New Challenges in Turbulence Research VI
Les Houches, February 2021

Large Eddy Simulations of Turbulence and Insights generated regarding Wind Energy

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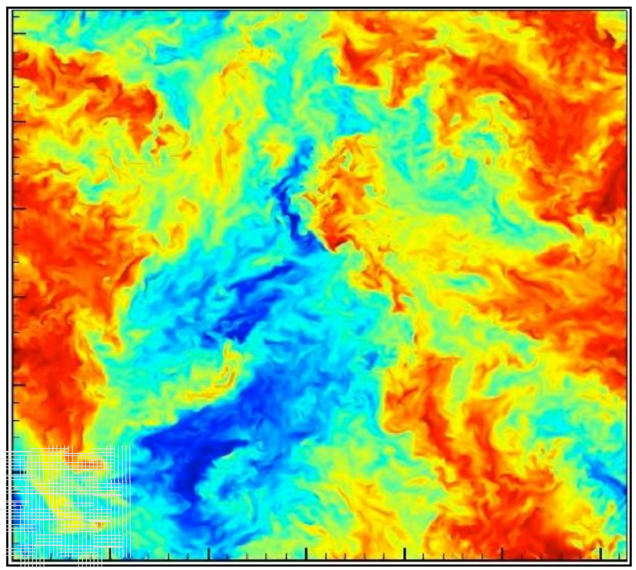
Coarse-graining - Large-Eddy-Simulation (LES):

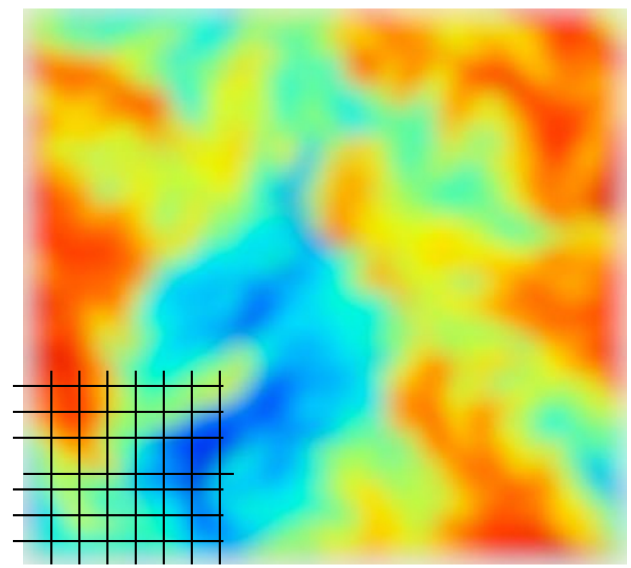
Coarse-graining for more affordable simulations

$$u_1(x, y, z_0, t_0)$$

4×10^9 d.o.f.



$$\tilde{u}_1(x, y, z_0, t_0)$$

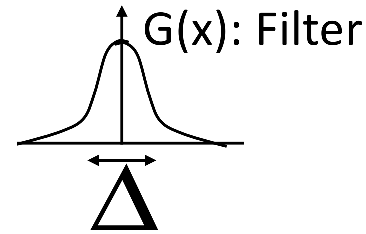


10^5 d.o.f.

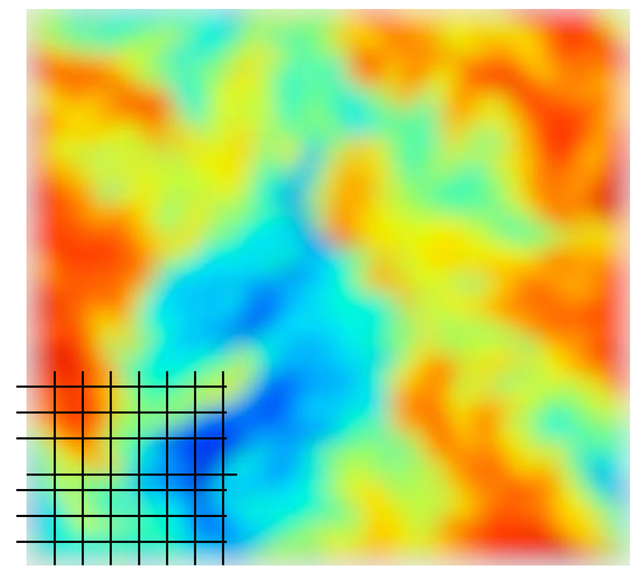
Large-eddy-simulation (LES) and filtering:

N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$



$$\tilde{u}_1(x, y, z_0, t_0)$$



Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

where SGS stress tensor is:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Most common modeling approach: eddy-viscosity

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

$$\nu_{sgs} = l \times vel \sim \Delta \times (\Delta\,|\tilde{\mathbf{S}}|)$$

$$\nu_{sgs} = (c_s\Delta)^2\,|\tilde{S}|$$

c_s: "Smagorinsky coefficient"

HISTORY: 1960s, 1970s

- J Smagorinsky
- DK Lilly
- J Deardorff

Effects of τ_{ij} upon resolved motions: **Energetics (kinetic energy):**

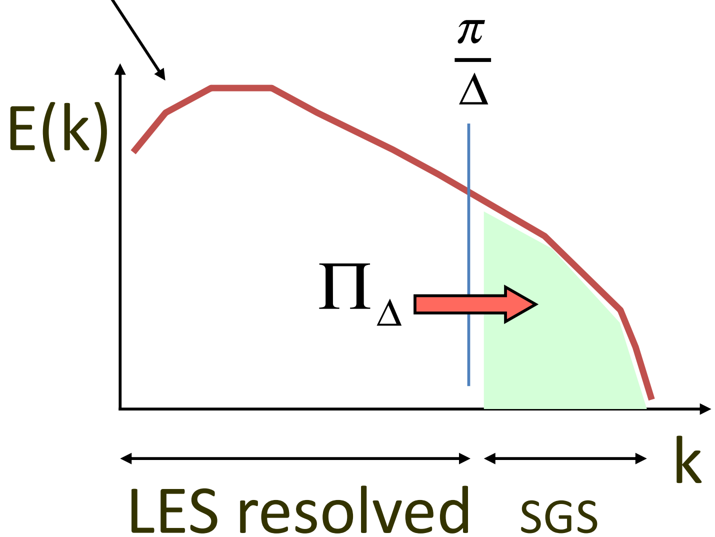
$$\frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial x_k} = -\frac{\partial}{\partial x_j}(\ldots.) - 2\nu \tilde{S}_{jk}\tilde{S}_{jk} - \left(-\tau_{jk}\tilde{S}_{jk}\right)$$

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

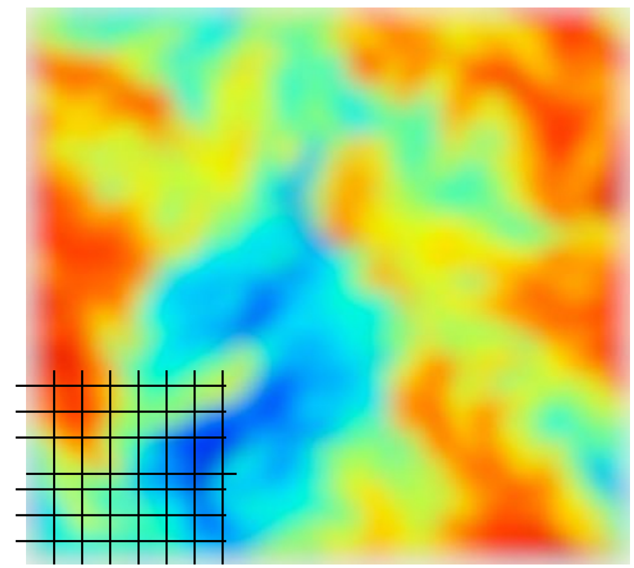
$$\Pi_\Delta = -\left\langle \tau_{jk}\tilde{S}_{jk} \right\rangle$$

Inertial-range flux

$$\varepsilon = \frac{u'^3}{L}$$



$\tilde{u}_1(x, y, z_0, t_0)$



Δ

Two-point structure of coarse-grained NS:

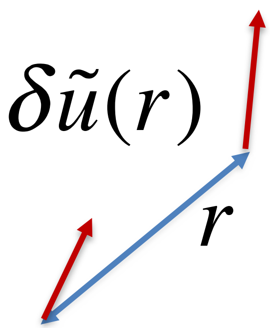
$$\frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial x_k} = -\frac{\partial}{\partial x_j}(....) - 2\nu \tilde{S}_{jk}\tilde{S}_{jk} - \left(-\tau_{jk}\tilde{S}_{jk}\right)$$

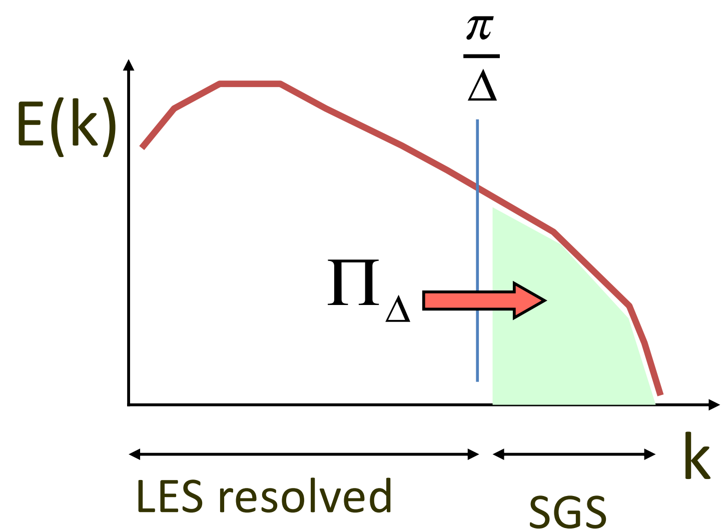
$$\Pi_\Delta = -\left\langle \tau_{jk}\tilde{S}_{jk} \right\rangle$$

Similarly to von Karman-Howarth and Kolmogorov equations,
For isotropic turbulence, in inertial range (CM PoF 1994):

“ **sufficient condition at $r >> \Delta$: predict Π_Δ** ” **correctly**

$$\left\langle \delta\tilde{u}(r)^3 \right\rangle + 6\left\langle \tau_{LL}(x)\delta\tilde{u}(r) \right\rangle = -\frac{4}{5}\Pi_\Delta r$$



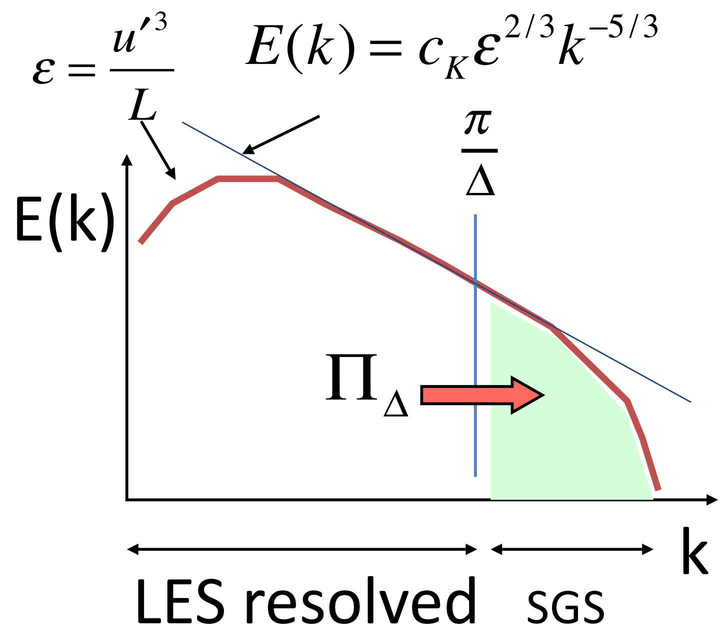


$$\tau_{ij} = -2(c_s\Delta)^2 \,|\tilde{S}|\, \tilde{S}_{ij}$$

Simplest model that can “control” the “dissipation”

But how much is c_s?

Theoretical calibration of c_s (D.K. Lilly, 1967) HIT:



$$\Pi_\Delta = \varepsilon = -\left\langle \tau_{ij} \tilde{S}_{ij} \right\rangle \qquad \tau_{ij} = -2(c_s\Delta)^2 |\tilde{S}| \tilde{S}_{ij}$$

$$\varepsilon = c_s^2 \Delta^2 2 \left\langle |\tilde{S}| \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle$$

$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle^{3/2}$$

$$\left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle = \frac{1}{2}\left\langle \frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)\right\rangle =$$

$$= \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k_j{}^2 \Theta_{ii}(\mathbf{k}) + k_i k_j \Theta_{ij}(\mathbf{k})] d^3\mathbf{k} = \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k^2 (\frac{E(k)}{4\pi k^2}(\delta_{ii} - \frac{k^2}{k^2})) + 0] d^3\mathbf{k}$$

$$= c_K \varepsilon^{2/3} \frac{1}{2} \int_0^{\pi/\Delta} k^{-5/3+2} \frac{3-1}{4\pi k^2} 4\pi k^2 dk = c_K \varepsilon^{2/3} \int_0^{\pi/\Delta} k^{1/3} dk = c_K \varepsilon^{2/3} \frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3}$$

$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left(c_K \varepsilon^{2/3} \frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3} \right)^{3/2}$$

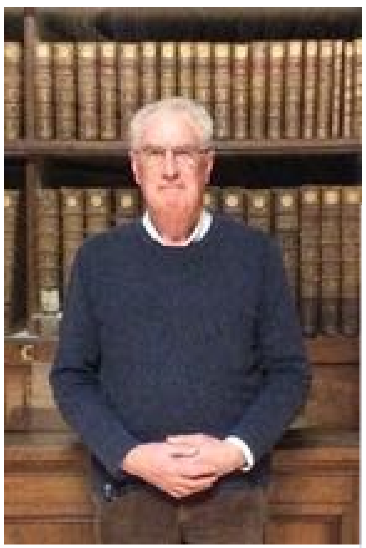
$$\Rightarrow 1 \approx c_s^2 \pi^2 \left(\frac{3c_K}{2}\right)^{3/2} \Rightarrow c_s = \left(\frac{3c_K}{2}\right)^{-3/4} \pi^{-1}$$

$$c_K = 1.6 \Rightarrow c_s \approx 0.16$$

How to avoid "tuning" and case-by-case adjustments of model coefficient in LES?

The Dynamic Model (30 years anniversary)

(Germano et al. Physics of Fluids, 1991)



Masimo Germano
(proposed G-identity)

A dynamic subgrid-scale eddy viscosity model

Massimo Germano,[a] Ugo Piomelli,[b] Parviz Moin, and William H. Cabot
Center for Turbulence Research, Stanford, California 94305

(Received 14 November 1990; accepted 7 March 1991)

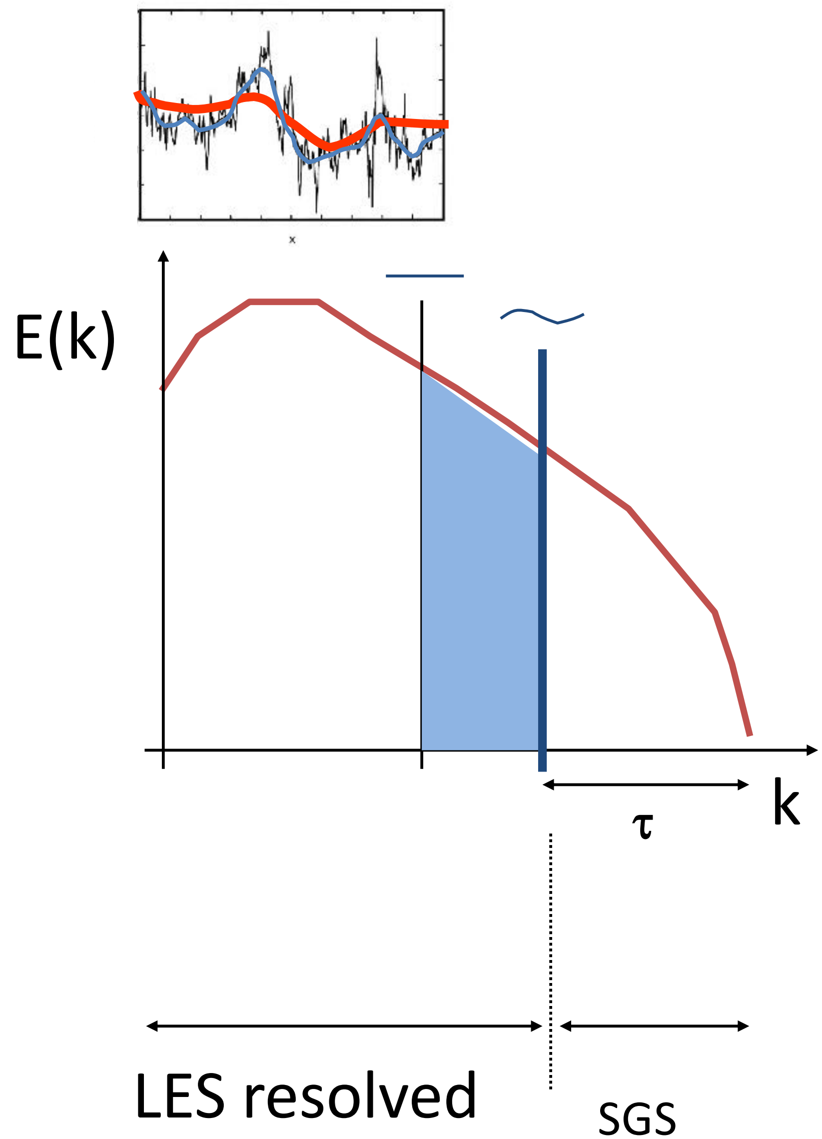
One major drawback of the eddy viscosity subgrid-scale stress models used in large-eddy simulations is their inability to represent correctly with a single universal constant different turbulent fields in rotating or sheared flows, near solid walls, or in transitional regimes. In the present work a new eddy viscosity model is presented which alleviates many of these drawbacks. The model coefficient is computed dynamically as the calculation progresses rather

Germano identity and dynamic model

(Germano et al. 1991):

Exact (“rare” in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \qquad\qquad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$



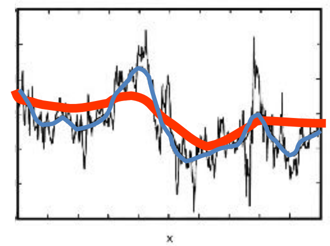
Germano identity and dynamic model

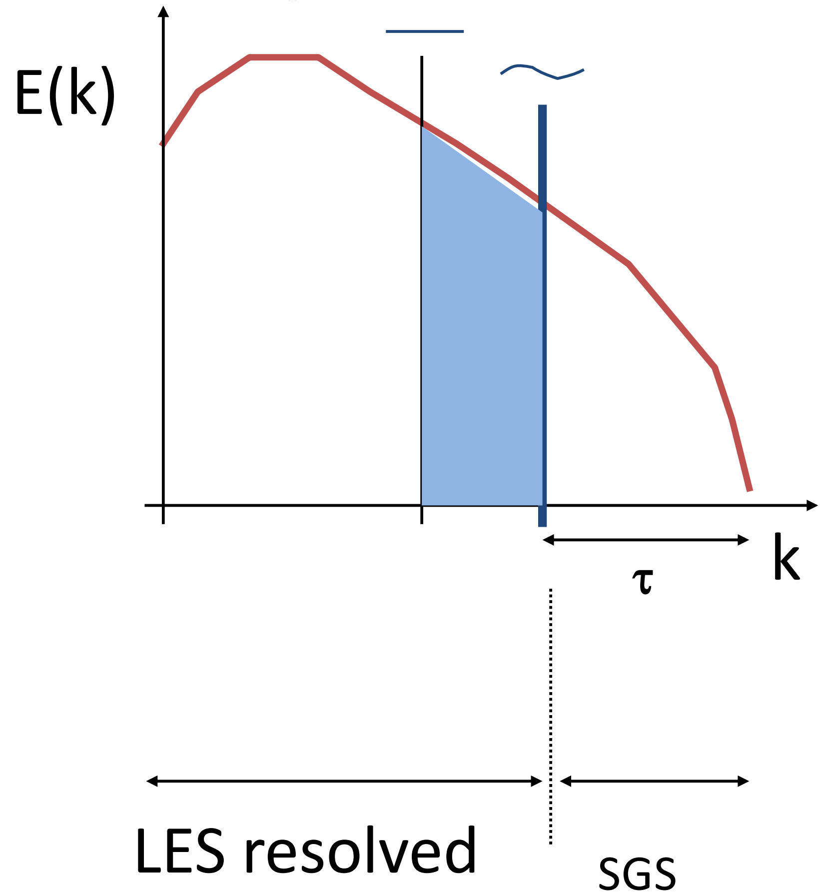
(Germano et al. 1991):

Exact ("rare" in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \quad - \overline{\tilde{u}_i \tilde{u}_j} \; + \overline{\tilde{u}_i \tilde{u}_j} \quad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$T_{ij} = \qquad \bar{\tau}_{ij} \quad + \quad L_{ij}$$





Germano identity and dynamic model

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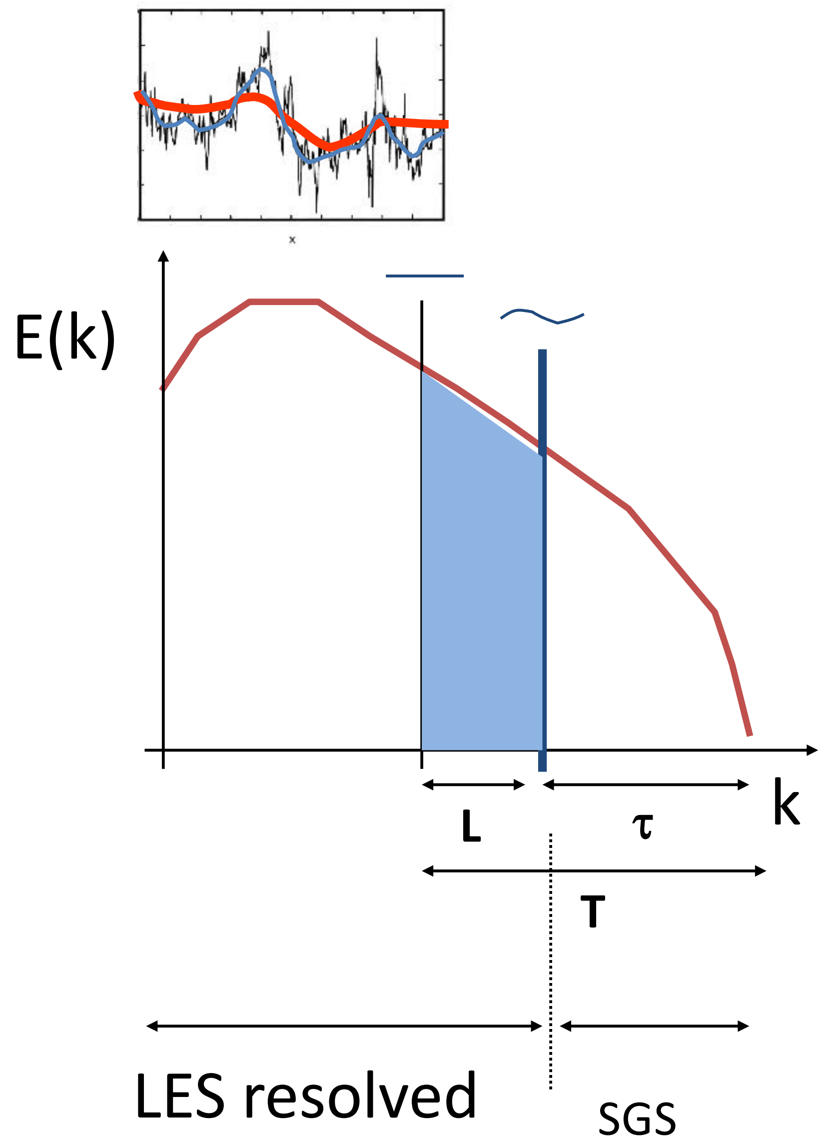
$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i \tilde{u}_j} + \overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$



Germano identity and dynamic model

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Exact ("rare" in turbulence):

$$\overline{\widetilde{u_iu_j}} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j = \overline{\widetilde{u_iu_j}} - \overline{\tilde{u}_i\tilde{u}_j} + \overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j$$

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

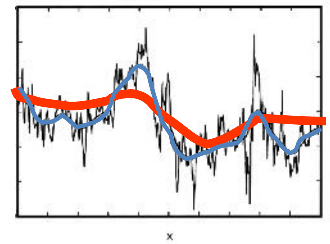
$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$

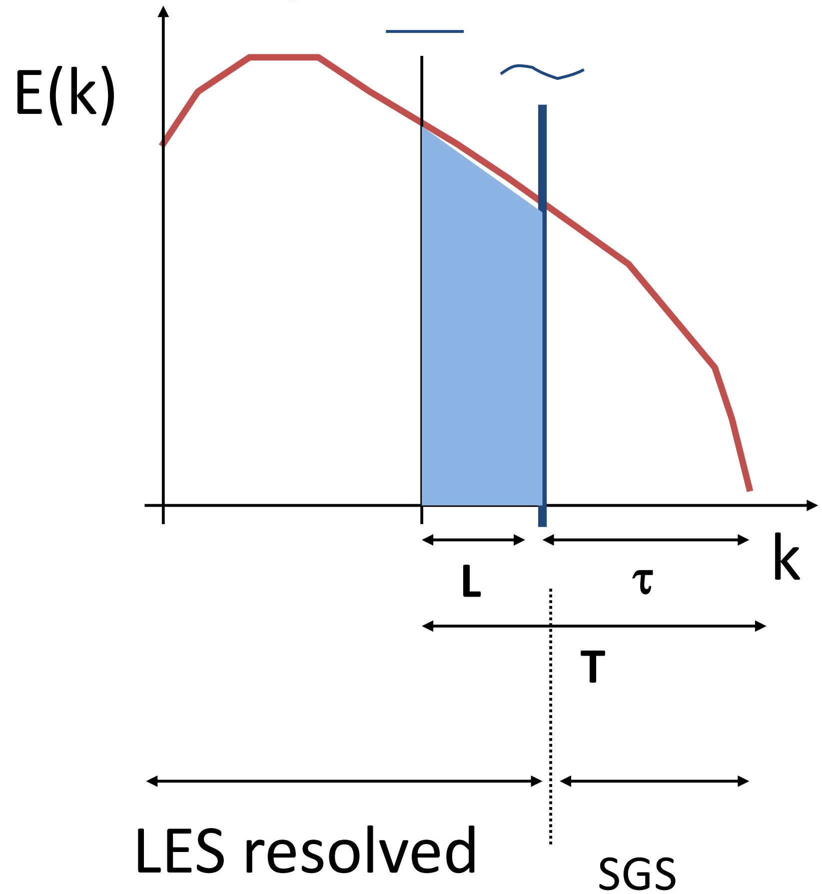
$$-2(c_s 2\Delta)^2 \,|\bar{\tilde{S}}|\,\bar{\tilde{S}}_{ij} \qquad -2(c_s\Delta)^2\, \overline{|\tilde{S}|\tilde{S}_{ij}}$$

Assumes scale-invariance:

$$L_{ij} - c_s^2 M_{ij} = 0$$

where $M_{ij} = 2\Delta^2\left(\overline{|\tilde{S}|\tilde{S}_{ij}} - 4\,|\bar{\tilde{S}}|\,\bar{\tilde{S}}_{ij}\right)$





Germano identity and dynamic model

(Germano et al. 1991):

$$L_{ij} - c_s^2 M_{ij} = 0$$

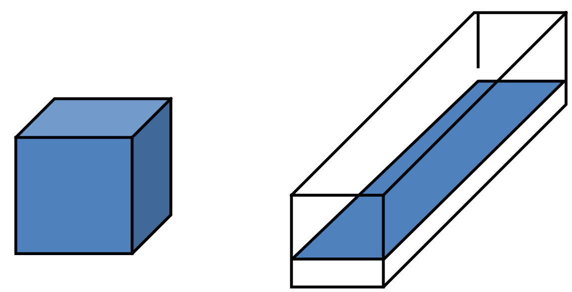
Over-determined system:
solve in "some average sense"
(minimize error, Lilly 1992):

$$\mathrm{E} = \left\langle \left(L_{ij} - c_s^2 M_{ij} \right)^2 \right\rangle$$

Minimized when:

$$c_s^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$

Averaging over regions of
statistical homogeneity
or fluid trajectories



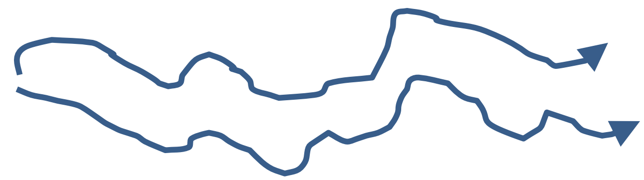
"Machine Learning":

Computer "learns" C_s by focusing
on the right statistics/physics

Lagrangian averaging

- How and from where to "learn" from large scales:
- Time averaging: should be in Lagrangian frame for Galilean invariance….
- Averaging backward in time along particle trajectory.

$$\mathcal{I}_f = \int_{-\infty}^{t} f\left(\mathbf{z}(t'),t'\right)\frac{1}{T}\exp\left(-\frac{t-t'}{T}\right)dt'$$



CM, Lund & Cabot (JFM, 1996)

According to this model, the Smagorinsky coefficient is evaluated as

$$c_S = \mathcal{J}_{LM}/\mathcal{J}_{MM}, \qquad (13.268)$$

where $\mathcal{J}_{LM}$ and $\mathcal{J}_{MM}$ represent the averages $(M_{ij}\mathcal{L}_{ij})_{ave}$ and $(M_{ij}M_{ij})_{ave}$. The simple relaxation equation

$$\frac{\bar{D}\mathcal{J}_{MM}}{\bar{D}t} = -(\mathcal{J}_{MM} - M_{ij}M_{ij})/T \qquad (13.269)$$

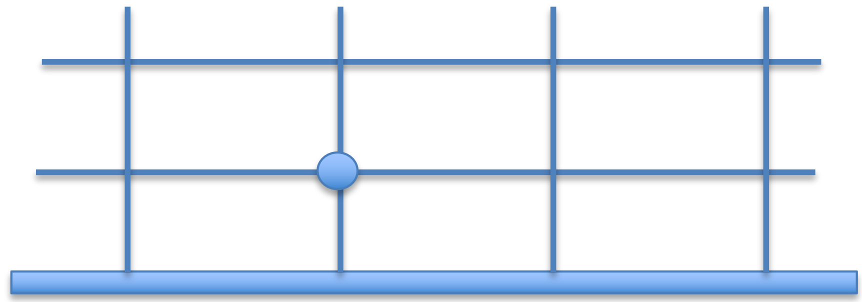
is solved for $\mathcal{J}_{MM}$, where T is a specified relaxation time. This is equivalent to averaging along the particle path, with relative weight $\exp[-(t-t')/T]$ at the earlier time t'. The similar equation that is solved for $\mathcal{J}_{LM}$ is

$$\frac{\bar{D}\mathcal{J}_{LM}}{\bar{D}t} = -I_0(\mathcal{J}_{LM} - M_{ij}\mathcal{L}_{ij})/T, \qquad (13.270)$$

Pope textbook

Scale dependence:
highly relevant for **wall-modeled LES**

$$\nu_T = [C_s(\Delta)\,\Delta]^2 \left|\tilde{S}\right|$$



$\Delta \sim y$

$\ell \sim \kappa y$

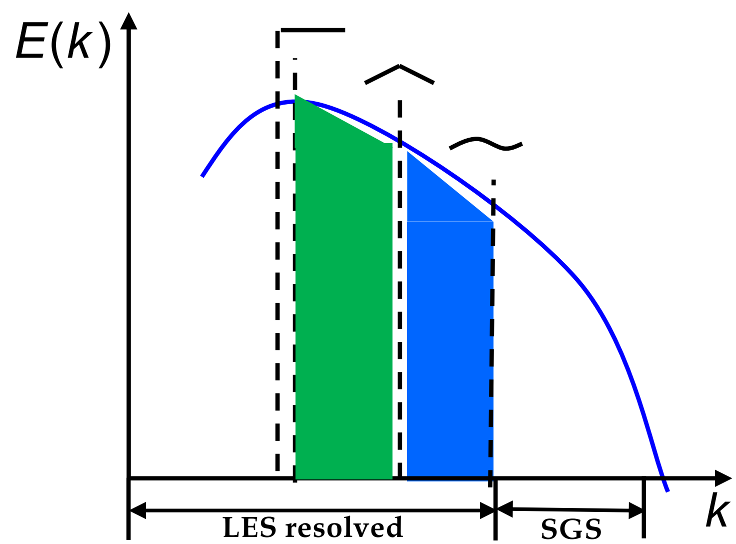
Δ and esp. 2Δ **not** in inertial range !!

- **Scale-dependence:**

$$C_s(\Delta) \sim \Delta^{\varphi}$$

$$c_s(\hat{\Delta}) = \gamma_c\, c_s(\Delta)$$

$$\gamma_c = c^2_{s,\hat{\Delta}} / c^2_{s,\Delta} = c^2_{s,\bar{\Delta}} / c^2_{s,\hat{\Delta}}$$

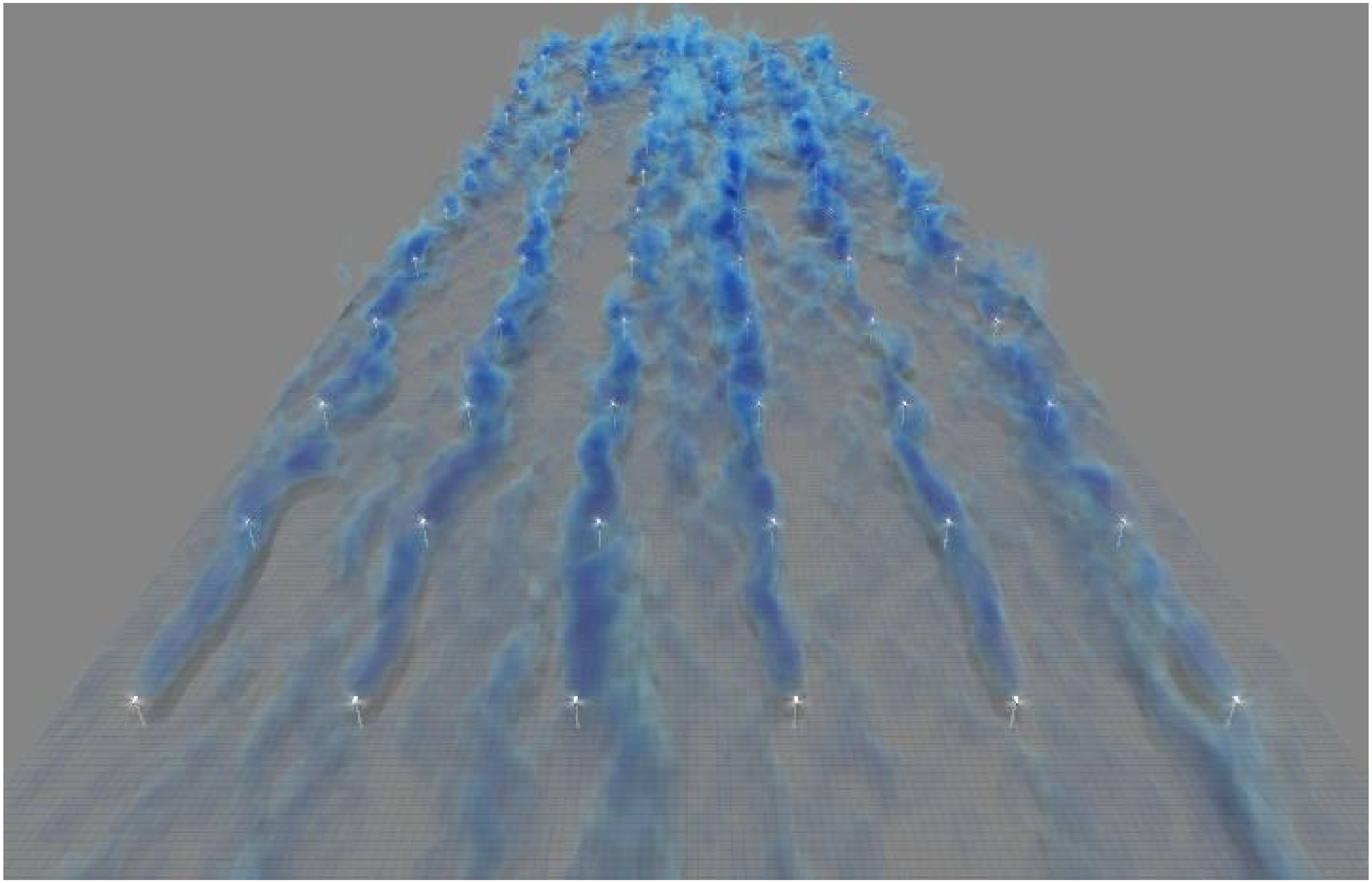


Test-filtering at 2 scales: e.g. 2Δ and 4Δ

Porte-Agel, CM & Parlange (JFM, 2000)

Example appliction of LES: Wind farm simulations using LASD SGS model (and ADM)

Stevens et al. (2016)



Code: LESGO

A pertinent new "canonical turbulent flow":
The windturbine-array boundary layer (WTABL)
(=WAKES + ATMOSPHERIC BOUNDARY LAYER)

Horns Rev 1: Photograph: Christian Steiness

Photo credit: Bel Air

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& Applied Fluid Mechanics

JHU Mechanical Engineering

WINDINSPIRE

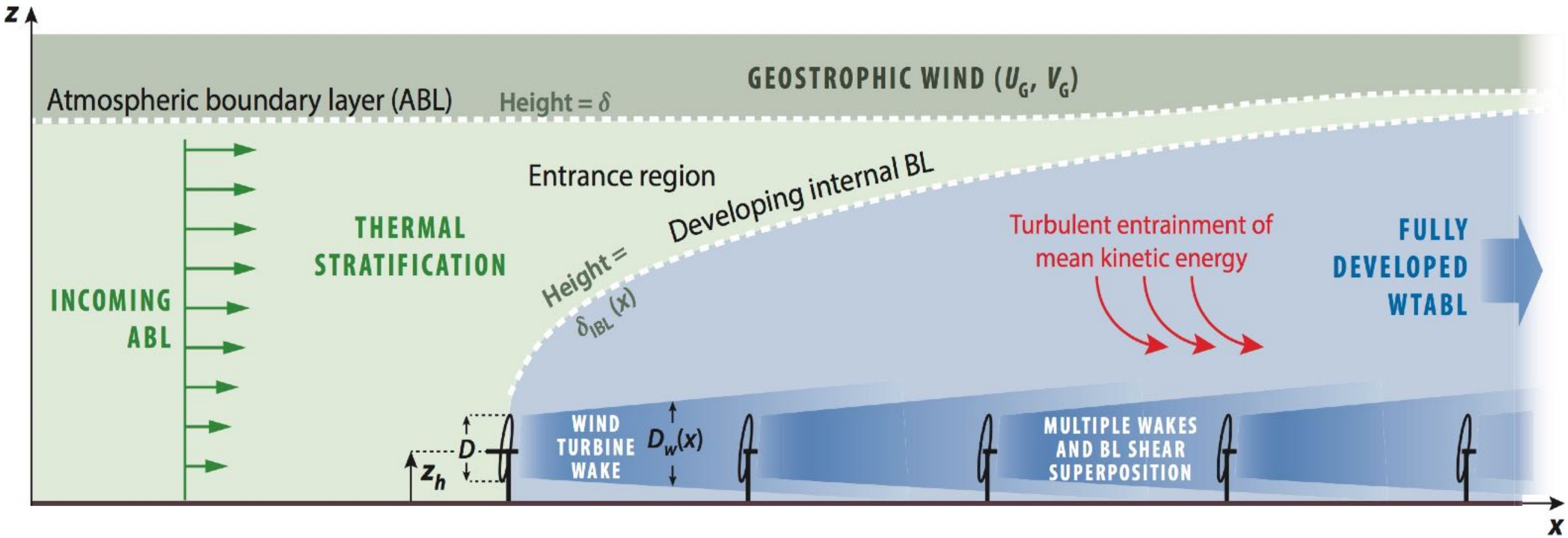
Collaborators:

- Richard J.A.M. Stevens (now Twente U., NL) – LES + CWBL
- Marc Calaf (now U. Utah) – LES + BL models
- Prof. Johan Meyers (KU Leuven, B) – LES
- Dennice Gayme (JHU) – reduced models + control
- Juliaan Bossuyt (KU Leuven, JHU visiting student, B) – windtunnel
- Michael Howland (JHU undergrad till 2016) – windtunnel
- Michael Wilczek (now MPI Göttingen, D) – spectral theory

Funding: NSF OISE-1243882 (WINDINSPIRE project)

Simulations: XSEDE, SARA (NL) & MARCC

Fluid mechanics of the wind turbine-array boundary layer (WTABL)



From: R.J.A.M. Stevens & C.M., “Flow structure and turbulence in wind farms”, (2017), Annu. Rev. Fluid Mech. **49**, 311-339.

Wind farm design & optimization:

great need for simple engineering (reduced) models

1. Mean Power optimization: mean velocity

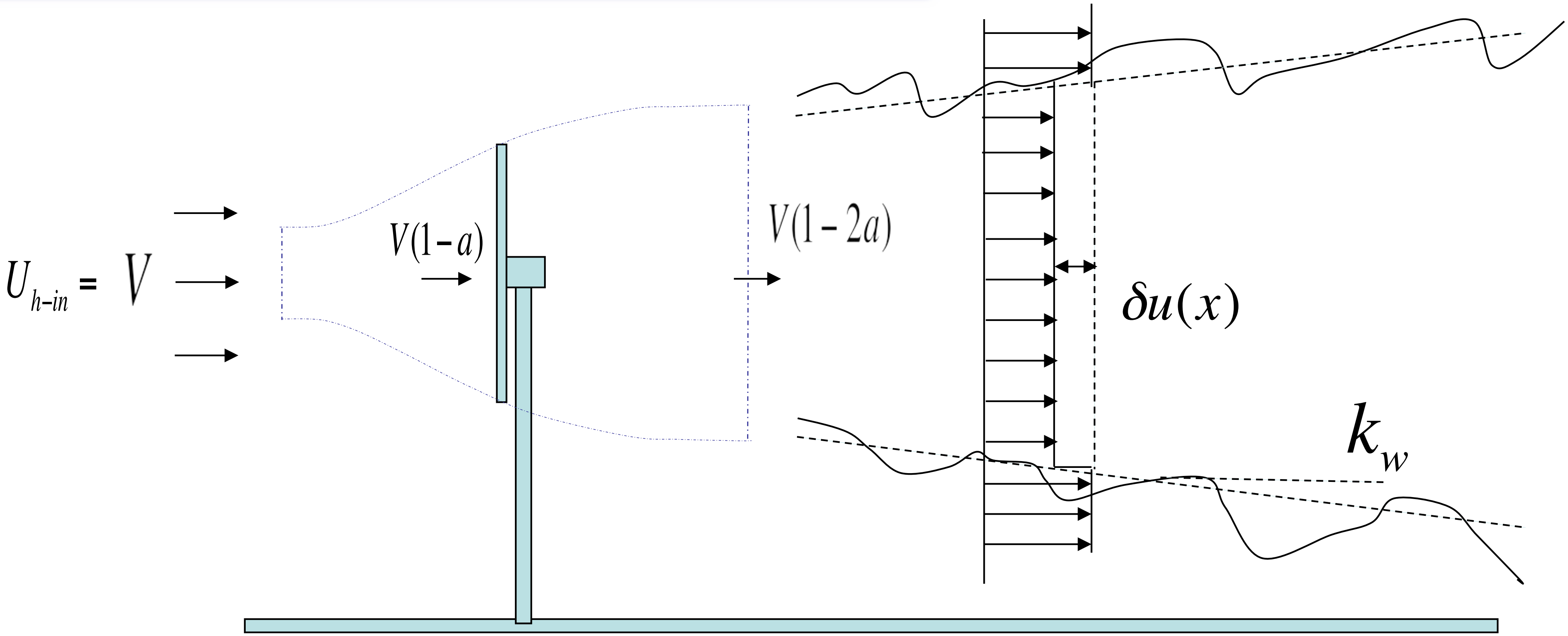
$$P_{\mathrm{turb}} = \frac{1}{2} C_P \rho \frac{\pi}{4} D^2 \; U_{\mathrm{turb}}^3$$

$$\frac{1}{P_{\max}} P_{tot}(s, D, z_h, C_T, \mathrm{layout..}) = \sum_{\text{all turbines}} \left(\frac{U_{turb}}{U_{h-in}} \right)^3$$

2. Fluctuations: power variability due to turbulence

Jensen model: The single wake

Lissaman (1979) / static Jensen (1984)



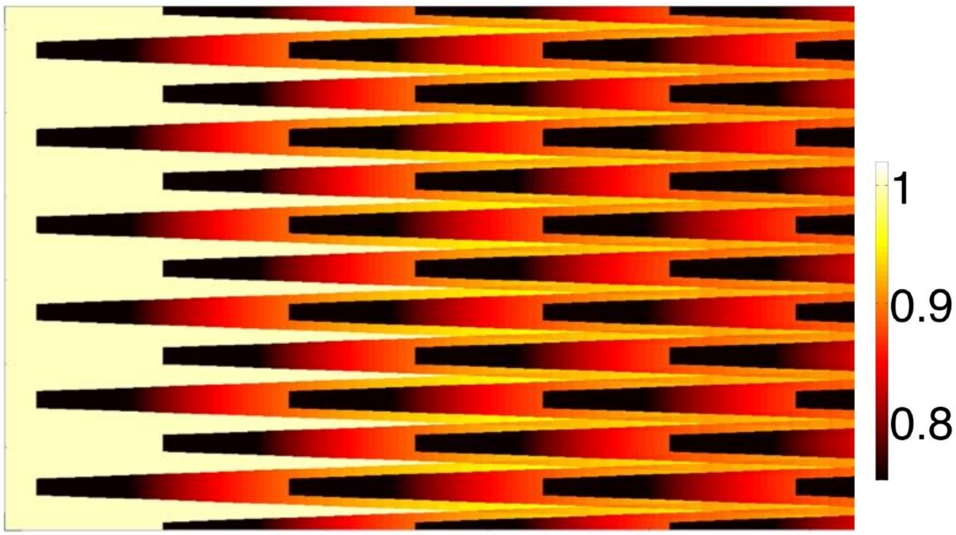
inviscid momentum theory gives "IC" for wake model

wake model: turbulence governs growth rate k_w of wake

$$a = \frac{1}{2}\left(1 - \sqrt{1 - C_T}\right)$$

$$\delta u(x,j) = U_{h0} - u(x) = \frac{2aU_{h0}}{\left(1 + 2k_w \frac{x - x_j}{D}\right)^2}$$

Jensen model: The wake superposition



Lissaman (1979) / Katic et al. (1986)

$$u'^2 \sim \delta u^2 \;\Rightarrow\; \Sigma\, u'^2 \sim \Sigma\, \delta u^2 \;\Rightarrow$$

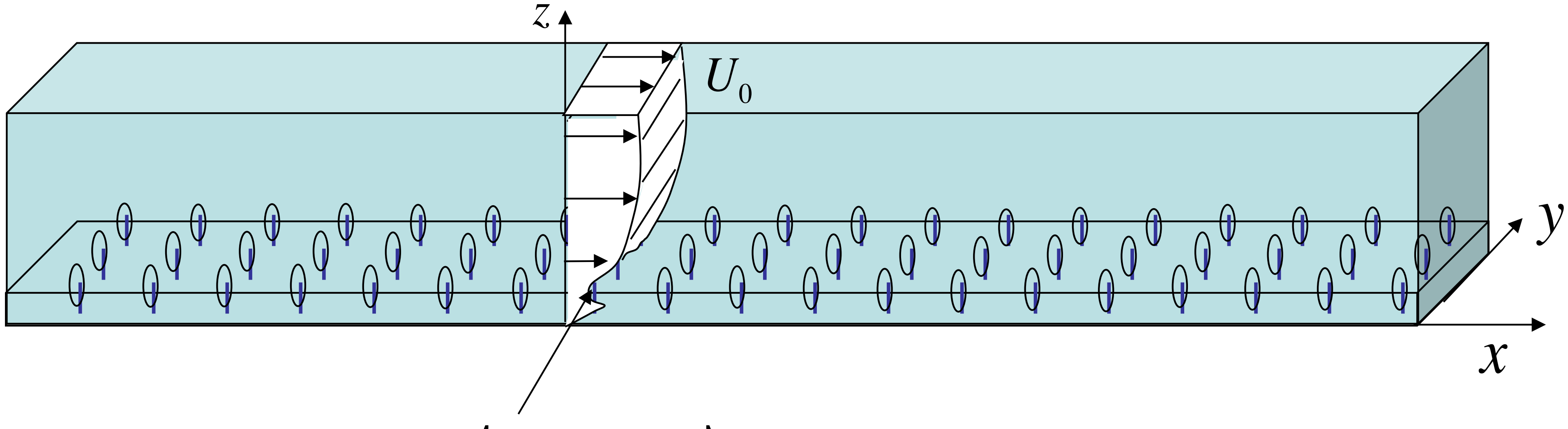
$$\Rightarrow \delta u_{net}^2 = \Sigma\, \delta u_j^{\,2} \;\Rightarrow\; 1 - \frac{U_h}{U_{h0}} = \left(\Sigma \left(\frac{\delta u_j}{U_{h0}} \right)^2 \right)^{1/2}$$

Superposition of squared velocity deficits, can be rationalized by assuming that kinetic energy is additive (independent turbulence fluctuations)

$$\frac{U_h(s, C_T, \ldots)}{U_{h0}} = 1 - \left(1 - \sqrt{1 - C_T}\right) \left(\sum_{j \in J_{Tk}} \left[1 + 2k_w \frac{x_{Tk} - x_j}{D} \right]^{-4} \right)^{1/2}$$

But no connection to ABL structure (OK for small farms)

A boundary layer (canopy flow) view: the mean velocity vertical profiles in fully developed WTABL



$$U(z) = \left\langle \overline{u}(x, y, z) \right\rangle_{xy}$$

horizontal (canopy) average

Data: from LES of WTABL typical simulation setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\nabla \tilde{p}^* - \nabla \cdot \boldsymbol{\tau} + \mathbf{f}$$

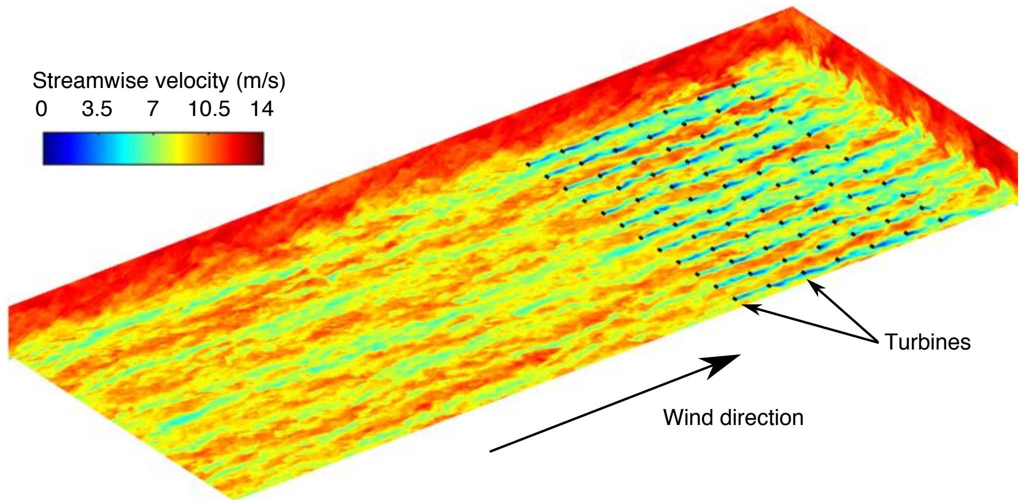
High-fidelity, but has large computational cost

Actuator disk forcing:

$$F = -\frac{1}{2}\rho A C_T' \langle u \rangle_d^2$$

Power:

$$P = \frac{1}{2}\rho A C_T' \langle u \rangle_d^3$$



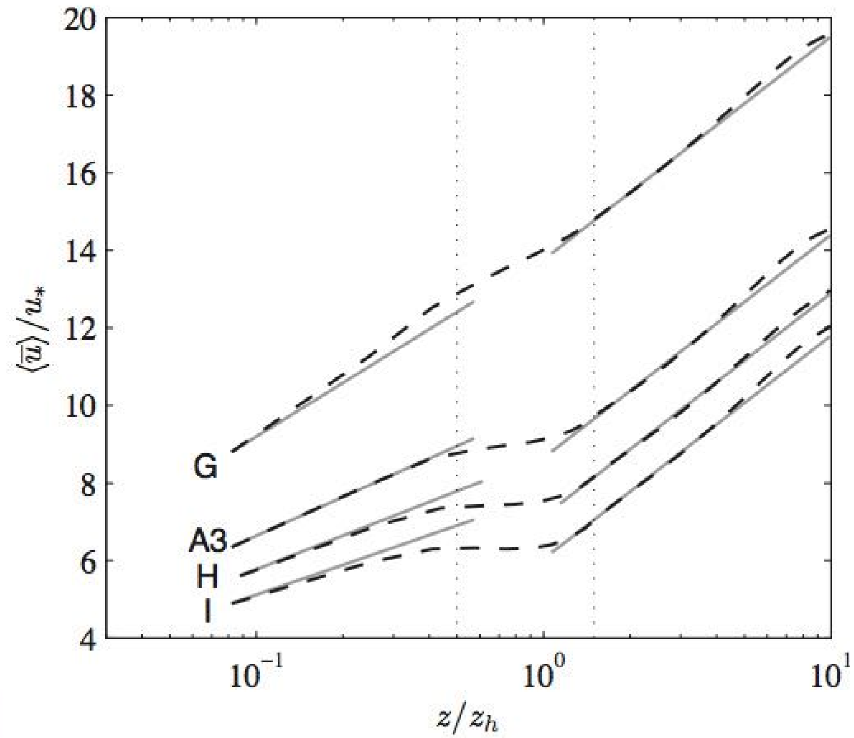
$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

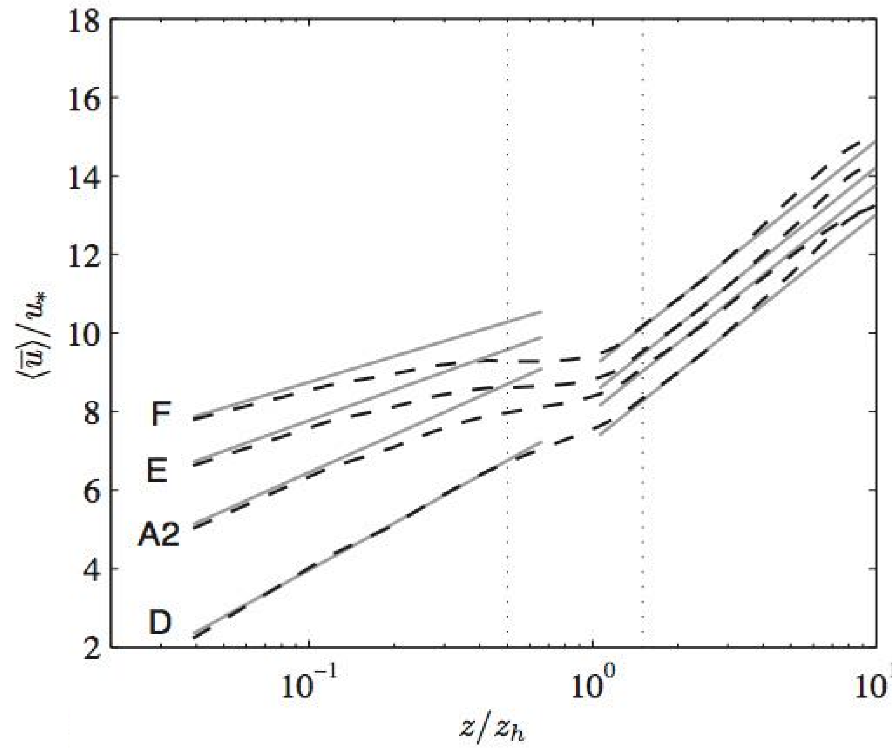
$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128 \quad \rightarrow \quad 1024 \times 512 \times 512$$

- Horizontal periodic boundary conditions (for FD-WTABL, or precursor for developing)
- Top surface: zero stress, zero w
- Bottom surface B.C.: w=0 + Wall stress: Standard wall function relating wall stress to $u(z_1)$
- Scale-dependent dynamic Lagrangian model eddy-viscosity (*no* adjustable parameters
- More details: Calaf et al. Phys. Fluids. **22** (2010) 015110

Horizontal mean velocity in WTABL from LES (ADM):

Calaf et al 2010 (confirming hypothesis by Frandsen 1992): 2 log-laws



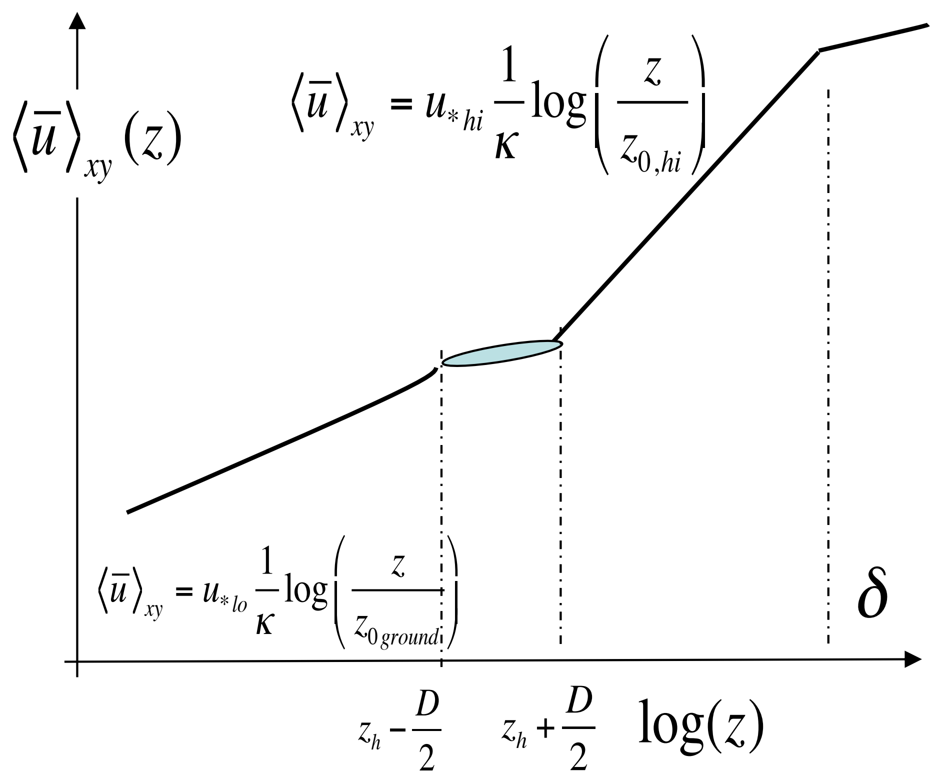


Other studies of WTABL velocity distributions:

- Cal et al. (JRSE 2010)
- Johnstone & Coleman (J Wind Eng & Ind A, 2012)
- Yang, Kang & Sotiropoulos (PoF 2012)
- Chamorro & Porté-Agel (2013)
- Chatterjee & Peet (Pys Rev Fluids 2018)
- Ghate & Lele (J Fluid Mech, 2017)

Top down model:

S. Frandsen 1992, Frandsen et al. 2006, Calaf et al 2010, Stevens 2015..:



Two "constant stress" layers with:

$$u_{*\,hi}^2 \approx u_{*\,lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

In wake layer, reduced slope:

$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

Effective wind farm roughness:

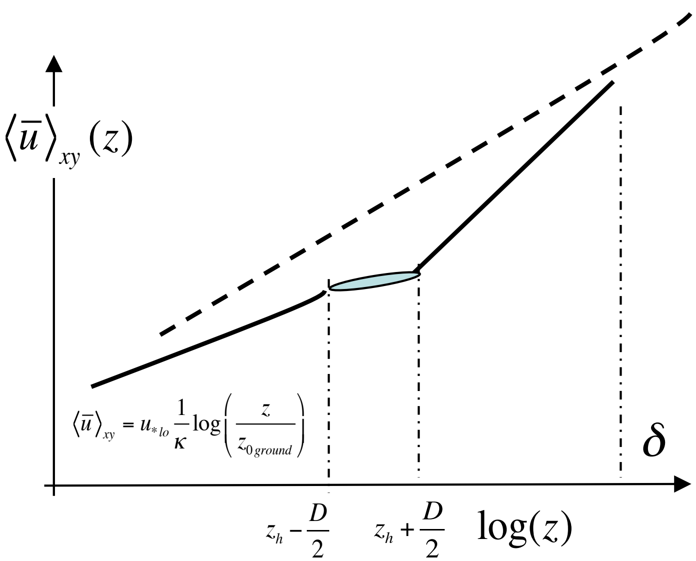
$$z_{0,hi} = z_h \left(1 + \frac{D}{2 z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8 \kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}} \left(1 - \frac{D}{2 z_h}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

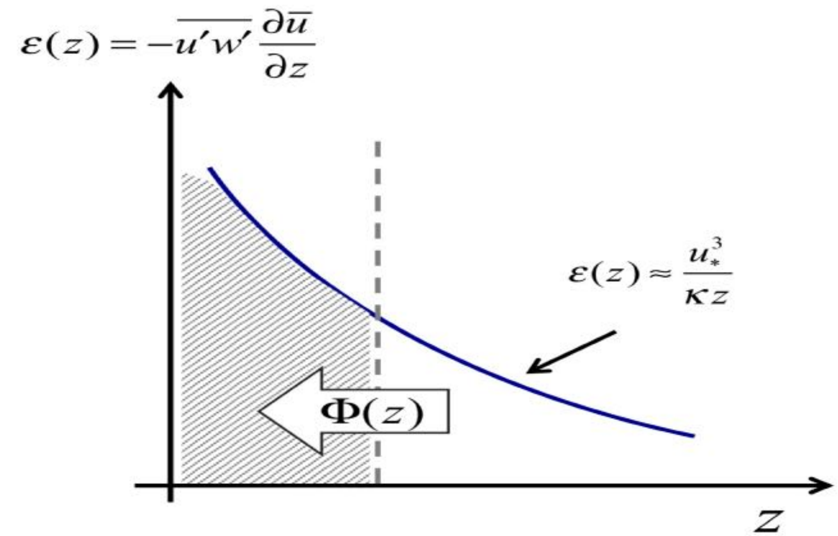
Mean velocity at hub height, normalized by ABL unperturbed inflow :

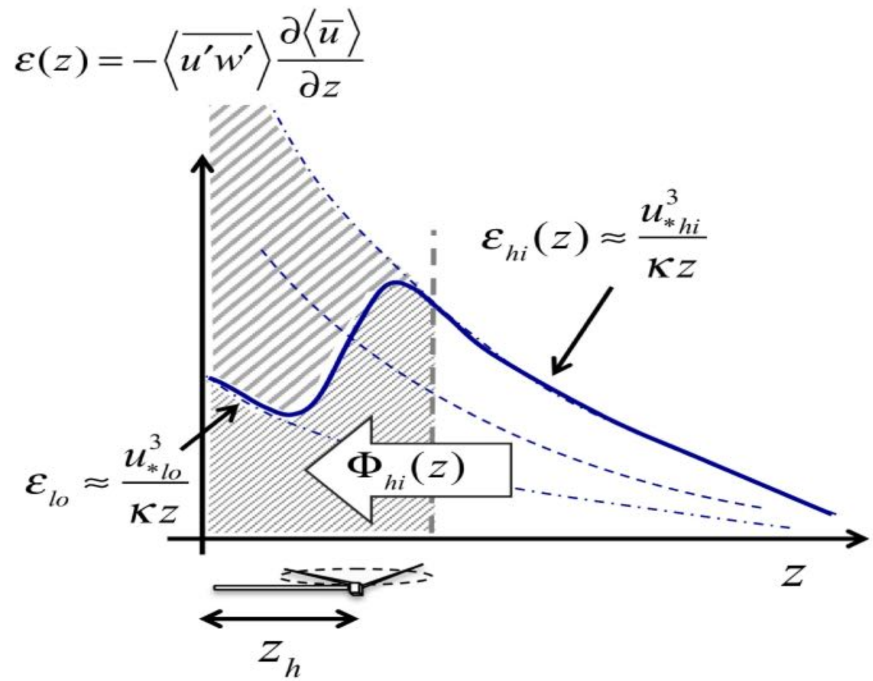
$$\frac{U_h(s, C_T, \ldots)}{U_{h0}} = \frac{\ln(\delta / z_{0,\mathrm{lo}})}{\ln(\delta / z_{0,\mathrm{hi}})} \ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}} \right) \left(1 + \frac{D}{2 z_\mathrm{h}} \right)^{\beta} \right] \left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}} \right) \right]^{-1}$$

Top down model:

SIDE NOTE: Top-down model enables us to understand fate of **mean kinetic energy in the WTABL**: $\frac{1}{2}\langle\bar{u}\rangle_{xy}^2$



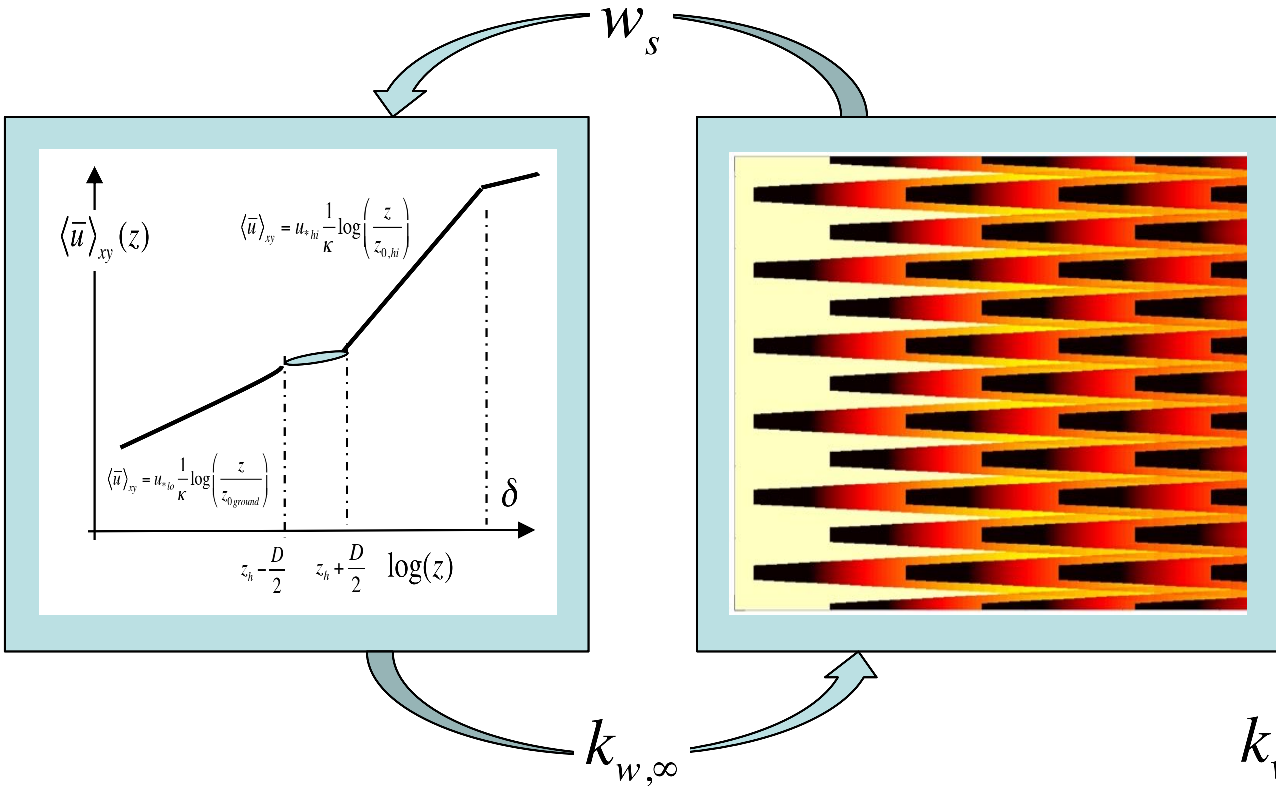




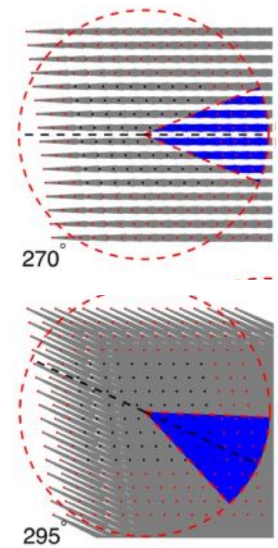
Instead of being dissipated entirely in BL, mean KE extracted by turbines and dissipated

Coupled wake boundary layer model (CWBL)

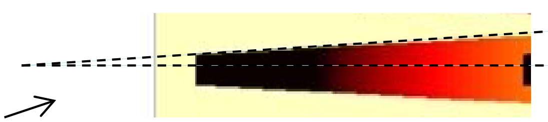
Stevens, Gayme & CM, Wind Energy **19**, 2023-2040 (2016)



$$w_s = \frac{A_w}{A_T}$$



$k_{w,\infty}$

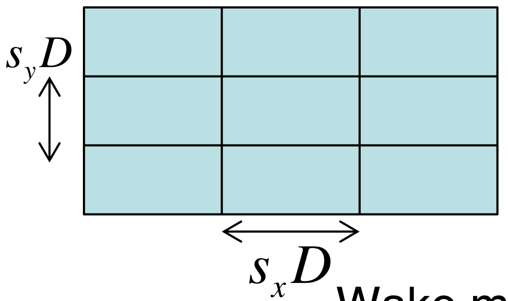


$$\frac{\ln(\delta/z_{0,\mathrm{lo}})}{\ln(\delta/z_{0,\mathrm{hi}})}\ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}}\right)\left(1+\frac{D}{2z_\mathrm{h}}\right)^\beta\right]\left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}}\right)\right]^{-1} = \frac{1}{N_d}\sum_{k=1}^{N_d}\left[1-2a\left(\sum_{j\in J_{A,k}}\left[1+k_{w,\infty}\frac{x_{T,k}-x_j}{R}\right]^{-4}\right)^{1/2}\right]$$

$$z_{0,hi} = z_h\left(1+\frac{D}{2z_h}\right)^\beta \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 w_s s_x s_y}+\left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^\beta\right]\right)^{-2}\right]^{-1/2}\right)$$

Model comparisons with LES (fully developed)

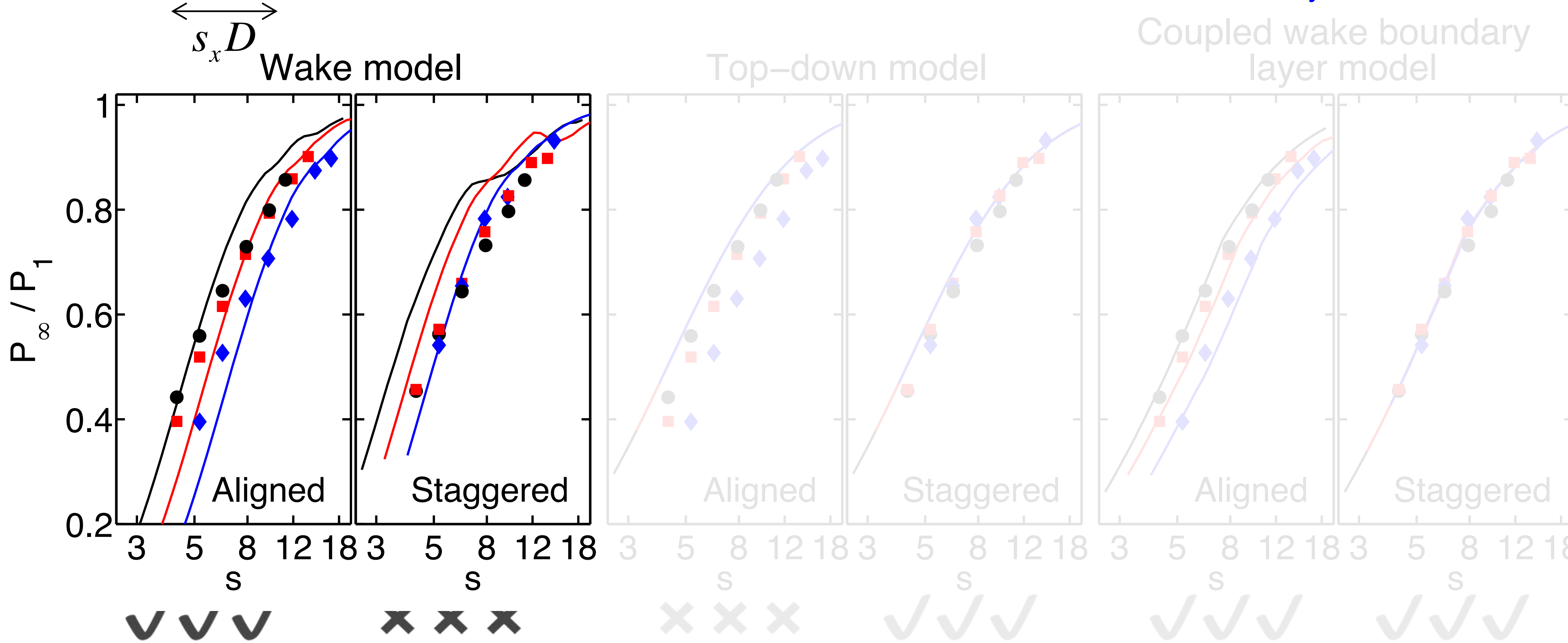
CWBL model: distinguishes well between aligned & staggered cases



$$\left(\frac{U_h(\infty)}{U_{h-0}}\right)^3$$

$$s = \sqrt{s_x s_y}$$

Color indicates spanwise spacing
s_y=3.49
s_y=5.23
s_y=7.85

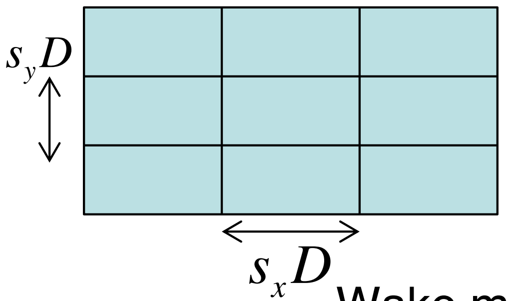


O : LES
------: CWBL

Stevens, Gayme, Meneveau, JRSE **7**, 023115 (2015)

Model comparisons with LES (fully developed)

CWBL model: distinguishes well between aligned & staggered cases



$$\left(\frac{U_h(\infty)}{U_{h-0}}\right)^3 \qquad s = \sqrt{s_x s_y}$$

Color indicates spanwise spacing
s_y=3.49
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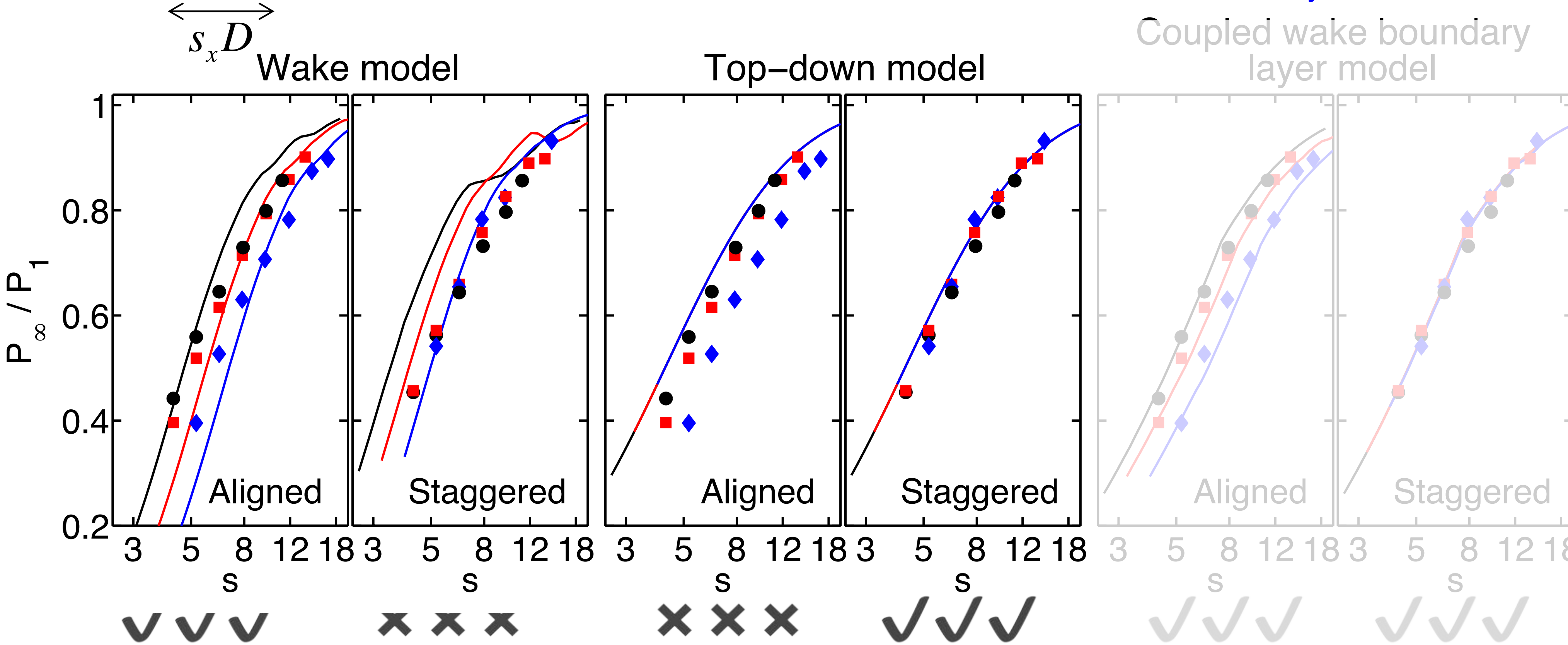

Stevens, Gayme, Meneveau, JRSE **7**, 023115 (2015)

O : LES
------: CWBL

Model comparisons with LES (fully developed)

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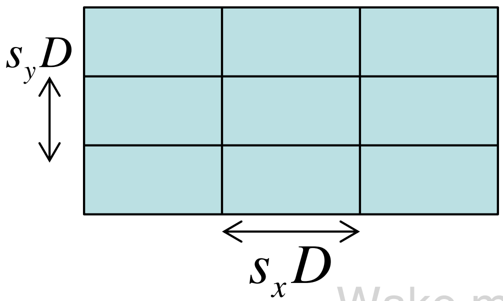

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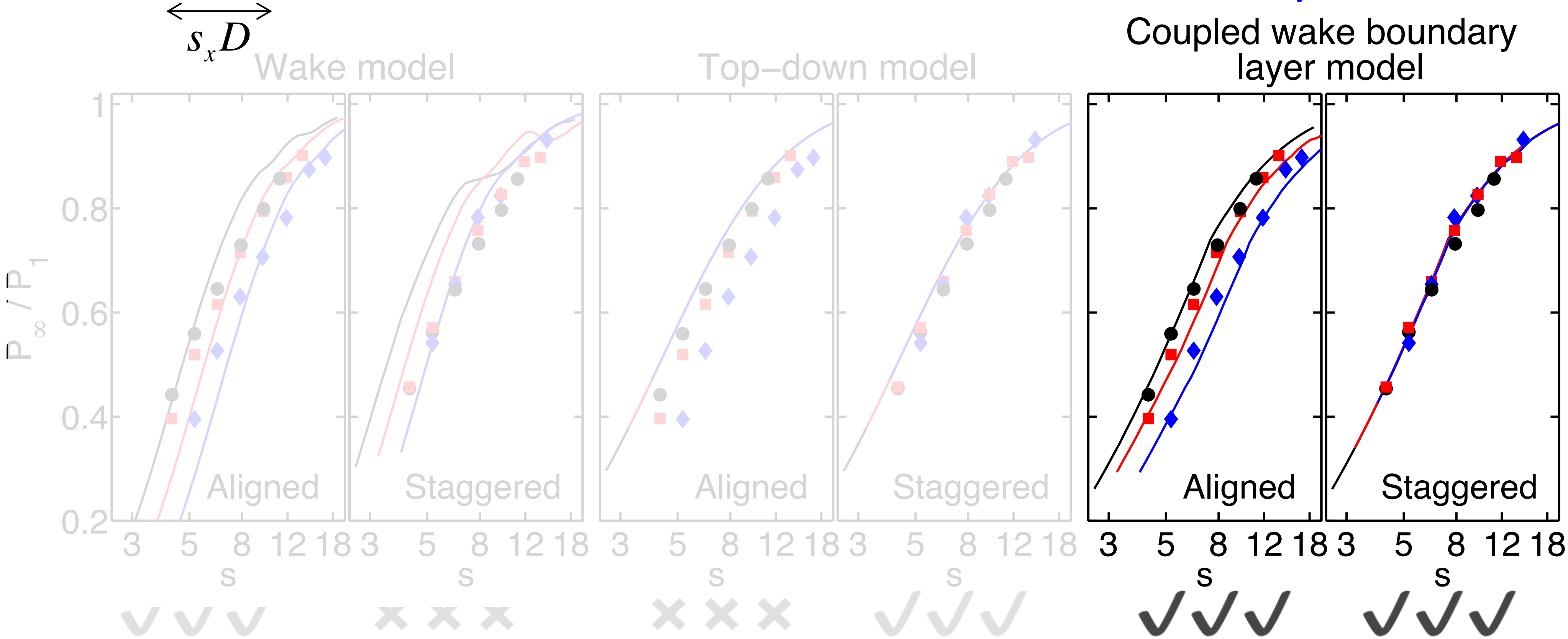


O : LES

------: CWBL

Stevens, Gayme, Meneveau, JRSE **7**, 023115 (2015)

Model comparisons with LES and field data (Horns Rev)

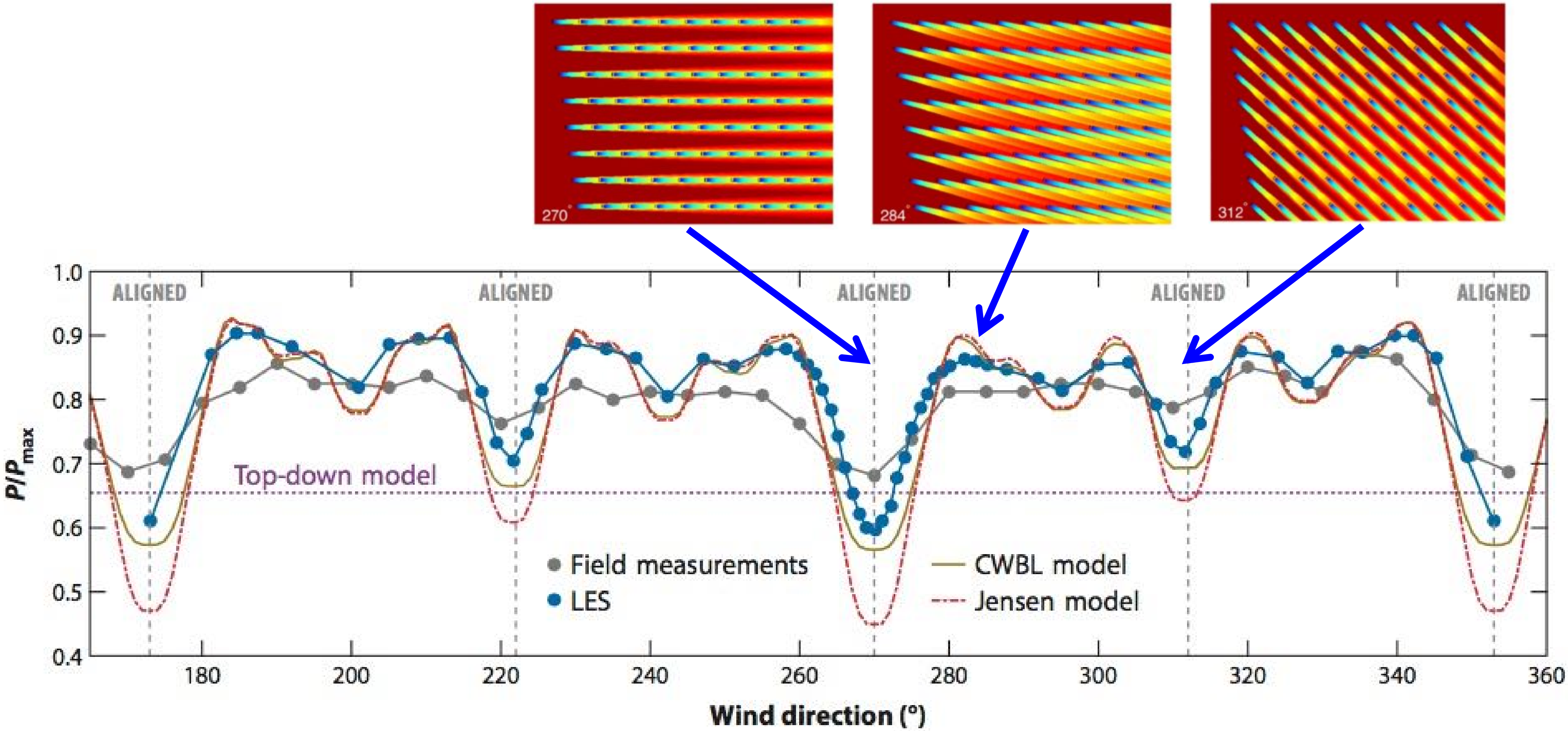


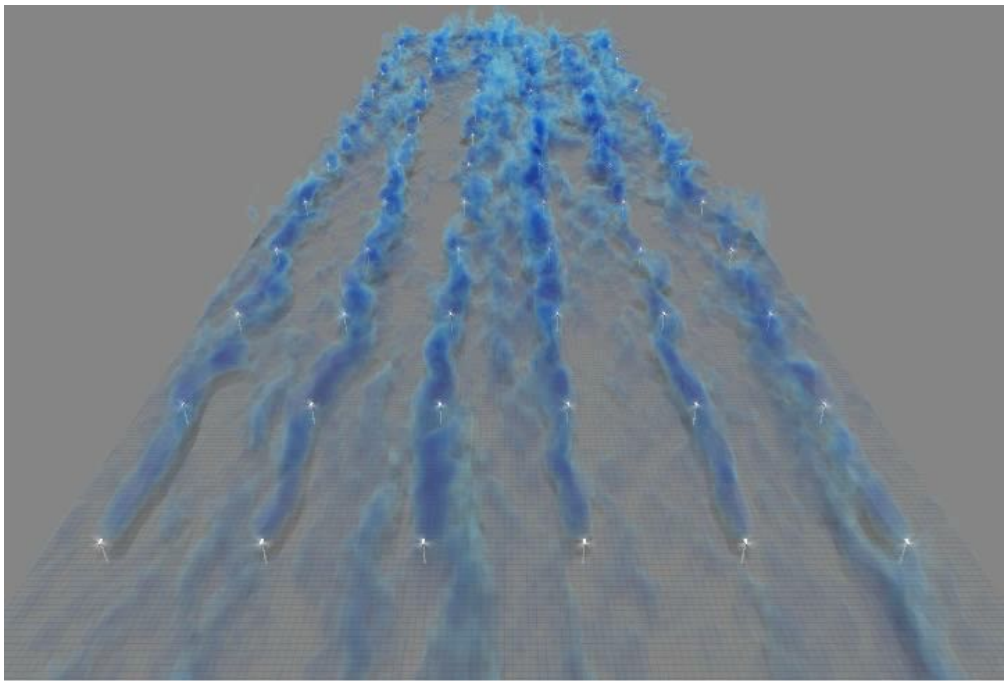
Figure 8

Normalized total power output $P/P_{\max}$ of Horns Rev wind farm as a function of the incoming wind direction, where $P_{\max}$ is the power of a non-wake-affected turbine times the number of turbines. The field measurement data are digitally extracted from the figure on page 25 of Peña et al. (2013). The large-eddy simulation (LES) results are from Porté-Agel et al. (2013); the coupled wake boundary layer (CWBL), Jensen, and top-down results are from Stevens et al. (2016b). Figure adapted with permission from Stevens et al. (2016b, figure 8).

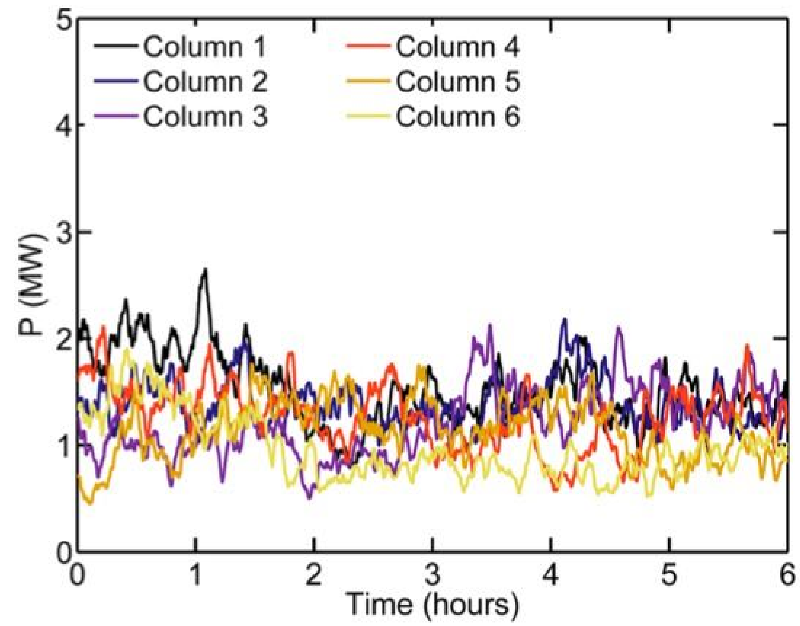
Stevens & M, Annu Rev Fluid Mech (2017)

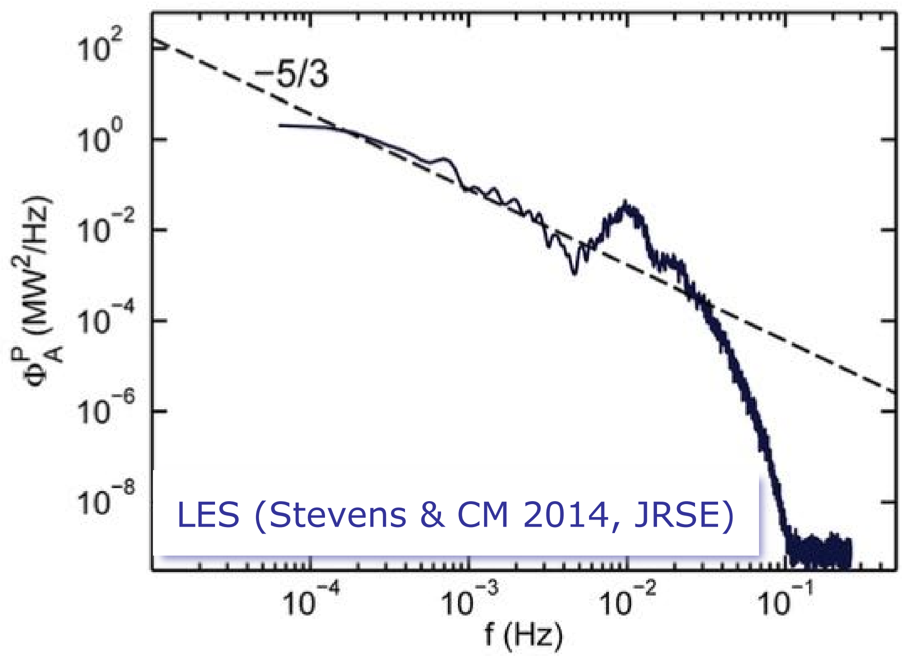
Temporal fluctuations in power:

Simulations: R.J.A.M. Stevens et al, JRSE **6**, 023105 (2014), using ADM in JHU-LES code



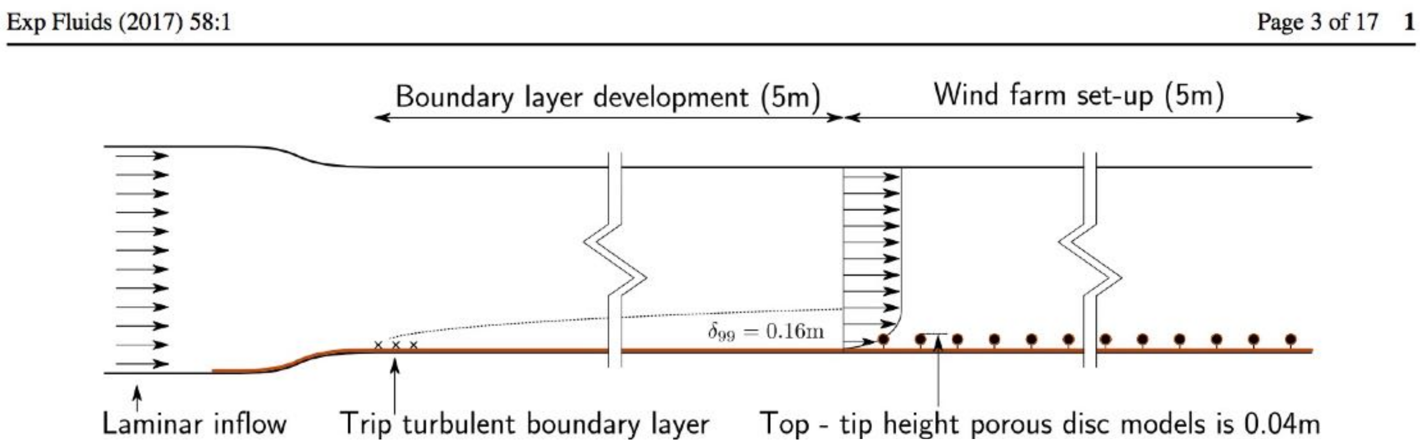
Visualization courtesy of D. Brock (Extended Services XSEDE)

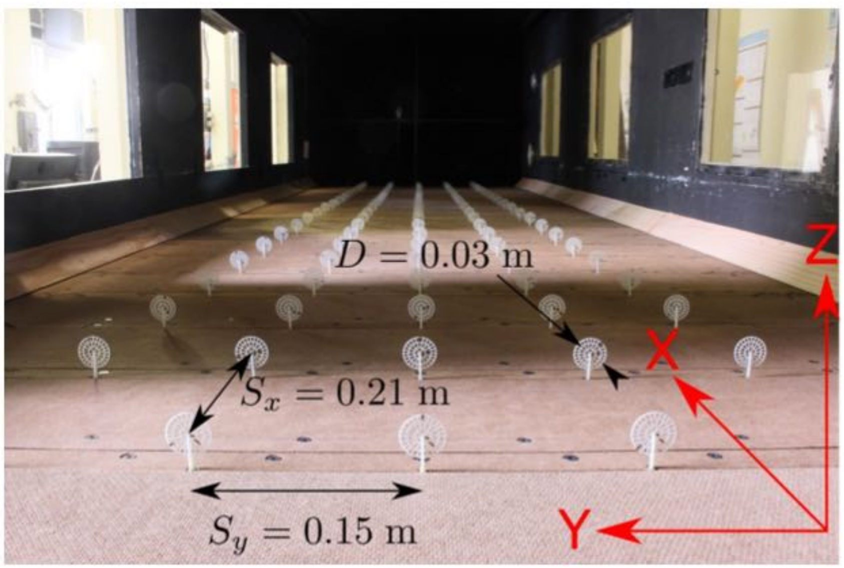


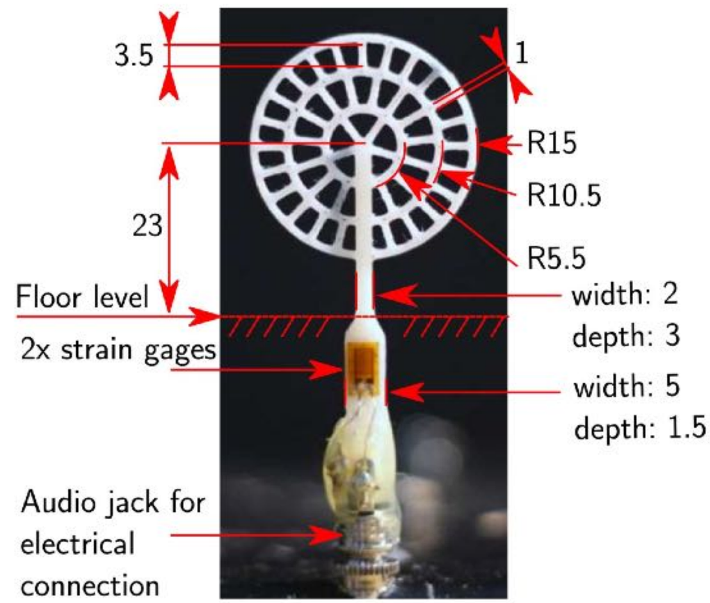


Wind tunnel tests in a "micro-windfarm" in the Corrsin wind tunnel: (Juliaan Bossuyt's thesis, KU Leuven)

J. Bossuyt, CM & J. Meyers (Exp Fluids 2016, JFM 2017)





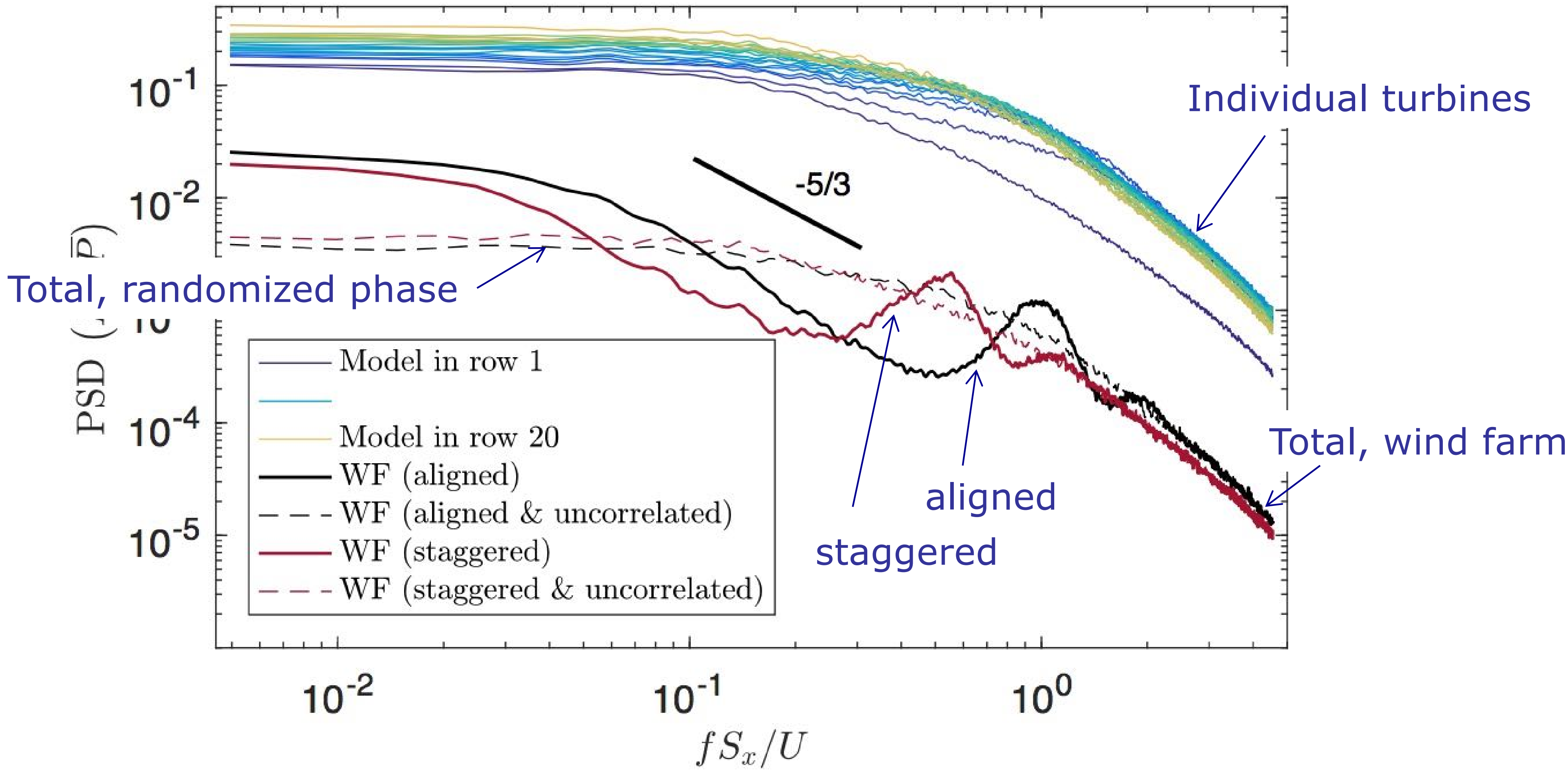


Flyby over micro-windfarm in Corrsin wind tunnel at JHU (staggered)

Power-spectral density of "power" fluctuations:

$$F_i(t) = \frac{1}{2}\rho C_T A\langle u(t)\rangle_i^2 \qquad P_i(t) = \frac{1}{2}\rho C_P A\langle u(t)\rangle_i^3 \approx \frac{C_P}{C_T}(\tfrac{1}{2}\rho A)^{-1/2} F(t)_i^{3/2}$$

(~ valid in region II)



Bossuyt et al. JFM (2017)

Temporal fluctuations in power:

$$P_{tot}(t;s,D,z_h,C_T,layout..) = \sum \tfrac{1}{2} c_p \rho A \left(U_h(t)\right)^3$$

- Spectral properties of fluctuations delivered to grid

$$E_P(\omega;s,D,z_h,C_T,layout..) = PSD(P')$$

We seek analytical models of spectral properties as function of wind farm design parameters (spacing, etc..)

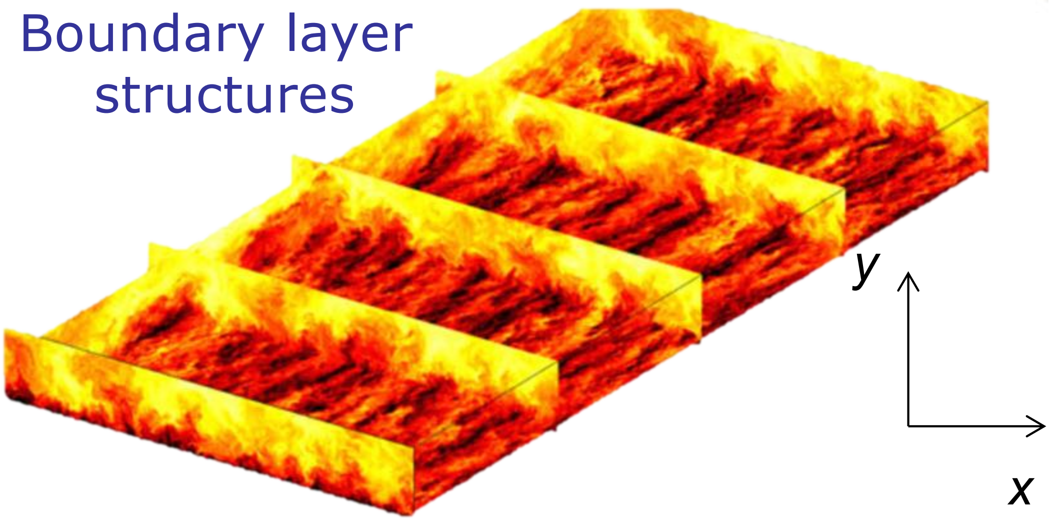
Interpret power as discrete sampling of TBL:

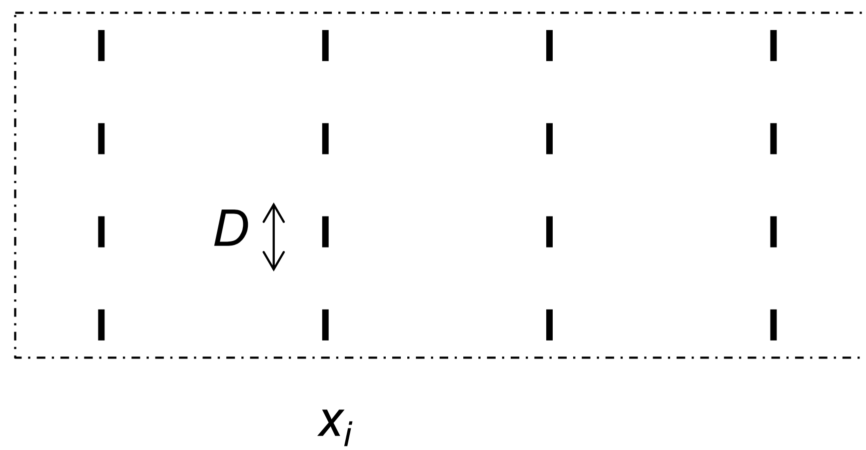
$$P_i(t) = \overline{P}_i + P_i'(t) \approx U^3 + 3U^2 u'(t) + h.o.t. \qquad P'_{\mathrm{WF}} = \sum P'_i \approx C_2 \sum \langle u \rangle'_i$$

(linearlization, but see Bandi (PRL 2017)

"Transfer function": $g(x,y) = \sum_{i=1}^{N} \delta(x - x_i) \frac{1}{D} H\left(\frac{D}{2} - |y - y_i|\right).$

Boundary layer structures





$$E_{P'}(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| \hat{g}(k_1, k_2) \right|^2 E_{11}(k_1, k_2, \omega, z_h)\, dk_1 dk_2$$

Needed:

JOHNS HOPKINS
Center for Environmental
& Applied Fluid Mechanics

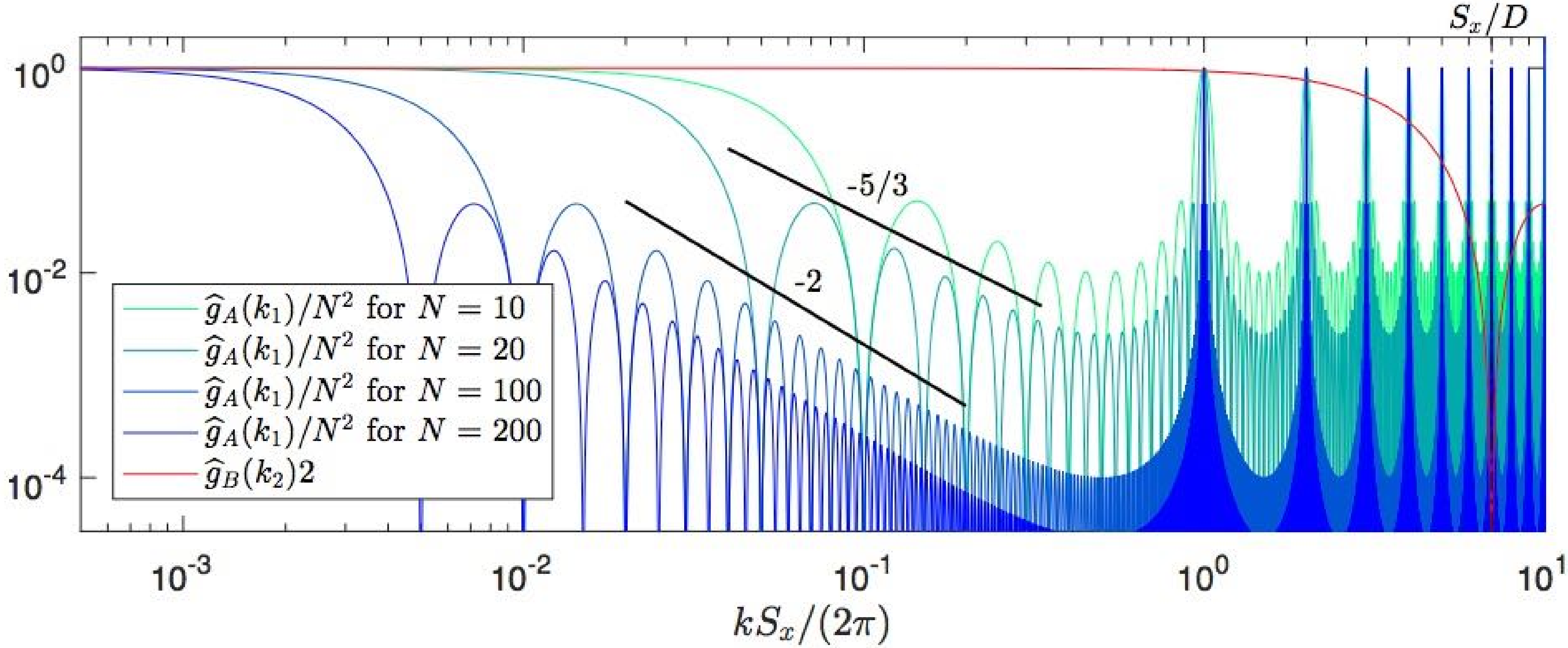
JHU Mechanical Engineering

WINDINSPIRE

Transfer function of turbine array (spacing, layout)

$$g(x,y) = \sum_{i=1}^{N} \delta(x - x_i) \frac{1}{D} H\left(\frac{D}{2} - |y - y_i|\right).$$

$$|\widehat{g}(k_1,k_2)|^2 = \left(\frac{\sin\left(k_2 \frac{D}{2}\right)}{k_2 \frac{D}{2}}\right)^2 \left(\sum_{i=1}^{N}\sum_{j=1}^{N} \cos\left(k_1\left(x_i - x_j\right) + k_2\left(y_i - y_j\right)\right)\right).$$



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JHU Mechanical Engineering

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Analytical model for wave#-freq spectrum of BL turbulence $E_{11}(k_x,k_y,\omega;z_h)$:

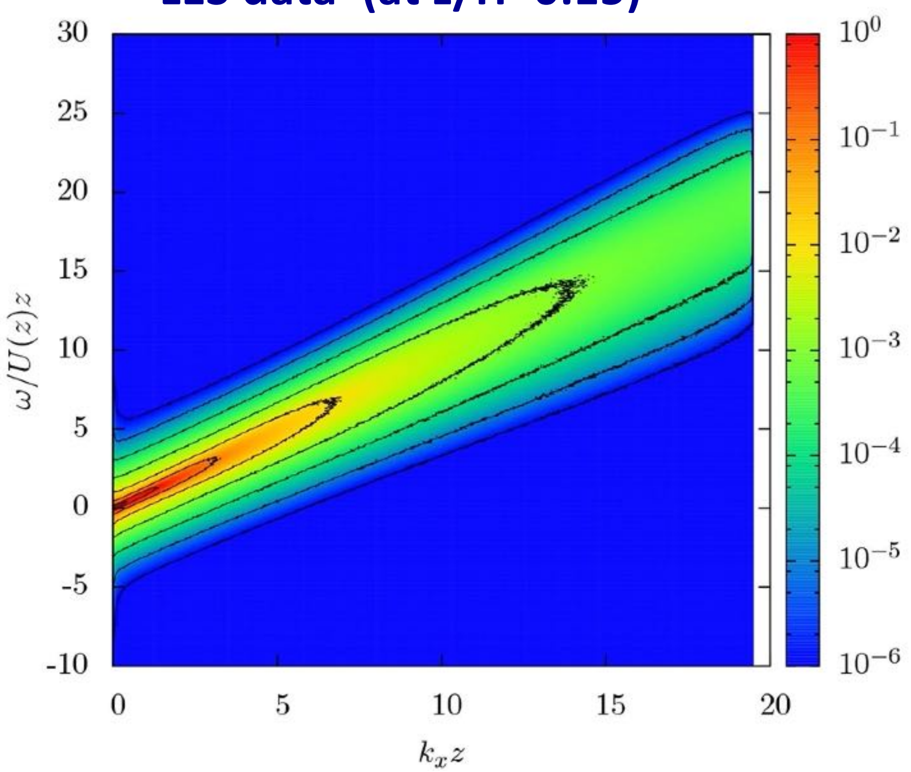
$$E_{11}(k_x,k_y,\omega;z)=\left\{[1-\theta_\alpha]A\left[\left(\frac{1}{H}\right)^4+k_x^4\right]^{-1/4}+\theta_\alpha\frac{\Gamma\left(\frac{1}{3}\right)}{5\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}C_K\varepsilon^{2/3}\left[1-\frac{8}{11}\frac{k_x^2}{k^2}\right]k^{-8/3}\right\}\left[2\pi\sigma^2(z)\right]^{-1/2}\exp\left[-\frac{(\omega-\mathbf{k}\cdot\mathbf{U})^2}{2\sigma^2(z)}\right]$$

$$U(z)=\frac{u_*}{\kappa}\log\left(\frac{z}{z_0}\right) \qquad \left\langle v_x^2\right\rangle=u_*^2\left[B-A\log\left(\frac{z}{H}\right)\right]$$

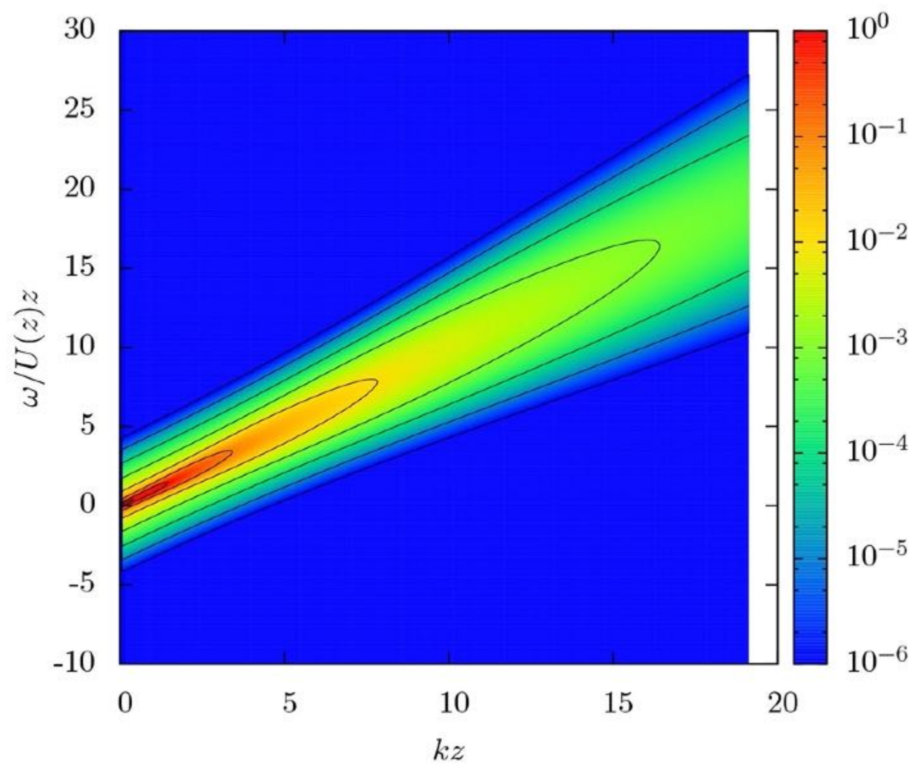
Wilczek et al. JFM 2015

$$\sigma^2(z)=\left\langle(\mathbf{v}\cdot\mathbf{k})^2\right\rangle=\left\langle v_x^2\right\rangle k_x^2+\left\langle v_y^2\right\rangle k_y^2 \qquad \sigma^2(z)=\left\langle v_x^2\right\rangle\left[k_x^2+Ck_y^2\right],\quad \text{A}=0.96,\ \text{B}=2.41,\ C=0.33,\ \kappa=0.4$$

LES data (at z/H=0.15)



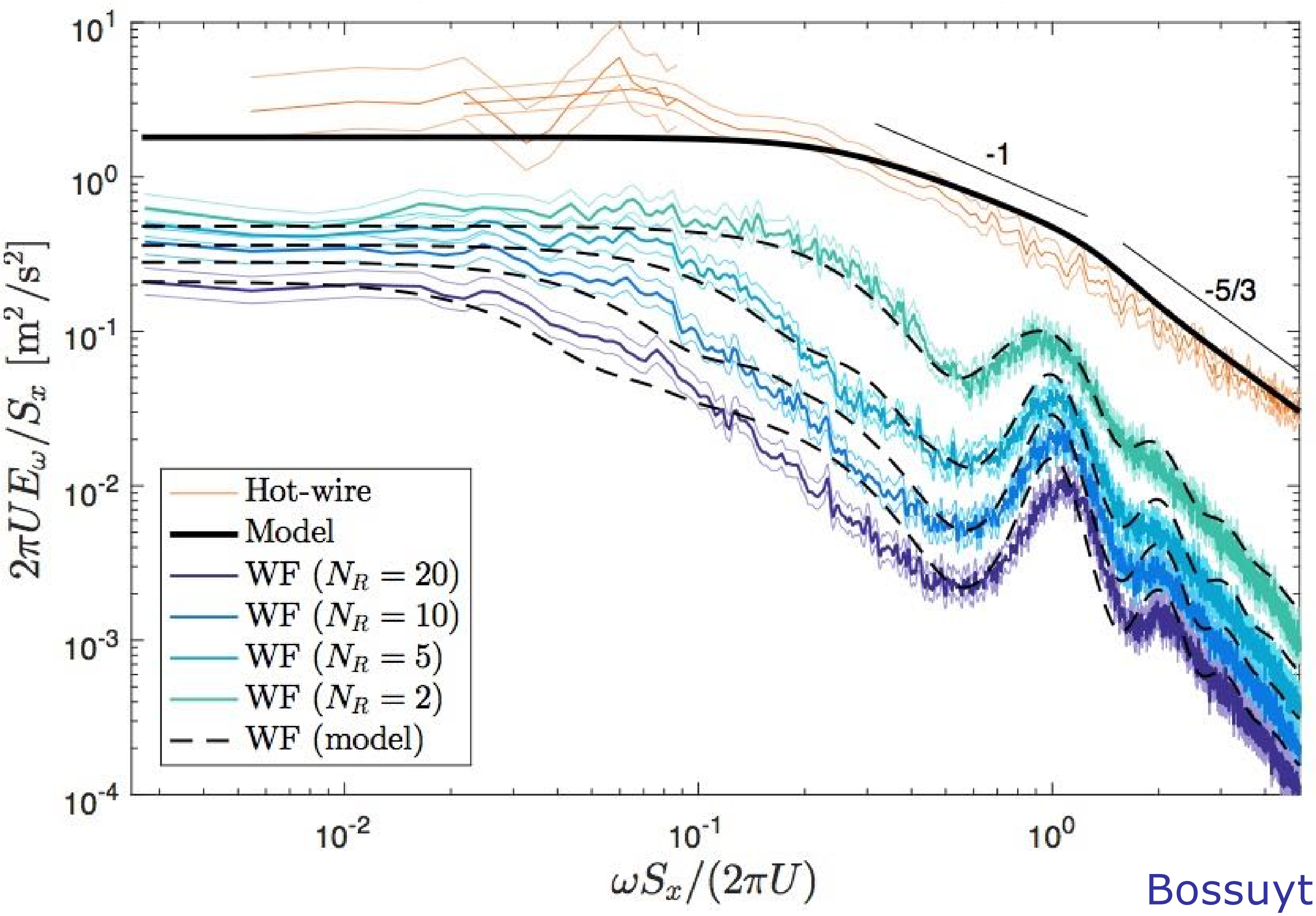
Model



Comparisons of measured and model spectra

$$E_{P'}(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\hat{g}(k_1,k_2)\right|^2 E_{11}(k_1,k_2,\omega,z_h)\; dk_1 dk_2$$

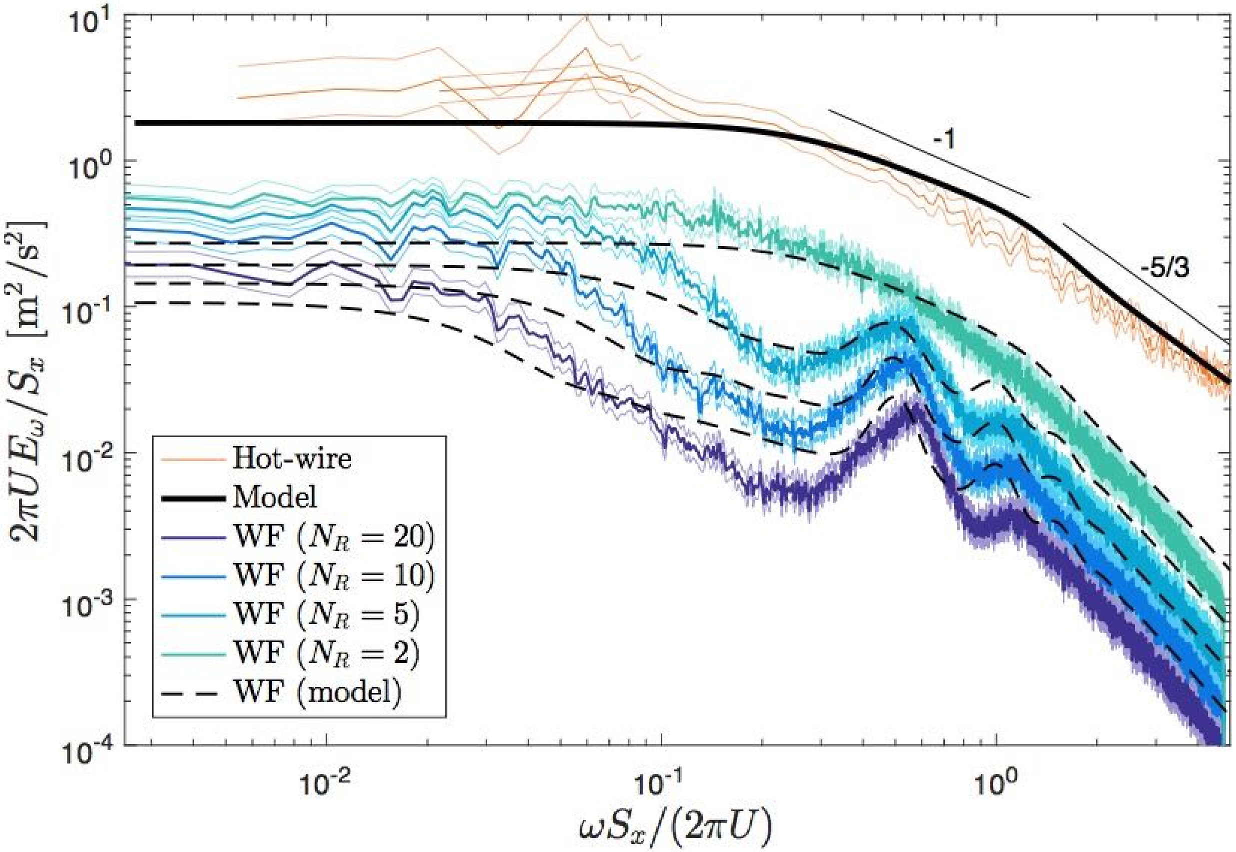
Aligned array (lines: data, dashed line: model)



Bossuyt et al. JFM (2017)

Comparisons of measured and model spectra

Staggered array (lines: data, dashed line: model)



$$E_{P'}(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\hat{g}(k_1,k_2)\right|^2 E_{11}(k_1,k_2,\omega,z_h)\, dk_1 dk_2$$

With this model, fluctuations can be included in cost function (e.g. power smoothing properties of various array configurations)

Closing:

- We are in the process of a major energy infrastructure shift – 50% wind electricity in a few decades possible – must get it right

- LES provides unprecedented fidelity of complex processes (Dynamic model: "applied RNG method", using scale-invariance)

- Still too expensive for design – not used routinely, even RANS

- At least we can provide simulation-informed simpler models (e.g. analytical or reduced dimension) to improve design/control tools

$$\frac{\ln(\delta/z_{0,\mathrm{lo}})}{\ln(\delta/z_{0,\mathrm{hi}})}\ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}}\right)\left(1+\frac{D}{2z_\mathrm{h}}\right)^{\beta}\right]\left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}}\right)\right]^{-1}=\frac{1}{N_d}\sum_{k=1}^{N_d}\left[1-2a\left(\sum_{j\in J_{A,k}}\left[1+k_{w,\infty}\frac{x_{T,k}-x_j}{R}\right]^{-4}\right)^{1/2}\right]$$

$$E_{P'}(\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|\hat{g}(k_1,k_2)\right|^2 E_{11}(k_1,k_2,\omega,z_h)\,dk_1dk_2$$