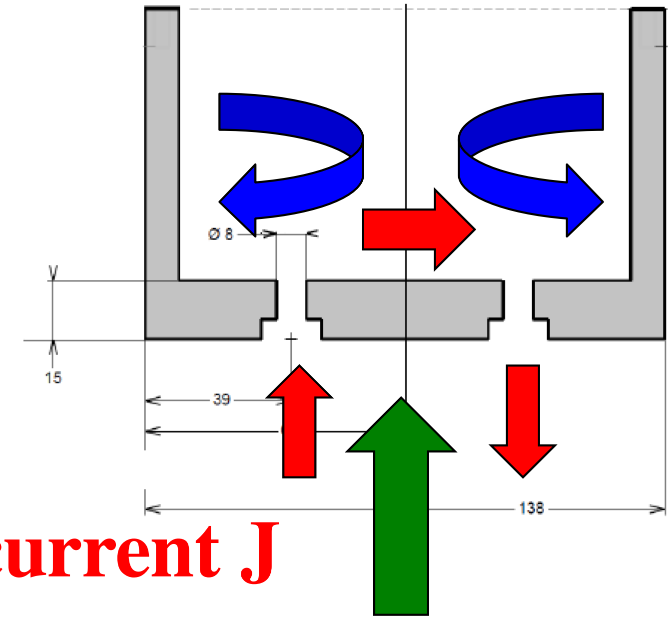
  - a turblent flow with a bimodal velocity pdf ; random reversals of the large scale flow

  - condensation of the kinetic energy in the largest scale
- Similar observations with numerical simulations of the 2D Navier-Stokes equation
- Description of these transitions and of the velocity pdfs using the truncated Euler equation

A nearly two-dimensional spatially forced flow

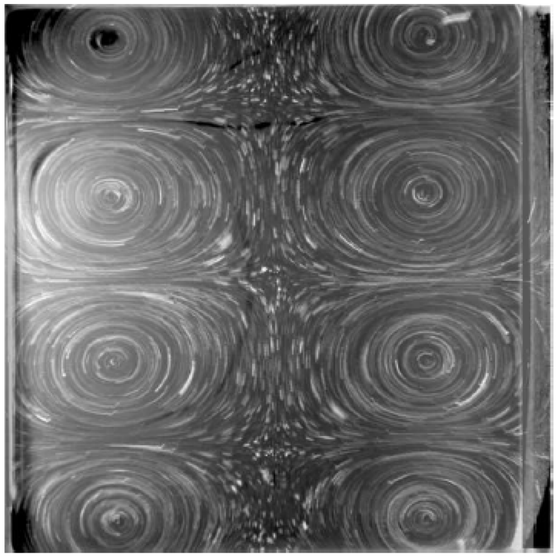
J. Herault, F. Pétrélis, S. Fauve, EPL 111, 44002 (2015).





current J

magnetic field B



Generation of an array of vortices by the Lorentz force in a layer of liquid gallium

The limit of two-dimensional flows

- Small magnetic Reynolds number: Rm = $\mu\sigma UL \sim 0.01$ ($U < 0.1$ m/s)
- High Hartman number: Ha = $hB\sqrt{(\sigma/\rho\nu)} \sim 100$

- Velocity independent of z except in a boundary layer of size $\delta \sim h/\mathrm{Ha}$
- Add a linear friction term, $-u/\tau$ in the Navier-Stokes equation for the velocity averaged on the height of the layer ($\tau \sim 10$ s)

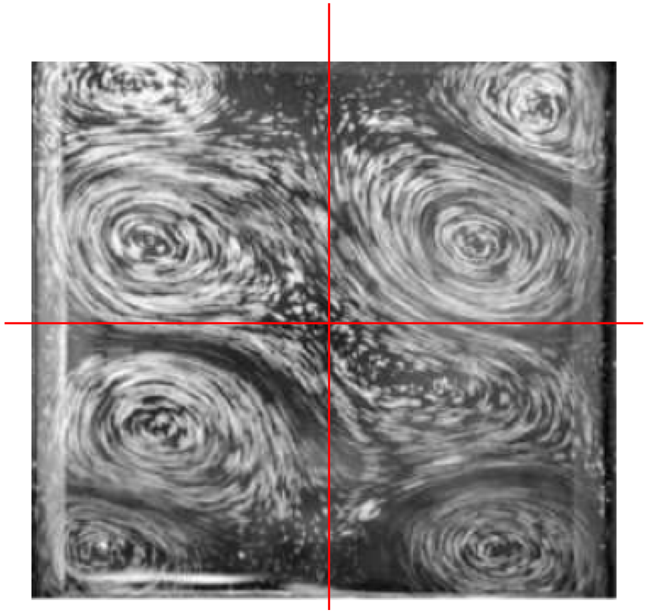
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla\pi + \frac{\mathbf{1}}{\mathbf{Re}}\nabla^{\mathbf{2}}\mathbf{u} - \frac{\mathbf{1}}{\mathbf{Rh}}\mathbf{u} + \mathbf{f}(\mathbf{x},\mathbf{y})$$

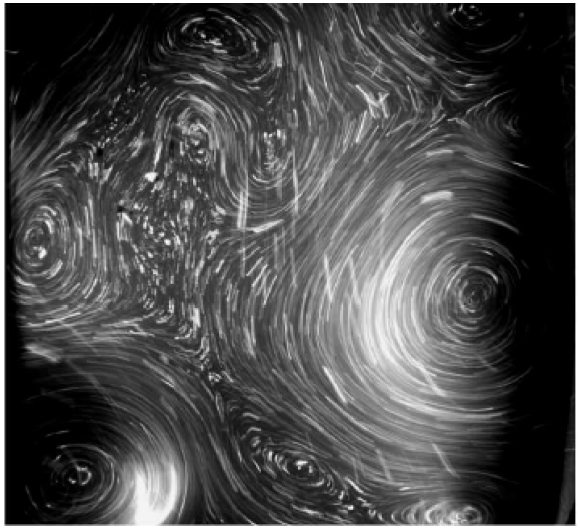
$$Re = \frac{UL}{\nu} \sim \frac{L}{\nu}\sqrt{\frac{IB}{\rho h}}\ ;\ Rh = \frac{u\tau}{L} \sim \frac{1}{L}\sqrt{\frac{Ih}{B\sigma\nu}}$$

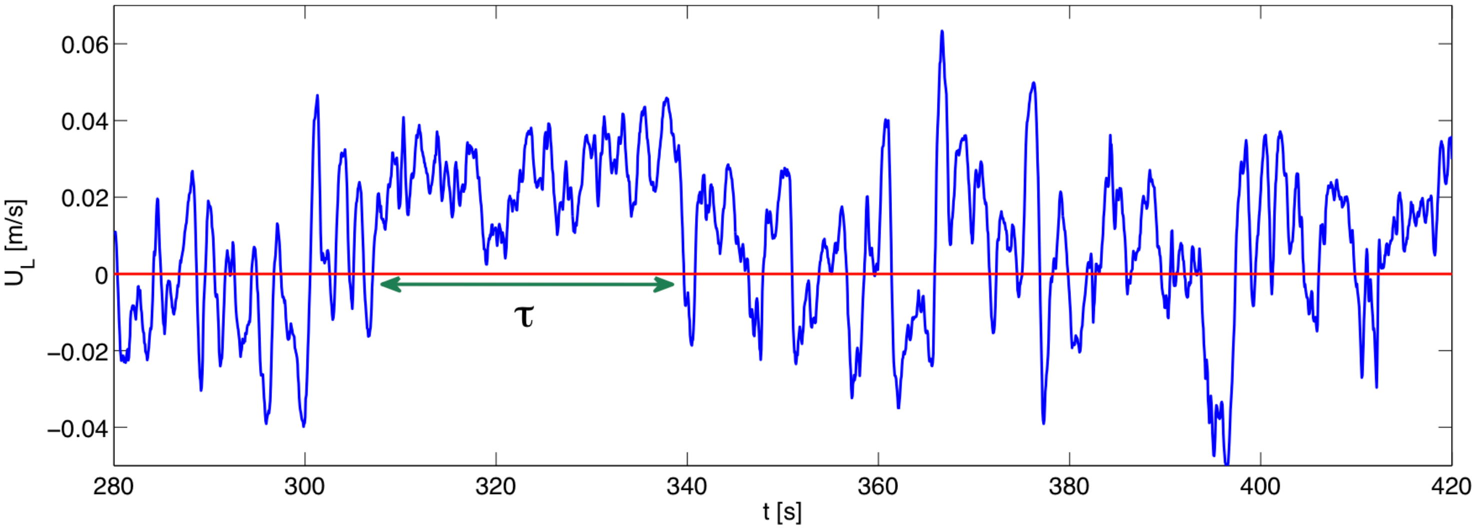
Re large ~ 10^4 .

The control parameter is Rh (inverse of large scale friction)

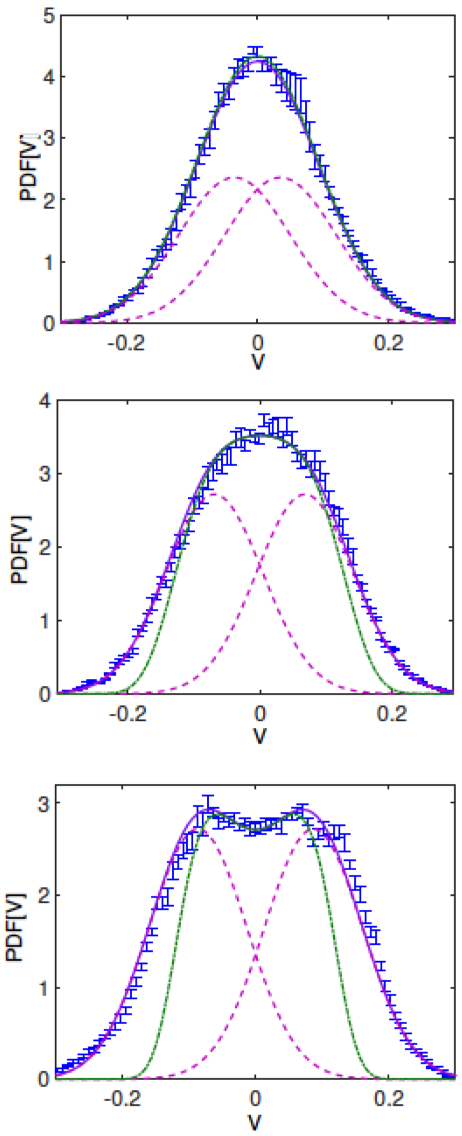
Transition to turbulence and generation of large scales when friction is decreased



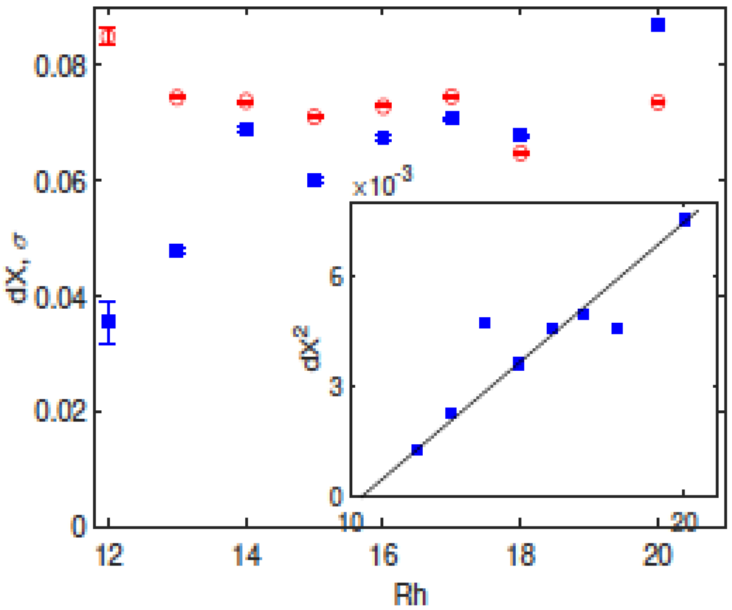




Transition from Gaussian to bimodal velocity probability density function

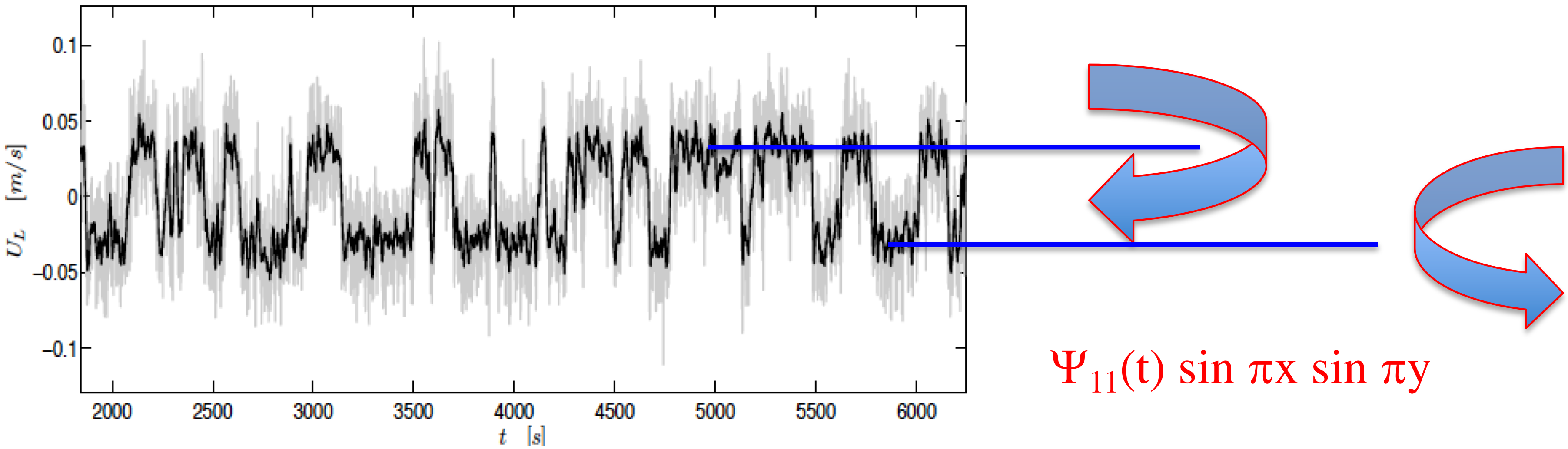


$$P[V] = (2\sqrt{2\pi}\sigma)^{-1}(exp(-(V-dX)^2/(2\sigma^2)) \\ +exp(-(V+dX)^2/(2\sigma^2)))\,.$$

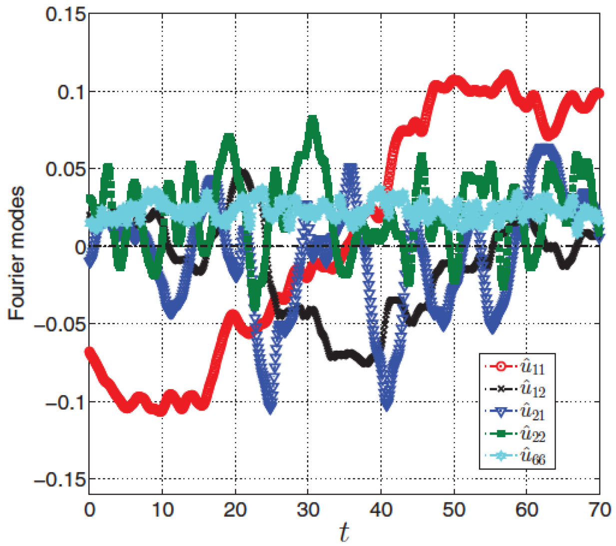


A continuous bifurcation from a turbulent regime with a Gaussian PDF to a bimodal regime with two most probable opposite values of the large scale circulation: Random reversals

Random reversals of the large scale circulation



As Rh is increased further, the mean waiting time between reversals increases and reversals stop.



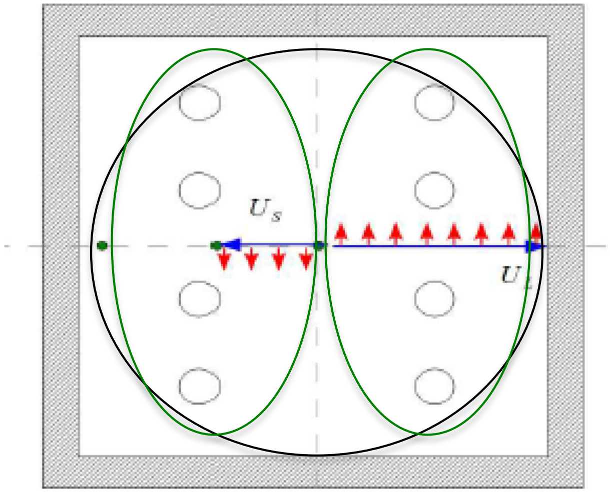
Higher velocity modes involved in the dynamics of the reversal of the large scale circulation: reversals cannot be described with a low dimensional dynamical system in general

(different from reversals of the magnetic field in a dynamo experiment)

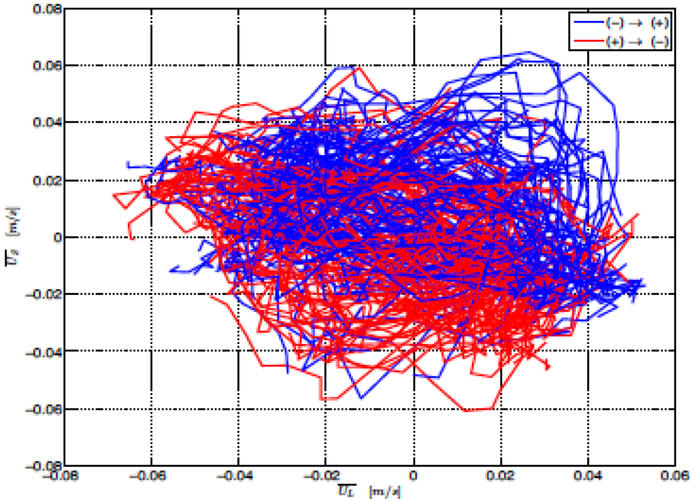
A low dimensional description of reversals ?

Definition of the two lowest modes:

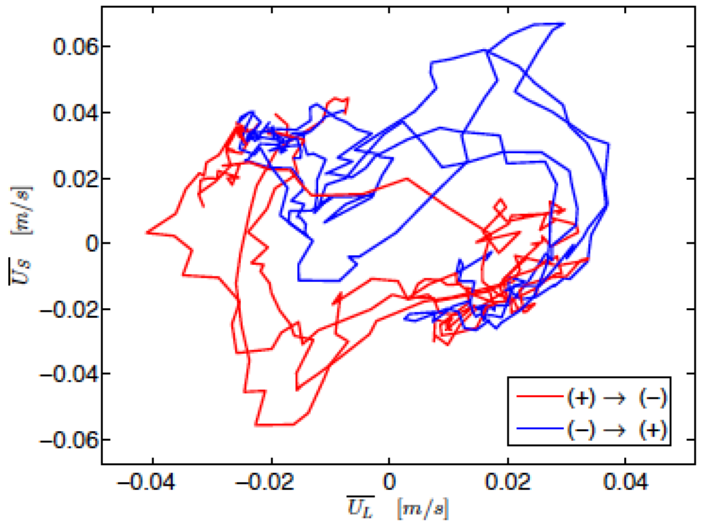
- Large scale mode (1, 1) ~ dipole
- Next mode (2,1) ~ quadrupole



Rh=38



Rh=48



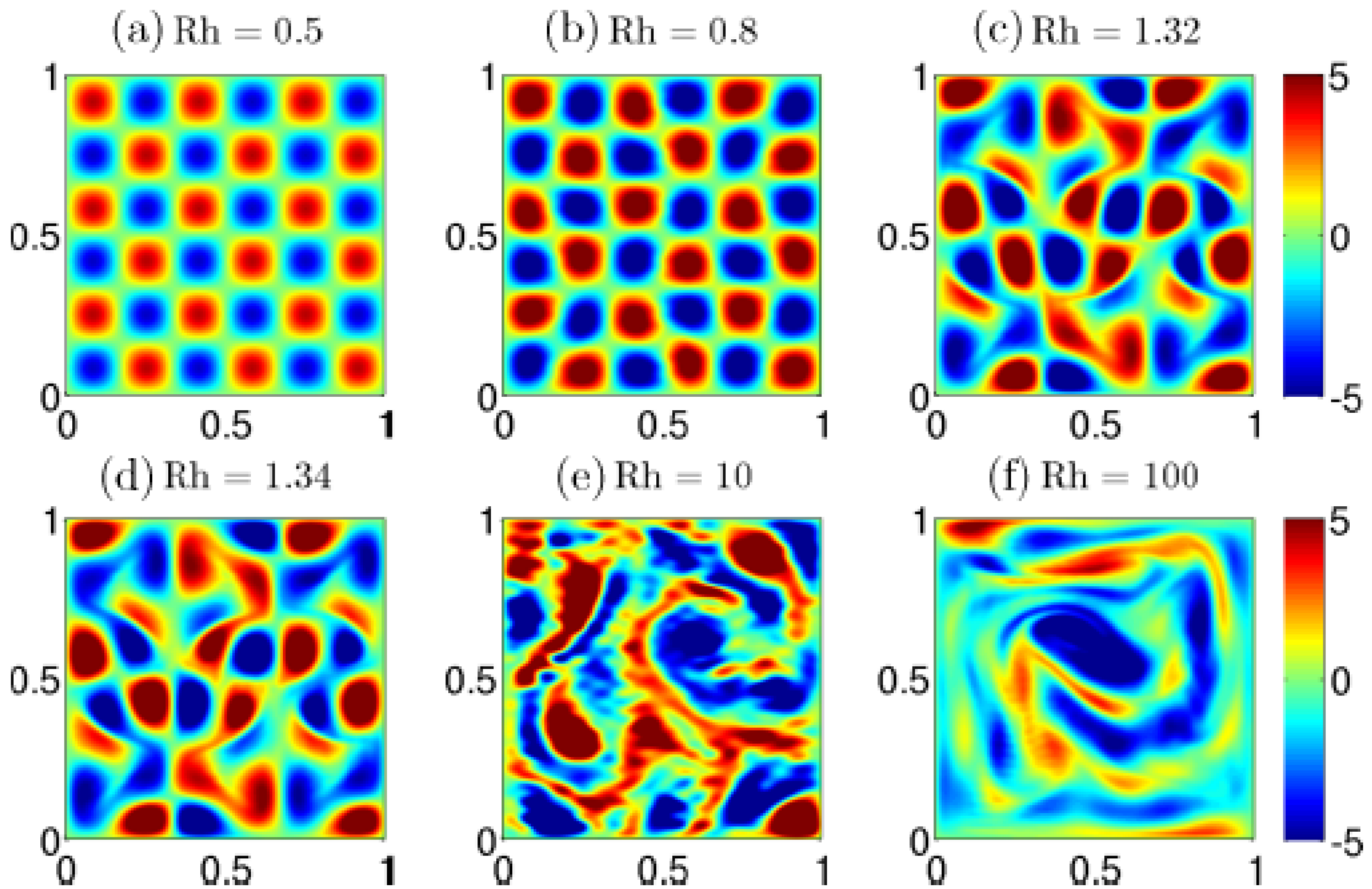
No simple trajectories related to reversals except close to the transition to the condensed regime (non reversing dominant large scale circulation)

Numerical simulations in the 2D limit

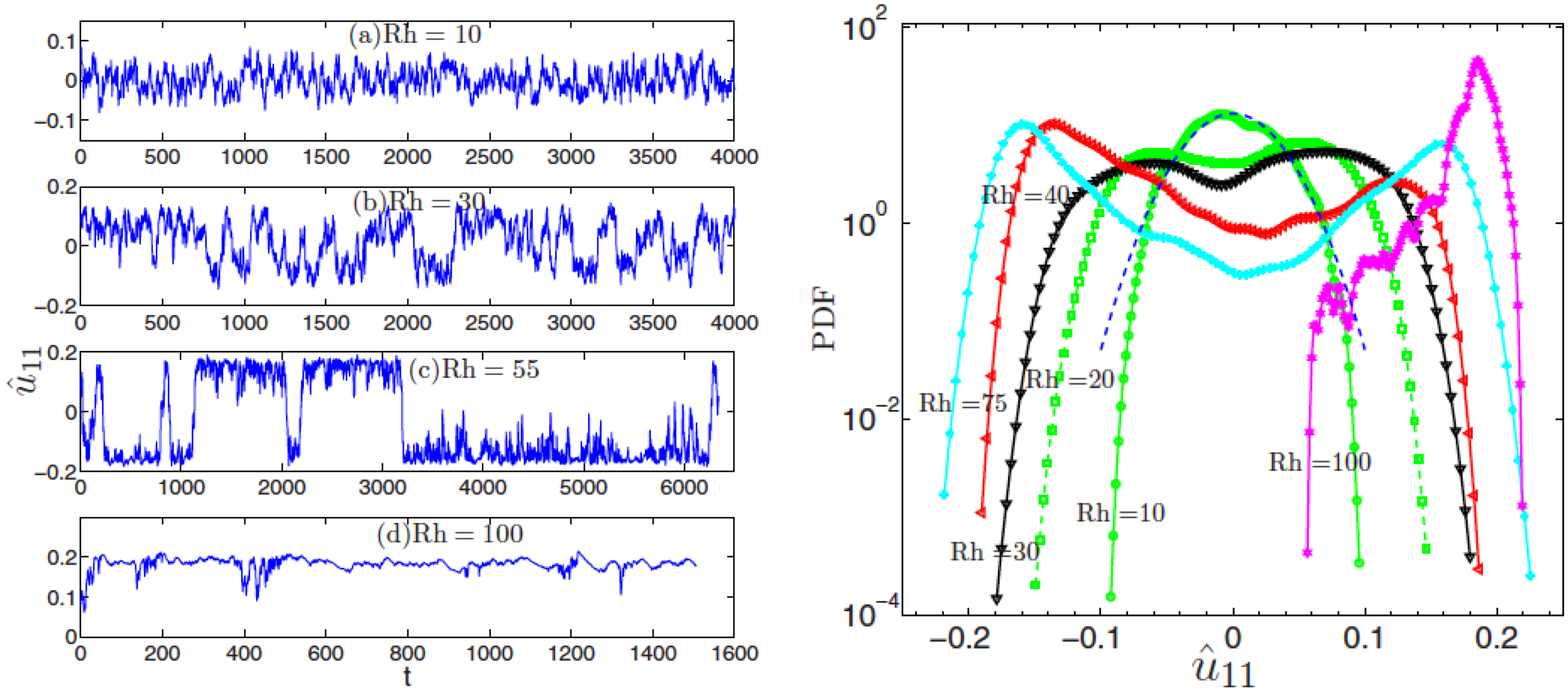
(P. Mishra et al., PRE 91, 053005 (2015))

Spatially periodic forcing of a planar flow: u=-$\partial_y\psi$, v= $\partial_x\psi$

$$\partial_t \Delta\psi + J(\Delta\psi, \psi) = -\frac{1}{Rh}\Delta\psi + \frac{1}{Re}\Delta^2\psi + 6\pi\sin(6\pi x)\sin(6\pi y)$$



Transitions within the turbulent regime: transition to bimodal PDF and reversals



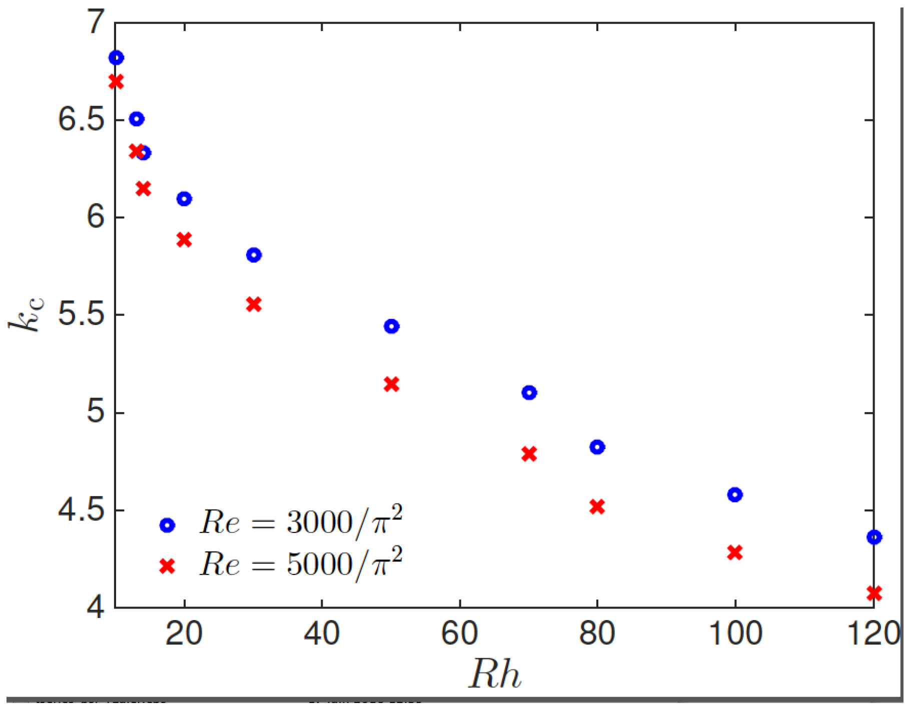
Rh controls the scale at wich the inverse cascade stops: $l_c \sim L\ Rh^{3/2}$

- Rh ~ 100 (small friction) : most of the kinetic energy in a large scale circulation
- Rh ~ 50 : large scale eddy unstable : reversals i.e. bimodal velocity PDF
- Rh ~ 10 : Gaussian velocity PDF

Transitions within the turbulent regime described by equilibrium statistical mechanics

V. Shukla, S. Fauve, M. Brachet, PRE 94, 061101 (2016)

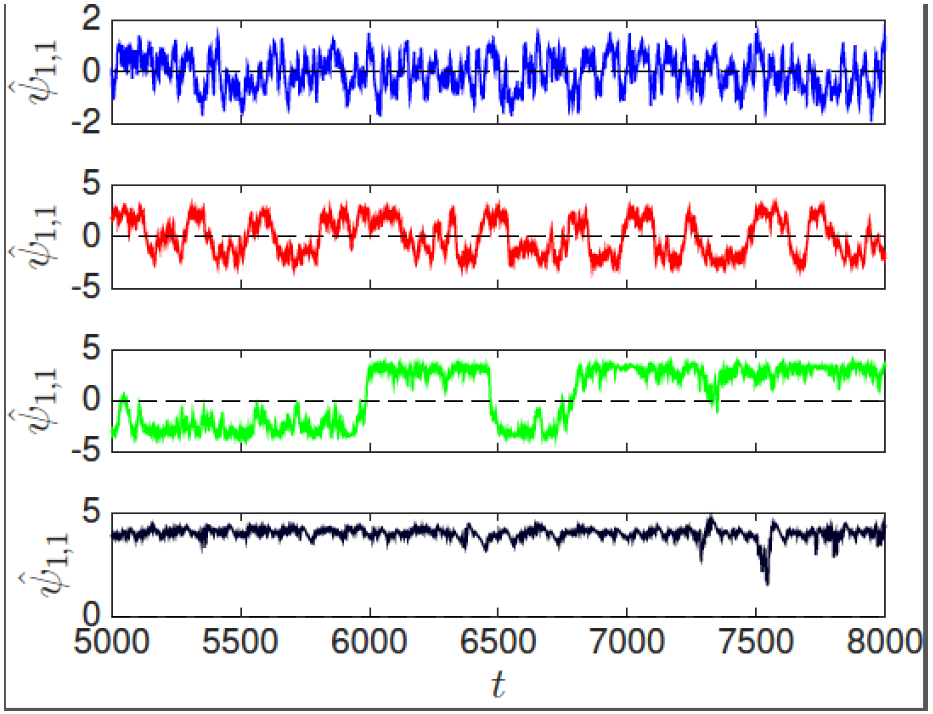
- Consider large scale modes up to the forcing wave number following the 2D Euler equation truncated at k_f
- Two quadratic invariants: energy E and enstrophy Ω
- In the full simulation, Rh determines the value of the wavenumber k_c at which the inverse cascade stops: $k_c^2 = \Omega / E$



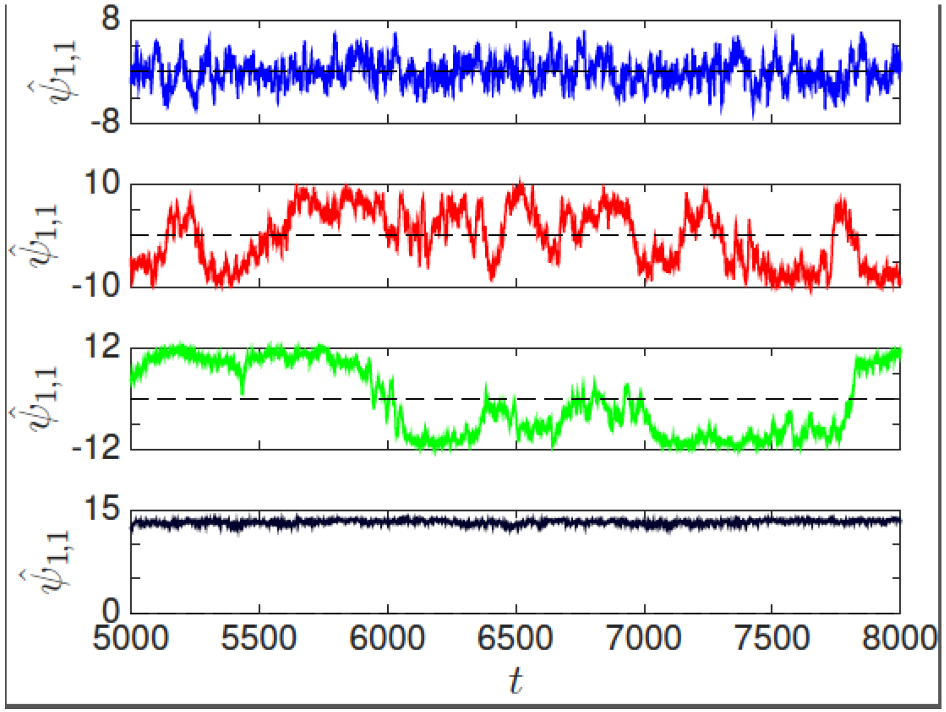
Direct simulations versus truncated Euler equation

Initial conditions for the truncated Euler equation with decreasing values of k_c .

This corresponds to increase the width of the large scales [k_c k_f]

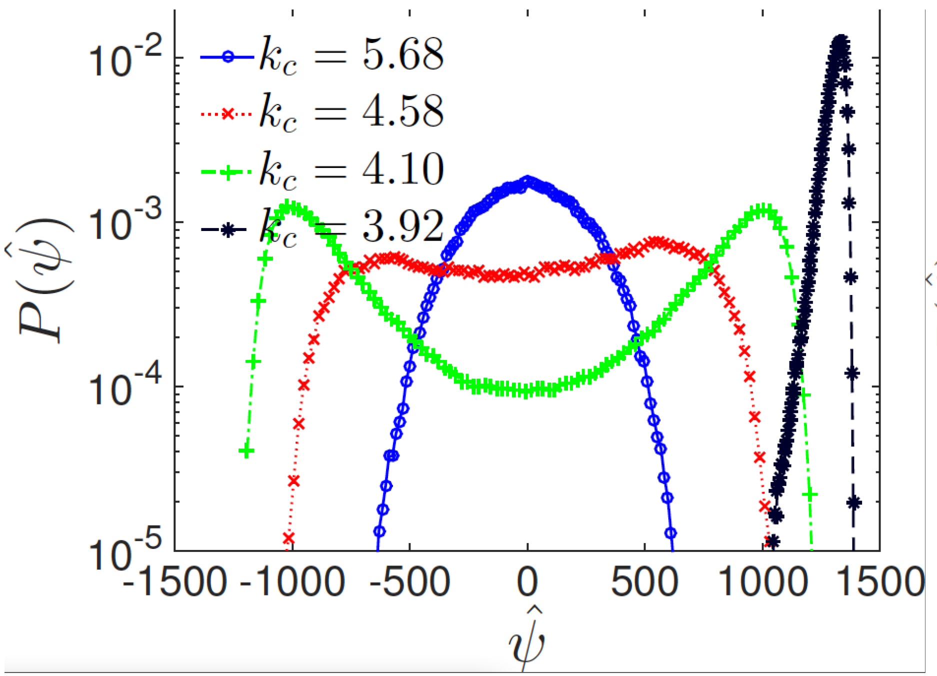


DNS 2D Navier-Stokes

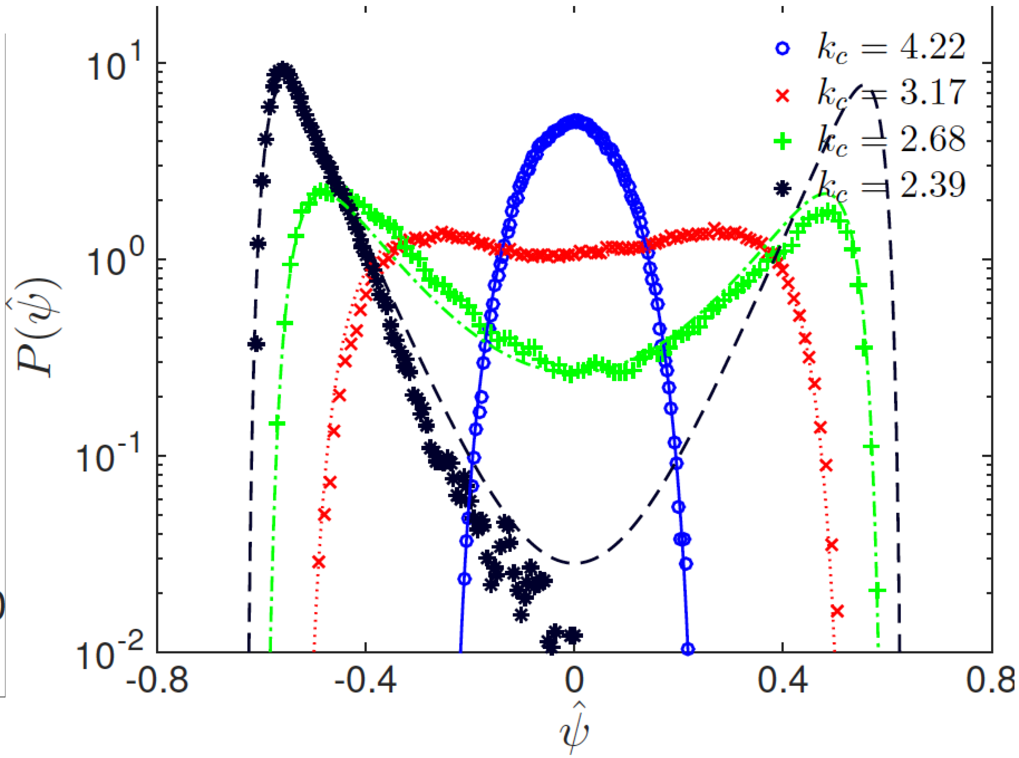


Truncated Euler equation

Truncated Euler equation and microcanonical ensemble

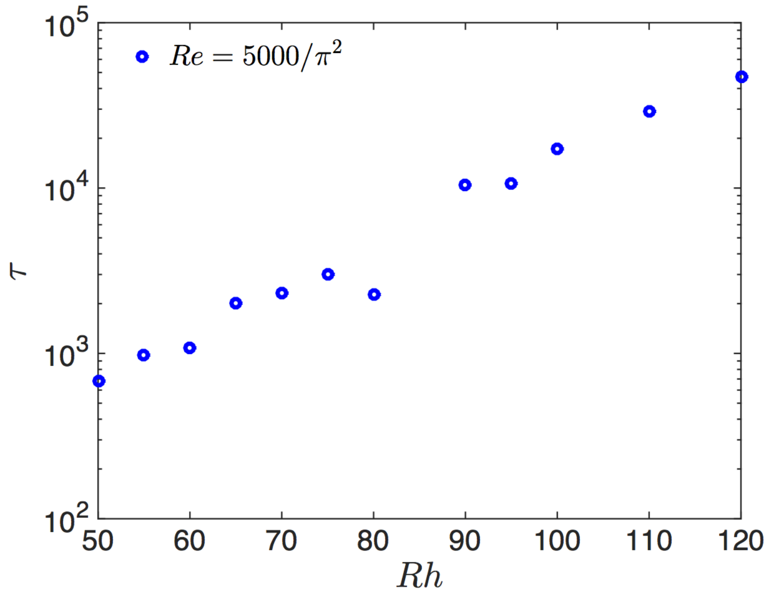


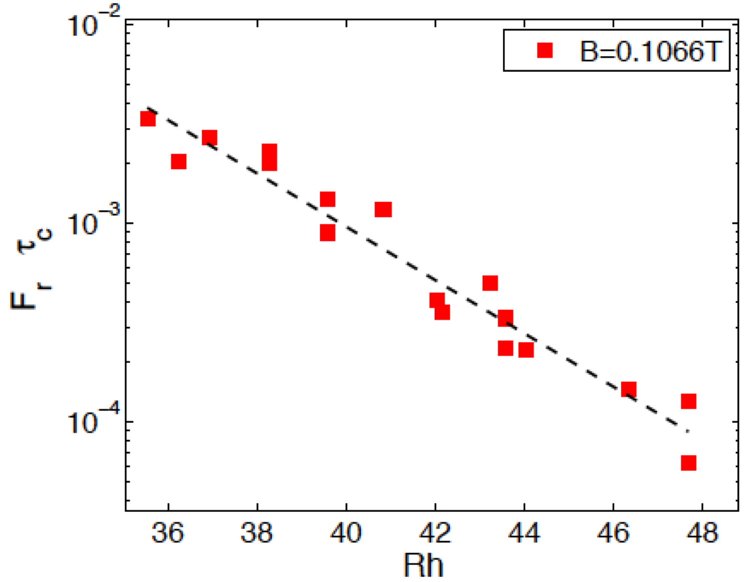
Euler equation truncated at the forcing wavenumber

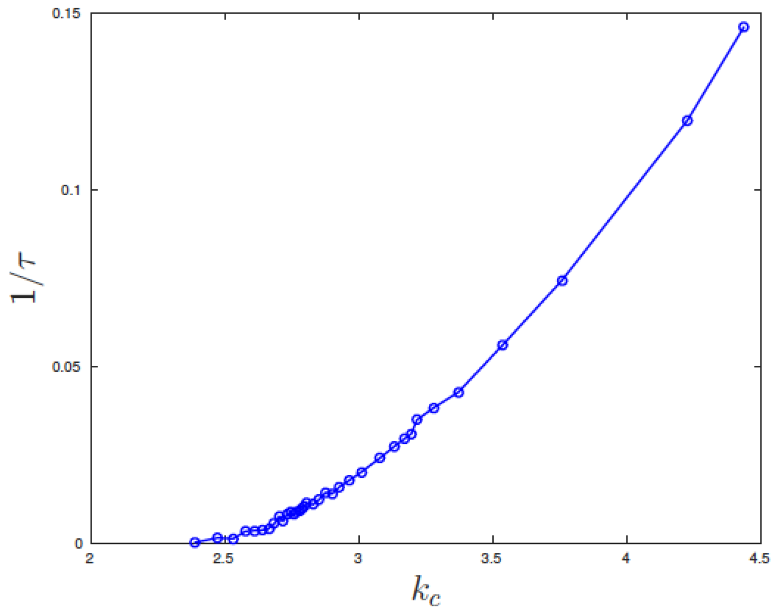


Minimal model and PDF obtained from the microcanonical ensemble

Divergence of the mean waiting time between reversals toward the condensed regime







Experiments and DNS of the 2D Navier-Stokes equation

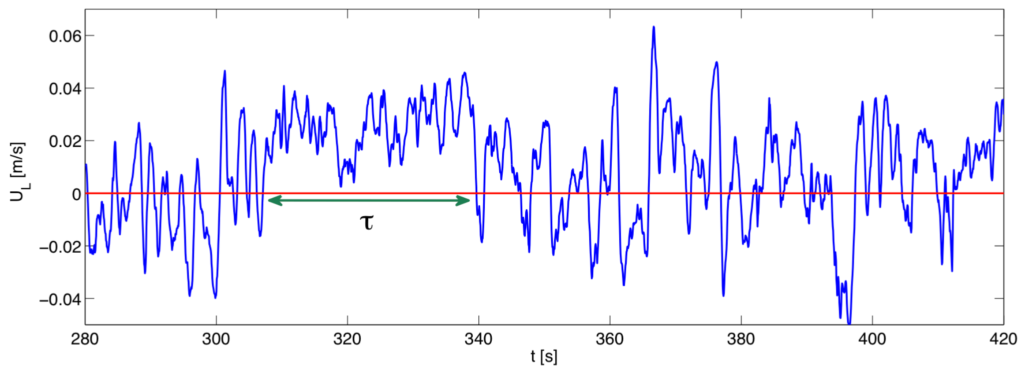
$\tau \sim \exp Rh$

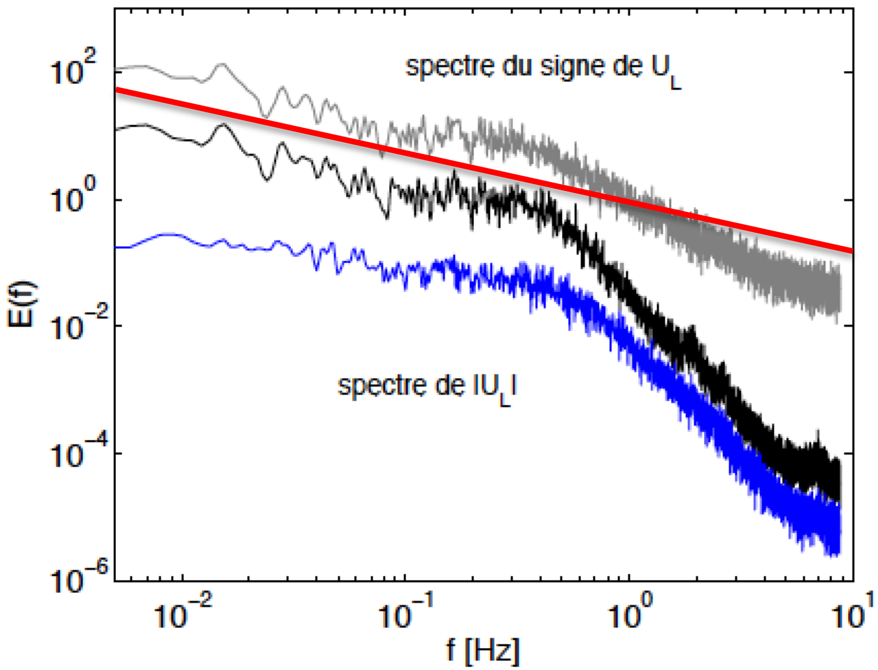
Truncated Euler

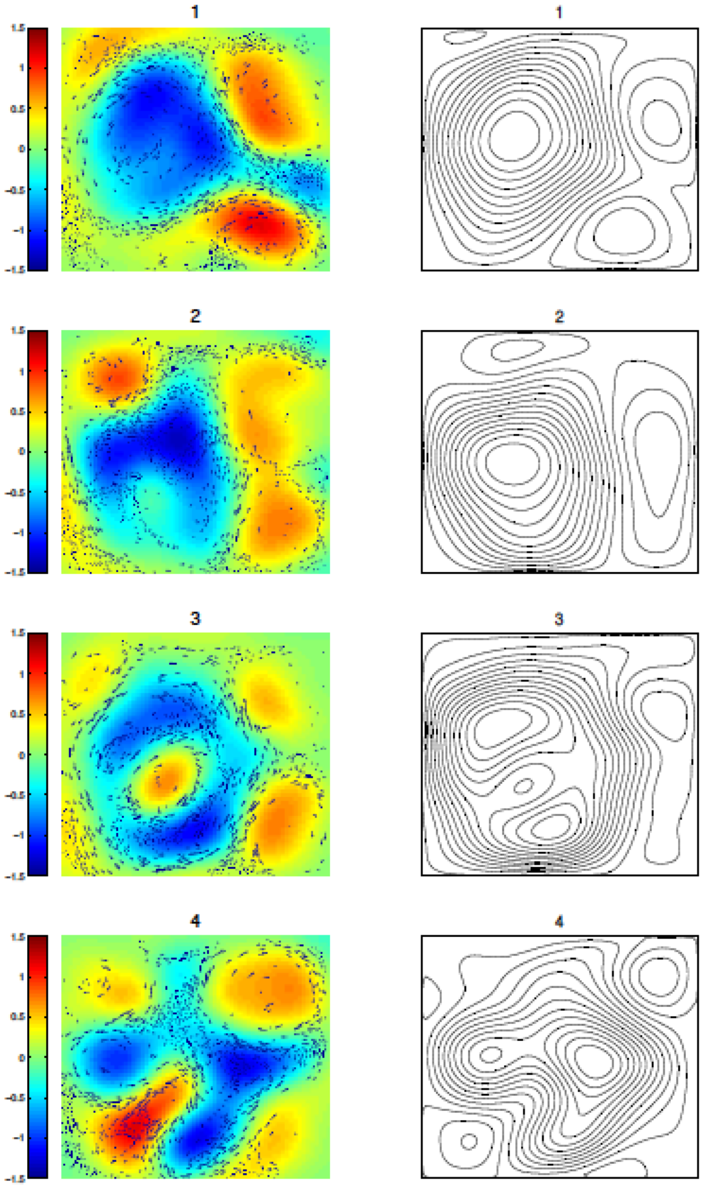
$1/\tau \sim k_c - k_0$

Correlations and 1/f noise in the reversal regime

J. Herault, F. Pétrélis, S. Fauve (2015) EPL 111, 44002, J. Stat. Phys. 161, 1379





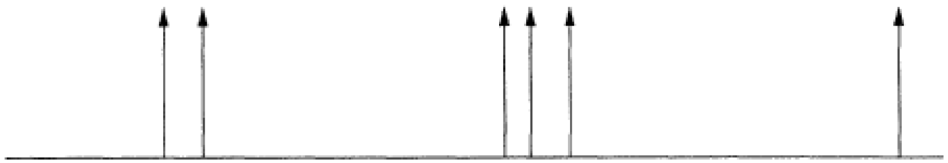


Low frequency spectra and PDF of waiting times

Lowen and Teich, PRE 47, 992 (1993)

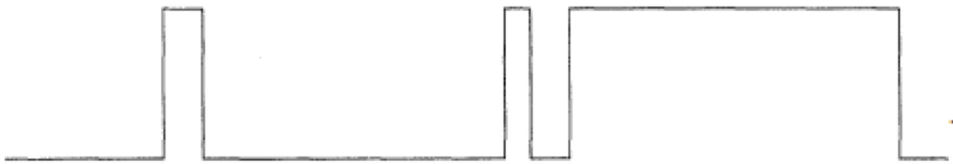
Distribution of waiting time between successive events

$P(\tau) \approx \tau^{\beta}$ for $\tau_i << \tau << \tau_e$



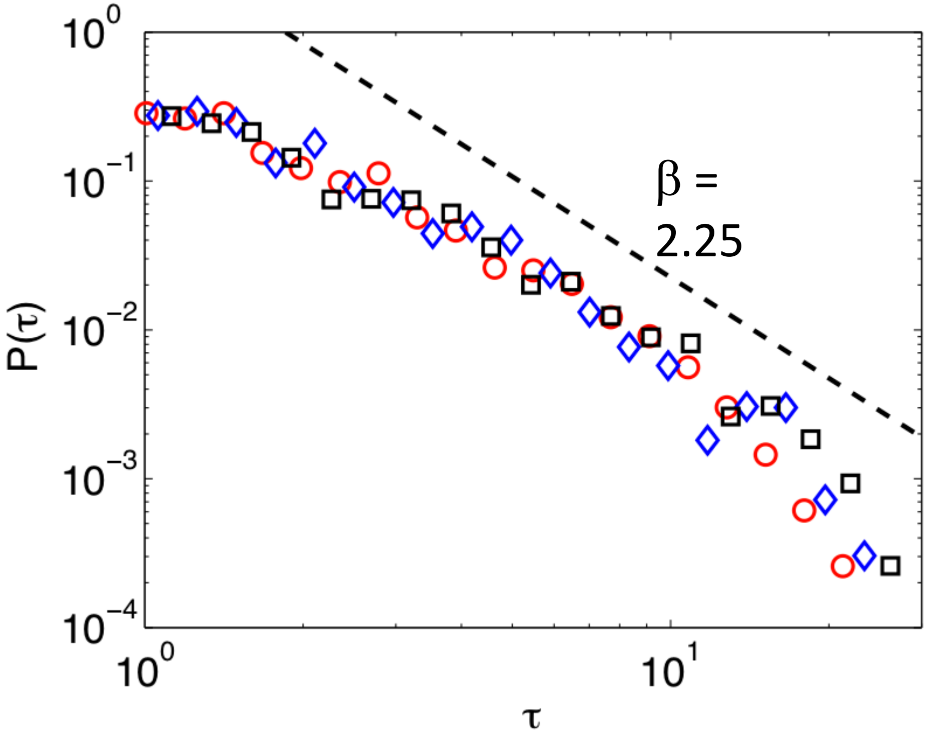
$$E(f) \sim \begin{cases} f^{-(\beta-1)} & \text{for} \quad 1 < \beta < 2 \\ \ln(f\tau_i)^{-2} f^{-1} & \text{for} \quad \beta = 2 \\ f^{-(3-\beta)} & \text{for} \quad 2 < \beta < 3 \\ \ln(\tau_i f) & \text{for} \quad \beta = 3 \end{cases}$$

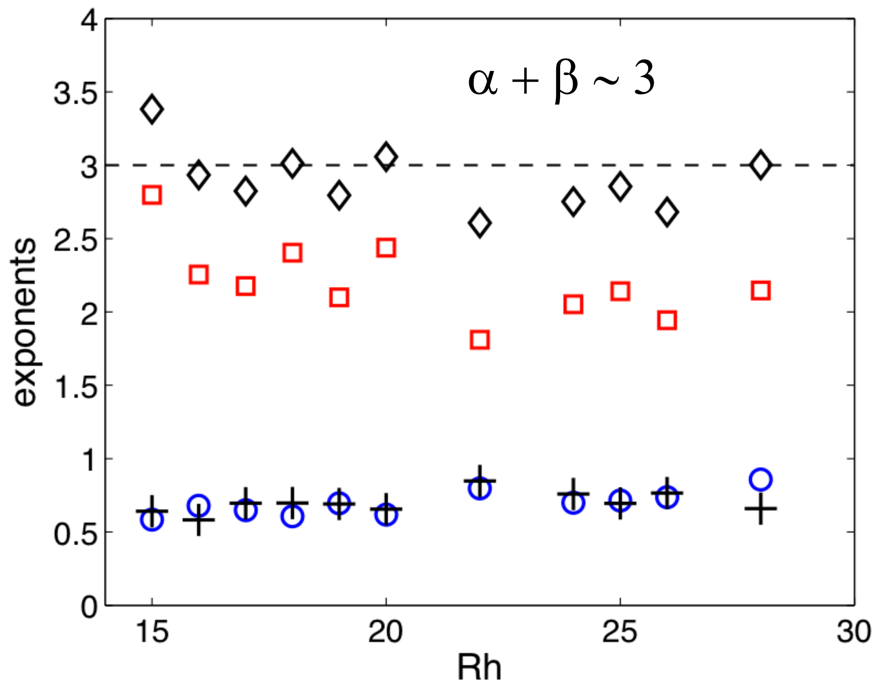
(b)



$$E(f) \sim \begin{cases} f^{-(3-\beta)} & \text{for} \quad 1 < \beta < 3 \\ \ln(\tau_i f) & \text{for} \quad \beta = 3 \end{cases}$$

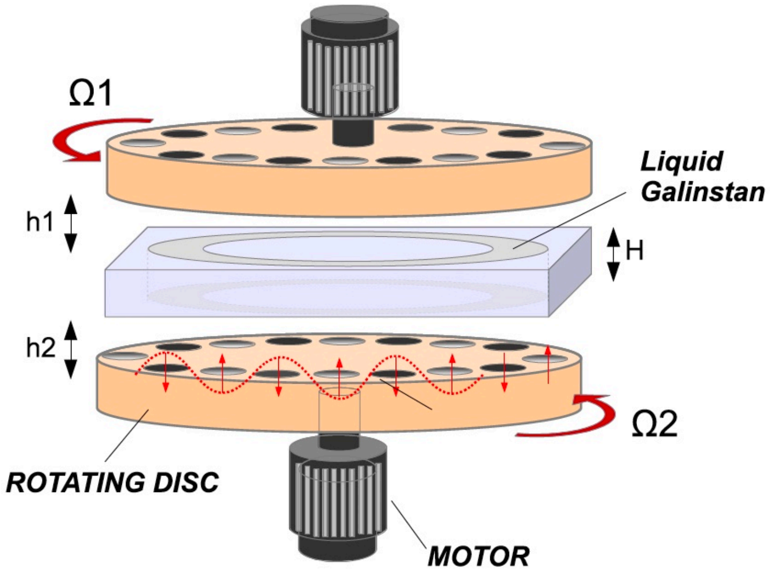
PDF of waiting times

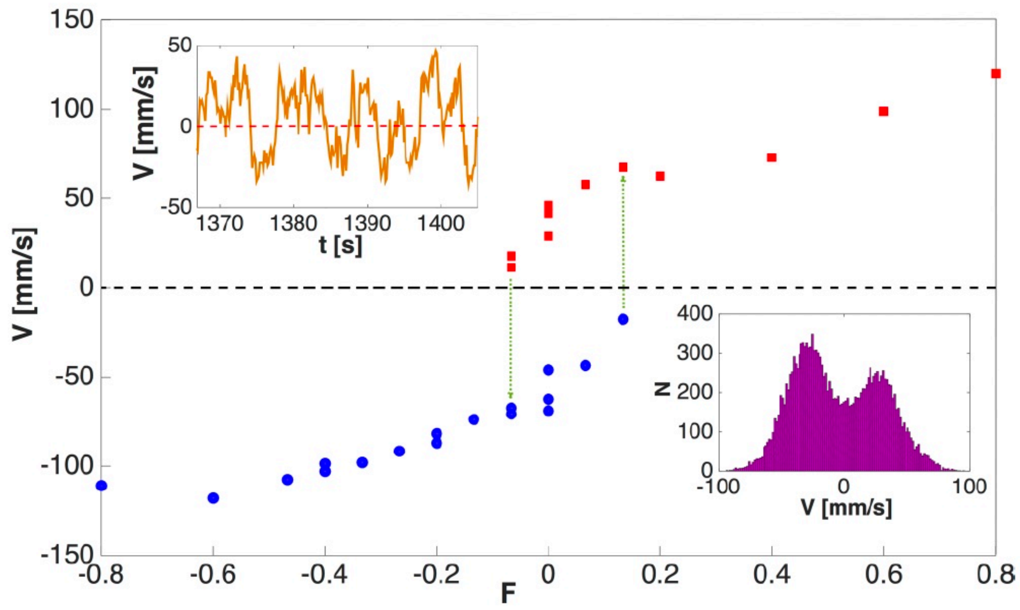




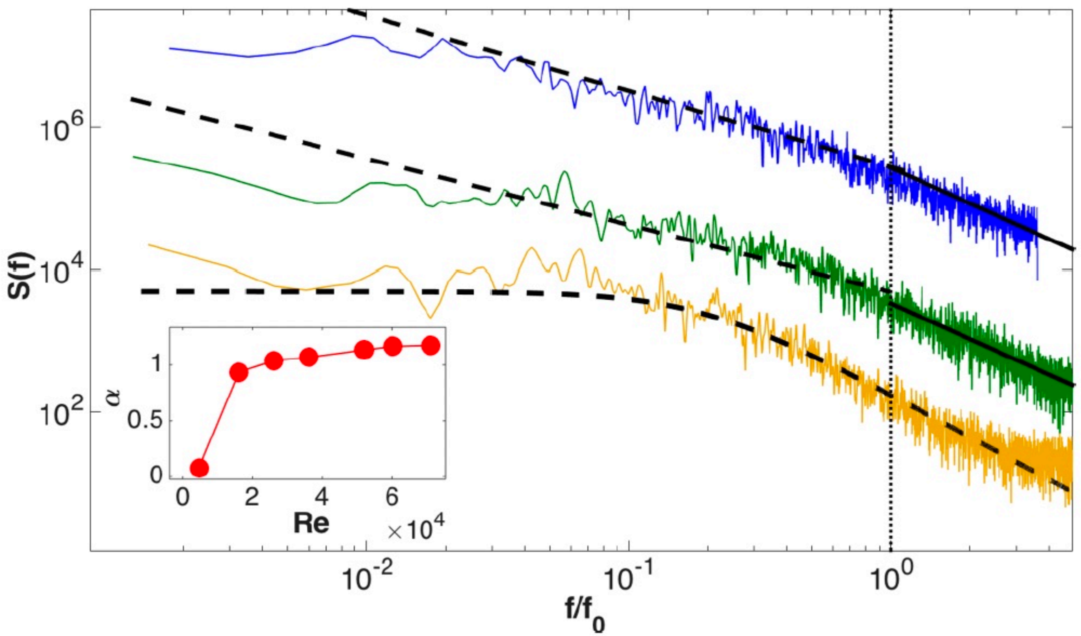
A MHD induction driven shear flow

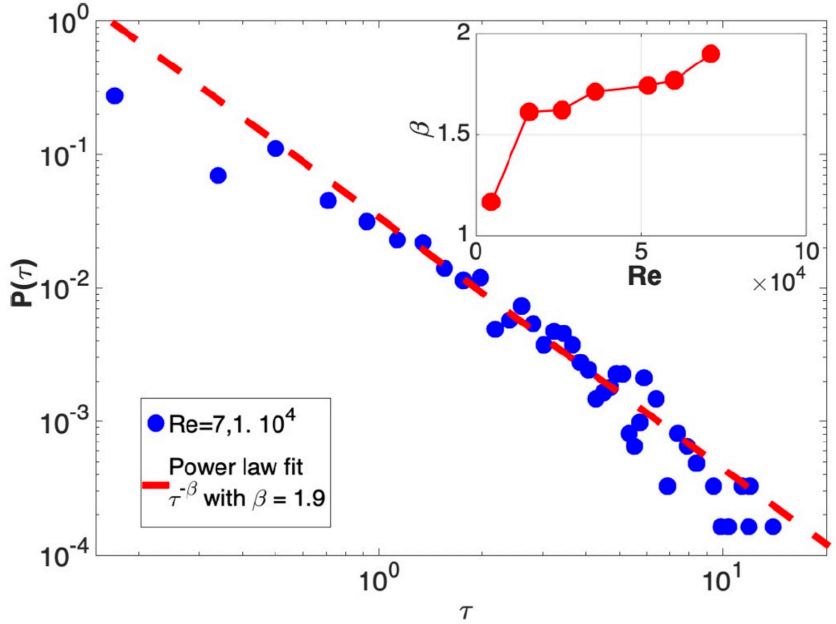
M. Pereira, C. Gissinger, S. Fauve, PRE 99, 023106 (2019)

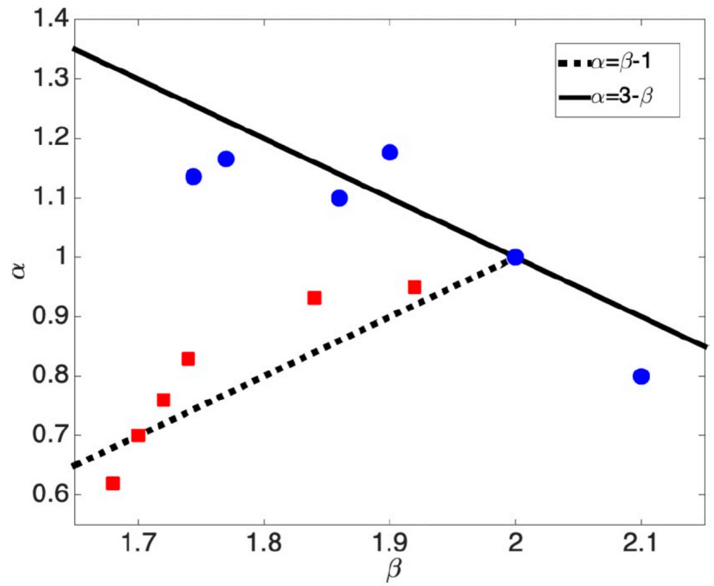




1/f noise in shear flows

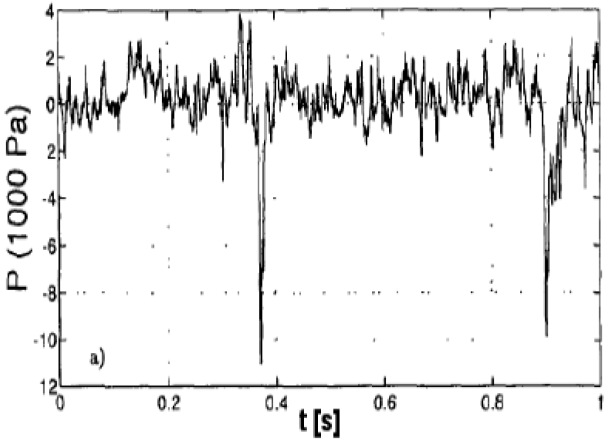


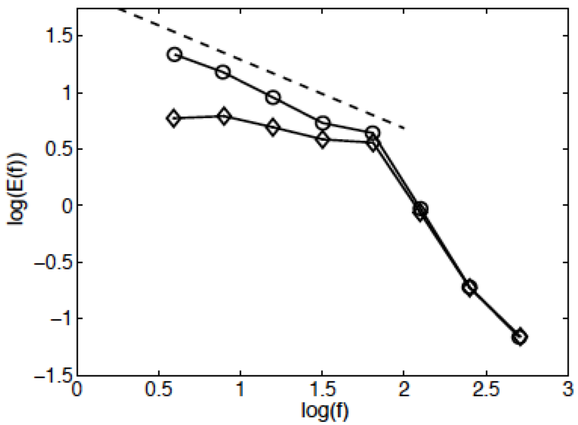


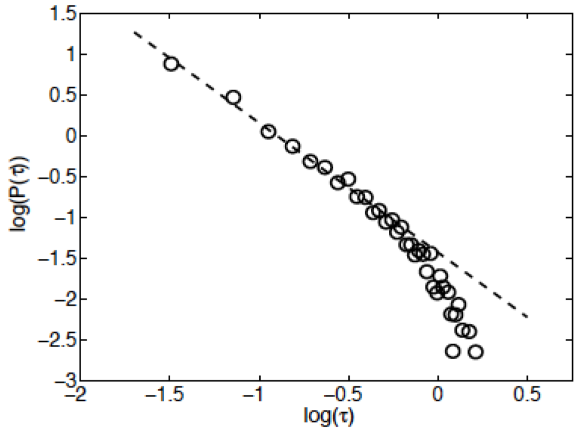


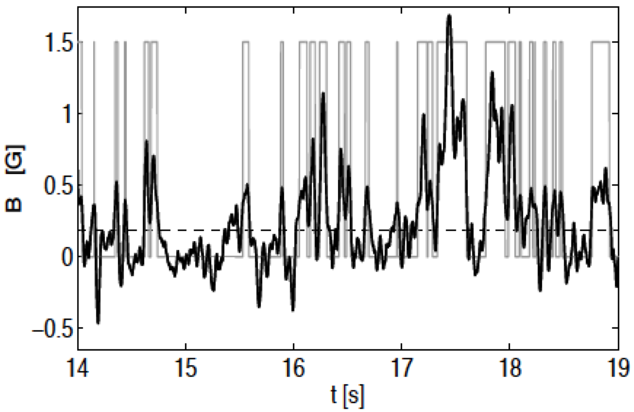
1/f noise and coherent structures in turbulence

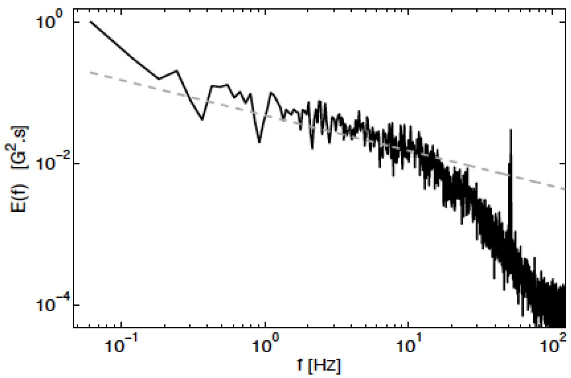
System	Variable	Process	α	β	Relation
Two-dimensional turbulence	flow rate	symmetric	0.7	2.25	$\alpha = 3 - \beta$
Von Kárman Sodium Dynamo	magnetic field	bursting	0.5	2.5	$\alpha = 3 - \beta$
Von Kárman flow	pressure	bursting	0.6	1.58	$\alpha = \beta - 1$

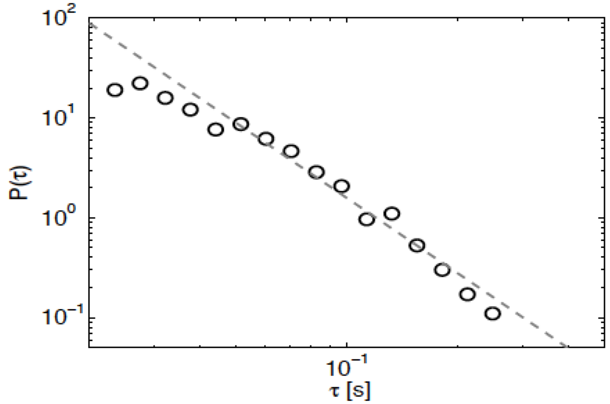












Conclusions

- Scales larger than the forcing scale of turbulent flows are in statistical equilibrium.
- Transitions betweeen different turbulent regimes can be studied as « bifurcations » of the probability density function and modeled using the truncated 2D Euler equation.
- Low frequency behavior of turbulent flows often related to the dynamics of coherent structures