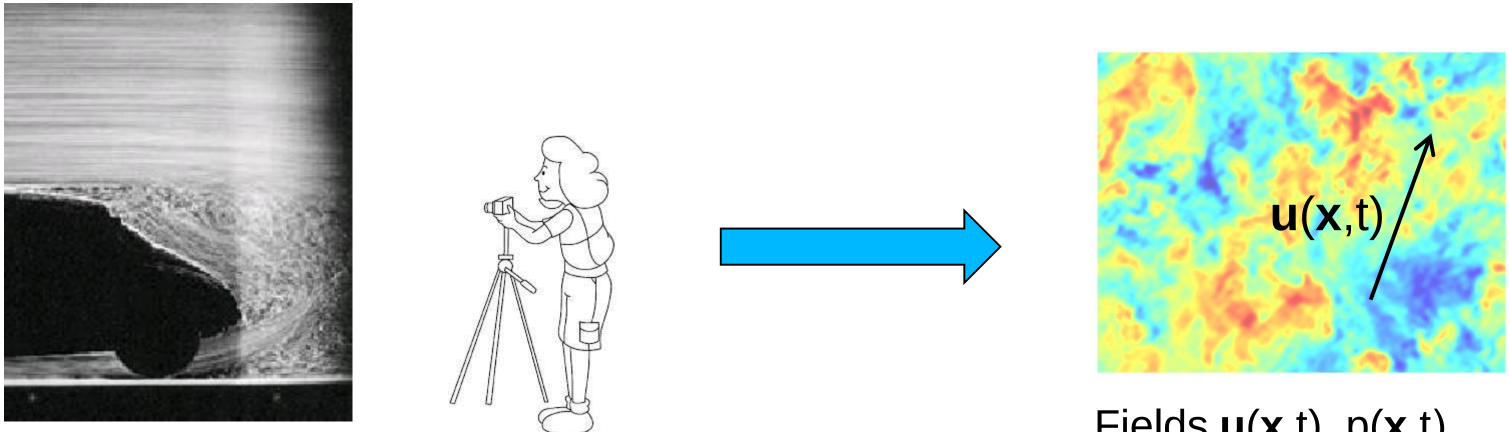
Particles in turbulence: from tracers to inertial objects

Aurore Naso

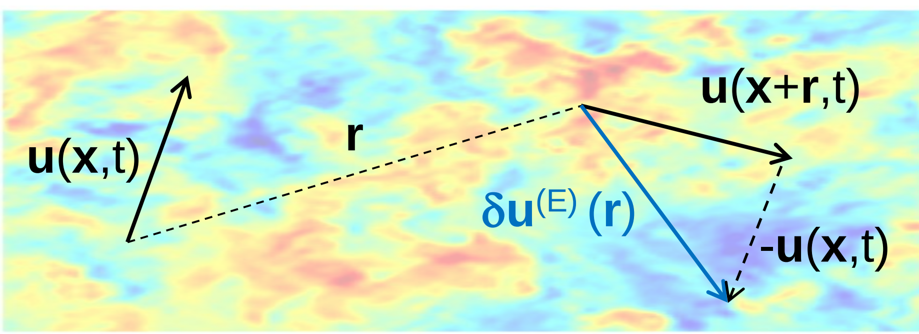
Laboratoire de **M**écanique des **F**luides et d'**A**coustique
CNRS, École Centrale de Lyon, UCB Lyon 1, INSA de Lyon
France

New Challenges in Turbulence Research VI – 10/02/2021

- Turbulence has been for a long time studied in the **Eulerian framework**



Fields **u**(**x**,t), p(**x**,t), …

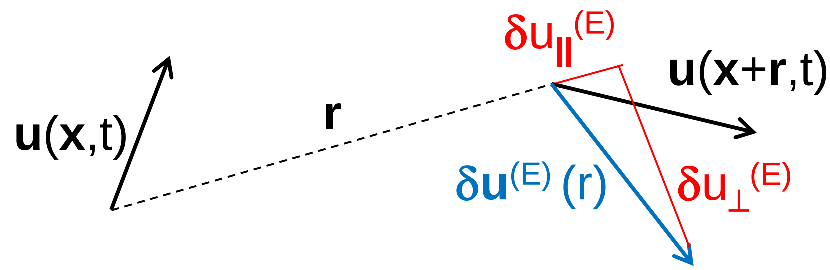


→ velocity increments:

$$\delta \mathbf{u}^{(E)} = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$$

- $\delta\mathbf{u}^{(E)}$ can be profitably projected onto preferential directions, **along** and **perpendicular** to **r** :

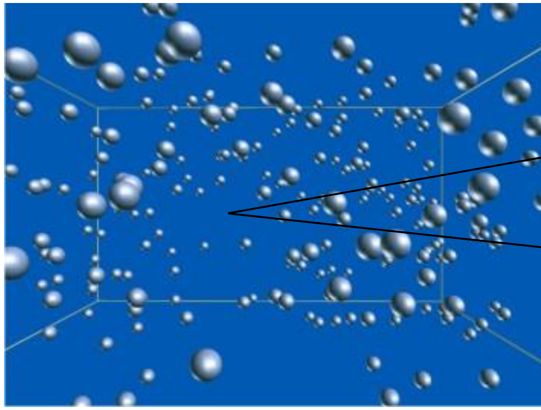
Longitudinal	$\delta u_{\parallel}^{(E)} = \delta\mathbf{u}^{(E)} \cdot \hat{\mathbf{r}}$
Transverse	$\delta u_{\perp}^{(E)} = \lvert\delta\mathbf{u}^{(E)} \times \hat{\mathbf{r}}\rvert \cos\theta$



$\hat{\mathbf{r}} = \mathbf{r}/r$; $\theta \in [0,2\pi[$

- In statistically stationary, homogeneous and isotropic turbulence, the statistics of these quantities only depend on r = |**r**|.

- In the past decades, growing interest in examining turbulence from a **Lagrangian point of view**

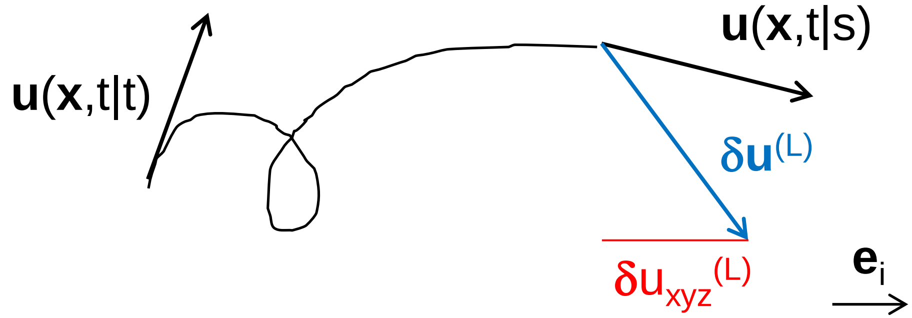


« Particles » trajectories
$\rightarrow \mathbf{u}(\mathbf{x},t|s)$

- Natural formulation for turbulent transport (1 part.) and mixing (>1 part.)

- Lagrangian velocity increments:

$$\boxed{\delta \mathbf{u}^{(L)} = \mathbf{u}(\mathbf{x},t|s) - \mathbf{u}(\mathbf{x},t|t)}$$



- Standard projection on a fixed coordinate system:

$$\boxed{\delta u_{xyz}^{(L)} = \delta \mathbf{u}^{(L)} \cdot \mathbf{e}_i}$$

- In statistically stationary, homogeneous and isotropic turbulence, the statistics of $\delta u_{xyz}^{(L)} = \delta u_x^{(L)}$ only depend on τ=s-t.

Expected scaling laws of velocity increments in the inertial range of scales (in HIT=homogeneous and isotropic turbulence):

Eulerian:

✓ Structure functions:

$$S_p^E(r) = \langle (v(\mathbf{x}+\mathbf{r}) - v(\mathbf{x}))^p \rangle$$

✓ Dimensional prediction ($\eta << r << L$):

$$S_p^E(r) \sim (\varepsilon r)^{\xi_p}; \quad \xi_p = p/3$$

✓ In particular:

$$S_3^E(r) = -\frac{4}{5}\varepsilon r$$

→ derived exactly from the Kármán-Howarth equation

→ time-irreversibility of turbulence

Lagrangian:

✓ Structure functions:

$$D_p^L(\tau) = \langle (v(t+\tau) - v(t))^p \rangle$$

✓ Dimensional prediction ($\tau_\eta << \tau << T_L$):

$$D_p^L(\tau) \sim (\varepsilon \tau)^{\zeta_p}; \quad \zeta_p = p/2$$

✓ In particular:

$$D_2^L(\tau) = C_0 \varepsilon \tau$$

(not exact !)

✓ Zero odd moments

Important quantities in Lagrangian turbulence: acceleration and pressure gradient

- **Acceleration** of a fluid particle = natural parameter.

- *Yeung, PoF 1997:* the accelerations of a pair of particles initially close to each other can stay correlated over times >> τ_η.

- *Vedula & Yeung, PoF 1999:*

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} = \mathbf{a_p} + \mathbf{a_v}$$

→ if Re large: $\mathbf{a} \sim \mathbf{a}_p$

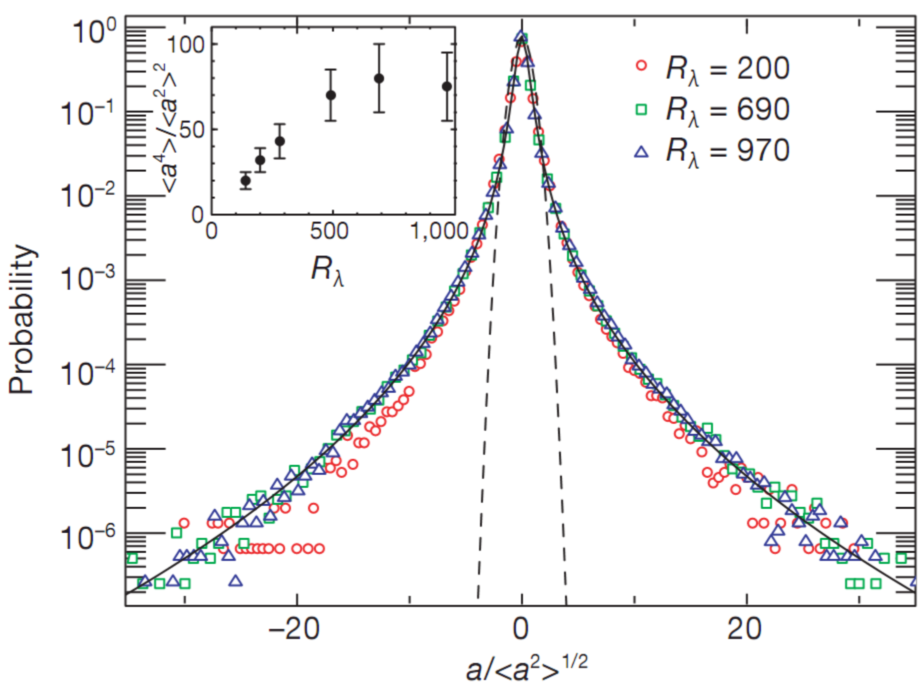
The **pressure gradient** fluctuations have Eulerian length scales >> η (nonlocal quantity !!!)

- K41 pressure spectrum is obtained for $R_\lambda > 600$ only (*Tsuji & Ishihara, PRE 2003*).

Acceleration statistics

Cornell: *La Porta et al, Nature 2001* « silicon strip detectors »

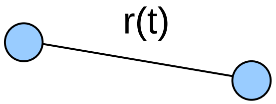
and Lyon: *Volk, Mordant, Verhille & Pinton, 2008* laser Doppler



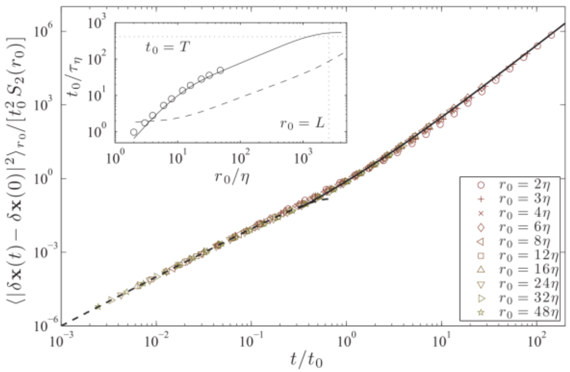
- Acceleration PDF: symmetric and highly nonGaussian (very large accelerations).
- Asymptotic shape at high Re ($R_\lambda \geq 600$).
- a_0 ~ 6:

$$\langle a_i a_j \rangle = a_0 \varepsilon^{3/2} \nu^{-1/2} \delta_{ij}$$

Pair statistics



Richardson's law $\langle r^2 \rangle = g\varepsilon t^3$ not clearly observed.

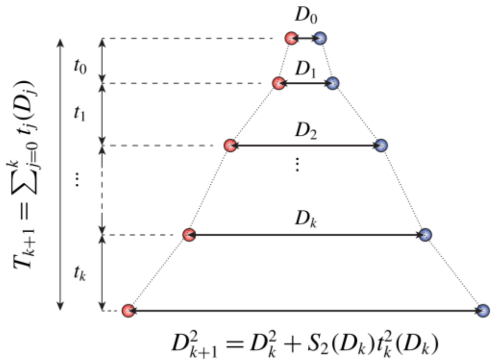


Bitane, Homann & Bec, 2012

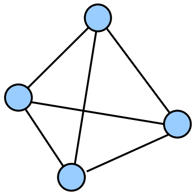
See also:
Bourgoin, Ouellette et al, Science 2006
Ott & Mann, JFM 2000

Bourgoin, JFM 2015:

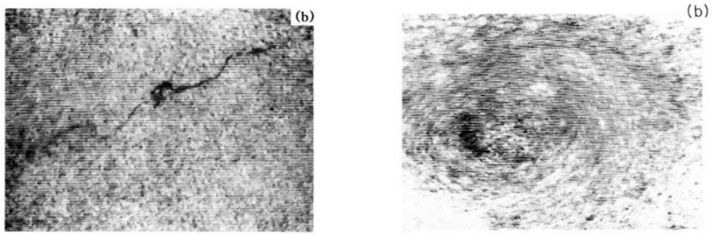
Physical phenomenology for the time evolution of the mean-square relative separation, based on a scale-dependent ballistic scenario.



Tetrad statistics



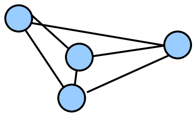
- Following a set of four particles gives access to the local flow topology (*Chertkov, Pumir & Shraiman, PoF 1999*; *Naso & Pumir, PRE 2005*).



Douady, Couder & Brachet, PRL 1991

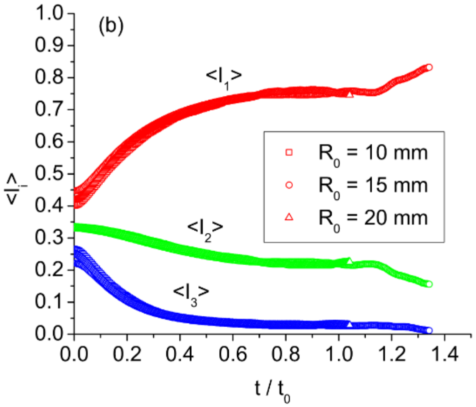
- New point of view on turbulence: energy transfer (*Pumir, Shraiman & Chertkov, EPL 2001*), small scales universality (*Naso, Chertkov & Pumir, JoT 2006*), refined LES schemes (*van der Bos, Tao, Meneveau & Katz, PoF 2002*; *Pumir & Shraiman, JSP 2003*; *Chevillard, Li, Eyink & Meneveau, 2008*).

- Geometry:
  Tetrahedra flatten (I = moment-of-inertia tensor).



- Statistics and dynamics of the perceived velocity gradient tensor:
  Alignment between perceived vorticity and perceived strain

Pumir, Bodenschatz & Xu, PoF 2013



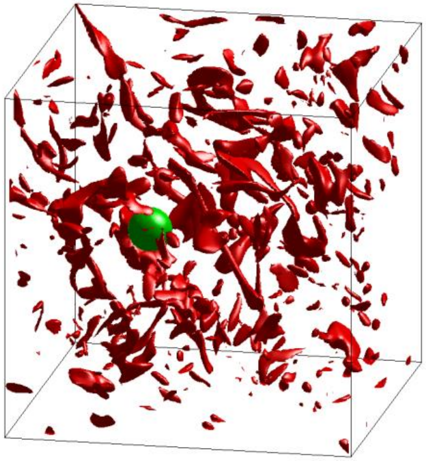
Xu, Ouellette & Bodenschatz, NJP 2008

- Lagrangian turbulence deals with (ideal) fluid particles.
  Transport of **real particles** (solid inclusions, drops, bubbles) ?




- Investigations of the turbulent transport of **small** (< η) **particles**:
  → Lagrangian description of particles motion by a **force balance**

- More recent investigations on **large inclusions**:



→ solid particles: Exp. *Qureshi et al, 2007*; *Zimmermann et al, 2011*; *Bellani & Variano, 2012*; *Klein et al, 2013*; *Zimmermann et al, 2013*; *Mathai et al, 2015*; …

Num. *Lucci et al, 2010*; *Naso & Prosperetti, 2010*; *Cisse, Homann & Bec, 2013*; *Chouippe & Uhlmann, 2015*; *Chouippe & Uhlmann, 2019*; …

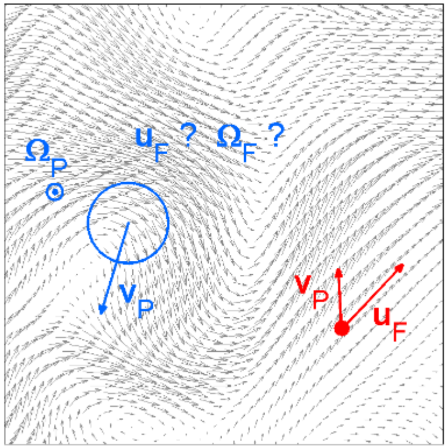
→ bubbles: Exp. *Ravelet et al, 2011*; *Prakash et al, 2012*; *Alméras et al, 2017*; …

Num. *Merle, Legendre & Magnaudet, 2005*; *Lu & Tryggvason, 2006, 2013*; *Loisy & Naso, PRF 2017*; …

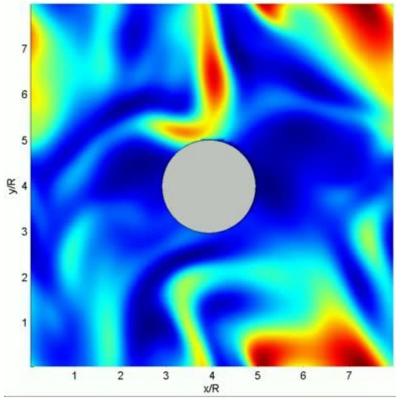
Turbulent transport of finite-size ($d >> \eta$) inclusions

- **Modeling: many open questions:**

→ equations of motion (translation, rotation) ?
→ "slip" velocity ?
→ backreaction on the carrier phase ?

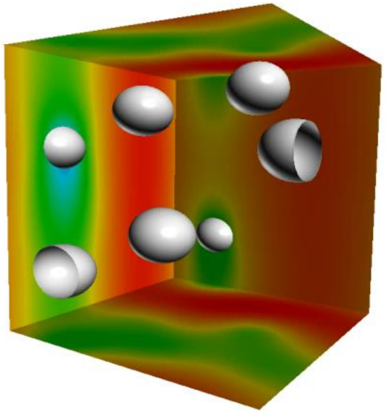


- **Severe difficulties for numerics**:
Need of fully resolved simulations



Naso & Prosperetti, NJP 2010

(solid particle)



Loisy, Naso & Spelt, JFM 2017

(bubbles)

Turbulent transport of small (d < η) inclusions

Lagrangian description of particles motion by a force balance:

$$m_p \frac{d\mathbf{V}}{dt} = m_p\mathbf{g} + m_f\left(\frac{D\mathbf{U}}{Dt} - \mathbf{g}\right) + \mathbf{F_P}$$

$\mathbf{F_P}$ (force due to the particle) = ???

Turbulent transport of small (d < η) inclusions

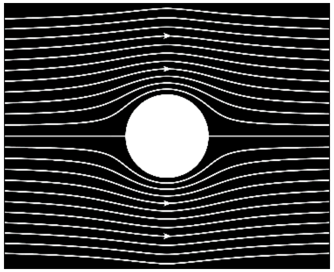
Basset-Boussinesq-Oseen equation (spherical solid particle)

Boussinesq , C. R. Acad. Sci. Paris 1885; Basset,1888;
Gatignol, J. Mech. Theor. Appl. 1983; Maxey & Riley, PoF 1983

$$m_p \frac{d\mathbf{v}_p}{dt} = m_f \frac{D\mathbf{u}}{Dt} + 6\pi r \mu_f (\mathbf{u} - \mathbf{v}_p) + \frac{1}{2} m_f \left(\frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{v}_p}{dt} \right)$$
$$+ 6r^2 (\pi \mu_f \rho_f)^{1/2} \int_0^t \frac{d(\mathbf{u} - \mathbf{v}_p)/d\tau}{(t-\tau)^{1/2}} d\tau + (m_p - m_f)\mathbf{g}$$

= Fluid acceleration + Stokes drag + Added mass + History force + Buoyancy

Expression derived assuming a Stokes flow at the particle scale: $Re_p = 2r|u-v|/\nu << 1$!!!

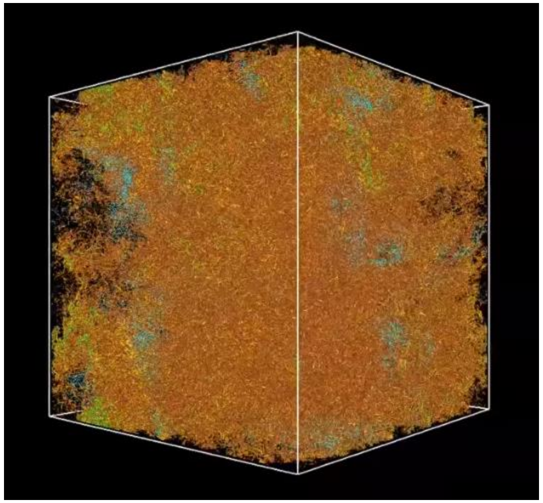


Turbulent transport of small (d < η) inclusions

In the limit of small, spherical, very dense particles, $\rho_p/\rho_f \gg 1$

(e.g., sand in air, water droplets in air, …)

$$m_p \frac{d\mathbf{v}_p}{dt} = 6\pi r \mu_f (\mathbf{u} - \mathbf{v}_p) \quad \Rightarrow \quad \boxed{\frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u} - \mathbf{v}_p}{St}}$$



Ireland & Collins

$$St = \frac{2r^2}{9\nu\tau_\eta}\frac{\rho_p}{\rho_f} = \tau_p / \tau_\eta$$

Stokes number

- St<<1: particle ~ fluid tracer
- St>>1: very inertial particle

Turbulent transport of small ($d < \eta$) inclusions

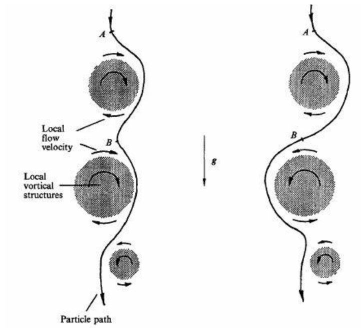
Introduction

In the limit of small, spherical, very dense particles, $\rho_p/\rho_f \gg 1$
and in the presence of gravity

$$\frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u} - \mathbf{v}_p}{St} + \mathbf{g}$$

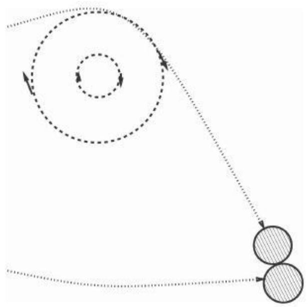
Turbulence tends to:

increase the settling velocity



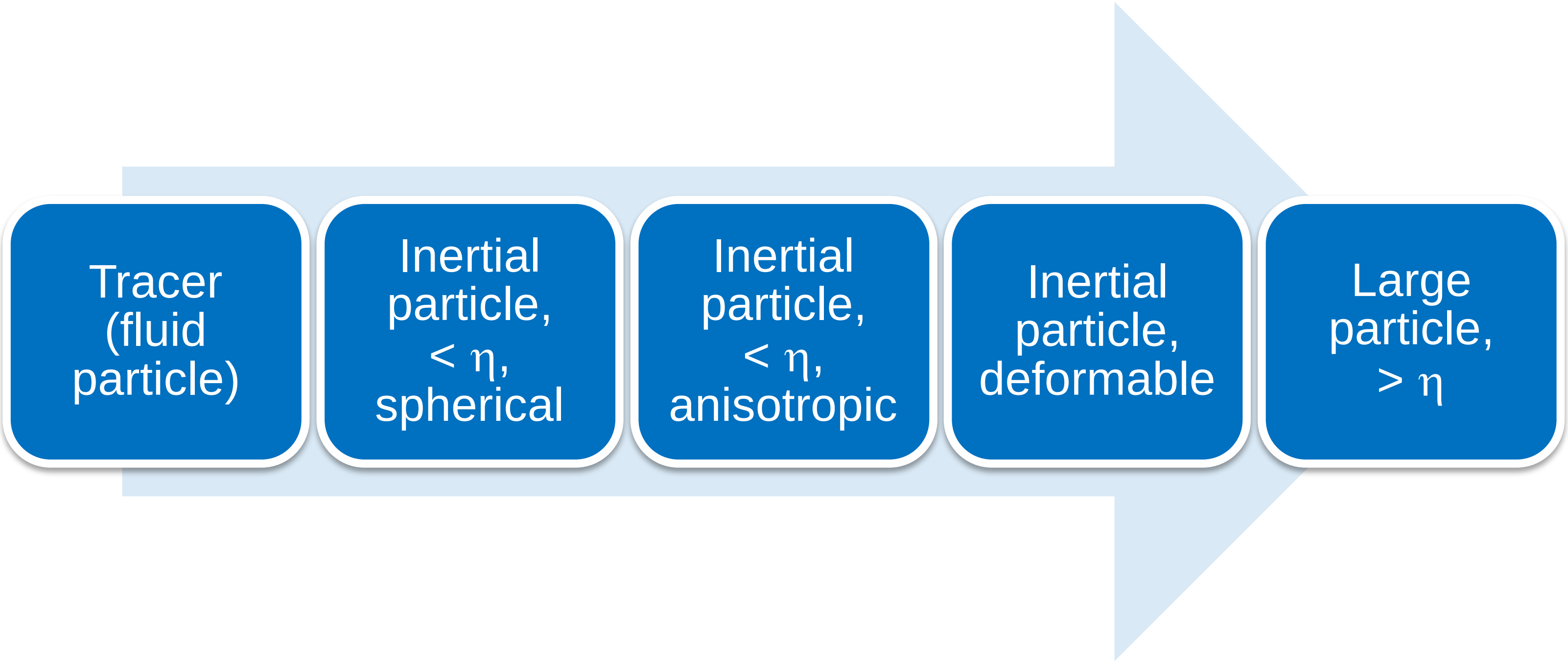
Wang & Maxey, JFM 1993

increase the collision rate



Falkovich & Pumir, J. Atm. Sci. 2007

Increasing complexity



Outline

Tracer (fluid particle)

Inertial particle, $< \eta$, spherical

Inertial particle, $< \eta$, anisotropic

Inertial particle, deformable

Large particle, $> \eta$

Time irreversibility of turbulence in the Lagrangian frame

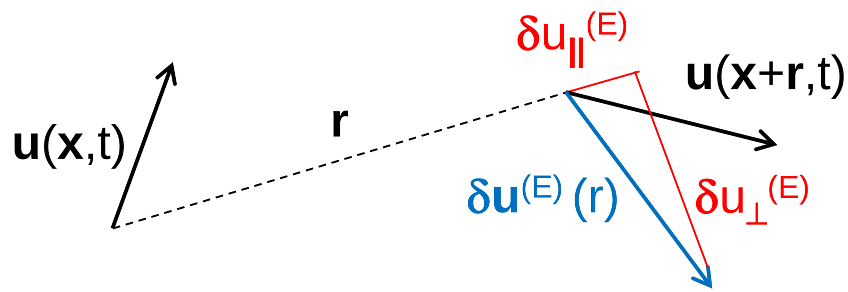
Settling, orientation and collisions of spheroidal particles

Time irreversibility of turbulence in the Lagrangian framework

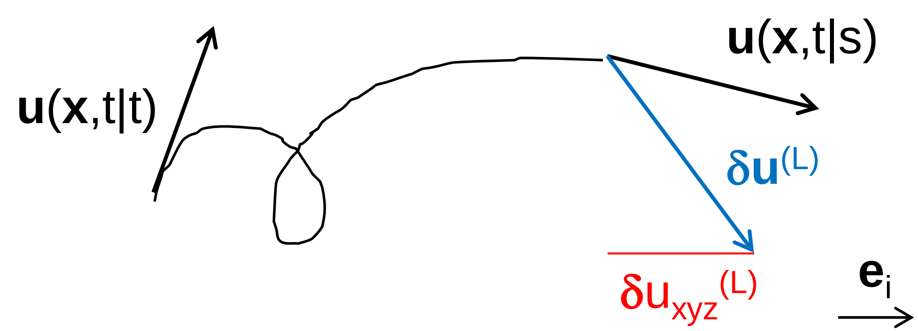
In coll. with: Emmanuel Lévêque (LMFA)

Turbulence extensively studied with a particular focus on velocity increments

Eulerian framework:

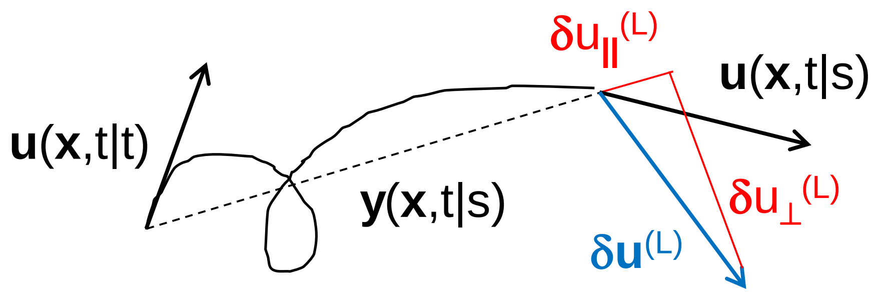


Lagrangian framework:



- **Consensus:** the statistics of turbulence should connect explicitly to its peculiar geometric properties (see, e.g., *Chertkov et al, PoF 1999*; *Li & Meneveau, PRL 2005; Naso & Pumir, PRE 2005; Chevillard & Meneveau, PRL 2006;* …), not captured by the standard velocity increments

- **Idea:** try to recast such properties in the classical phenomenology of turbulence, by introducing longitudinal and transverse Lagrangian velocity increments:



$$\delta u_{\|}^{(L)}(\mathbf{x},t|s) = \delta \mathbf{u}^{(L)}(\mathbf{x},t|s) \cdot \hat{\mathbf{y}}(\mathbf{x},t|s)$$

$$\delta u_{\perp}^{(L)}(\mathbf{x},t|s) = |\, \delta \mathbf{u}^{(L)}(\mathbf{x},t|s) \times \hat{\mathbf{y}}(\mathbf{x},t|s) \,| \cos\theta$$

$$\hat{\mathbf{y}} = \mathbf{y}/\|\mathbf{y}\|$$

$$\theta \in [0,2\pi[$$

- natural extension of their Eulerian counterparts
- alternative path to the description of Lagrangian statistics

- In stationary, homogeneous and isotropic turbulence, the statistics of $\delta u_{\|}^{(L)}(\mathbf{x},t|s)$ and $\delta u_{\perp}^{(L)}(\mathbf{x},t|s)$ only depend on the time interval $\tau = s - t$

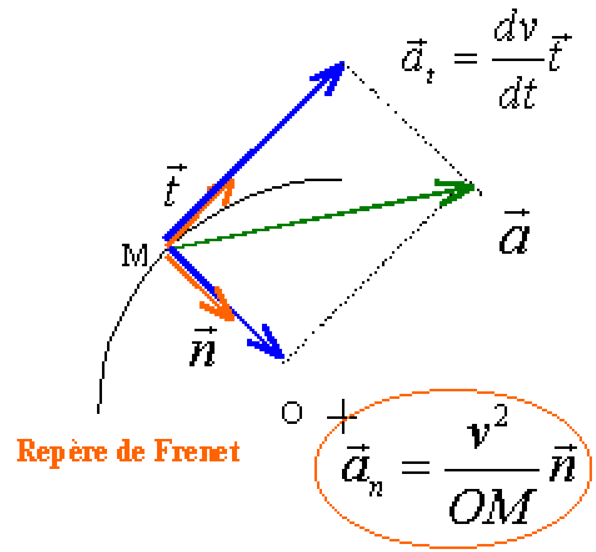
- In the limit $\tau \rightarrow 0$,

$$\delta u_{\|}^{(L)}(\tau) \sim \tau\, a_{\|}$$

$a_{\|}$: **tangential acceleration**, quantifying the variation of velocity magnitude

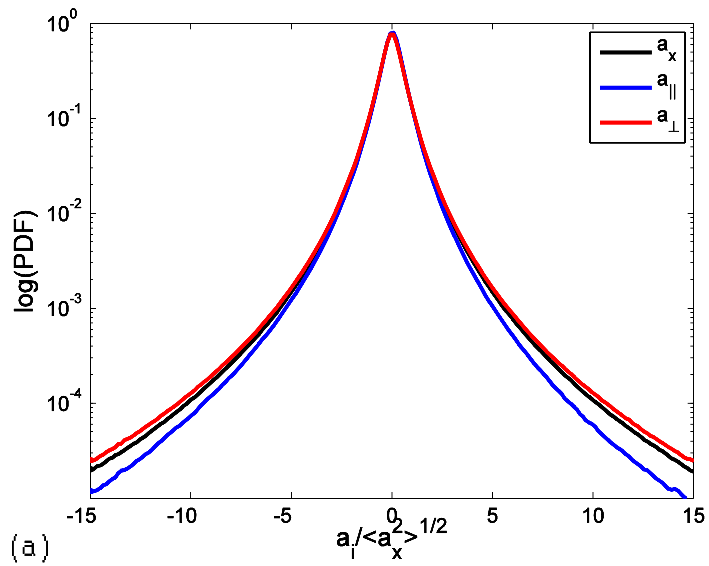
$$\delta u_{\perp}^{(L)}(\tau) \sim \tau\, a_{\perp}$$

$a_{\perp}$: **normal acceleration**, sensitive to the trajectory curvature



- **Investigation, by DNS, of the statistics** of these increments at different R_λ.

Acceleration PDF ($\tau \to 0$)
(R_λ = 280)



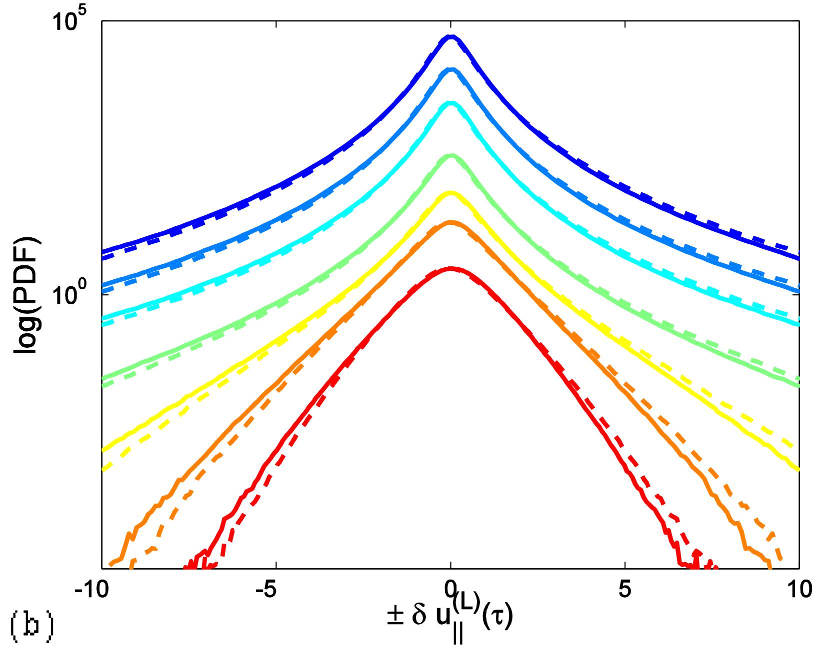
Lévêque & Naso, EPL 2014

- PDF of $a_{||}$ negatively skewed (Sk ≈ -0.4) => **time-irreversibility of turbulence**

deceleration > (positive) acceleration

(see also power fluctuations and kinetic energy increments; *Xu et al, PNAS 2014; Mordant, 2001*)

PDF of the longitudinal velocity increments $\delta u_{\|}^{(L)}$ (R_λ = 280)



Lévêque & Naso, EPL 2014

Same properties satisfied at any τ:

➢ $$S_{\|}(\tau) = \frac{\langle \delta u_{\|}(\tau)^3 \rangle}{\langle \delta u_{\|}(\tau)^2 \rangle^{3/2}} < 0$$

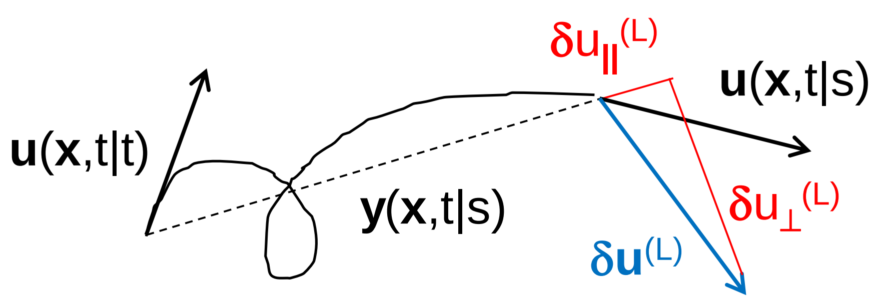
Statistics conditioned on the flow topology

- Longitudinal increment exactly zero in the case of pure (constant-speed) rotation

  → should be of higher magnitude when the trajectory is straight (in flow regions of high strain)

- Transverse increment vanishes for a straight trajectory

  → should be of higher magnitude when the trajectory twists itself (in flow regions of high vorticity)

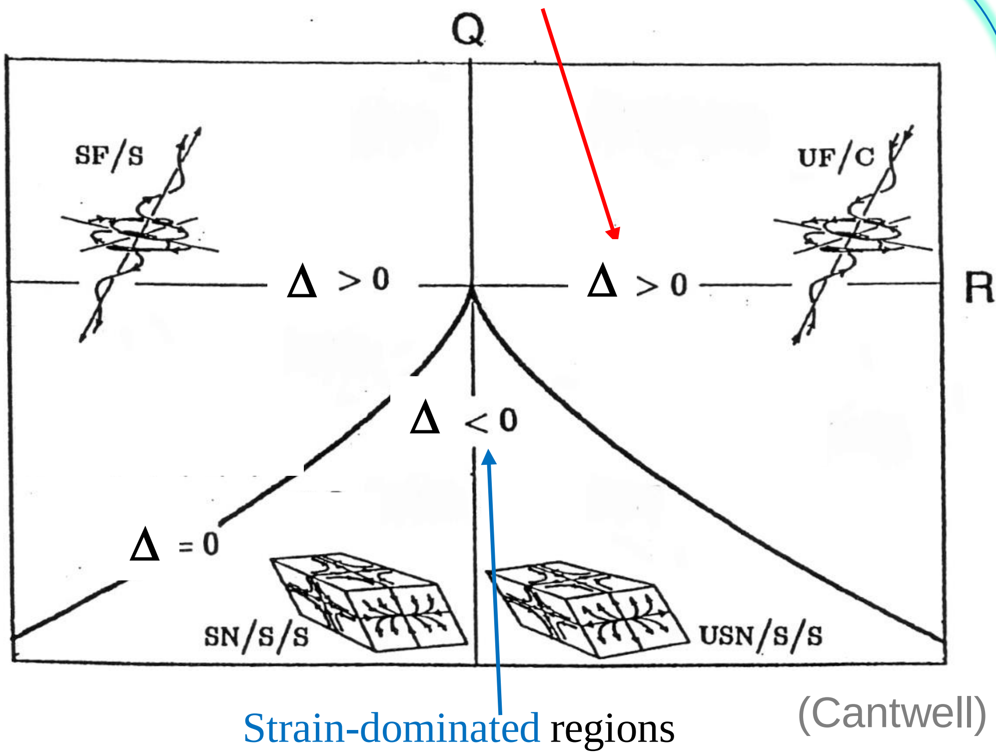


Statistics conditioned on the flow topology

- The eigenvalues of $m_{ab}=\partial_a u_b$ depend on 2 parameters only:

$$Q = -\frac{1}{2}Tr(m^2)$$
$$R = -\frac{1}{3}Tr(m^3)$$

- $\Delta = 27R^2 + 4Q^3$:
  - $\Delta > 0$: vorticity-dominated
  - $\Delta < 0$: strain-dominated



- Variances of the **acceleration components** conditioned on the **sign of Δ** ?

Statistics conditioned on the flow topology

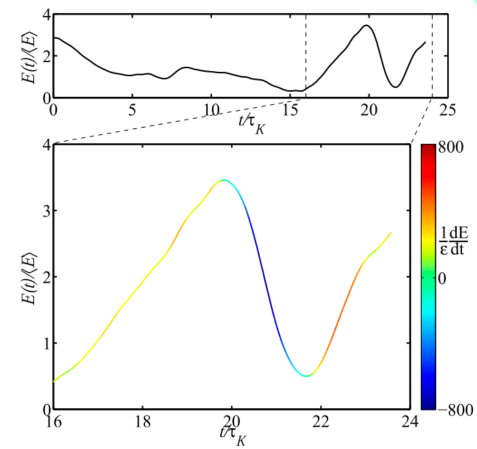
- Variances of the **acceleration components** conditioned on the **sign of Δ**

$$\boxed{1 < \frac{\langle a_{\parallel}^2 | \Delta > 0 \rangle}{\langle a_{\parallel}^2 | \Delta < 0 \rangle} < \frac{\langle a_{xyz}^2 | \Delta > 0 \rangle}{\langle a_{xyz}^2 | \Delta < 0 \rangle} < \frac{\langle a_{\perp}^2 | \Delta > 0 \rangle}{\langle a_{\perp}^2 | \Delta < 0 \rangle}}$$

- Therefore:
  - Higher variances in vorticity-dominated regions than in strain-dominated ones, for all the components.
  - Effect more pronounced for the transverse component than for the longitudinal one.

Further studies

- Time irreversibility of turbulence also evidenced in the Lagrangian framework, both in 2D and 3D, through measurement of Sk(p=**u.a**) and Sk(δE):
  Xu, Pumir, Falkovich, Bodenschatz et al, PNAS 2014



- *Pumir, Xu, Boffetta, Falkovich & Bodenschatz, PRX 2014*:
  In 3D, [$<p^3> < 0$] essentially due to cross-correlation of pressure and dissipation.

- *Pumir, Xu, Bodenschatz & Grauer, PRL 2016*:
  In 3D, [$<p^3> < 0$] linked with positive sign of vortex stretching.

- Time irreversibility of the statistics of a single particle also investigated in compressible (*Grafke, Frishman & Falkovich, PRE 2015*) and in rotating (*Maity, Govindarajan & Ray, PRE 2019*) turbulence.

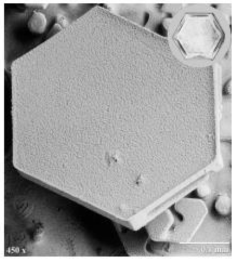
Settling, orientation and collisions of spheroidal particles:

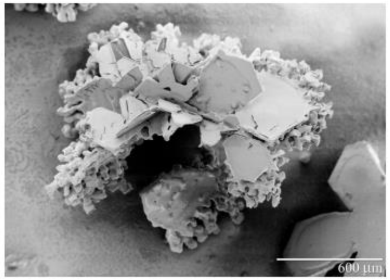
application to cloud microphysics

In coll. with: Alain Pumir, Muhammad Z. Sheikh, Jennifer Jucha,
Facundo Cabrera, Nicolas Plihon, Mickael Bourgoin
(Laboratoire de Physique, ÉNS Lyon)
Bernhard Mehlig, Kristian Gustavsson (Gothenburg University, Sweden)
Emmanuel Lévêque, Diego Lopez (LMFA)

Settling, orientation, collisions and aggregation of particles in cold clouds

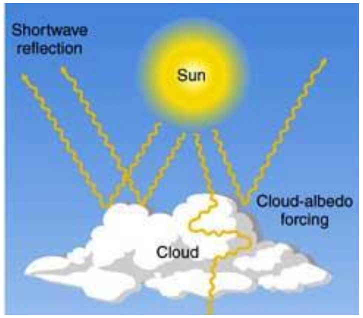
- Collision and aggregation of ice crystals
  → formation of graupels





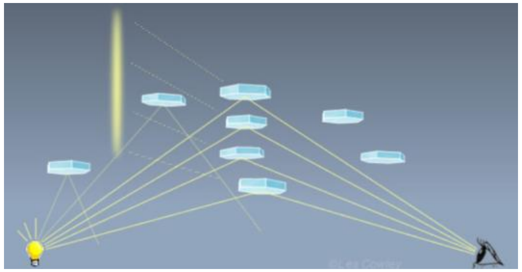
Electron and confocal microscopy laboratory,
US Agriculture Research Center

- Ice crystals orientation
  → EM waves (light) reflexion



See also *Bréon & Dubrulle, JAS 2004*.

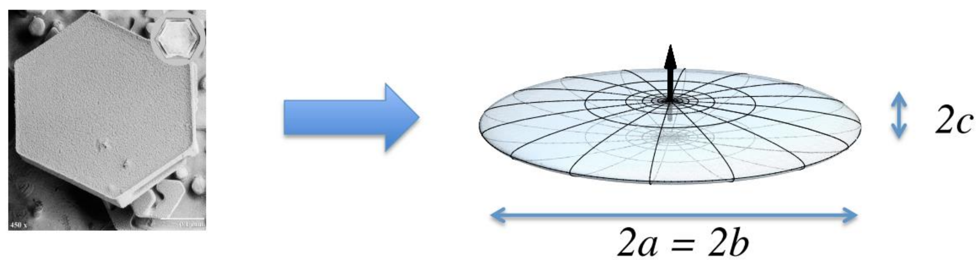
https://www.atoptics.co.uk/halo/lpil.htm



Crystals motion

Spheroidal particles

- Assume that the crystal is a thin oblate ellipsoid of revolution (spheroid): c << b=a.



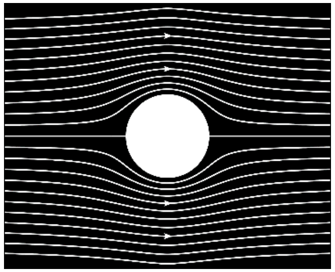
- Small ($a < \eta$), heavy ($\rho_p >> \rho_f$) and spheroidal particles.

- The force and torque acting on the spheroid must be determined by solving the (Navier-)Stokes equations around the object.

First effect of fluid inertia on the translational motion

- Translational and rotational dynamics of spheroidal particles in turbulence:
  For particles ≠ fluid tracers, need to write equations of motion

$m_p\, d\mathbf{v}/dt$ = **hydrodynamic force** + buoyancy
$I_p\, d\omega/dt$ = **hydrodynamic torque**



- Assuming Stokes flow: hydrodynamic force = Stokes force
  hydrodynamic torque = Stokes (Jeffery's) torque

For a spherical object:

$$\mathbf{F}_{Stokes} = 6\pi r \mu_f (\mathbf{u} - \mathbf{v}_p)$$

First effect of fluid inertia:

$$\mathbf{F}_{Oseen} = \mathbf{F}_{Stokes} \times \left(1 + \frac{3Re_p}{16}\right) \sim \mathbf{F}_{Stokes} \text{ if } Re_p \ll 1$$ (in practice if r << η)

For a spheroidal object:

* Similar correction (~ Re_p) available for translational motion (*Brenner, JFM 1961*).

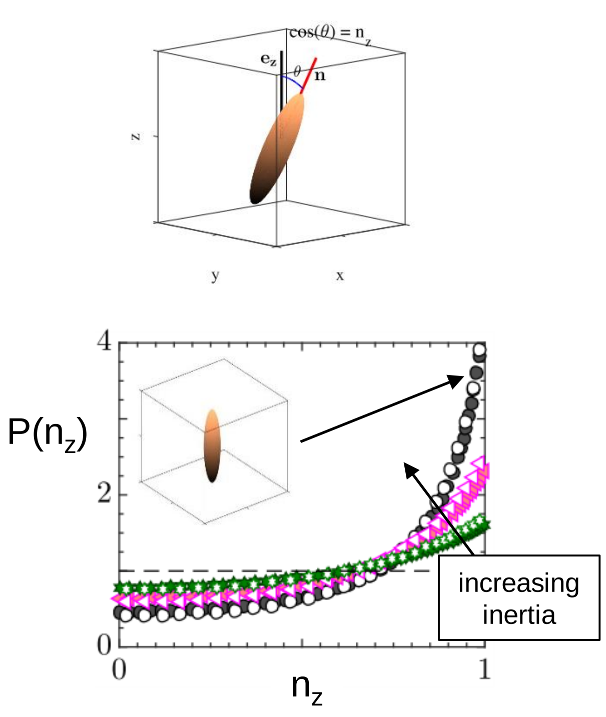
* What about rotational motion ???

Settling of spheroids in a turbulent flow: orientation distribution calculated numerically using **Stokes torque**

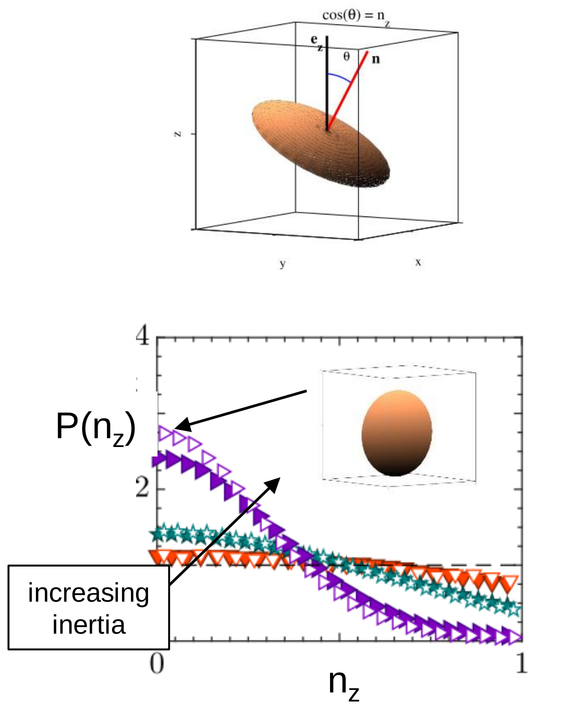
- Integration of the resulting set of equations for particles suspended in a turbulent flow (*Gustavsson et al, 2014*; *Siewert et al, 2014a,b*; *Gustavsson et al, 2017*; *Jucha et al, 2018*; *Naso et al, 2018*):

If W_s (settling velocity) > U_0 (fluid velocity), the orientation distribution is biased ("vertical"):

<u>Prolate (rod-like):</u>

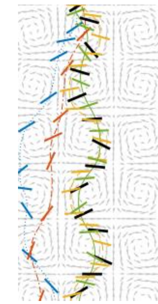


<u>Oblate (disk-like):</u>

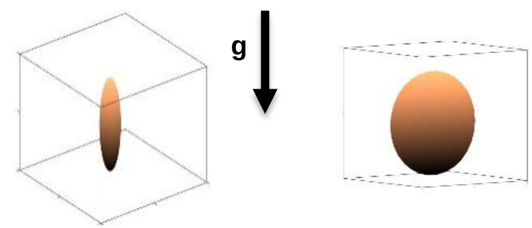


First effect of fluid inertia on the rotational motion of settling spheroids

- Experimental results:
  - * *Lopez & Guazzelli, PRF 2017*: slender rods in a 2D laminar flow
    → "horizontal" settling



  - * *Klett, J. Atmos. Sci. 1995* (~spheres); *Kramel, PhD 2017* (slender rods) in turbulence:
    → "horizontal" settling



Results opposite to those obtained by DNS in turbulent flows using Stokes approximation !

- Fluid inertia correction on rotational motion of spheroids first derived for nearly spherical objects (*Cox, JFM 1965*) and slender bodies (*Khayat & Cox, JFM 1989*), and recently for arbitrary aspect ratios (*Dabade et al, JFM 2015*).

Problem: fluid-inertia torque ~ Stokes torque !!!

This inertial correction → "horizontal" settling.

- <u>Idea:</u> Determine the conditions under which fluid inertia can be neglected for the angular dynamics of spheroids settling in turbulence.

Angular motion of spheroids

- Angular equation of motion, in the particle frame:

$$\frac{d\hat{\boldsymbol{\omega}}}{dt} = \begin{pmatrix} \hat{T}_{xx}/\hat{I}_{xx} \\ \hat{T}_{yy}/\hat{I}_{yy} \\ \hat{T}_{zz}/\hat{I}_{zz} \end{pmatrix} + \begin{pmatrix} \hat{\omega}_y \hat{\omega}_z \frac{\hat{I}_{yy}-\hat{I}_{zz}}{\hat{I}_{xx}} \\ \hat{\omega}_z \hat{\omega}_x \frac{\hat{I}_{zz}-\hat{I}_{xx}}{\hat{I}_{yy}} \\ \hat{\omega}_x \hat{\omega}_y \frac{\hat{I}_{xx}-\hat{I}_{yy}}{\hat{I}_{zz}} \end{pmatrix}$$

, with

$\hat{\boldsymbol{\omega}}$: angular velocity

$\hat{\mathbf{I}}$: moment-of-inertia tensor

$\boxed{\hat{\mathbf{T}} = \hat{\mathbf{T}}_{St} + \hat{\mathbf{T}}_{I}}$: hydrodynamic torque

"Stokes" contribution
(*Jeffery, 1922*)

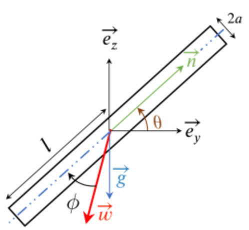
Contribution due to fluid inertia
for a particle moving steadily
in a uniform flow
(*Dabade et al, 2015*)

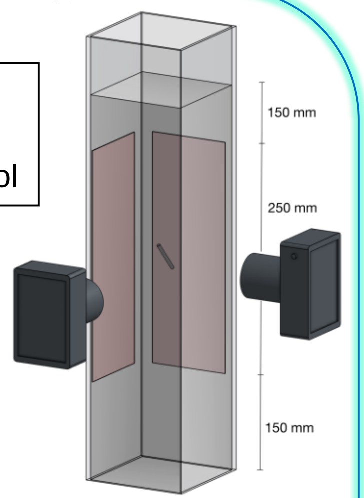
- NB: Fluid inertia can also induce corrections due to shear (*Candelier, Mehlig & Magnaudet, JFM 2019*) and unsteadiness.

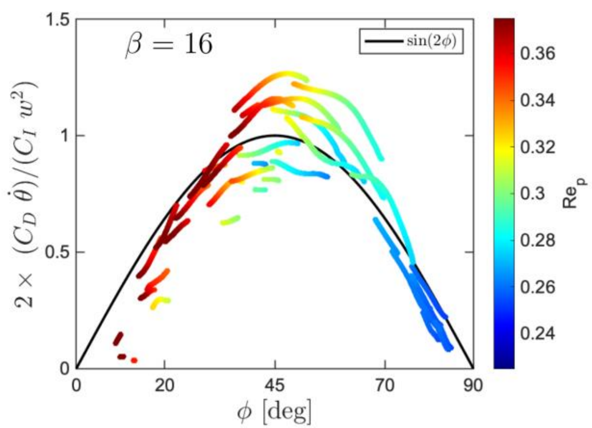
Spheroidal particles

Inertial correction to the torque: confrontation with experiments and resolved DNS

Experiments:
Rods of aspect ratios 8 and 16 dropped in glycerol

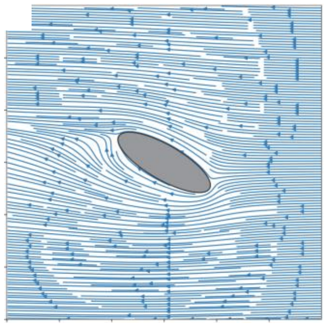


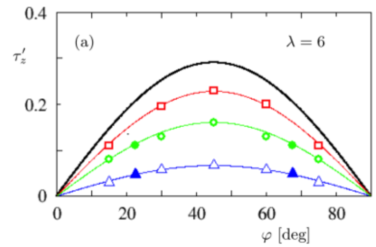


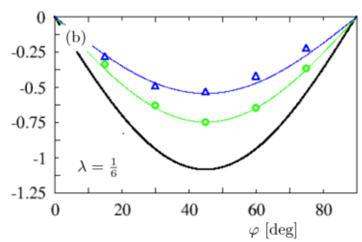


Cabrera, Sheikh, Naso, Plihon, Bourgoin & Pumir, in prep., 2021

Particle-resolved DNS:
Spheroid set fixed in a uniform flow







Jiang, Zhao, Andersson, Gustavsson, Pumir & Mehlig, submitted, 2021

Angular motion of spheroids

Hydrodynamic torque: $\hat{\mathbf{T}} = \hat{\mathbf{T}}_{St} + \hat{\mathbf{T}}_{I}$

"Stokes" contribution
(*Jeffery, 1922*)

Contribution due to fluid inertia
for a particle moving steadily
in a homogeneous flow
(*Dabade et al, 2015*)

- Estimating the ratio $\boxed{|\hat{\mathbf{T}}_I|/|\hat{\mathbf{T}}_{St}|}$:

  * $|\hat{\mathbf{T}}_I|/|\hat{\mathbf{T}}_{St}| \ll 1$: Stokes expected to dominate ("vertical" settling)

  * $|\hat{\mathbf{T}}_I|/|\hat{\mathbf{T}}_{St}| \gg 1$: inertia expected to dominate ("horizontal" settling)

Evaluation of the ratio $|\mathbf{T}_{St}|/|\mathbf{T}_I|$

- For very flat disks (aspect ratio β << 1) and for thin rods (5 ≤ β ≤ 100):

$$|\hat{\mathbf{T}}_I|/|\hat{\mathbf{T}}_{St}| \sim \mathscr{R}, \quad \text{with} \quad \mathscr{R} \equiv \frac{u_s^2}{\nu s}$$

$\mathbf{u}_s = \mathbf{v} - \mathbf{u}$: slip velocity
$\mathbf{v}$: particle velocity
$\mathbf{u}$: fluid velocity at the particle position
s : inverse of a characteristic time scale ~ flow velocity gradients
ν : fluid viscosity

- Estimating the slip velocity as $u_s \sim W_s$, where $W_s \approx g\tau_p$ is the settling velocity of the particles, and using standard estimates for evaluating the turbulent velocity gradients leads to:

$$\boxed{\mathscr{R} \sim \left(\frac{W_s}{U_0}\right)^2 Re_f^{1/2}}$$

$Re_f = U_0 L/\nu$: large scale Reynolds number of the flow

Discussion of the ratio $|\mathbf{T}_{St}|/|\mathbf{T}_I|$

$$\boxed{\mathscr{R} \sim \left(\frac{W_s}{U_0}\right)^2 Re_f^{1/2}}$$

- Therefore, in the high Re_f regime, the fluid-inertia torque can be neglected (i.e., $\mathscr{R}$ can be small) only if W_s/U_0 is small
  → orientation distribution nearly uniform (see *Gustavsson et al, 2017*; *Jucha et al, 2018*).

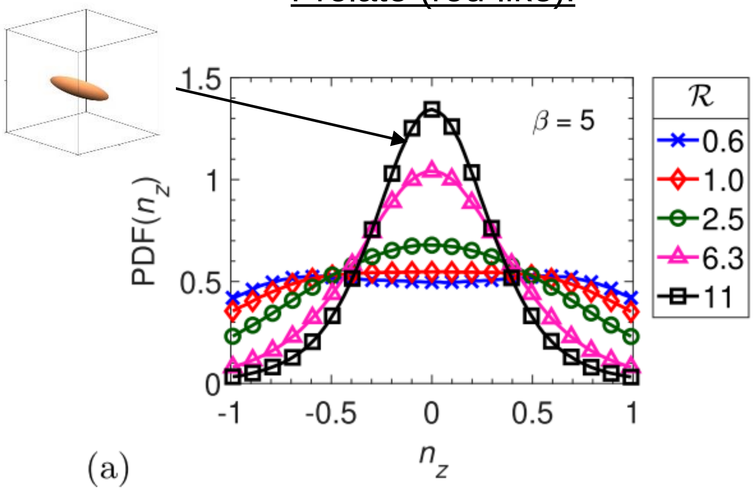
- The Stokes contribution can be neglected ($\mathscr{R} \gg 1$) simultaneously with a large ratio W_s/U_0
  → biased “horizontal” distribution

- **Therefore the orientation bias obtained in numerical works which neglect the fluid-inertia torque (biased “vertical” distribution) cannot be observed at large Re_f !!!**

Sheikh, Gustavsson, Lopez, Lévêque, Mehlig, Pumir & Naso, JFM 2020

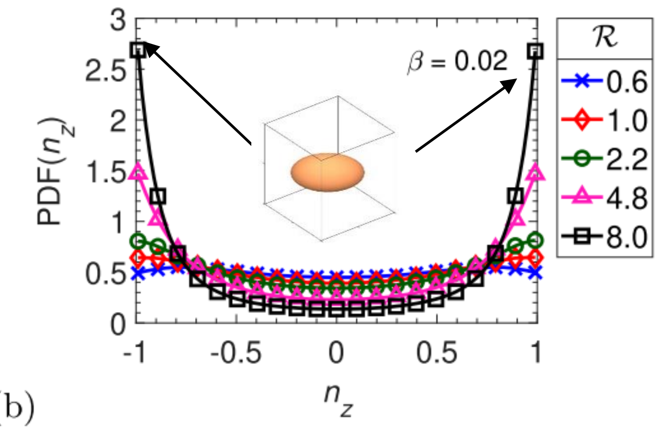
Numerical results in homogeneous and isotropic turbulence

<u>Prolate (rod-like):</u>



(a)

<u>Oblate (disk-like):</u>



(b)

- Transition from a uniform to a biased "horizontal" distribution observed at increasing $\mathscr{R}$.
- Biased "vertical" distribution never observed.
- Analysis seems to be valid for any β.

- NB: theory for the orientation distribution at large $\mathscr{R}$ derived in *Gustavsson et al, NJP 2019* and *Gustavsson et al, submitted, 2021*.

Settling, orientation and collisions of ice crystals: numerical setup

- Generation of an idealized stationary, homogeneous and isotropic turbulent flow in a cubic box with periodic boundary conditions (pseudo-spectral method).

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho_f} + \nu \nabla^2 \mathbf{u} + \mathbf{f},$$
$$\nabla \cdot \mathbf{u} = 0,$$

- 3 values of R_λ (or ε) considered:

Flow	I	II	III
ε (cm^2/s^3)	0.976	15.62	246.4
Re_λ	55.8	94.6	151.2
τ_K (s)	0.341	0.085	0.021
T_L (s)	1.96	0.70	0.26
u_{rms} (cm/s)	2.18	5.72	14.4
N	384	768	1576

Physical parameters

- Simulations designed for representing at best realistic situations in cloud conditions. Physical parameters at $T = -20\,°\mathrm{C}$ (*Pruppacher & Klett, 1997*).

- Parameters common to all runs:

Fluid			Ice crystals		Gravity
ρ_f (g cm^{-3})	ν (cm^2 s^{-1})	μ (g cm^{-1} s^{-1})	ρ_i (g cm^{-3})	a (µm)	g (cm s^{-2})
1.413×10^{-3}	0.1132	1.599×10^{-4}	0.9194	150	981

TABLE 1. Values of the physical parameters common to all runs. The fluid is moist air, whose volumetric mass, kinematic and dynamic viscosities are ρ_f, ν and μ, respectively. The density of the water droplets is ρ_l. The ellipsoidal ice crystals have a volumetric mass ρ_i and a semi-major axis a. The gravitational acceleration is denoted g.

Physical parameters

Runs	Flows	β	N_c	T_{run} (s)	$\langle U_s \rangle$ (cm/s)	St	Sv	$\langle \varphi^2 \rangle^{1/2}$	K	K^0
1	I	0.005	100^3	98	1.84	$8.5\,10^{-3}$	5.13	0.049	$1.2\,10^{-4}$	$4.3\,10^{-5}$
2	I	0.01	100^3	112	3.08	$1.7\,10^{-2}$	10.3	$3\,10^{-3}$	$5.9\,10^{-5}$	$4.2\,10^{-5}$
3	I	0.02	70^3	126	5.48	$3.4\,10^{-2}$	20.5	$3\,10^{-4}$	$4.9\,10^{-5}$	$4.1\,10^{-5}$
4	II	0.005	100^3	24	2.12	$3.4\,10^{-2}$	2.56	0.48	$6.7\,10^{-4}$	$1.6\,10^{-4}$
5	II	0.01	70^3	30	3.50	$6.8\,10^{-2}$	5.13	$5.7\,10^{-2}$	$4.2\,10^{-4}$	$2.0\,10^{-4}$
6	II	0.02	70^3	36	5.78	0.137	10.3	$5\,10^{-3}$	$2.7\,10^{-4}$	$3.0\,10^{-4}$
7	II	0.05	70^3	31.5	11.5	0.342	25.6	$5\,10^{-3}$	$2.4\,10^{-4}$	$8.6\,10^{-4}$
8	III	0.005	100^3	5.28	2.4	0.137	1.31	0.90	$1.4\,10^{-3}$	$1.0\,10^{-3}$
9	III	0.01	100^3	5.28	4.5	0.274	2.61	0.44	$2.4\,10^{-3}$	$1.9\,10^{-3}$
10	III	0.02	100^3	5.28	7.4	0.547	5.22	0.12	$3.6\,10^{-3}$	$4.7\,10^{-3}$

w or w/o gravity.

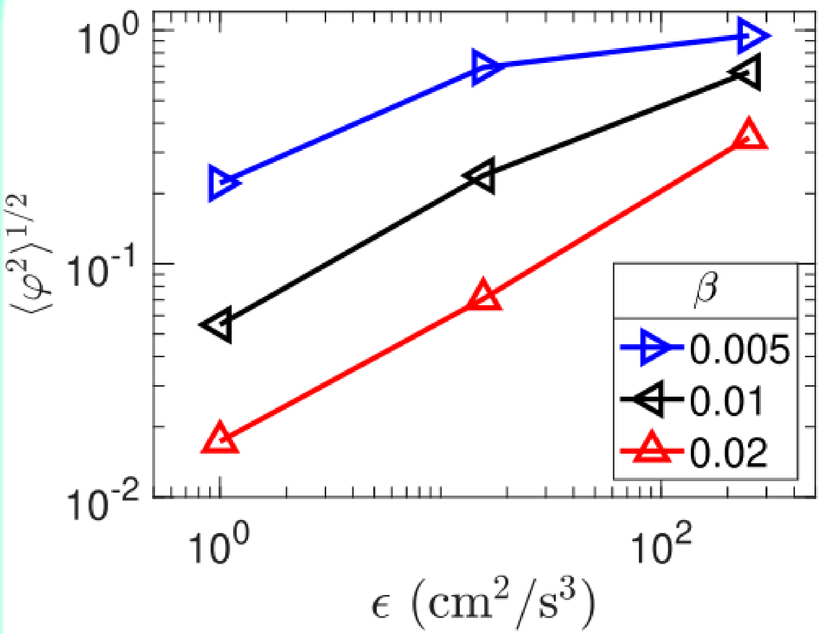
Translational motion of spheroids

$$\frac{d\mathbf{v}}{dt} = \mathbf{g} + \underbrace{\frac{6\pi\mu\tilde{a}}{m_p}\mathbf{M}_{St}\cdot(\mathbf{u}-\mathbf{v})}_{\text{Stokes drag}} + \underbrace{\frac{1}{m_p}\frac{9\pi}{8}\rho_f\tilde{a}^2|\mathbf{u}-\mathbf{v}|\mathbf{M}_I\cdot(\mathbf{u}-\mathbf{v})}_{\text{Fluid inertia correction}}$$

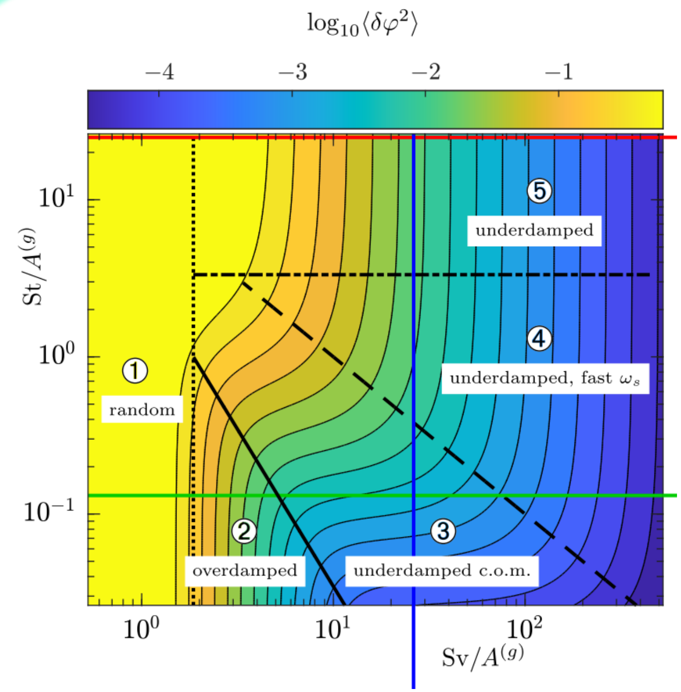
- $\mathbf{M}_{\mathrm{St}}$, $\mathbf{M}_I$: anisotropic resistance tensors, diagonal in the particle eigenframe
  + **R** : rotation matrix (particle frame → laboratory frame)

- **u**: fluid velocity
  v: particle velocity

Crystal settling: orientation statistics



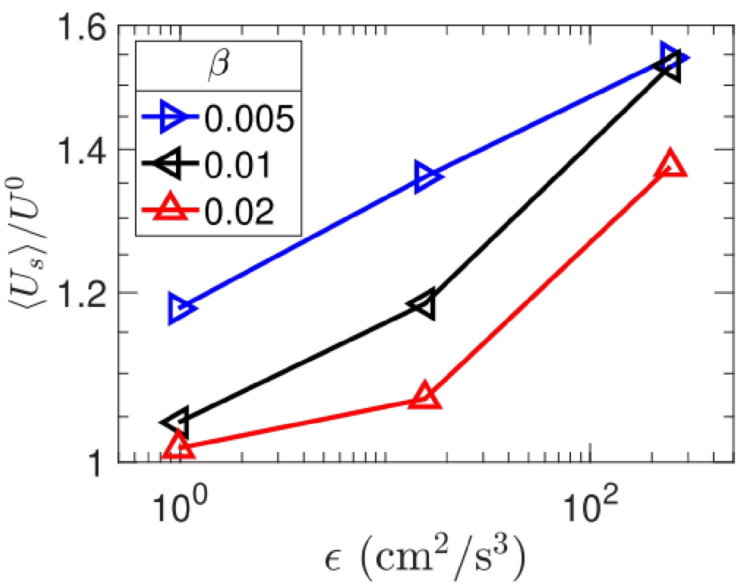
Broader orientation distribution at high ε and small β.

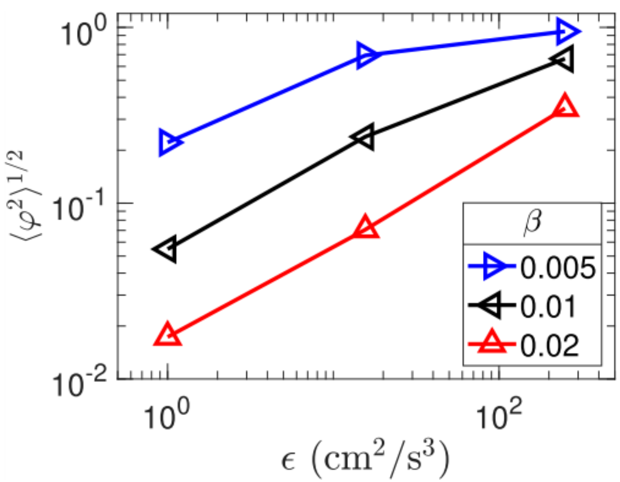


Theoretical modeling of orientation statistics as a function of β, St = τ_p / τ_η (Stokes number) and Sv = $g\tau_p$ / u_η (settling number).

Gustavsson et al, submitted, 2021

Settling velocity

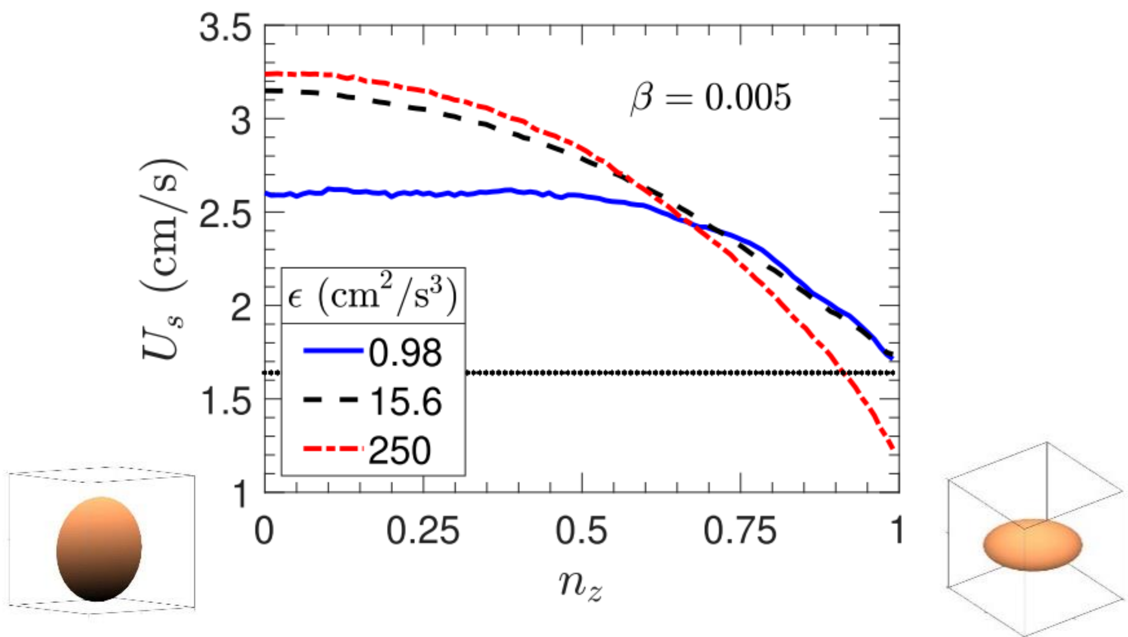




Settling velocity:

- increased by turbulence
- strongly correlated to particle orientation

Settling velocity conditioned on orientation



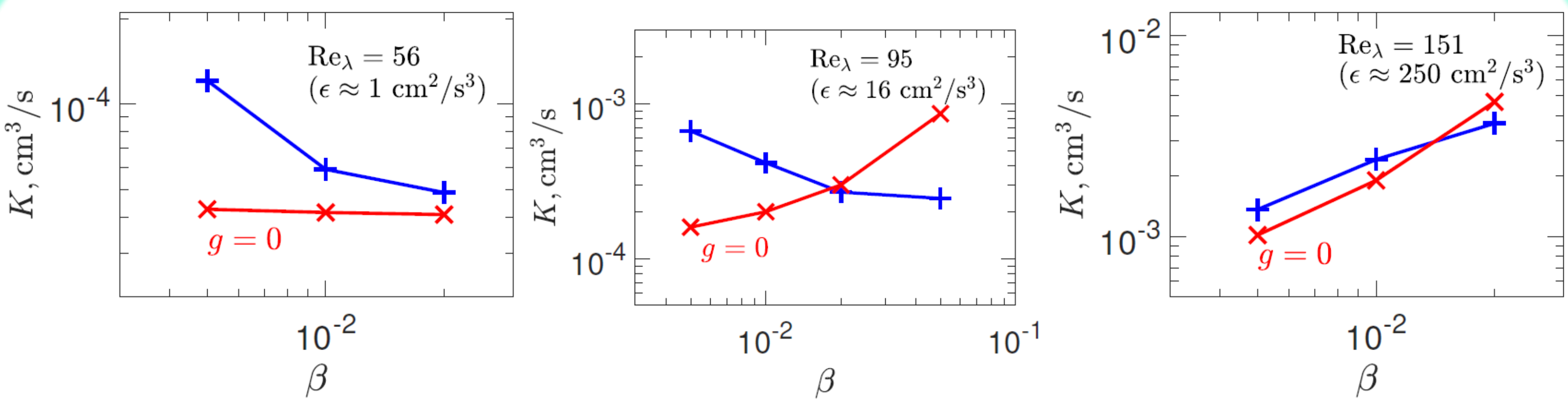
Two particles very close to each other may have a significant velocity difference, provided that they have different orientations → consequences for the collision rate…

Collision kernel

$$N_c = \frac{1}{2} K \times \frac{N^2}{V} \times T$$

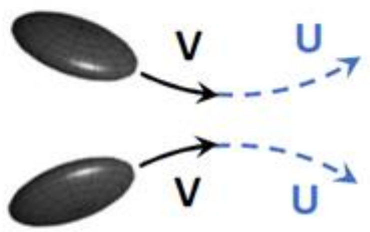
N_c: number of collisions.
K: collision kernel.
N: number of particles.
V: volume of the domain.
T: simulation time.

Pumir & Wilkinson, ARCMP 2016



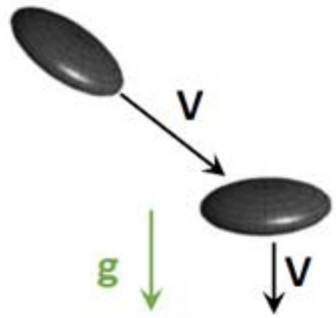
- Without gravity, K increases with β and ε (particle inertia), ~ spheres.
- Behavior less trivial in the presence of gravity.

Collision mechanisms for settling anisotropic particles



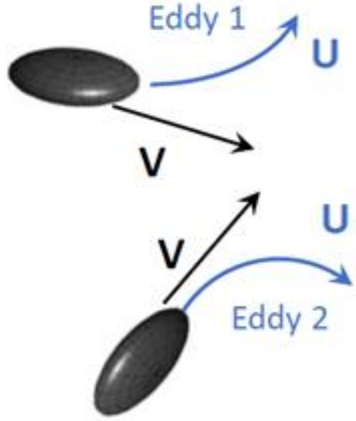
Turbulence: tracer particles brought together by velocity gradients.

Saffman & Turner, JFM 1956



Differential settling: faster spheroids fall on slower ones.

Jucha et al, PRF 2018



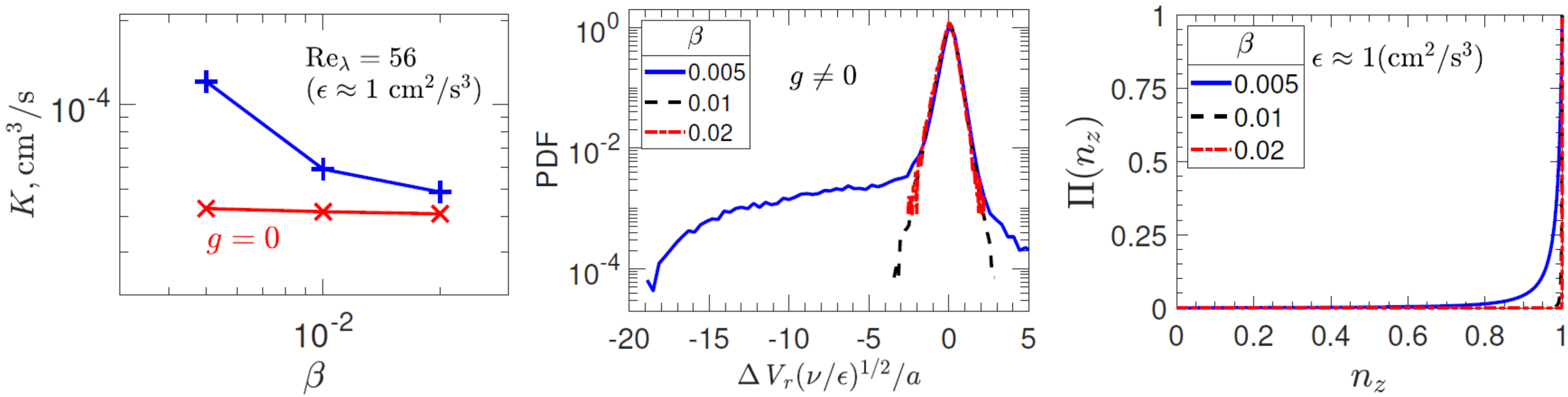
Particle inertia: particles from different locations collide due to the « sling effect ».

Falkovich & Pumir, JAS 2007

Collision kernel

ε = 1 cm^2/s^3; Re$_\lambda$ = 56 (St < 0.04)

$$\Delta V_r = (\mathbf{V_2} - \mathbf{V_1}) \cdot \frac{\mathbf{r_2} - \mathbf{r_1}}{|\mathbf{r_2} - \mathbf{r_1}|}$$

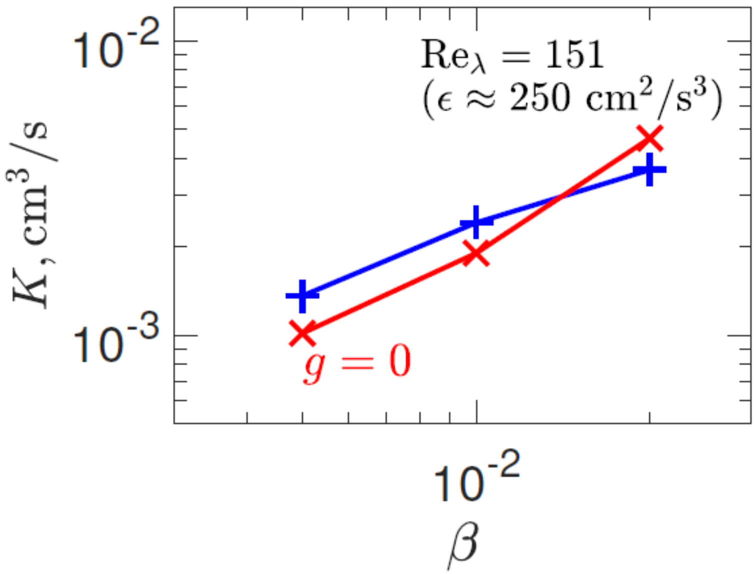


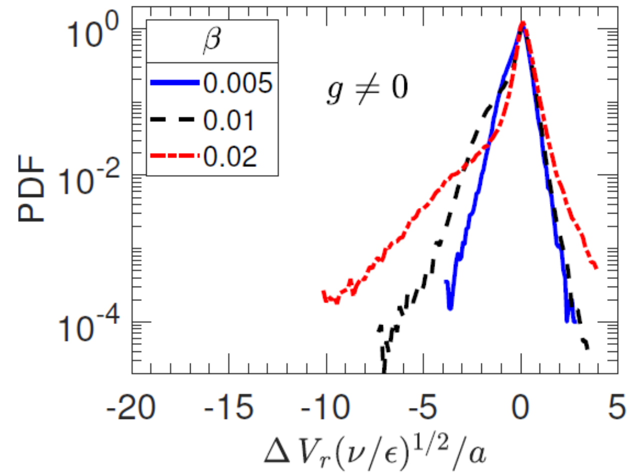
(A) $\epsilon = 0.98\ \mathrm{cm}^2/\mathrm{s}^3$

- Saffman-Turner (K ~ 3.10^{-5} cm^3/s; $\Delta v_r \sim a / \tau_\eta$) for $\beta \geq 0.01$.
- When g ≠ 0, differential settling for $\beta = 0.005$.

Collision kernel

ε = 246 cm^2/s^3; Re$_\lambda$ = 150 (St = 0.1-0.6)





(c) $\epsilon = 250$ cm^2/s^3

- Saffman-Turner ($\Delta v_r \sim a / \tau_\eta$) for β = 0.005.
- Inertial effects (St ~ 0.6) for β = 0.02.

Spheroidal particles

Summary - Discussion

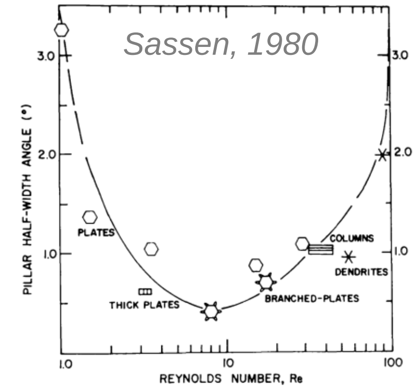
Take-home message (rotational motion of settling spheroids):

- In a turbulent flow, heavy spheroids can only settle either with a random orientation or preferentially horizontally. Neglecting the fluid-inertia torque may lead to wrong results !
- In laminar flows (not shown here), the three orientation regimes can be observed (uniform distribution, “vertical”, “horizontal”). The limit $Re_f \to 0$ requires some care.
- Our estimates were derived for very flat disks (aspect ratio $\beta \ll 1$) and for thin rods ($5 \leq \beta \leq 100$), but our numerical results show that they are also relevant for moderate values of β.

Take-home message (settling, orientation and collisions in clouds):

- Crystals can only settle horizontally or with a random orientation.
- Their differential settling can play a crucial role on collisions. In this case, gravity can increase the collision rate (opposite behaviour in monodisperse suspensions of spheres).
  Three mechanisms: turbulence, differential settling, particle inertia.
- Modelling of orientation and collision (ongoing work) statistics as a function of β, St and Sv.

Limitations of the present approach:



- Effects of fluid inertia due to shear and unsteadiness neglected.
- “Ghost collision” approximation, “one-way coupling”.
- Simplified crystal geometry.

Perspectives:

- Considering prolate spheroids ($-10\,^\circ\mathrm{C} \lesssim T \lesssim -5\,^\circ\mathrm{C}$).
- Collisions between crystals and supercooled water droplets.
- Varying the crystals aspect ratio simultaneously with their size, so as to keep their mass constant.
- Investigating further the collision mechanisms when particle inertia is dominant.

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* M. Z. Sheikh, K. Gustavsson, D. Lopez, E. Lévêque, B. Mehlig, A. Pumir and A. Naso, J. Fluid Mech. **886**, A9 (2020).