Stretching in isotropic turbulence: Polymers and elastic filaments

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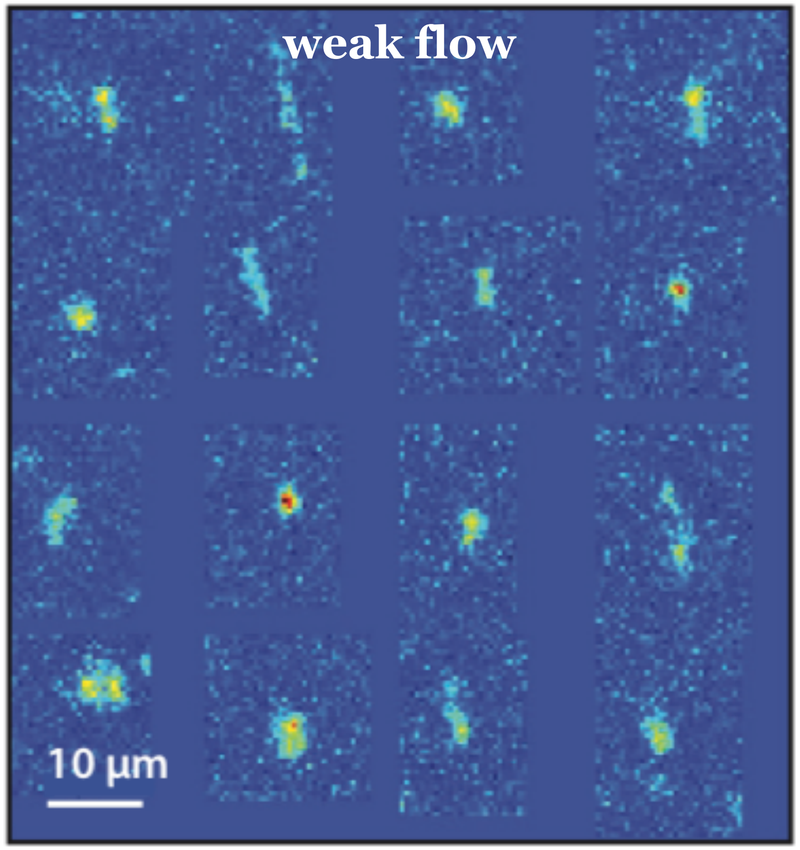
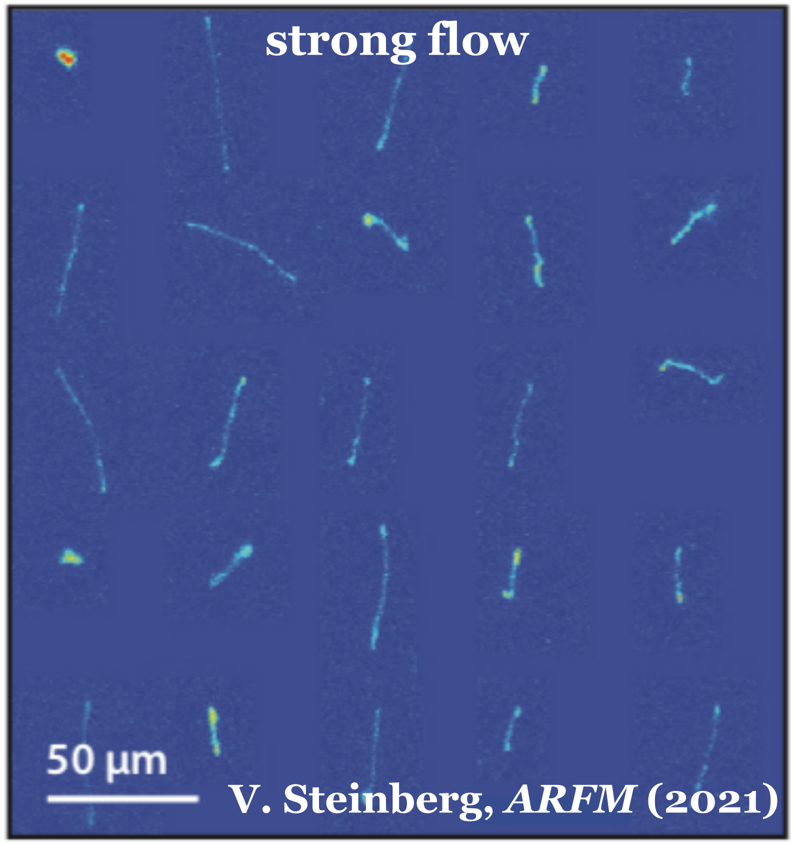
Outline

- Polymer modelling
- Coil-stretch transition in extensional flows
- Random and turbulent flows
  - PDF of the extension
    - Large deviations theory
    - Batchelor—Kraichnan model
    - Lagrangian simulations
    - Micro-fluidics experiments
  - Relaxation to steady state
  - Breakup
  - Stretching beyond the Kolmogorov scale
- Elastic filaments
- Perspectives

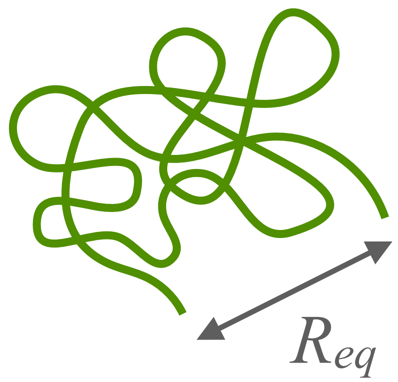
Polymer dynamics in a flow field



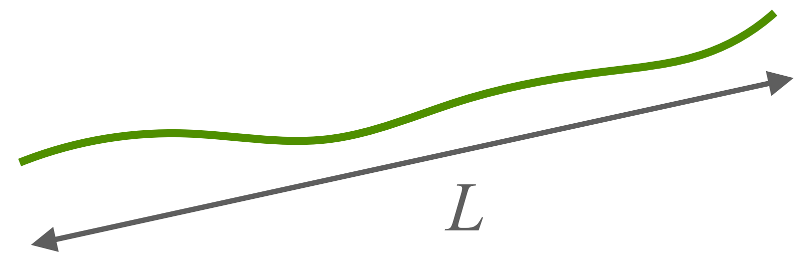


Polymers in a flow field: dimensionless parameters

Extensibility parameter $b = (L / R_{eq})^2$



equilibrium end-to-end separation



contour length

In experiments: polystyrene, PEO, PAM, and DNA; $10'\text{s}\,\mu\text{m} \lesssim L \lesssim 1\text{mm},\ 10^2 \lesssim b \lesssim 10^4$

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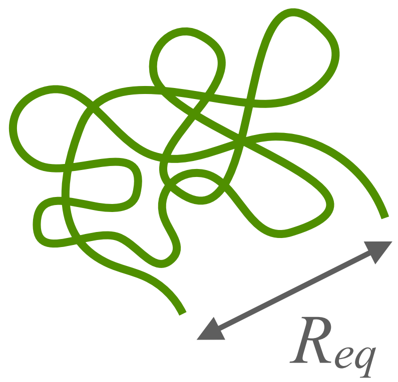

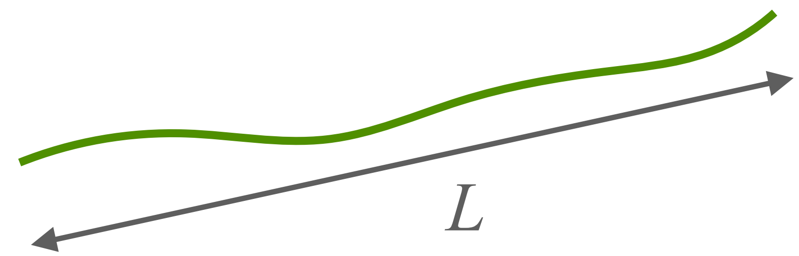

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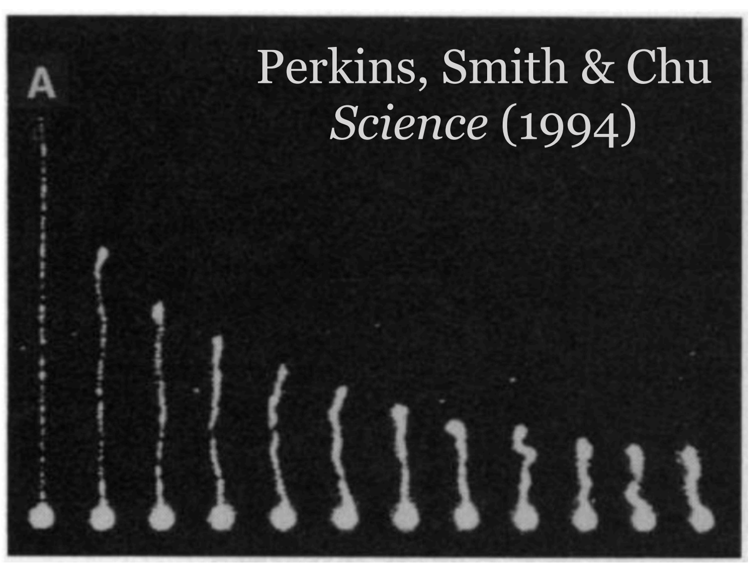
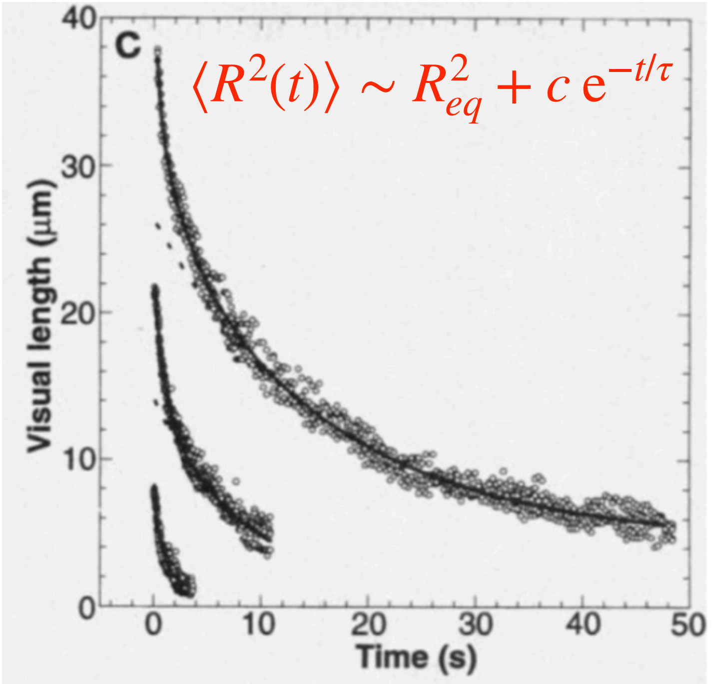
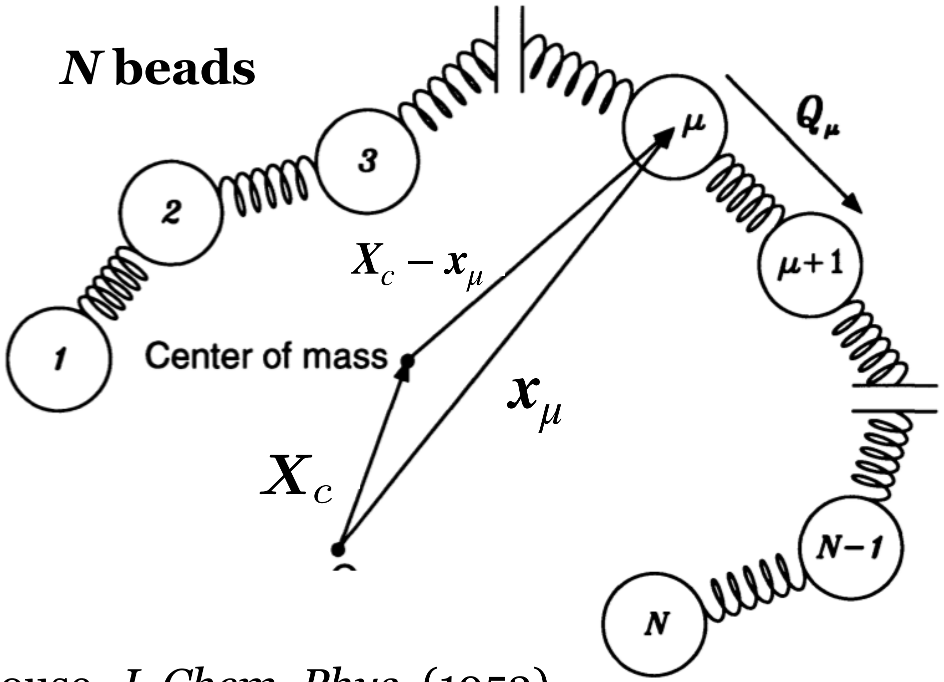
Weissenberg number $\mathrm{Wi} = \tau / T_{\text{flow}}$



DNA with $L \approx 40\mu\text{m}$, sucrose or glycerol solution



Polymer models: The bead–spring chain



Rouse, *J. Chem. Phys.* (1953)

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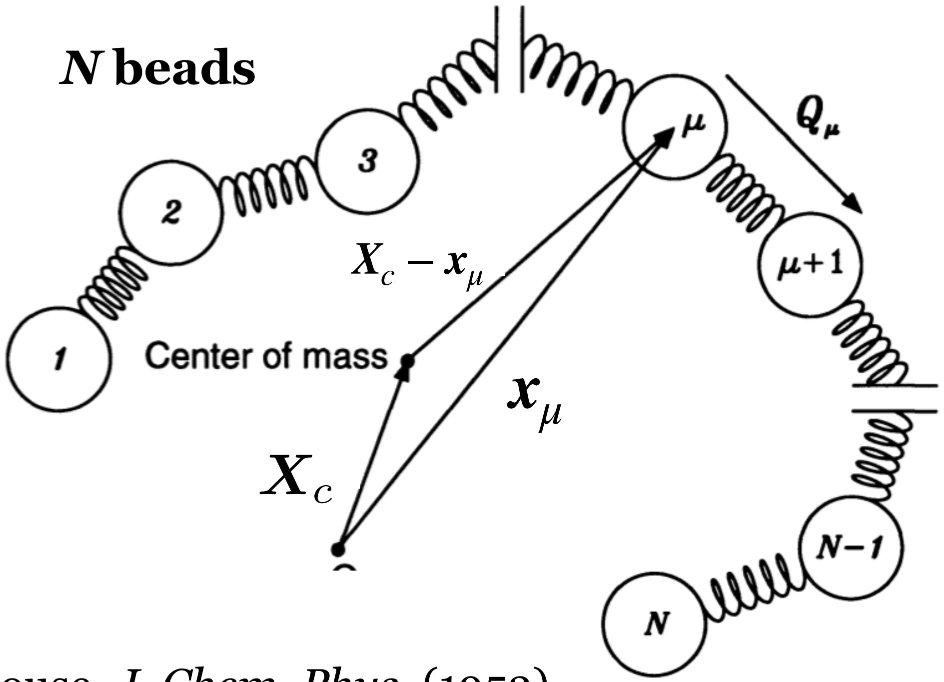

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Forces on the *i*-th bead

1. Stokes drag: $\boldsymbol{F}_i^d = -\zeta[\dot{\boldsymbol{x}}_i - \boldsymbol{u}(\boldsymbol{x}_i, t)]$

2. Thermal noise: $\boldsymbol{F}_i^B = \sqrt{2K_BT/\zeta}\ \boldsymbol{\xi}_i(t)$ with $\boldsymbol{\xi}_i(t)$ independent white noises

3. Elastic force: $\boldsymbol{F}_i^{\mathrm{el}} = k\, f_i\, (\boldsymbol{x}_{i+1} - \boldsymbol{x}_i) + k\, f_{i-1}\, (\boldsymbol{x}_{i-1} - \boldsymbol{x}_i)$

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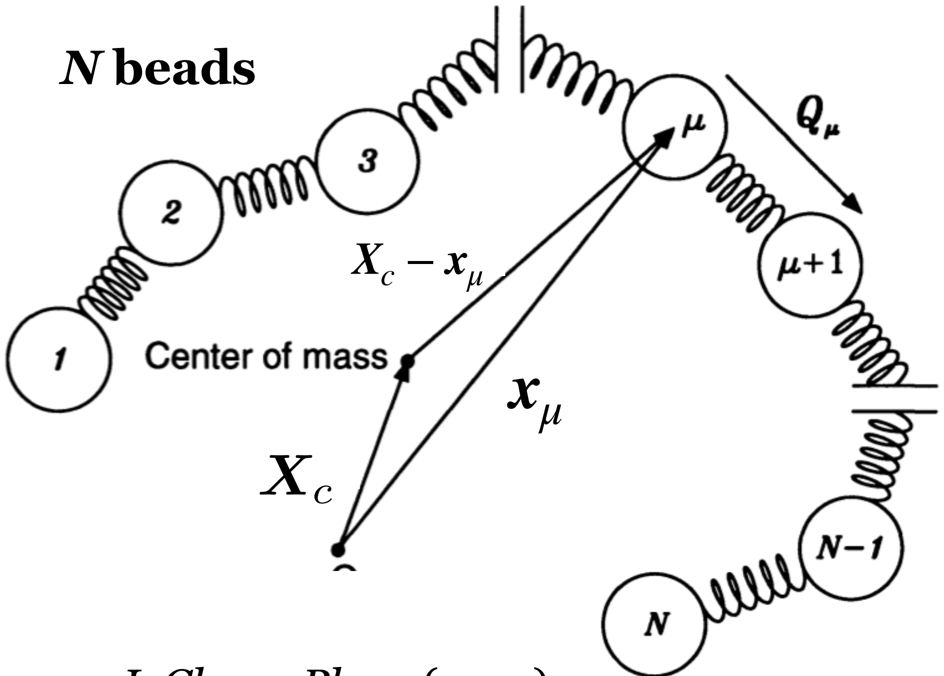

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Elastic coefficient

$$f_i = 1 \quad \text{(Hooke)}$$

$$f_i = \frac{1}{1 - Q_i^2/\ell^2}, \quad Q_i = |\boldsymbol{x}_{i+1} - \boldsymbol{x}_i| \quad \text{(FENE)}$$

$$f_i = \frac{2}{3} - \frac{\ell}{6Q_i} + \frac{\ell}{6Q_i(1 - Q_i/\ell)^2} \quad \text{(Marko—Siggia)}$$

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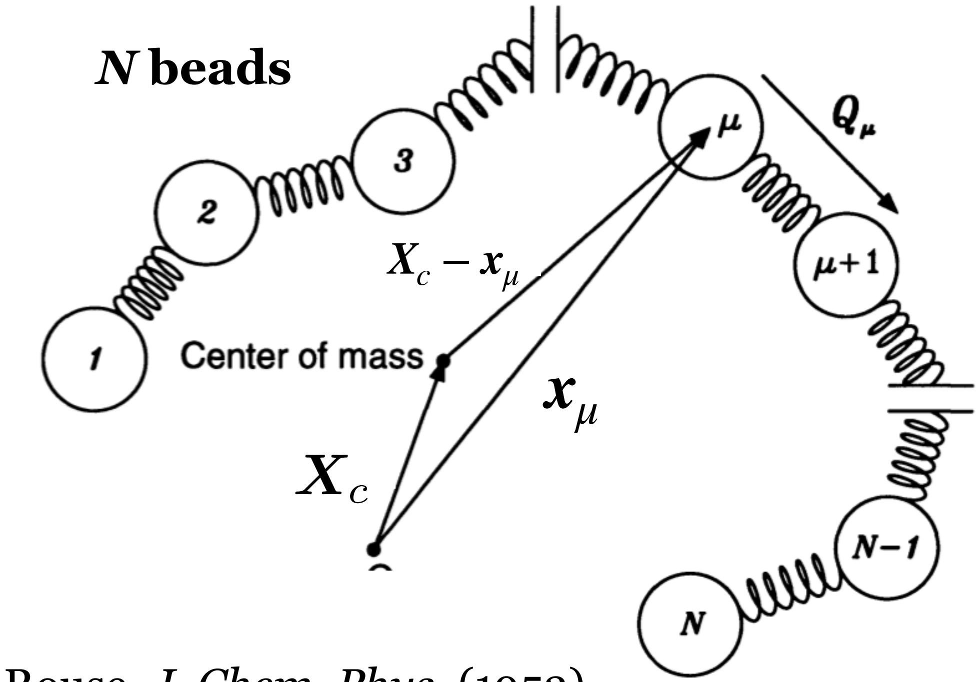

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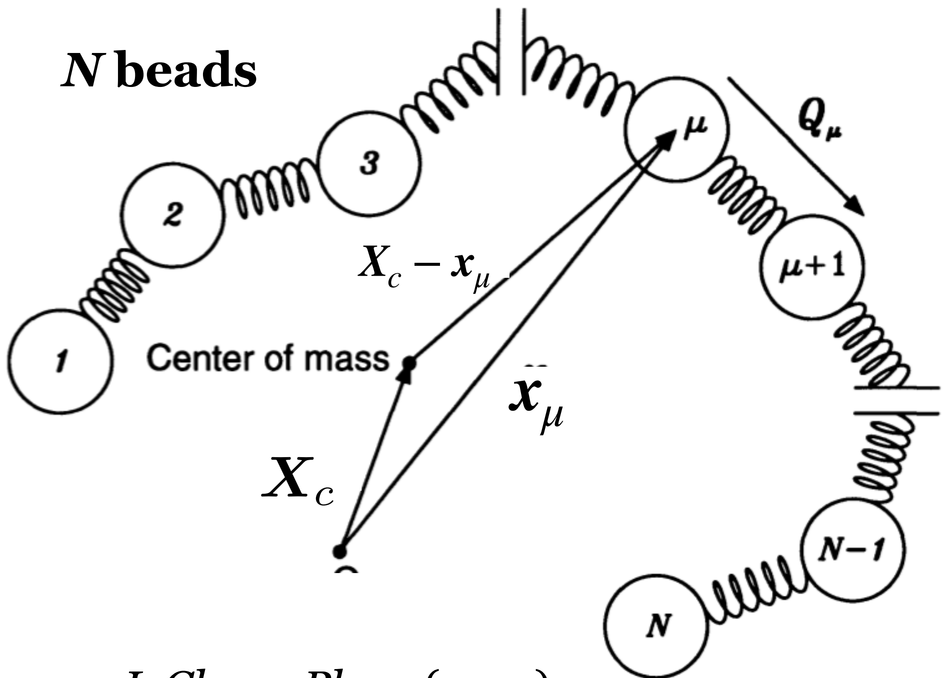
Assumptions

Negligible *hydrodynamic* and *excluded-volume* interactions

Negligible inertia: $m\ddot{\boldsymbol{x}}_i = 0$ and hence $\boldsymbol{F}_i^d + \boldsymbol{F}_i^{\text{el}} + \boldsymbol{F}_i^B = 0$

Linear velocity field: $\boldsymbol{u}(\boldsymbol{x}_{i+1}) - \boldsymbol{u}(\boldsymbol{x}_i) = \nabla \boldsymbol{u} \cdot (\boldsymbol{x}_{i+1} - \boldsymbol{x}_i)$

Polymer models: The bead–spring chain



Rouse, *J. Chem. Phys.* (1953)

$$\frac{\mathrm{d}\boldsymbol{X}_c}{\mathrm{d}t} = \boldsymbol{u}(\boldsymbol{X}_c, t) + \frac{1}{N}\sqrt{\frac{R_{eq}^2}{6\tau}}\,\sum_{i=1}^{N}\boldsymbol{\xi}_i(t)$$

$$\frac{\mathrm{d}\boldsymbol{Q}_i}{\mathrm{d}t} = \nabla\boldsymbol{u}\cdot\boldsymbol{Q}_i - \frac{1}{4\tau}\left(2f_i\boldsymbol{Q}_i - f_{i+1}\boldsymbol{Q}_{i+1} - f_{i-1}\boldsymbol{Q}_{i-1}\right) + \sqrt{\frac{R_{eq}^2}{6\tau}}\,[\boldsymbol{\xi}_{i+1}(t) - \boldsymbol{\xi}_i(t)], \qquad \tau = 4k/\zeta$$

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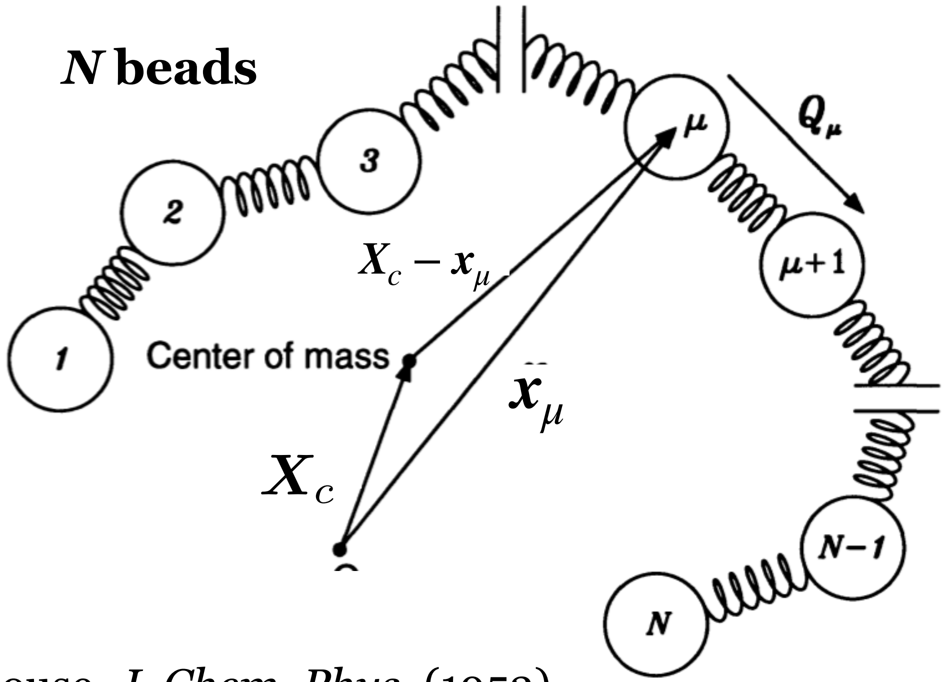

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$$\tau_{\text{chain}} = \frac{6\tau}{N(N+1)}$$

$$L = \ell(N-1)$$

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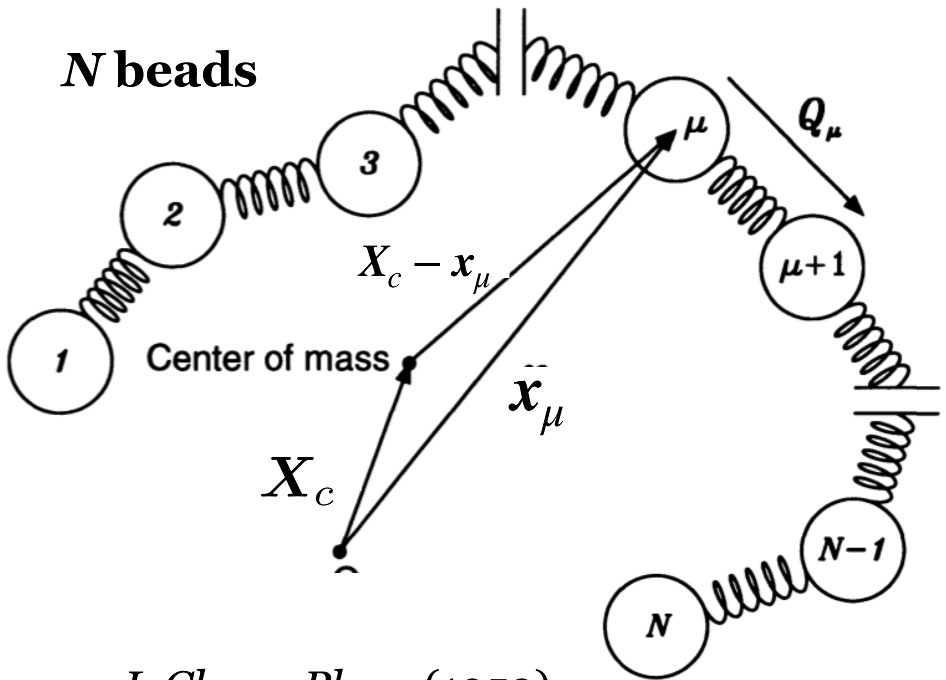

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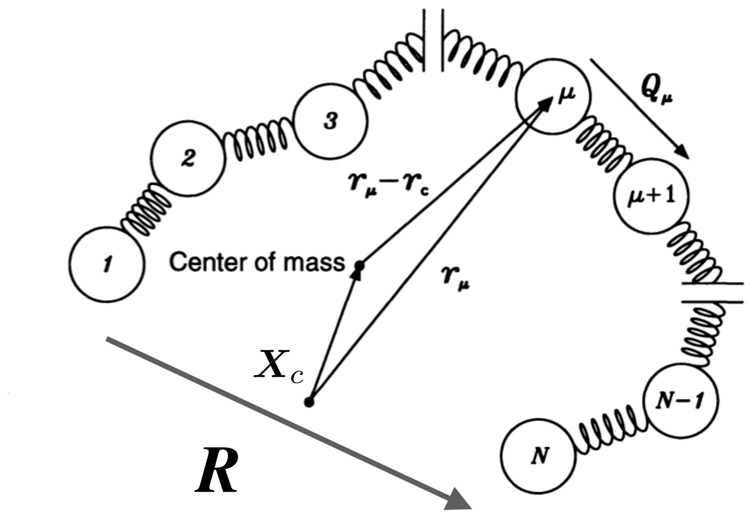
Doi & Edwards, *The Theory of Polymer Dynamics* (Oxford University Press, 1986)

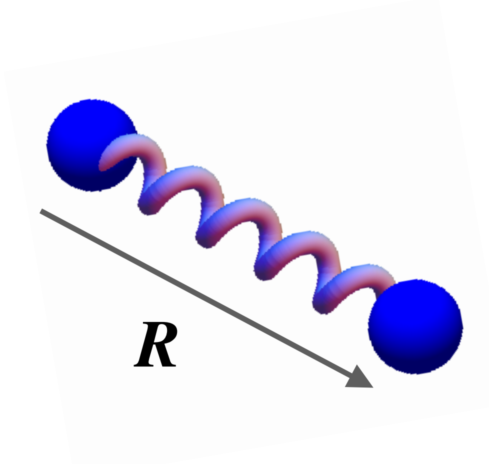
Bird, Curtiss, Armstrong & Hassager, *Dynamics of Polymeric Liquids*, Vol. 2 (Wiley, 1987)

Öttinger, *Stochastic Processes in Polymeric Fluids* (Springer, 1996)

Graham, *Microhydrodynamics, Brownian Motion, and Complex Fluids* (Cambridge University Press, 2018)

Polymer models: The dumbbell model ($N = 2$)





$$\frac{d\boldsymbol{R}}{dt} = \nabla \boldsymbol{u} \cdot \boldsymbol{R} - \frac{f(R)}{2\tau}\boldsymbol{R} + \sqrt{\frac{R_{\mathrm{eq}}^2}{3\tau}}\,\boldsymbol{\xi}(t)$$

Associated Fokker–Planck equation for the PDF of *R*: $\psi(\boldsymbol{R}, t)$

$$\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial R_i}\left\{\left[\partial_j u_i R_j - \frac{f(R)}{2\tau} R_i\right]\psi\right\} + \frac{R_{\mathrm{eq}}^2}{6\tau}\frac{\partial^2 \psi}{\partial R_i \partial R_i}$$

Continuum models of polymer solutions (Oldroyd-B, FENE-P, ...)

Polymer conformation tensor $\mathsf{C} = \langle \boldsymbol{R} \otimes \boldsymbol{R} \rangle_\xi$

Coil-stretch transition in an extensional flow

P.-G. De Gennes, *J. Chem. Phys.* (1974); E.J. Hinch, *Colloques Internationaux du CNRS* (1975)

2D extensional flow

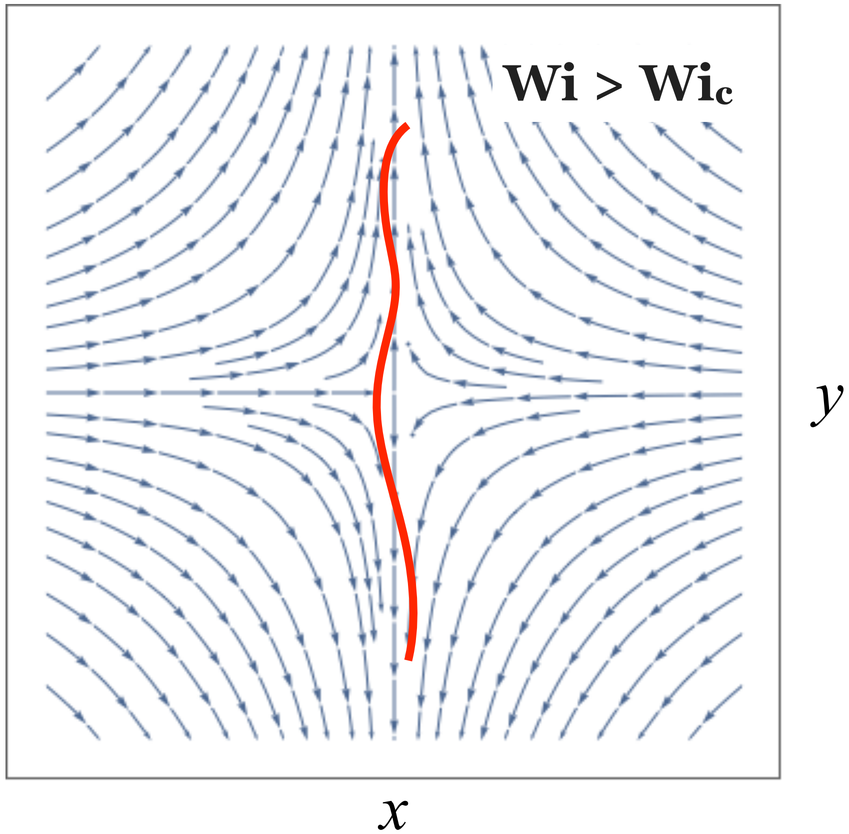
$$\boldsymbol{u} = \gamma(-x, y), \quad \mathrm{Wi} = \gamma\tau$$

$$\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \nabla\boldsymbol{u}\cdot\boldsymbol{R} - \frac{f(R)}{2\tau}\boldsymbol{R} + \sqrt{\frac{R_{\mathrm{eq}}^2}{3\tau}}\,\boldsymbol{\xi}(t)$$

Neglect the noise and consider linear elasticity:

$$\dot{R}_x = -\left(\gamma + \frac{1}{2\tau}\right)R_x$$

$$\dot{R}_y = -\left(\gamma - \frac{1}{2\tau}\right)R_y$$



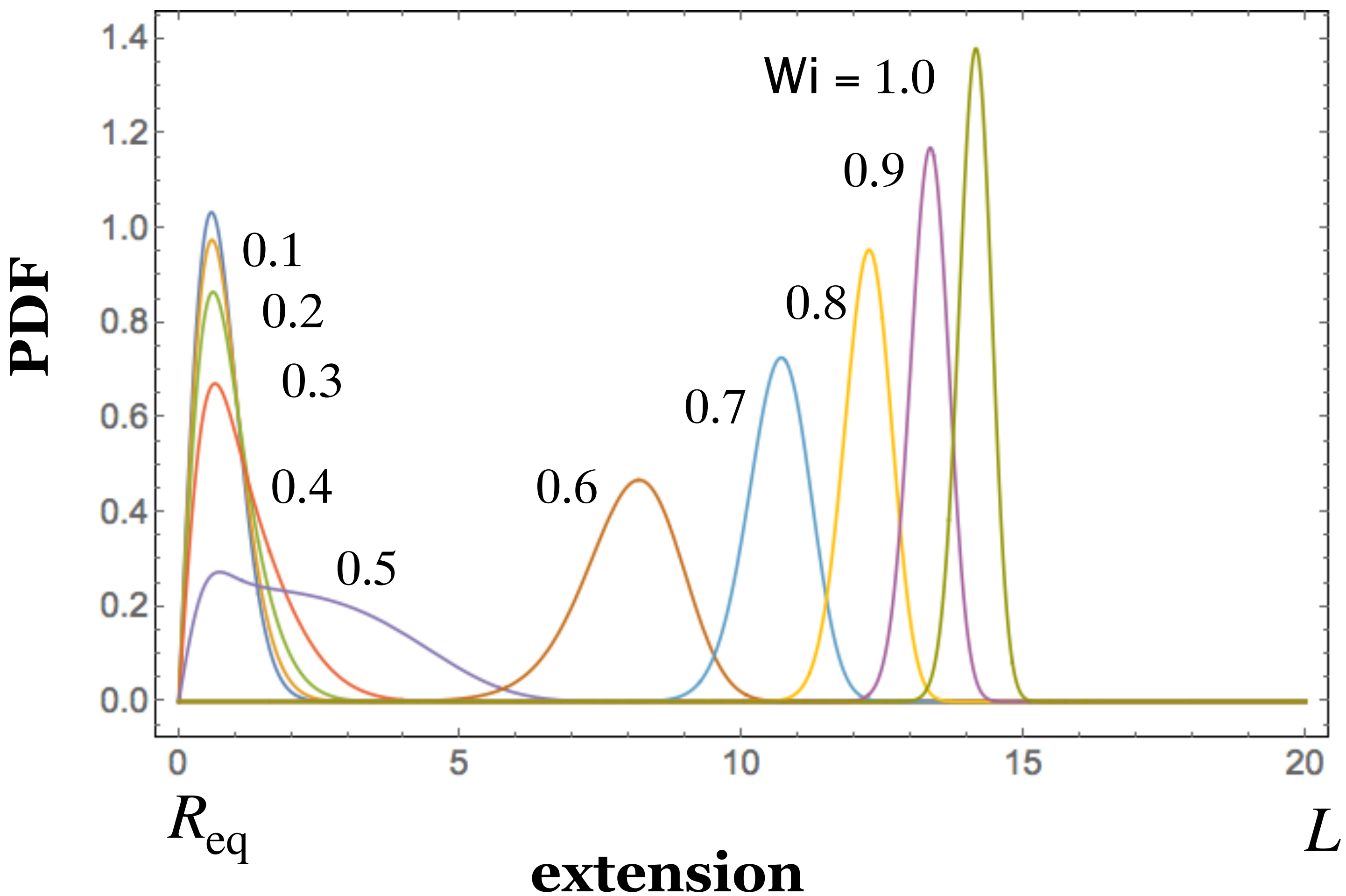
$$R \sim R_y \sim R_0\,\mathrm{e}^{(\mathrm{Wi}-\mathrm{Wi}_{\mathrm{crit}})\,t/\tau}, \quad \mathrm{Wi}_{\mathrm{crit}} = 1/2$$

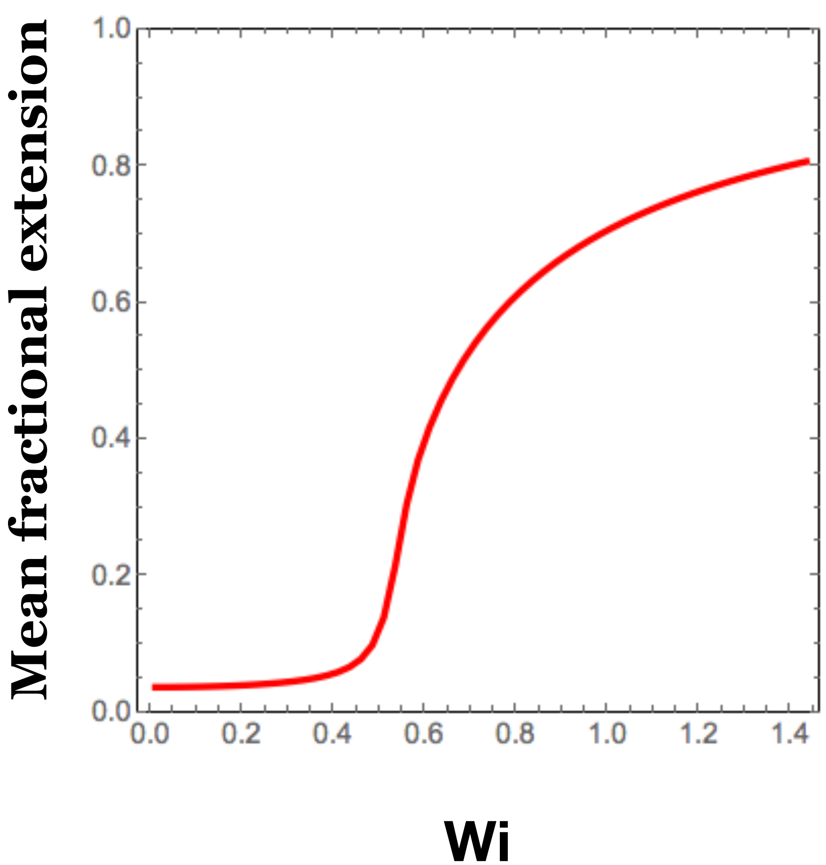
Coil-stretch transition in an extensional flow

FENE model (nonlinear elasticity) $\boldsymbol{F}^{\mathrm{el}} = \dfrac{\boldsymbol{R}}{1 - R^2/L^2}$

If $\boldsymbol{u}$ is steady, potential, and $\nabla \boldsymbol{u} = (\nabla \boldsymbol{u})^{\mathsf{T}}$:

$$\psi_{\mathrm{st}}(\boldsymbol{R}) \propto \mathrm{e}^{-3\Phi/R_{\mathrm{eq}}^2}\, \mathrm{e}^{3\tau R_{\mathrm{eq}}^{-2}(\nabla \boldsymbol{u} : \boldsymbol{R}\boldsymbol{R})} \quad \text{with} \quad \boldsymbol{F}^{\mathrm{el}} = -\nabla \Phi$$

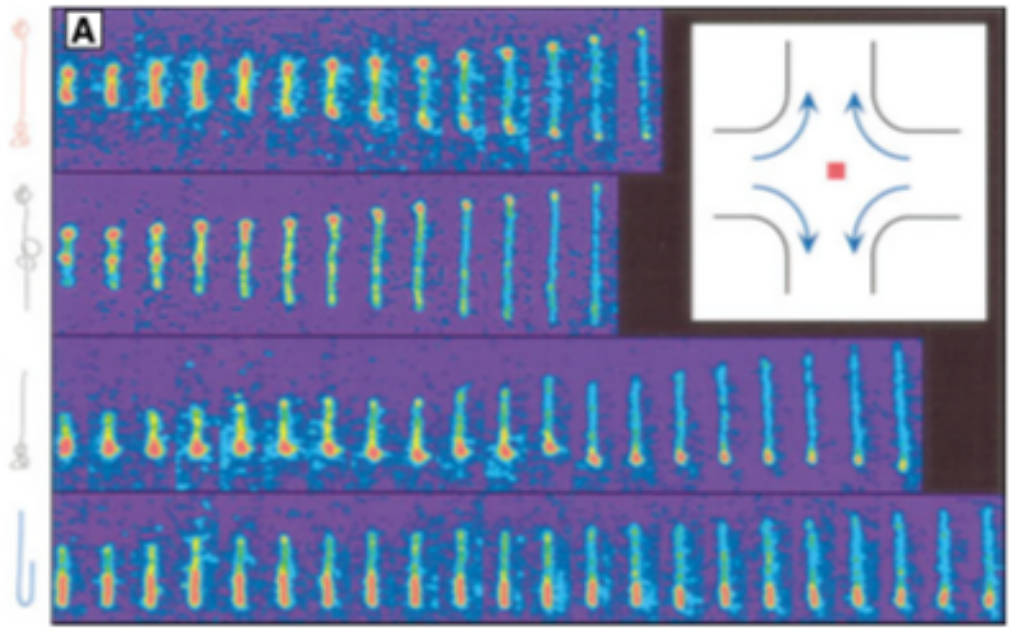


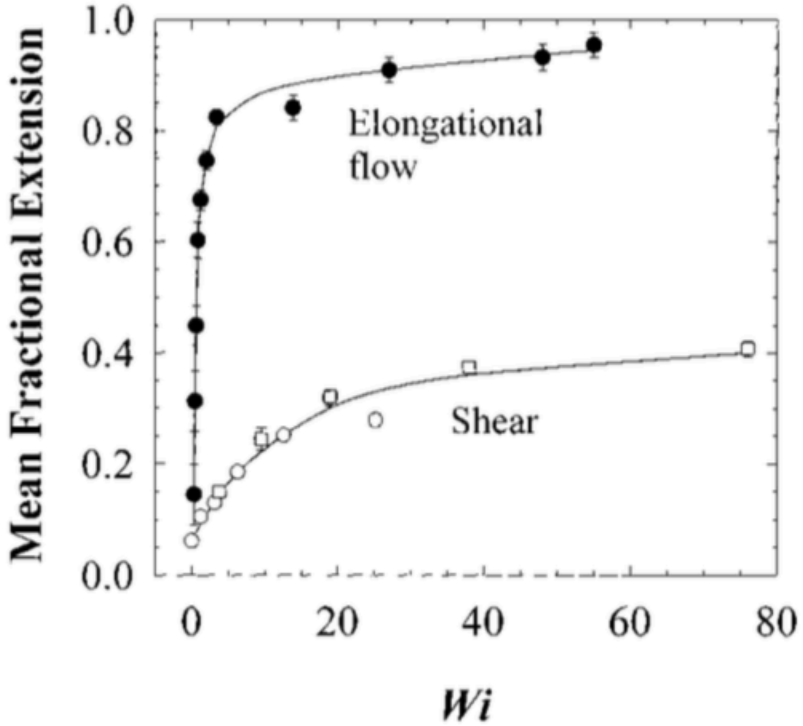


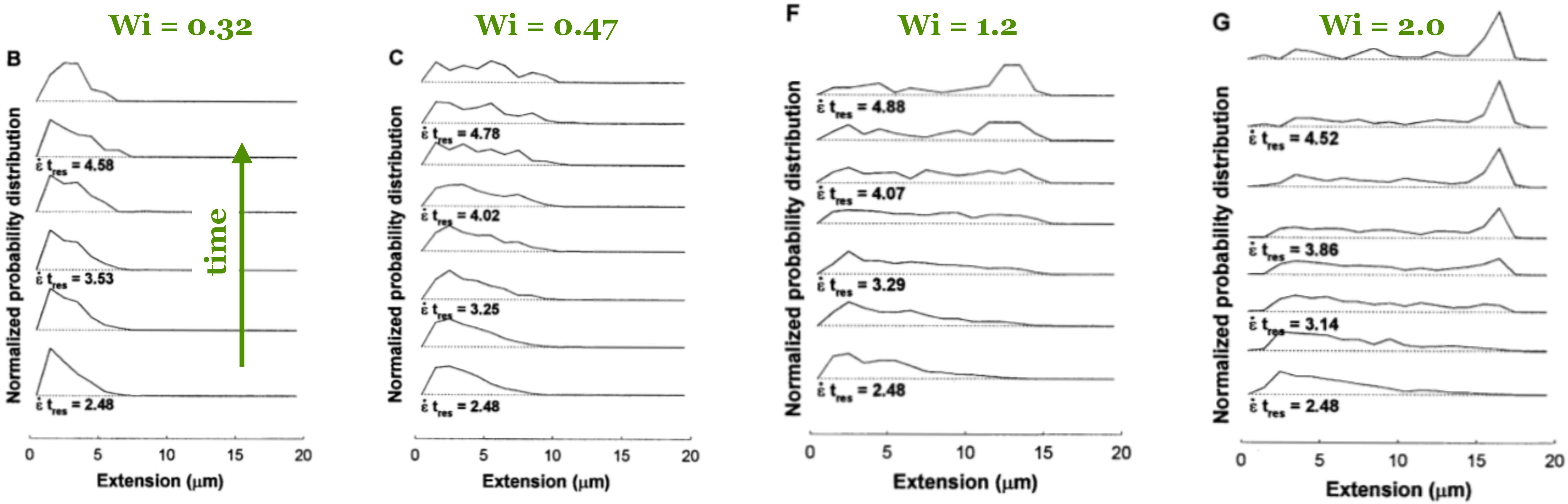
Coil-stretch transition in an extensional flow

Perkins, Smith & Chu, *Science* (1997)

Smith, Babcock & Chu, *Science* (1999)

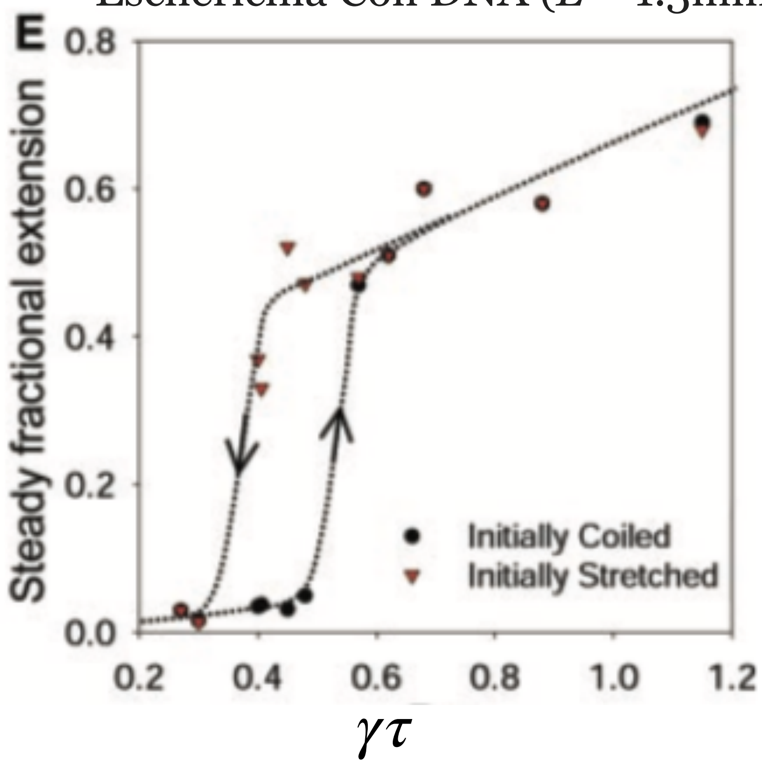






Hysteresis

Escherichia Coli DNA (L = 1.3mm)



Schroeder, Babcock, Shaqfeh & Chu, *Science* (2003)

Conformation dependent drag coefficient

$$\zeta_{\text{coil}} \propto \eta R_{eq}$$

$$\zeta_{\text{stretch}} \propto \frac{\eta L}{\ln(L/d)}$$

η solvent viscosity; d polymer diameter

Escherichia Coli DNA: $\zeta_{\text{stretch}}/\zeta_{\text{coil}} \approx 5$

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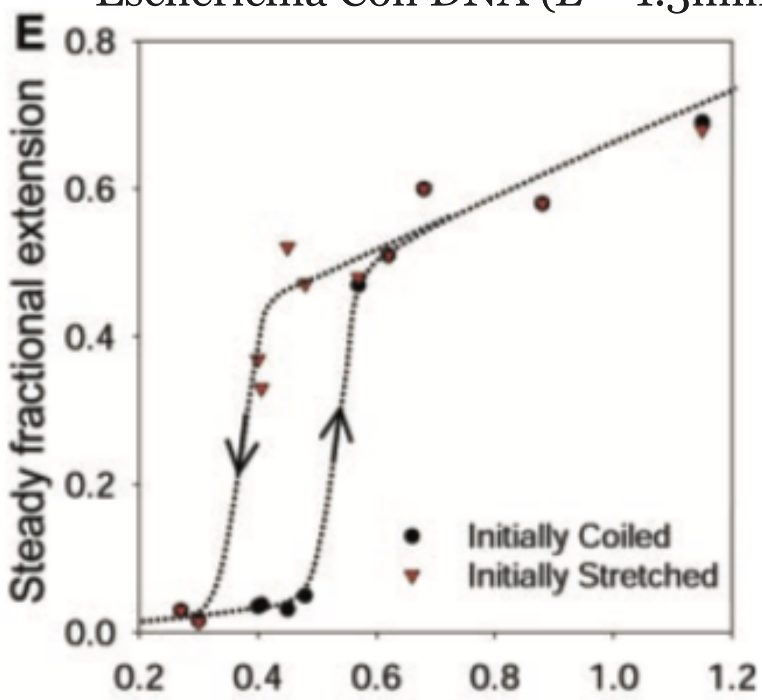

$\gamma\tau$

Schroeder, Babcock, Shaqfeh & Chu, *Science* (2003)

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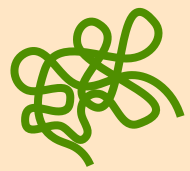

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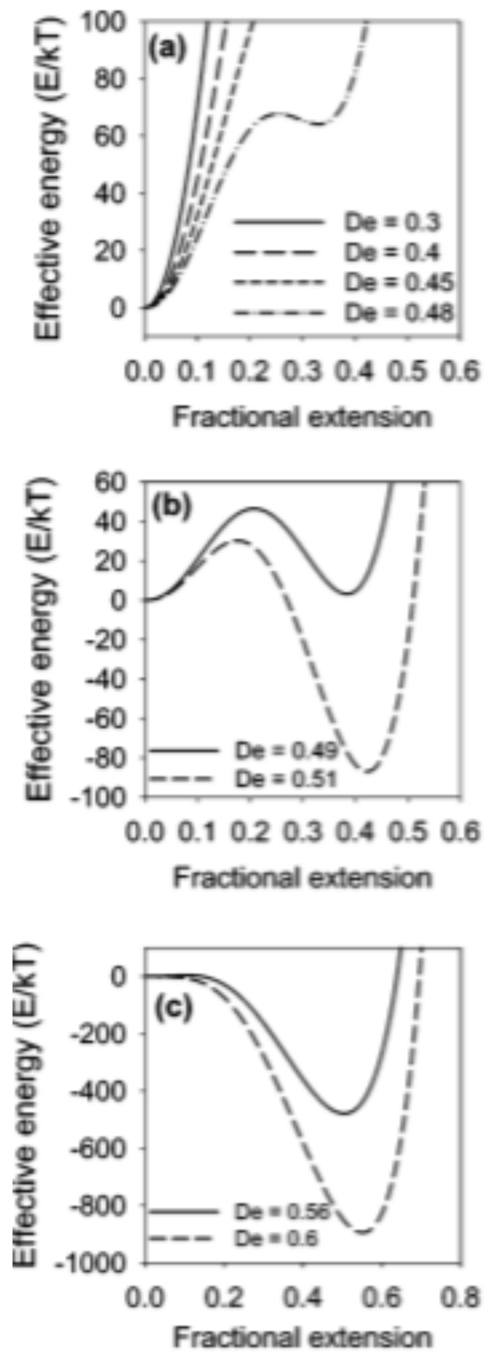

Schroeder, Shaqfeh & Chu
Macromolecules (2004)

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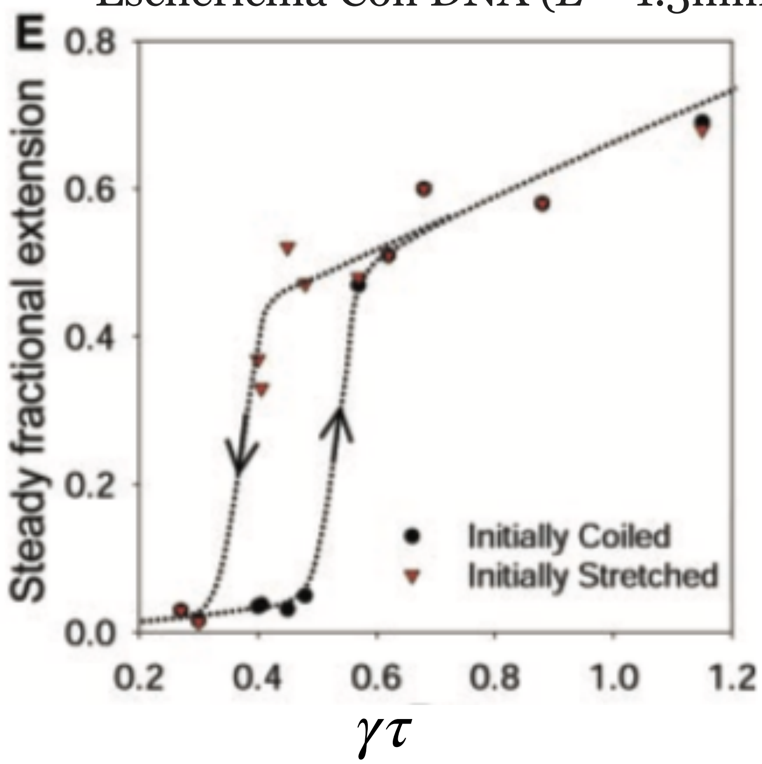

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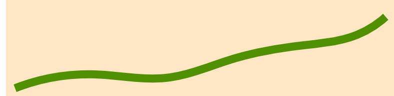

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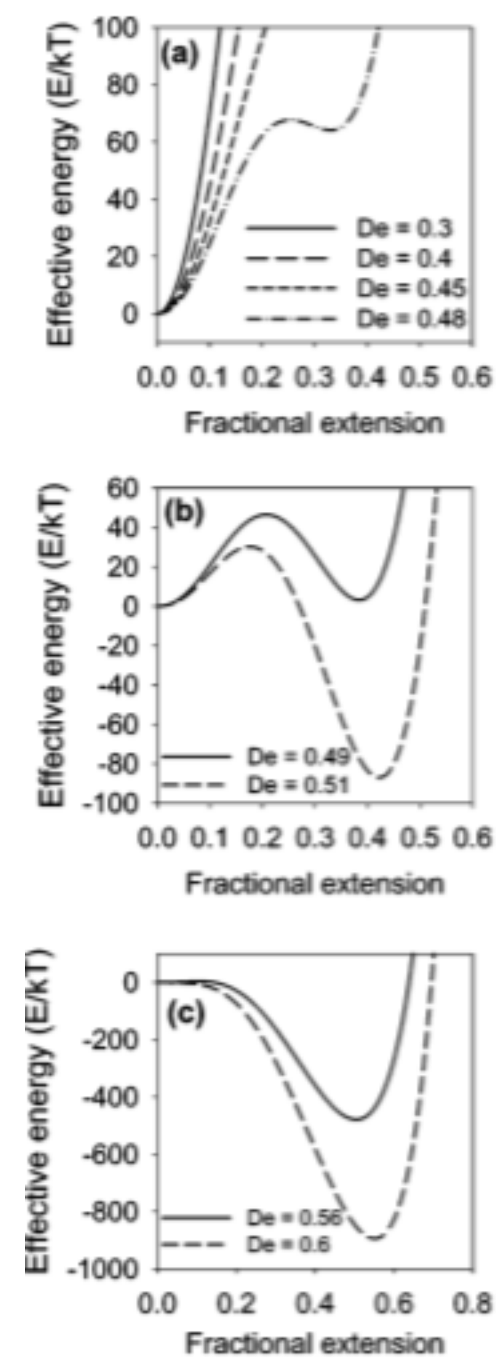


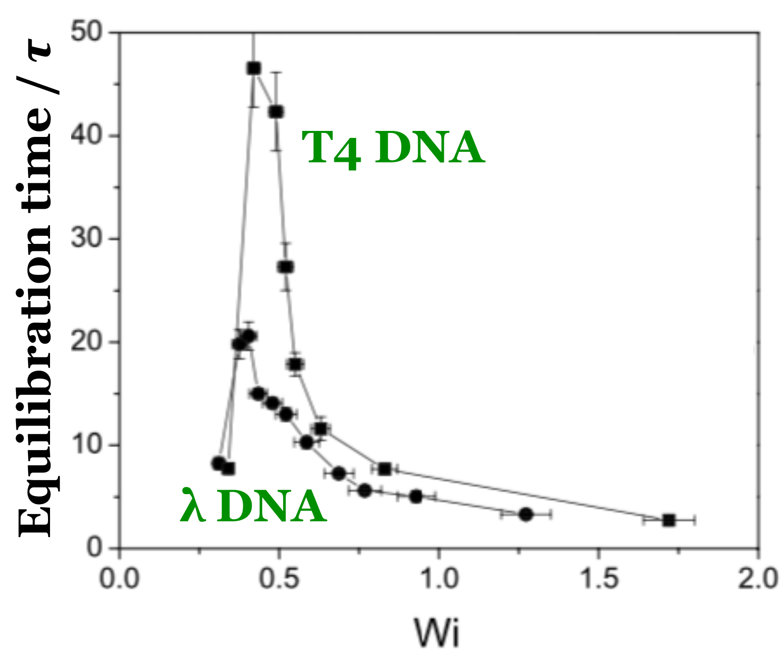
Schroeder, Shaqfeh & Chu
Macromolecules (2004)

Critical slowing down

Gerashchenko & Steinberg,
Phys. Rev. E (2008)

■ T4 DNA, L = 67μm,
$\zeta_s/\zeta_c = 2.1$

● λ DNA, L = 21μm,
$\zeta_s/\zeta_c = 1.6$

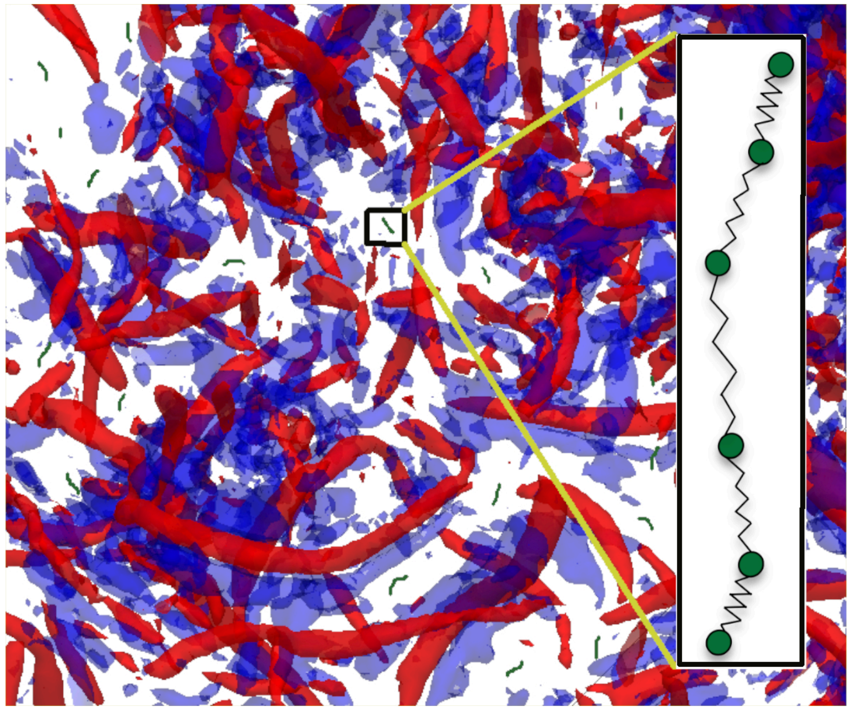


Single polymer dynamics in laminar flows

References

- Schroeder, *J. Rheol.* (2018)
- Larson, *J. Rheol.* (2005)
- Shaqfeh, *J. Non-Newtonian Fluid Mech.* (2005)
- Nguyen & Kausch (eds.), *Flexible Polymer Chain Dynamics in Elongational Flow* (Springer, 1999)

Turbulent flows



Lumley, *Symp. Math.* (1972); *J. Polym. Sci.: Macromol. Rev.* (1973)

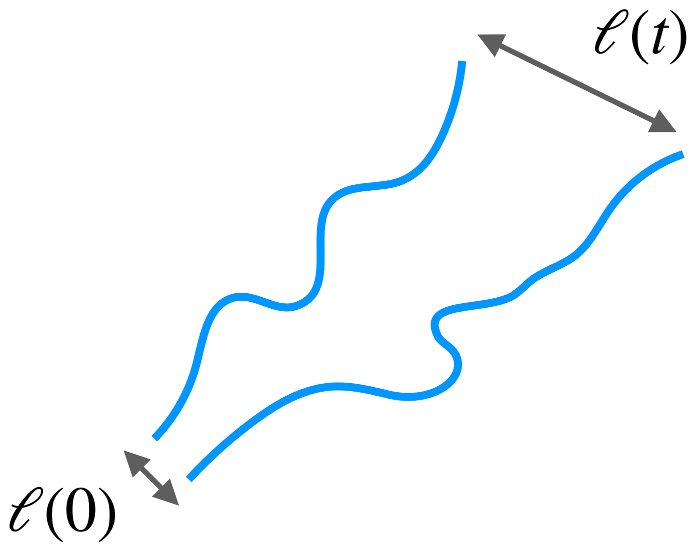
$$\langle R^2 \rangle \propto \exp\left\{ \left(2\langle S^2 \rangle T_L - \frac{1}{\tau} \right) t \right\}$$

$\langle S^2 \rangle$ mean-square strain rate

T_L Lagrangian correlation time scale of the strain rate

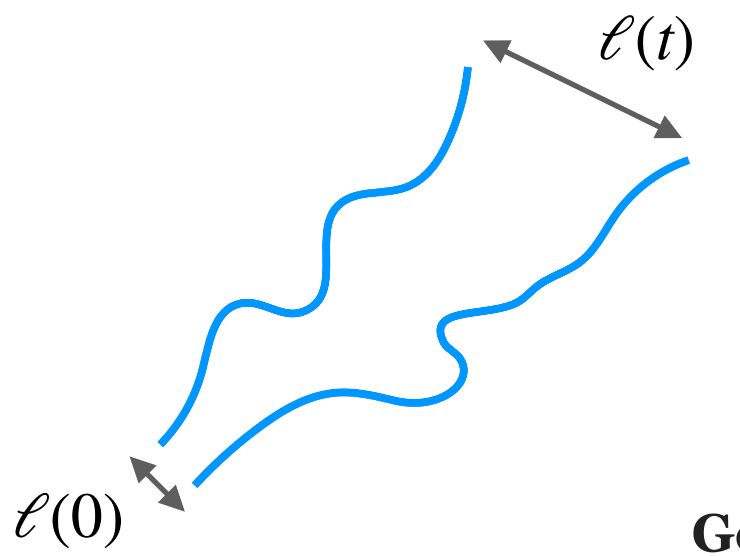
"If the mean-square strain rate exceeds a critical value related to the inverse of the terminal relaxation time but weighted by the persistence of the fluctuating strain rate regions, then the molecules begin to expand exponentially. The expansion of an individual molecule is, of course, not steady; as it moves from region to region, it will first expand, then shrink, etc.; however, when the criterion is exceeded, the expansion will win out, and it will gradually expand more and more."

Stretching in turbulent flows



Lyapunov exponent $$\lambda = \lim_{t\to\infty} \frac{1}{t} \left\langle \ln \left[\frac{\ell(t)}{\ell(0)} \right] \right\rangle$$

Stretching in turbulent flows



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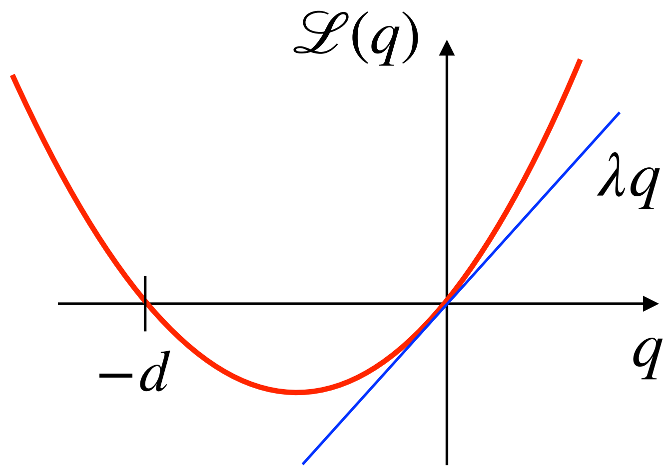
Generalized Lyapunov exponents $\mathscr{L}(q) = \lim_{t\to\infty} \frac{1}{t} \ln \left\langle \left[\frac{\ell(t)}{\ell(0)} \right]^q \right\rangle$

The function $\mathscr{L}(q)$ is convex and satisfies $\mathscr{L}(-d) = 0$ and $\mathscr{L}'(0) = \lambda$

If the PDF of $\ell(t)$ is log-normal
$\mathscr{L}(q) = \lambda q + \mu q^2$,
where μ is the variance of $\ln \ell(t)$



Paladin & Vulpiani, *Phys. Rep.* (1987)
Cecconi, Cencini & Vulpiani, *Chaos: from Simple Models to Complex Systems* (World Scientific, 2010)

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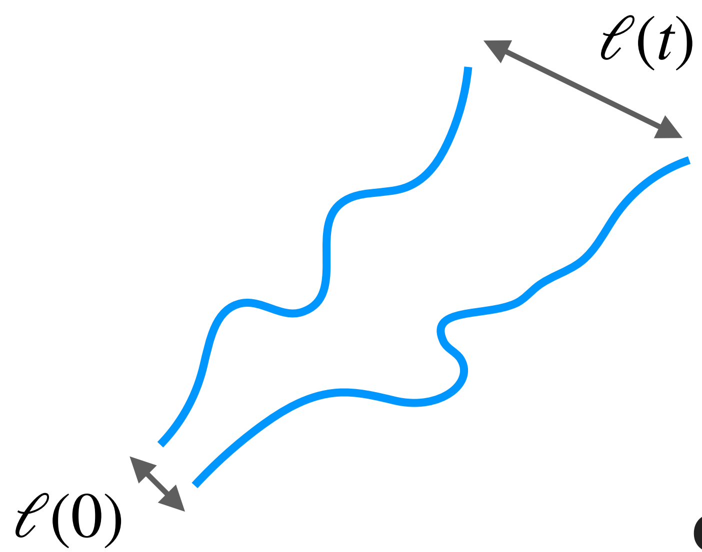

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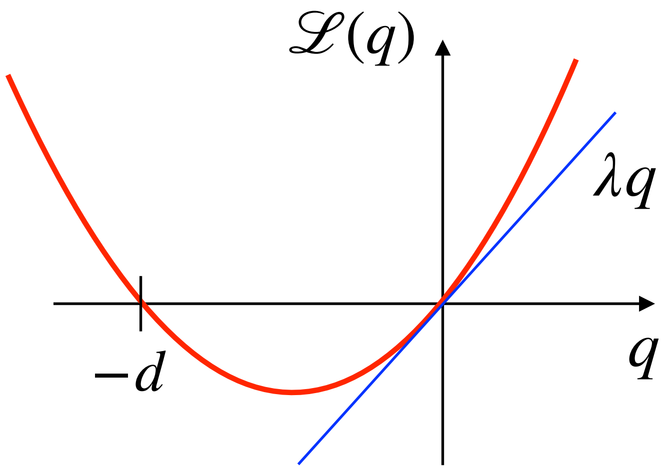
where μ is the variance of $\ln \ell(t)$



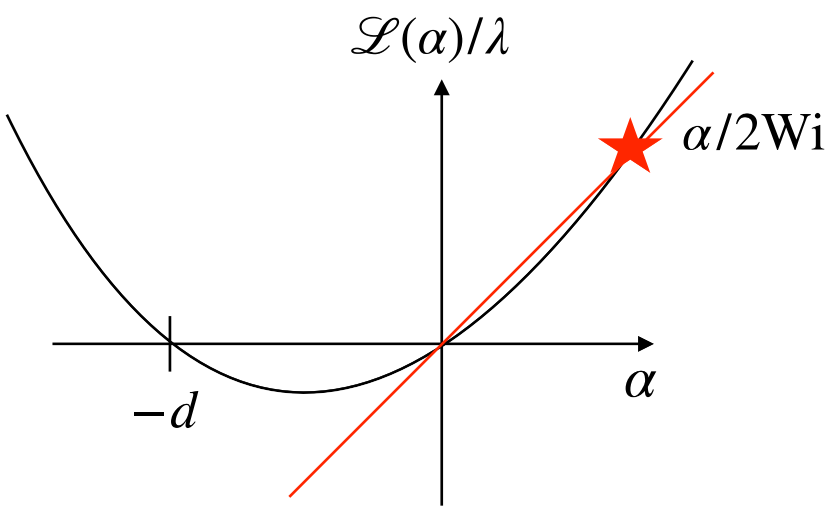
Paladin & Vulpiani, *Phys. Rep.* (1987)
Cecconi, Cencini & Vulpiani, *Chaos: from Simple Models to Complex Systems* (World Scientific, 2010)

3D turbulent flows
Bec, Biferale, Boffetta, Cencini, Musacchio & Toschi , *Phys. Fluids* (2006)
Bagheri, Mitra, Perlekar & Brandt, *Phys. Rev.* E (2012)
Johnson & Meneveau, *Phys. Fluids* (2016)

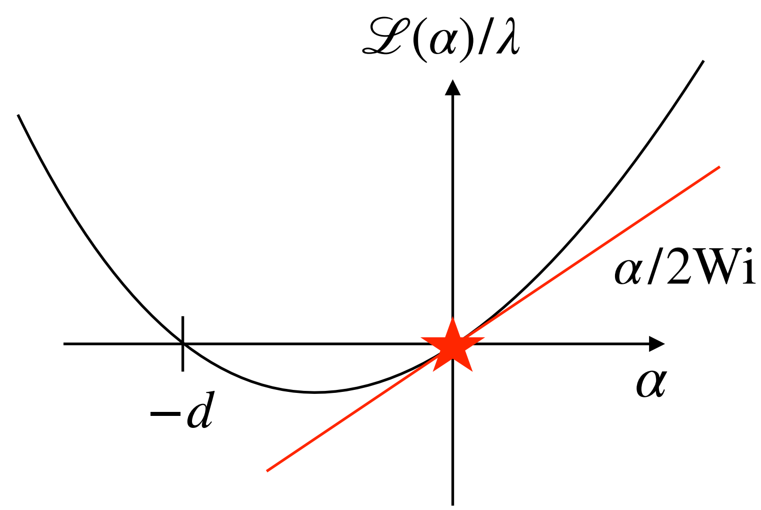
Large deviations theory

Balkovsky, Fouxon & Lebedev, *Phys. Rev. Lett.* (2000)
[here we use the formalism of Boffetta, Celani & Musacchio, *Phys. Rev. Lett.* (2003)]

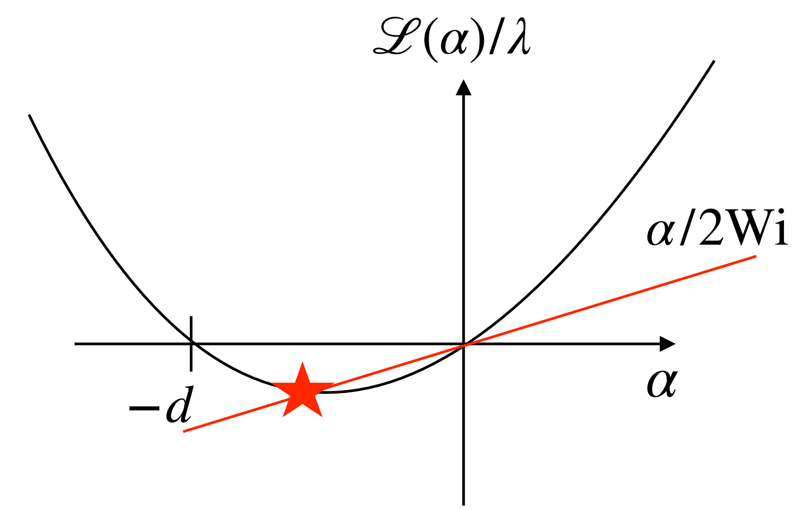
$$P_{\mathrm{st}}(R) \sim R^{-1-\alpha} \quad \text{for} \quad R_{eq} \ll R \ll L$$
$$\frac{\mathscr{L}(\alpha)}{\lambda} = \frac{\alpha}{2\mathrm{Wi}}, \qquad \mathrm{Wi} = \lambda\tau$$



$\mathrm{Wi} < 1/2 \rightarrow \alpha > 0$



$\mathrm{Wi} = 1/2 \rightarrow \alpha = 0$



$\mathrm{Wi} > 1/2 \rightarrow \alpha < 0$

For $\mathrm{Wi} = 1/2$ and $L \rightarrow \infty$, the PDF of R ceases to be normalisable $\Longrightarrow$ Coil—stretch transition at $\mathrm{Wi}_{\mathrm{crit}} = 1/2$

Near $\mathrm{Wi}_{\mathrm{crit}}$ the log-normal approximation $\mathscr{L}(q) = \lambda q + \mu q^2 + O(q^3)$ yields $\alpha = \dfrac{\lambda}{\mu}\left(\dfrac{1}{\mathrm{Wi}} - 2\right)$

Batchelor—Kraichnan random flow

FENE dumbbell model

stochastic differential equation

$$\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \nabla\boldsymbol{u}\cdot\boldsymbol{R} - \frac{f(R)}{2\tau}\boldsymbol{R} + \sqrt{\frac{R_{eq}^2}{\tau}}\,\boldsymbol{\xi}(t), \quad f(R) = \frac{1}{1-R^2/L^2}$$

Fokker—Planck equation for $\psi(\boldsymbol{R}, t)$

$$\partial_t\psi = -\nabla_{\boldsymbol{R}}\cdot\left\{\left[\nabla\boldsymbol{u}\cdot\boldsymbol{R} - \frac{f(R)}{2\tau}\boldsymbol{R}\right]\psi\right\} + \frac{R_{eq}^2}{2\tau}\Delta_{\boldsymbol{R}}\psi$$

Martins Afonso & DV, *J. Fluid Mech.* (2005)

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Kraichnan flow (Batchelor regime): $\nabla\boldsymbol{u}$ is a $d \times d$-dimensional isotropic white noise

$$\langle \nabla_j u_i(t)\,\nabla_l u_k(t')\rangle = \mathscr{K}_{ijkl}\,\delta(t-t'), \qquad \mathscr{K}_{ijkl} = 2\lambda[(d+1)\delta^{ik}\delta^{jl} - \delta^{ij}\delta^{kl} - \delta^{il}\delta^{jk}]/d(d-1)$$

Falkovich, Gawedzki & Vergassola, *Rev. Mod. Phys.* (2001)

Martins Afonso & DV, *J. Fluid Mech.* (2005)

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Falkovich, Gawedzki & Vergassola, *Rev. Mod. Phys.* (2001)

PDF of R with respect to both thermal noise and $\nabla\boldsymbol{u}$: $P(R, t)$

Thanks to isotropy,
1D Fokker—Planck eq.

$$\partial_t P = -\partial_R[D_1(R)P] + \partial_R^2[D_2(R)P]$$

$$D_1(R) = \frac{2(d+1)}{d}\,\mathrm{Wi}\,R - f(R)R + (d-1)\frac{R_{eq}^2}{R}$$

$$D_2(R) = \frac{2\mathrm{Wi}}{d} + R_{rq}^2$$

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$$D_2(R) = \frac{2\mathrm{Wi}}{d} + R_{rq}^2$$

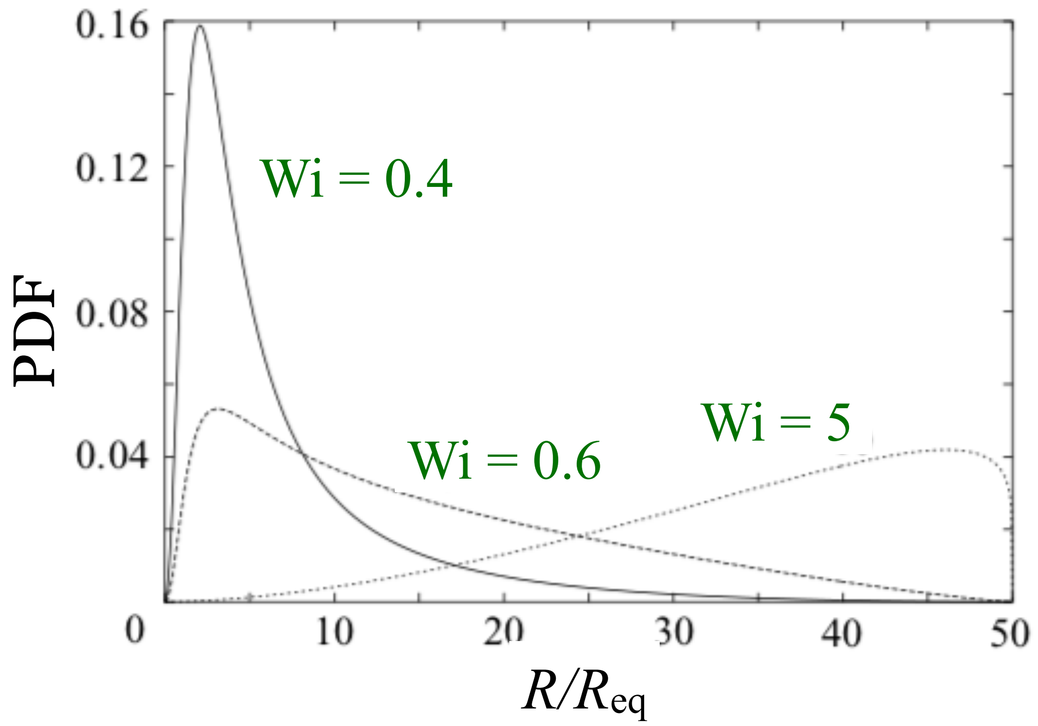
$$P_{\mathrm{st}}(R) \propto R^{d-1}\left(1+\frac{2\mathrm{Wi}}{d}\frac{R^2}{R_{eq}^2}\right)^{-h}\left(1-\frac{R^2}{L^2}\right)^{h} \qquad h = [2(b^{-1}+2\mathrm{Wi}/d]^{-1}$$

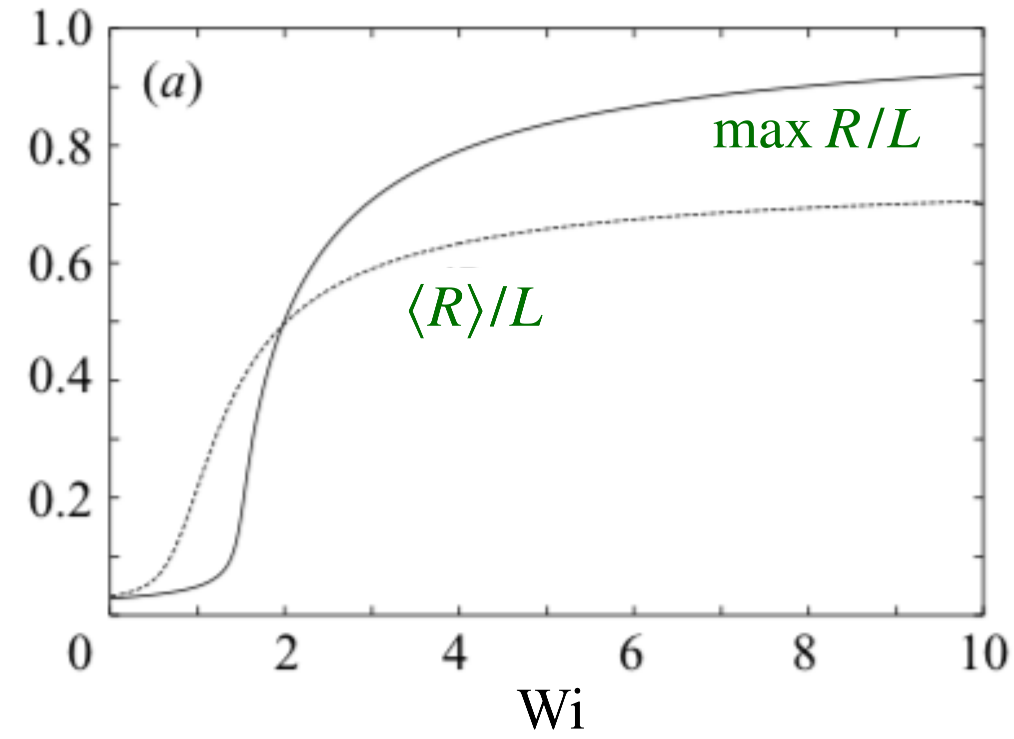
For $\quad R_{eq} \ll R \ll L: \quad P_{\mathrm{st}}(R) \sim R^{-1-\alpha} \quad$ with $\alpha = d\left(\frac{1}{2\mathrm{Wi}} - 1\right)$

In the Batchelor–Kraichnan flow, the statistics of $\ell(t)$ is log-normal with $\lambda/\mu = d$

Martins Afonso & DV, *J. Fluid Mech.* (2005)

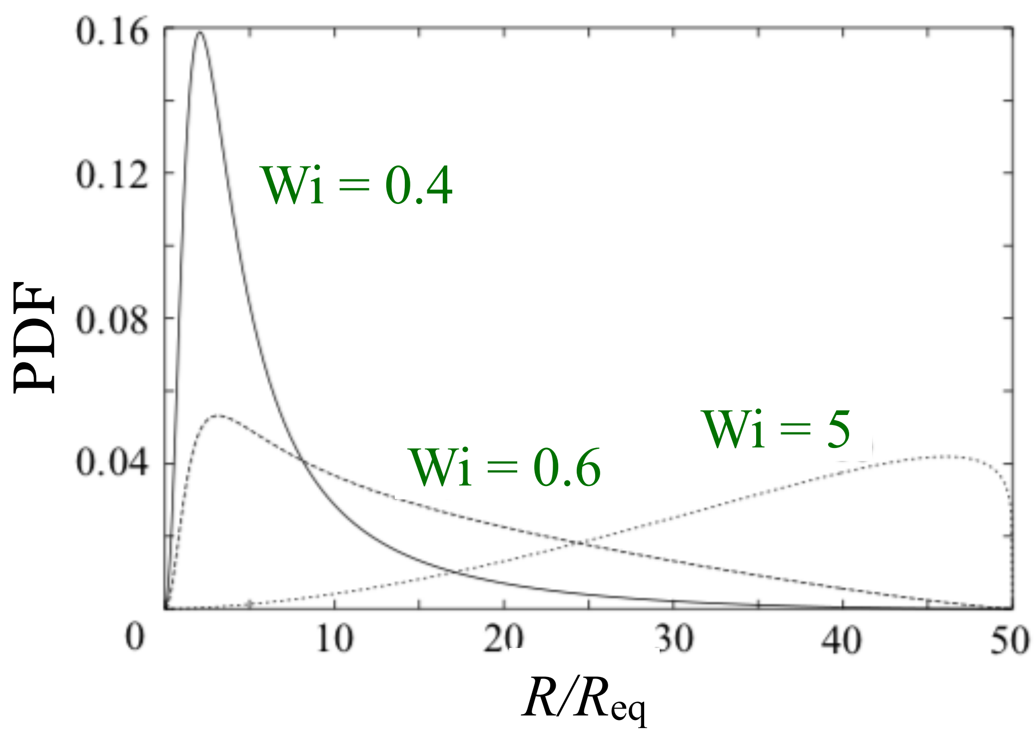
Batchelor—Kraichnan random flow

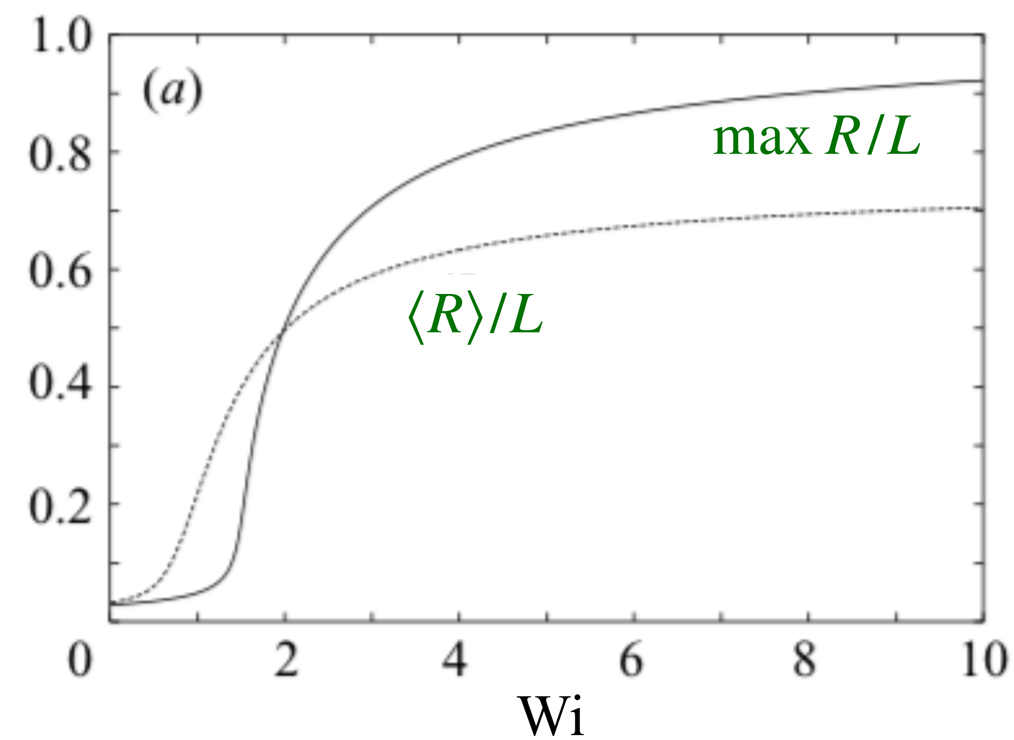




Martins Afonso & DV, *J. Fluid Mech.* (2005)

Batchelor—Kraichnan random flow





Martins Afonso & DV, *J. Fluid Mech.* (2005)

Polymers in the Batchelor—Kraichnan flow

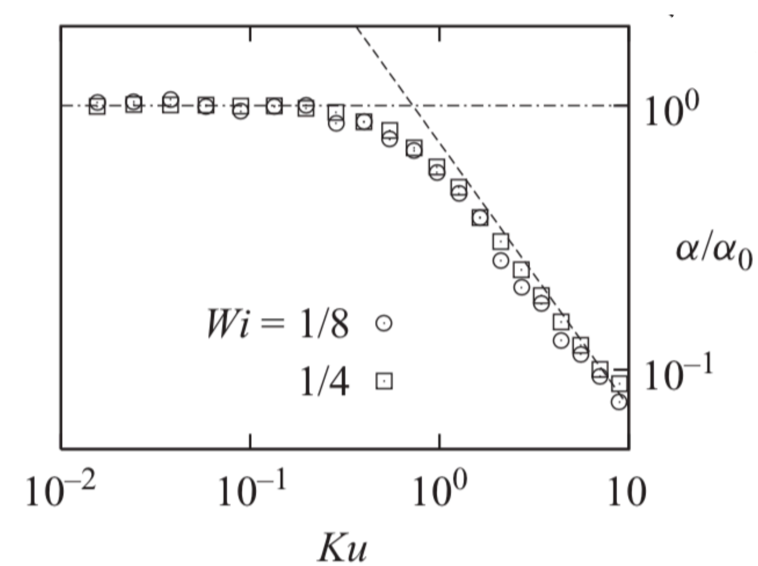
Chertkov, *Phys. Rev. Lett.* (2000)
Thiffeault, *Phys. Lett.* A (2003)
Celani, Musacchio & DV, *J. Stat. Phys.* (2005)
Celani, Puliafito & DV, *Phys. Rev. Lett.* (2006)
Plan, Ali & DV, *Phys. Rev.* E (2016)

2D random renewing flow

$\mathrm{Ku} = \lambda T_{\mathrm{corr}}$

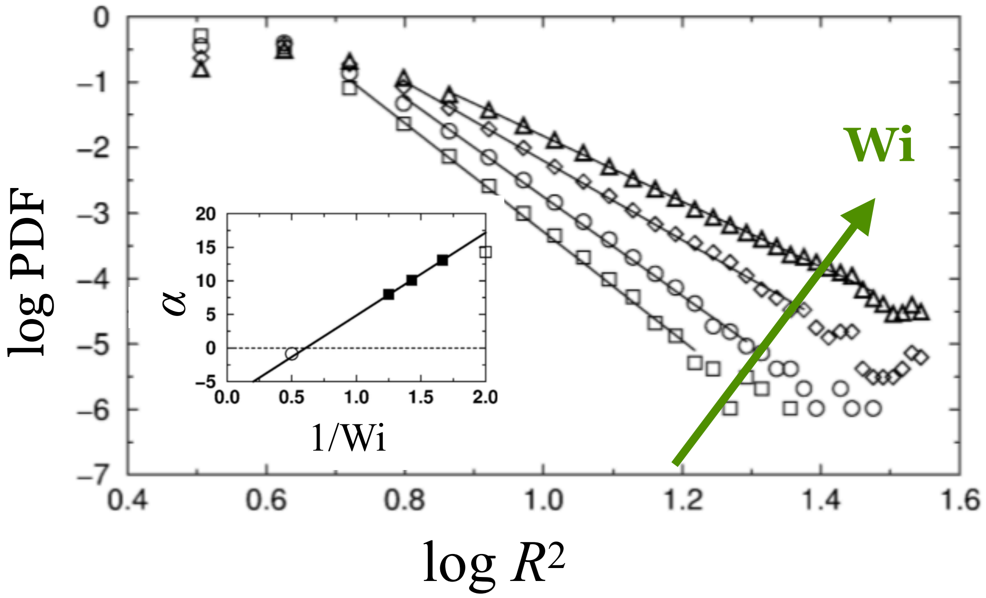
$\alpha_0 = \alpha(\mathrm{Ku} = 0)$

Musacchio & DV, *J. Fluid Mech.* (2011)



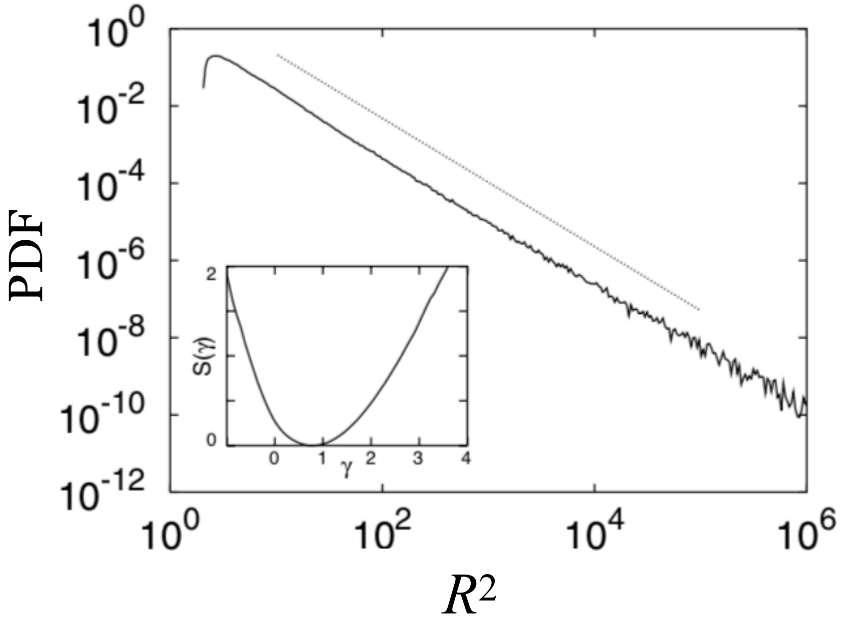
Numerical simulations

Turbulent shear – linear elasticity



Eckhardt, Kronjäger & Schumacher, *Comput. Phys. Commun.* (2002)

2D isotropic turbulence – linear elasticity



Boffetta, Celani & Musacchio, *Phys. Rev. Lett.* (2003)

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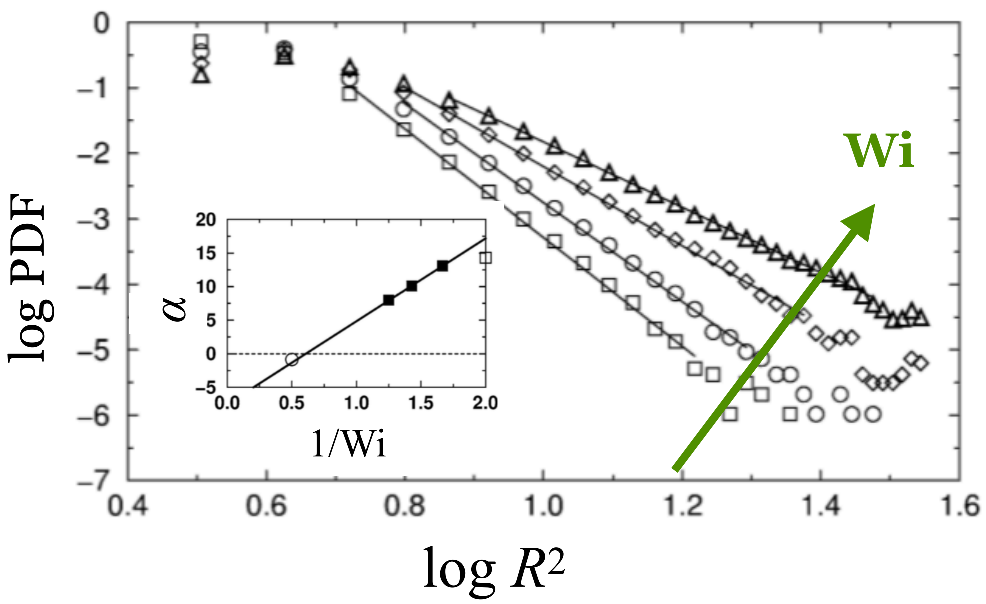

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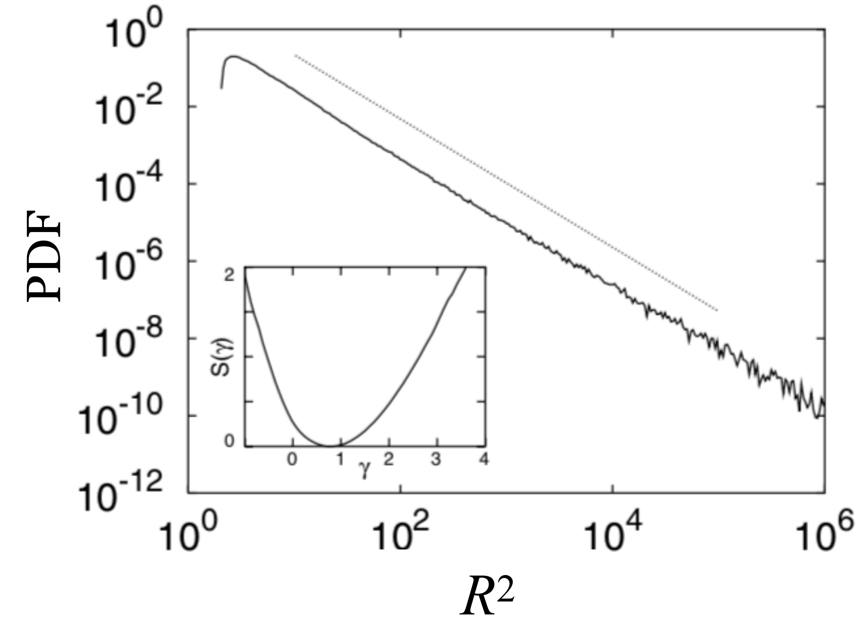
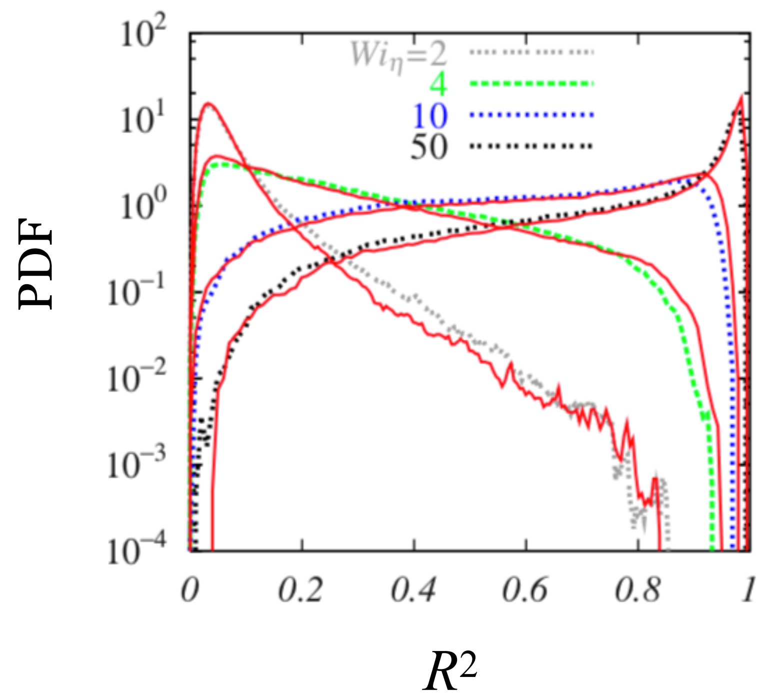


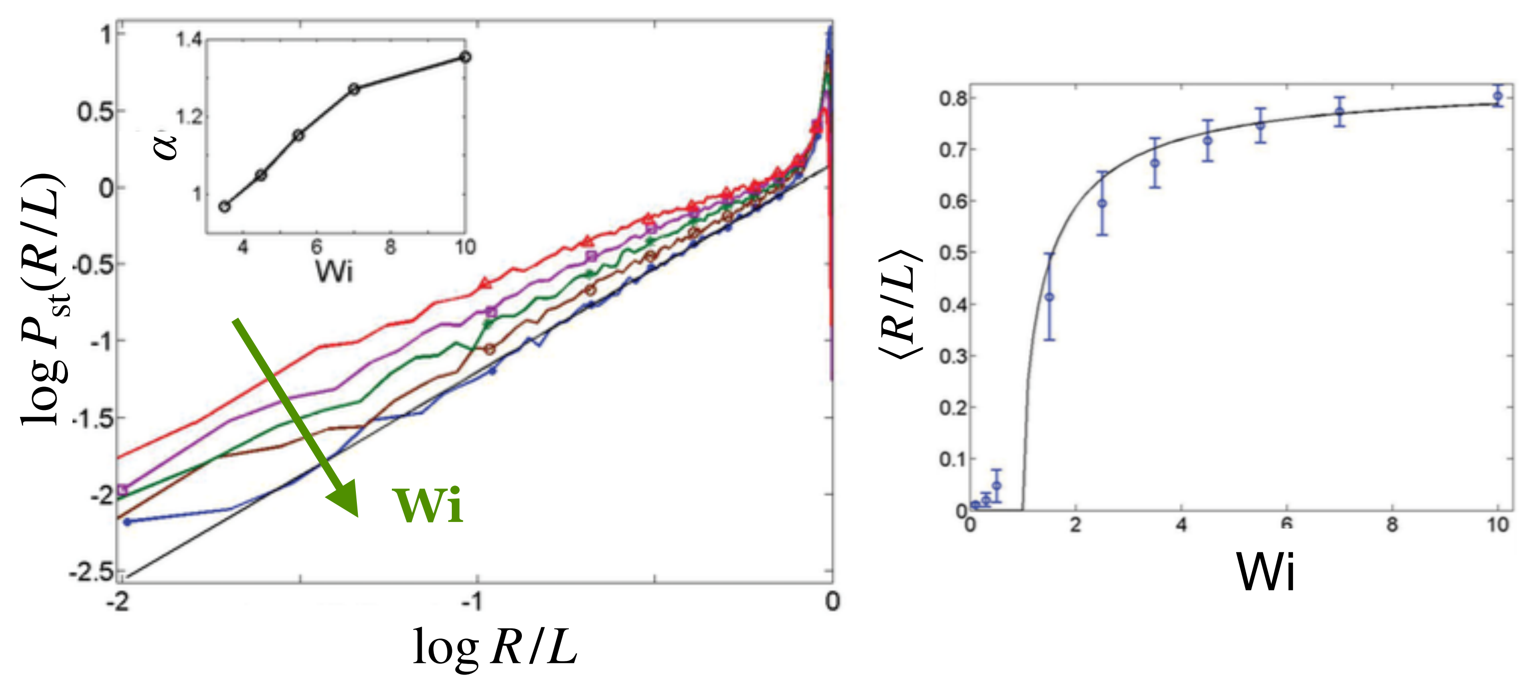
Boffetta, Celani & Musacchio, *Phys. Rev. Lett.* (2003)

3D isotropic – nonlinear elasticity



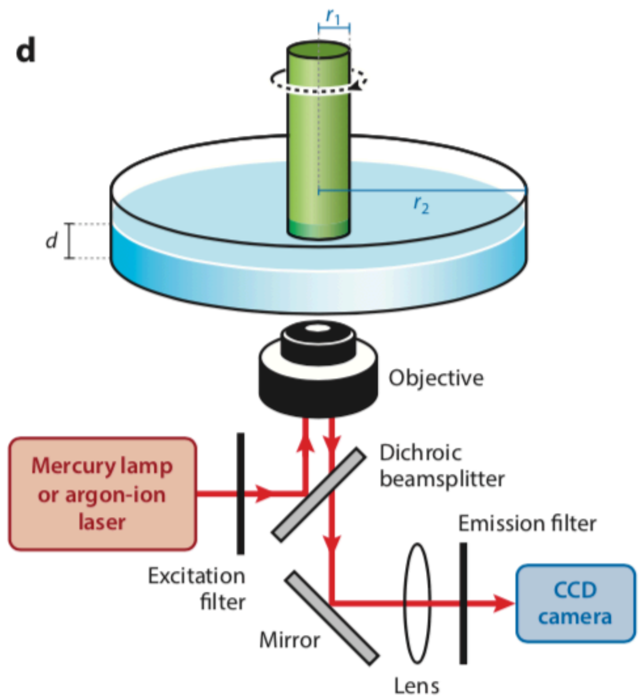
Watanabe & Gotoh, *Phys. Rev.* E (2010)

turbulent channel flow– nonlinear elasticity



Bagheri, Mitra, Perlekar & Brandt, *Phys. Rev.* E (2012)

Experiments

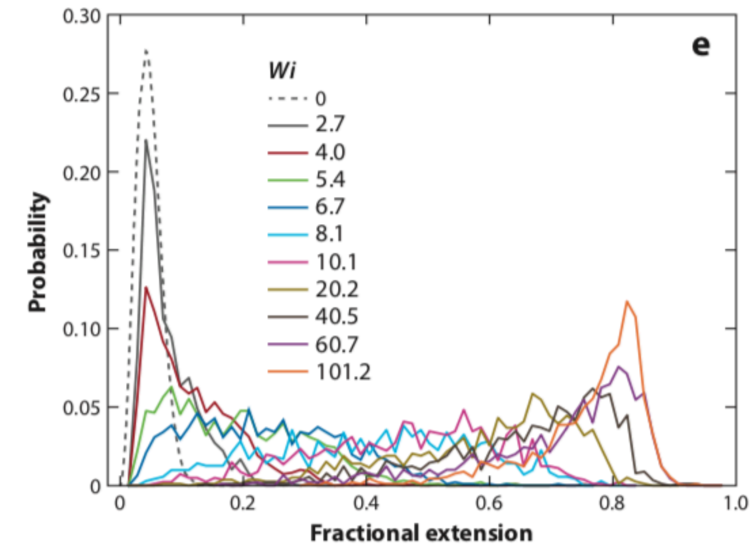


r_1 = 2.25mm, r_2 = 6mm
d = 675µm

λ DNA
(R_g = 0.73µm, L = 21µm)

T4 DNA
(R_{eq} = 3µm, L = 71.7µm)

Reynolds < 1



Chevallard, Gerashchenko & Steinberg, *EPL* (2005)
Steinberg, *C.R. Physique* (2009)
Liu & Steinberg, *EPL* (2010)

Liu & Steinberg, *Macromol. Symp.* (2014)
Steinberg, *Annu. Rev. Fluid Mech.* (2021)

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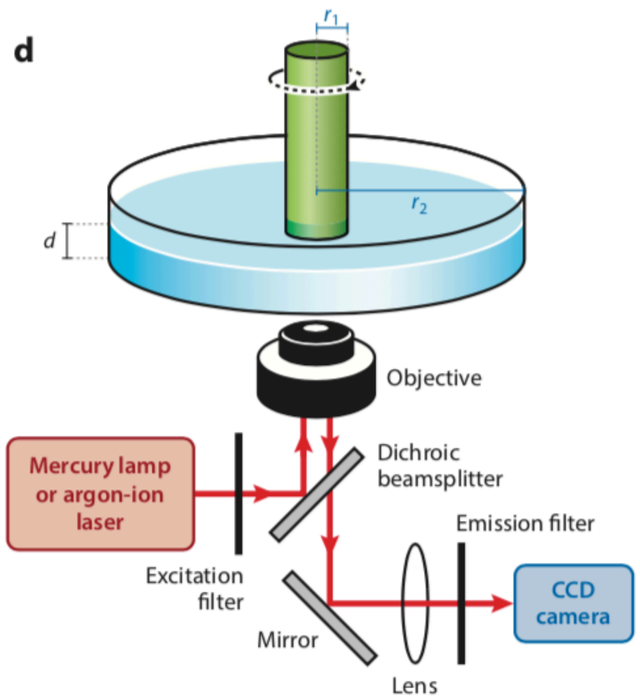

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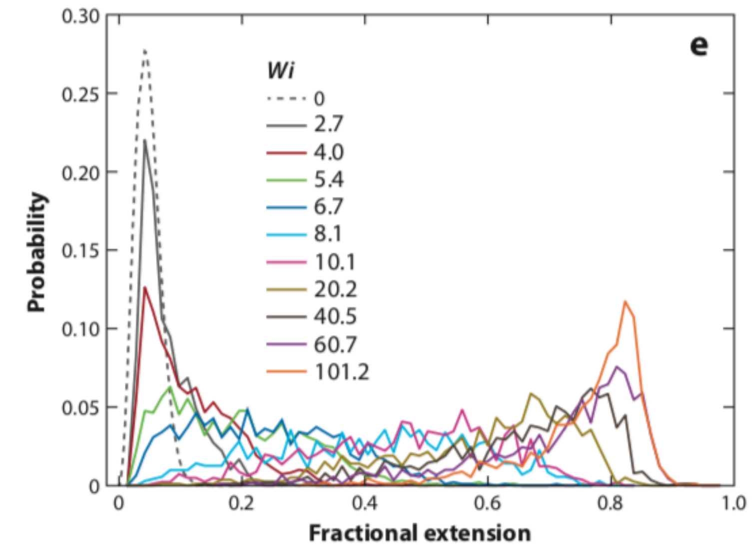

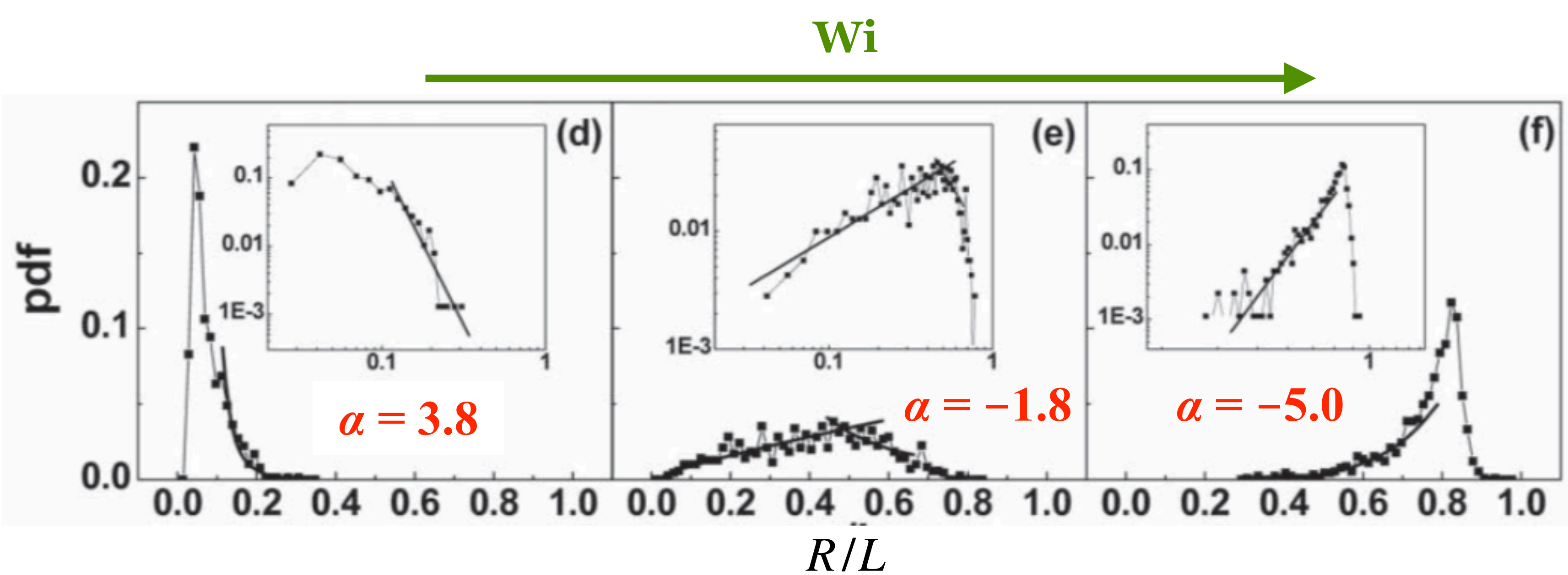

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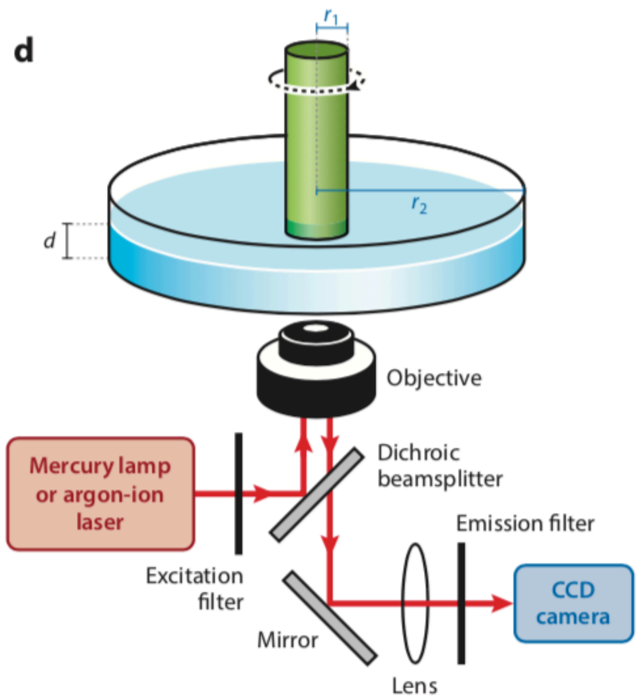

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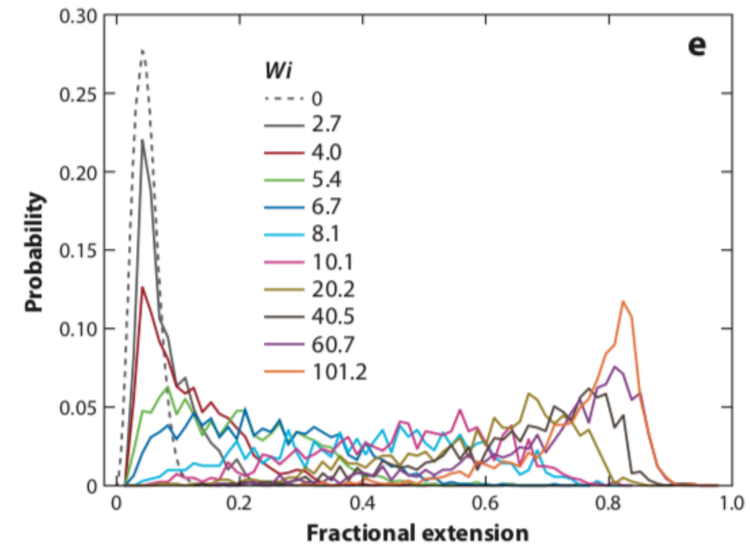

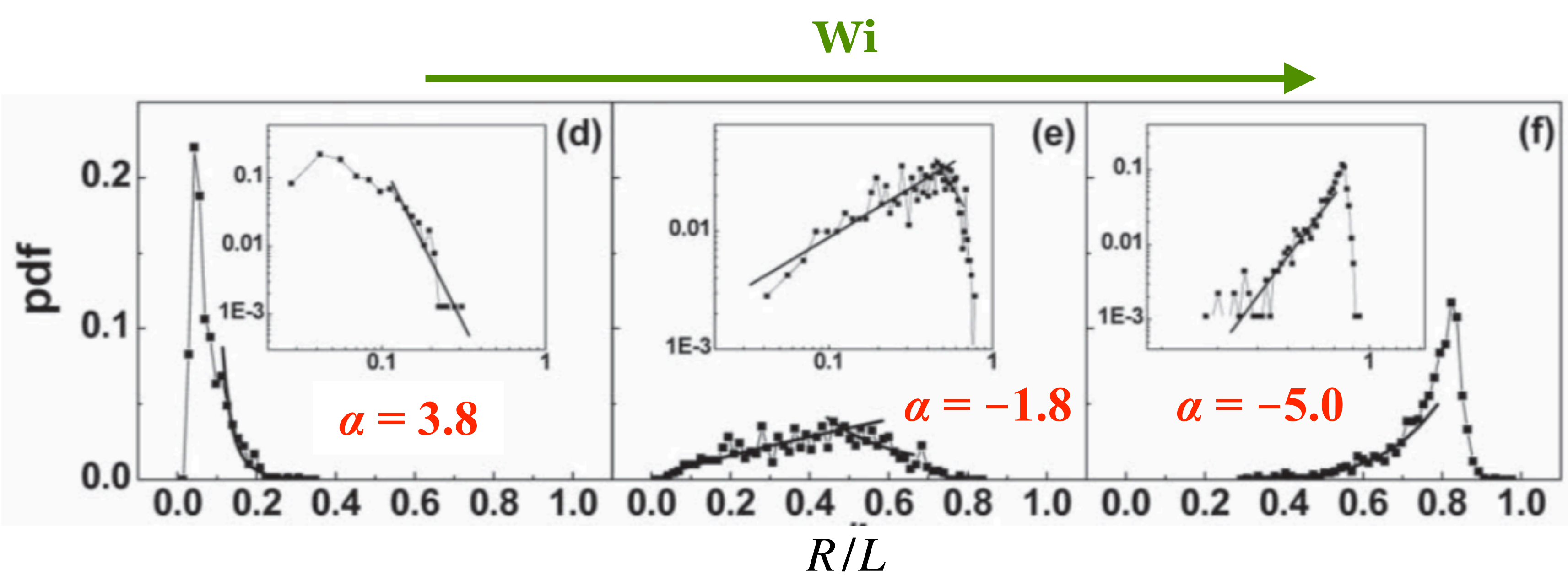

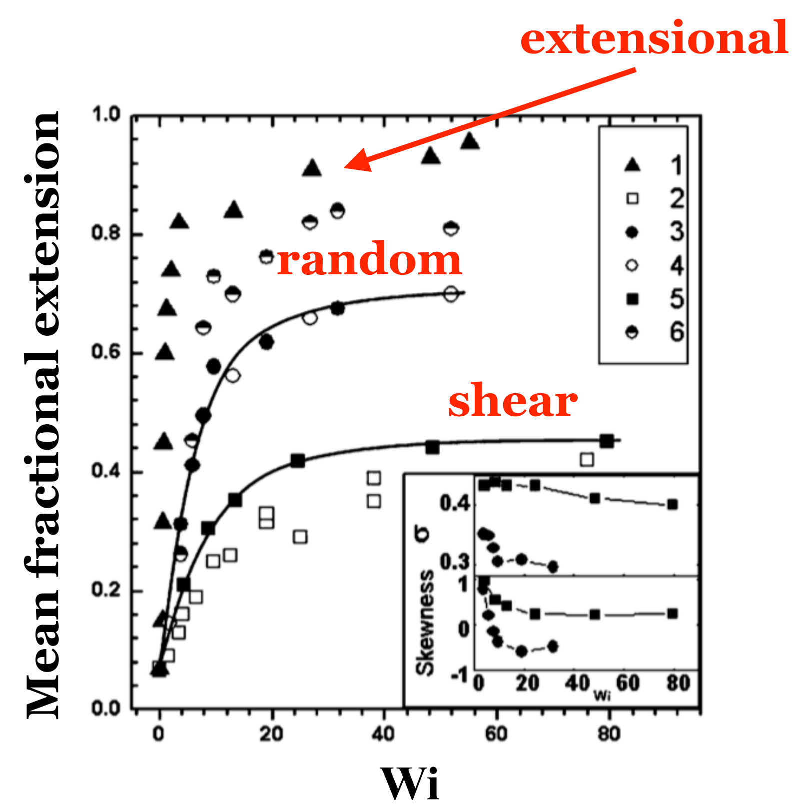

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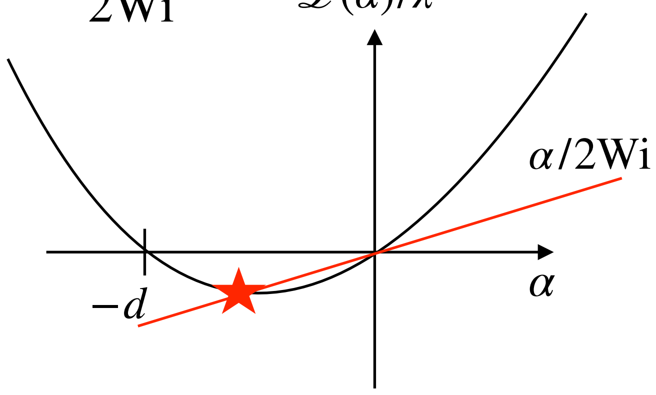
Effect of internal viscosity

$$\frac{\mathscr{L}(\alpha)}{\lambda} = \frac{\alpha}{2\mathrm{Wi}}$$



$\mathrm{Wi} > 1/2 \to \alpha < 0$

Theory

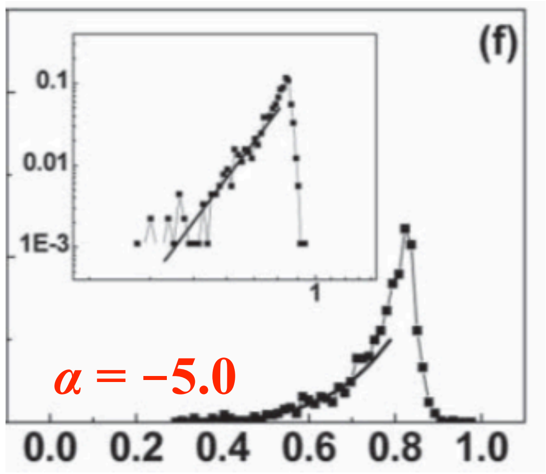
$$P_{\mathrm{st}}(R) \sim R^{-1-\alpha} \quad (R_{eq} \ll R \ll L)$$

$$\lim_{\mathrm{Wi}\to\infty} \alpha = -d$$

At large Wi: $P_{\mathrm{st}}(R) \sim R^{d-1}$

Experiment at large Wi:

$P_{\mathrm{st}}(R) \sim R^4$



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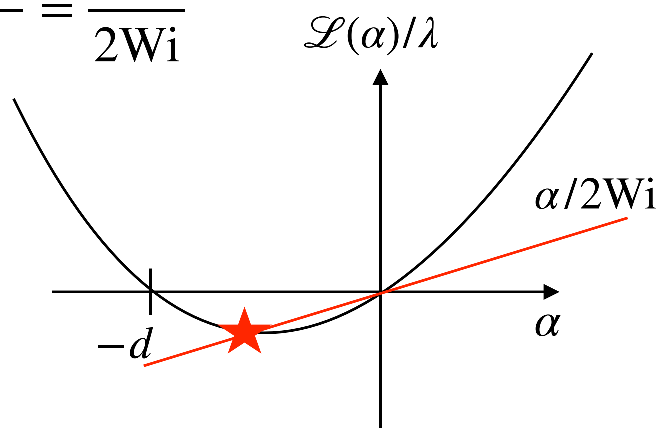

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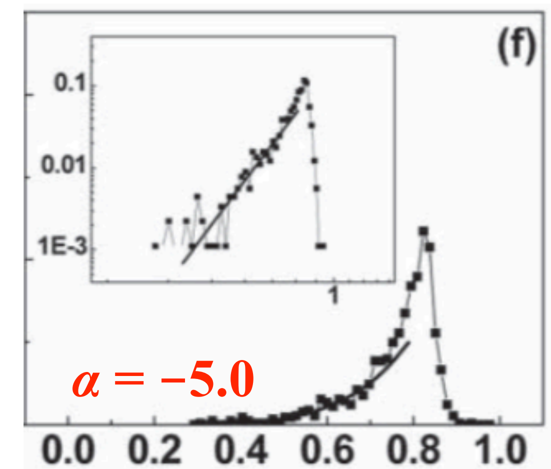

Liu & Steinberg, *Macromol. Symp.* (2014)

Internal viscosity

Kuhn & Kuhn, *Helv. Chim. Acta* (1945)

$$\boldsymbol{F}^{\mathrm{iv}} = -\phi(\boldsymbol{R}\cdot\dot{\boldsymbol{R}})\frac{\boldsymbol{R}}{R^2}$$

internal viscosity parameter

$$\epsilon = \frac{2\phi}{\zeta}$$

For recent applications: Kailasham, Chakrabarti & Prakash
J. Chem. Phys. (2018); *Phys. Rev. Res.* (2020)

Effect of internal viscosity

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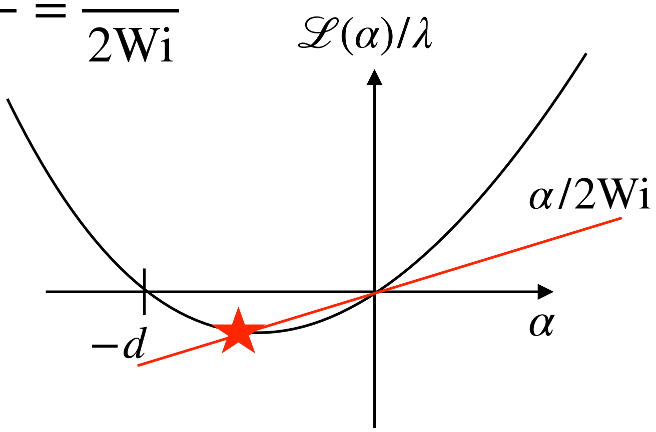

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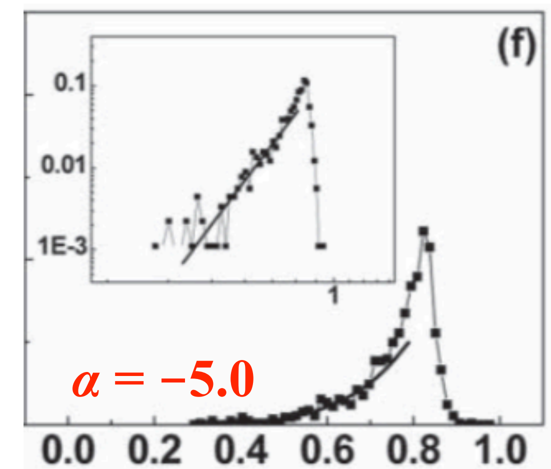

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internal viscosity parameter

$$\epsilon = \frac{2\phi}{\zeta}$$

For recent applications: Kailasham, Chakrabarti & Prakash *J. Chem. Phys.* (2018); *Phys. Rev. Res.* (2020)

$$\frac{1}{\lambda}\mathcal{L}\left(\frac{\alpha_\epsilon}{1+\epsilon}\right) = \frac{\alpha_\epsilon}{2(1+\epsilon)\mathrm{Wi}}$$

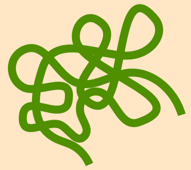
$$\alpha_\epsilon = \alpha(1+\epsilon)$$

At large Wi: $P_{\mathrm{st}}(R) \sim R^{d(1+\epsilon)-1}$

For $\epsilon = 2/3$, $P_{\mathrm{st}}(R) \sim R^4$

DV, *Soft Matter* (2021)

Temporal dynamics – Relaxation to steady state



$\zeta_{\text{coil}} \propto \eta R_{eq}$

$$\zeta_{\text{stretch}} \propto \frac{\eta L}{\ln(L/d)}$$

η solvent viscosity; d polymer diameter

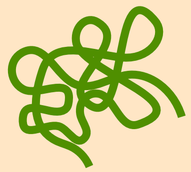
Conformation-dependent drag (Hinch, 1975)

$$\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \nabla \boldsymbol{u} \cdot \boldsymbol{R} - \frac{f(R)}{2\tau\, \nu(R)} \boldsymbol{R} + \sqrt{\frac{R_{eq}^2}{\tau}}\, \boldsymbol{\xi}(t)$$

$$\tau = \zeta_c / 4k, \quad \nu(R) = 1 + (\zeta_s/\zeta_c - 1) R/L$$

Celani, Puliafito & DV, *Phys. Rev. Lett.* (2006)

Temporal dynamics – Relaxation to steady state



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Batchelor—Kraichan flow

Expansion into eigenfunctions of the FP operator

$$\partial_t P = -\,\partial_R[D_1(R)P] + \partial_R^2[D_2(R)P] \qquad P(R,t) = P_{\rm st}(R) + \sum_{n=1}^{\infty} a_n p_n(r)\,\mathrm{e}^{-\sigma_n t}$$

$p_n(r)$ are the eigenfunctions of the FP operator and σ_n ($\sigma_n < \sigma_{n+1}$) the associated eigenvalues

$$T_{eq} = \sigma_1^{-1}$$

Celani, Puliafito & DV, *Phys. Rev. Lett.* (2006)

Temporal dynamics – Relaxation to steady state

Batchelor—Kraichnan flow

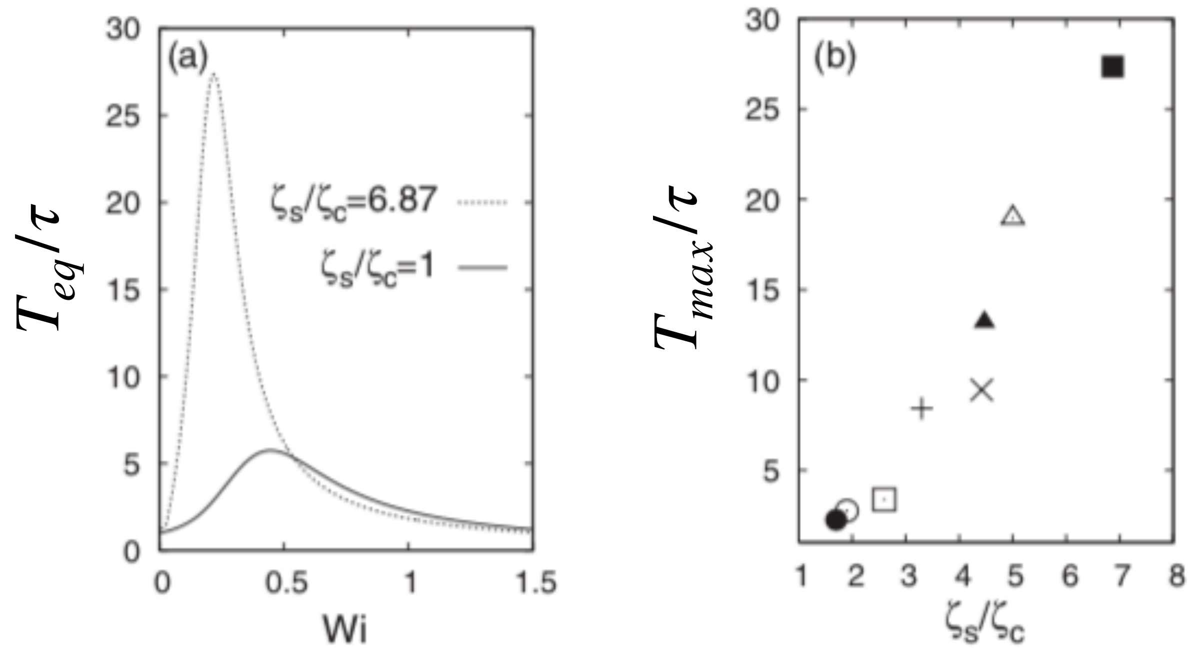
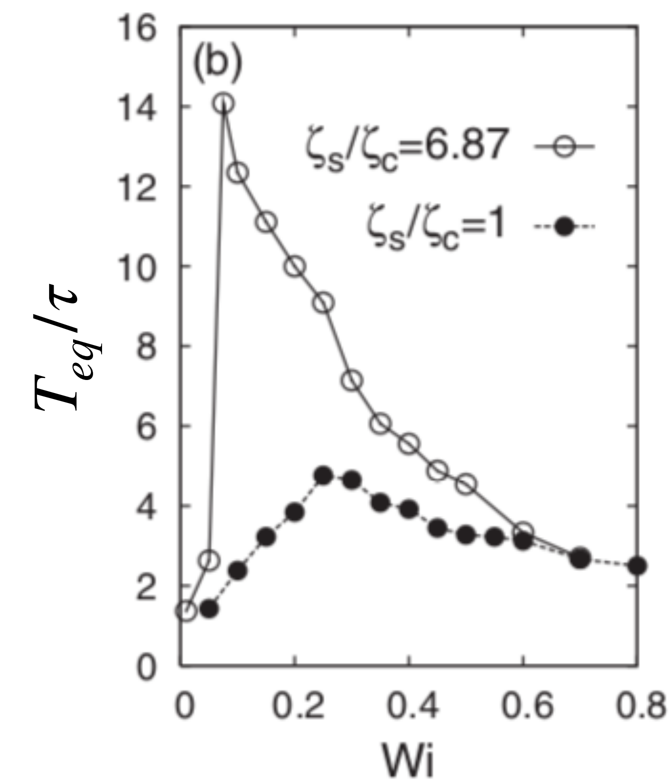


FIG. 3. Three-dimensional Batchelor-Kraichnan flow: (a) $t_{\rm rel}/\tau$ vs Wi for a PAM molecule ($b = 3953$); (b) $t_{\max}/\tau$ for the following polymers: DNA (●, $b = 191.5$; ○, $b = 260$; □, $b = 565$; +, $b = 2250$), polystyrene (×, $b = 673$), polyethyleneoxide (PEO) (▲, $b = 1666$), *Escherichia Coli* DNA (△, $b = 9250$), and PAM (■). Measures of b and ζ_s/ζ_c can be found in

stochastic flow with nonzero correlation time



Celani, Puliafito & DV, *Phys. Rev. Lett.* (2006)

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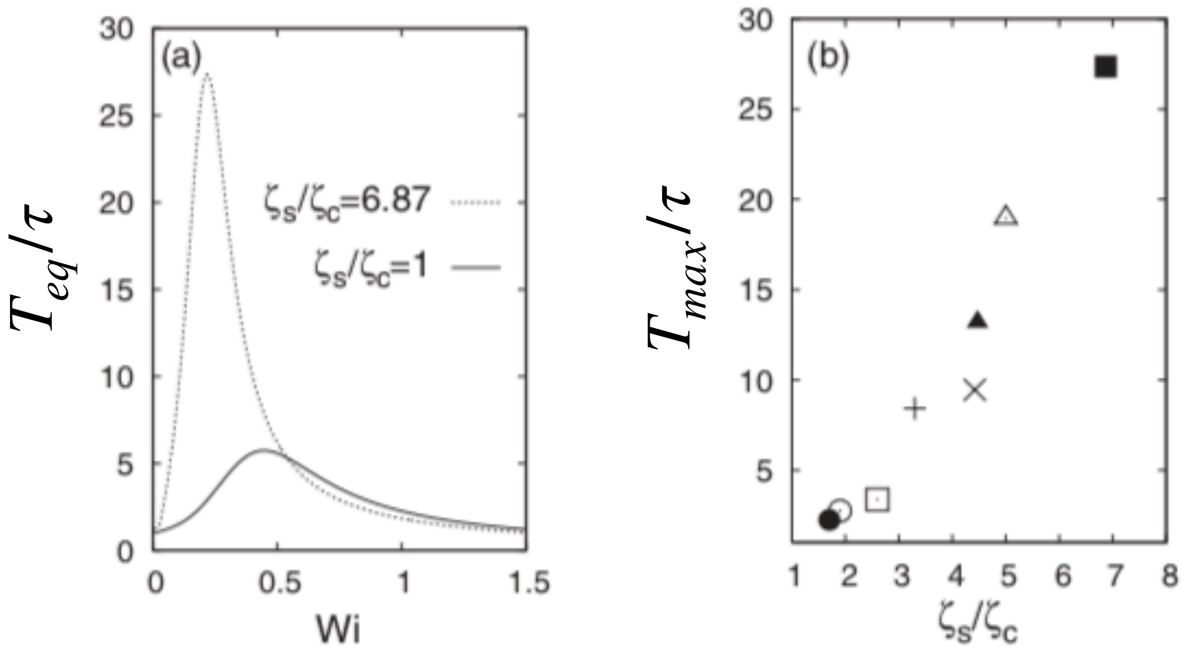

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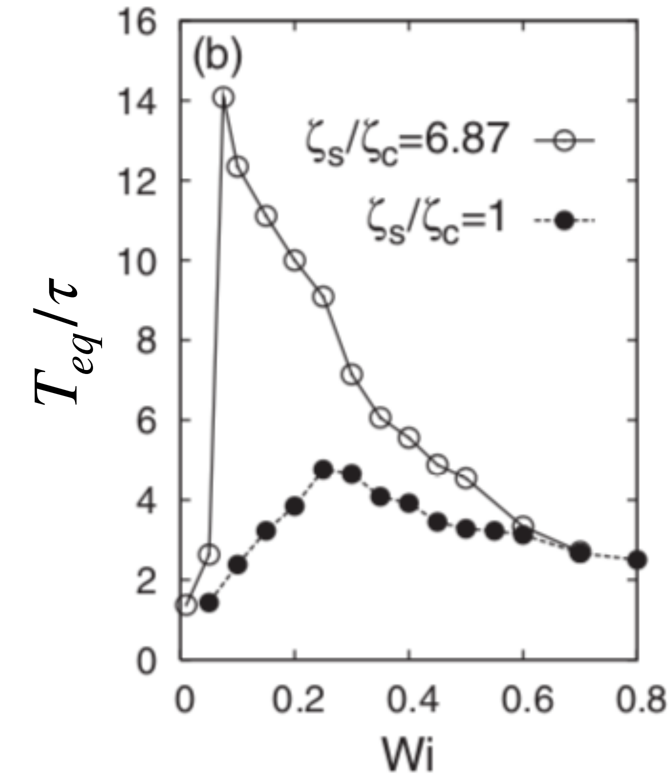
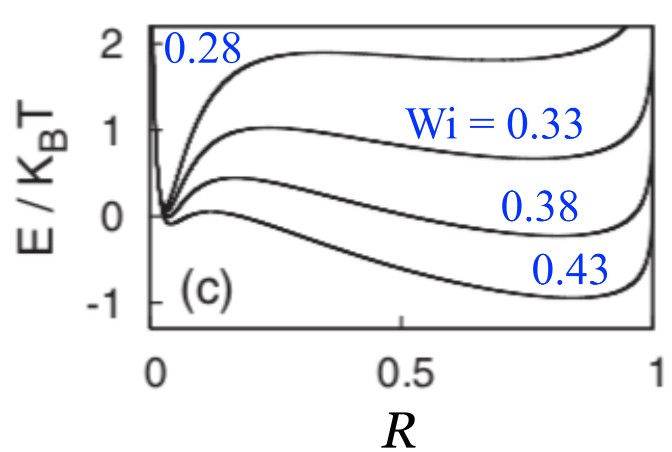


$P_{\rm st}(R) \propto e^{-E(R)/K_BT}$



- The slowing down of the dynamics is due to the heterogeneity of configurations near the CS transition
- No hysteresis is observed

Celani, Puliafito & DV, *Phys. Rev. Lett.* (2006)

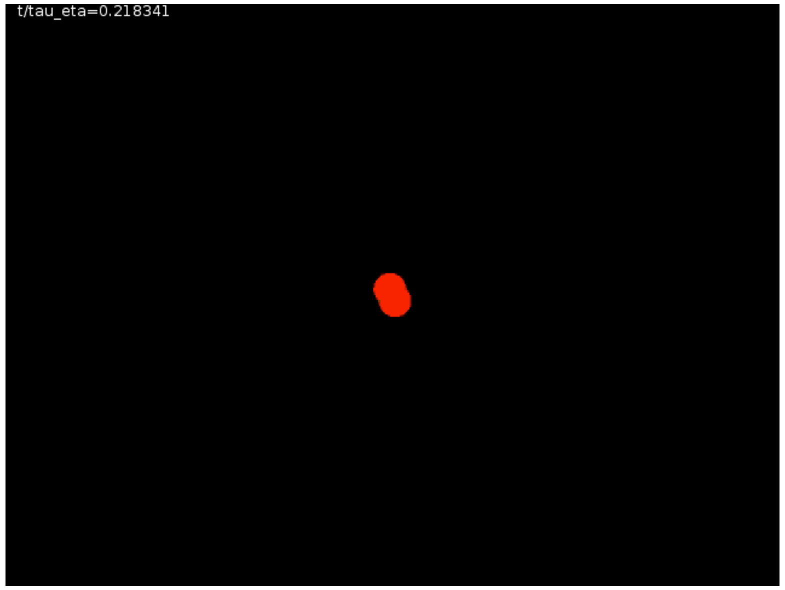
Validity of the dumbbell model

Parameter mapping (Jin & Collins, *New J. Phys.*, 2007)

N number of beads

$$L_{\text{chain}}^2 = \frac{L_{\text{dumb}}^2}{N-1} \qquad \tau_{\text{chain}} = \frac{6\tau_{\text{dumb}}}{N(N+1)}$$

DNS of 3D isotropic turbulence at $R_\lambda = 65$



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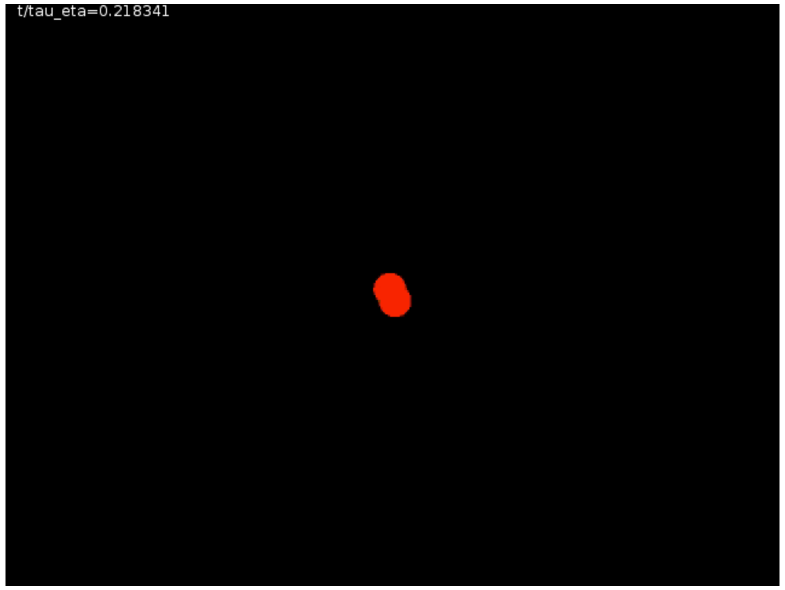

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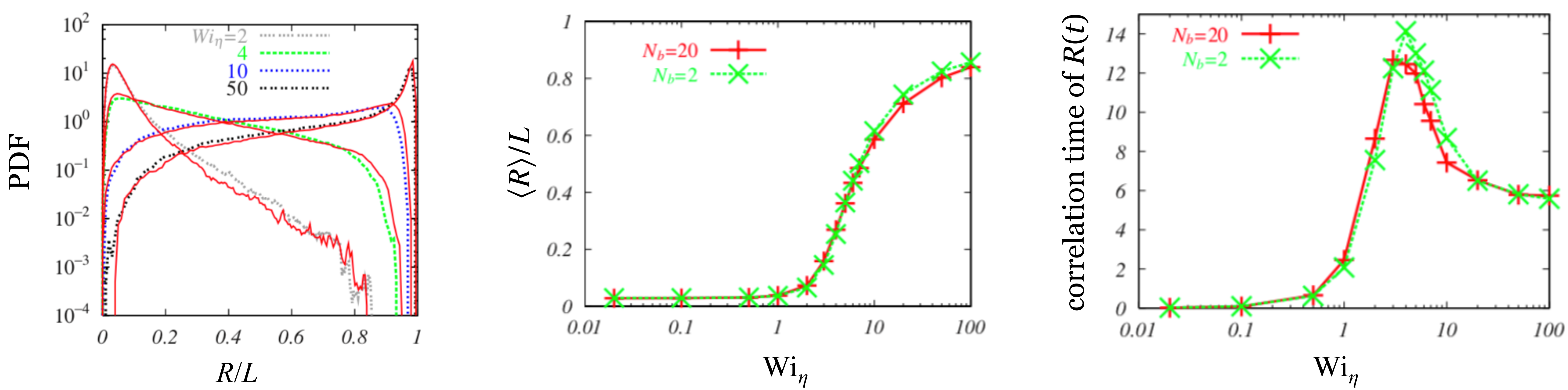
Comparison between a chain (*N*=20, red curves) and a dumbbell (*N*=2)

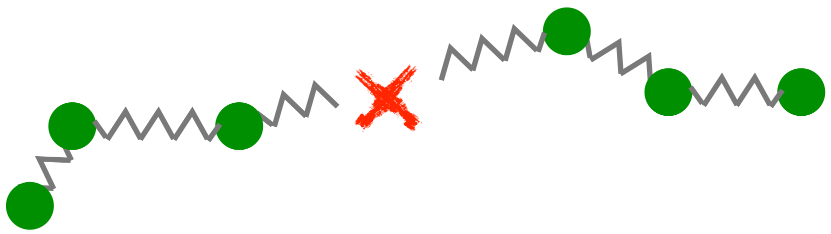
(Watanabe & Gotoh, *Phys. Rev.* E, 2010)

Lagrangian simulation in 3D isotropic turbulence at $R_\lambda = 47$ with 256 polymers



Breakup statistics

When the tension in one of the elastic links exceeds a critical value, the link breaks



$$|\boldsymbol{F}_i^{\mathrm{el}}| = \frac{R_i}{1 - R_i^2/\ell^2} \geqslant \mathcal{F}_{\mathrm{break}} \qquad \text{equivalent to:} \quad R_i \geqslant R_{\mathrm{break}}$$

$$R_{eq} \leqslant R_0 \leqslant R_{\mathrm{break}} \leqslant L \leqslant \eta_K \qquad\qquad R_0 = R(0)$$

DV, Watanabe, Ray, Picardo, *J. Fluid Mech.* (2021)

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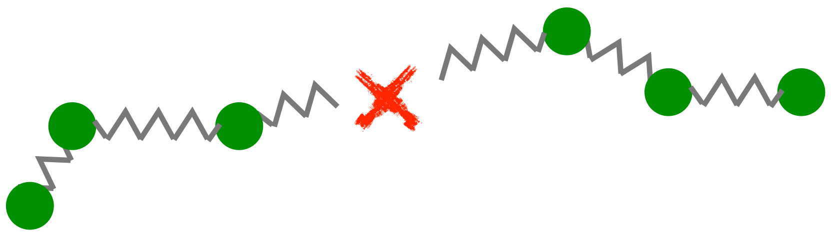

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$$R_{eq} \leqslant R_0 \leqslant R_{\rm break} \leqslant L \leqslant \eta_K \qquad R_0 = R(0)$$

Dumbbell model in the Batchelor–Kraichnan flow

$P(R,t)$ pdf of the extension: $\partial_t P = -\partial_R(D_1 P) + \partial_R^2(D_2 P)$

$$D_1(R) = \frac{2(d+1)}{d}\,\mathrm{Wi}\,R - f(R)R + (d-1)\frac{R_{eq}^2}{R}$$

$$D_2(R) = \frac{2\mathrm{Wi}}{d} + R_{eq}^2$$

Boundary conditions

reflecting at the origin: $-D_1 P + \partial_R(D_2 P)$ at $R = 0$

absorbing at the breakup length: $P(R_{\rm break}, t) = 0$

Initial condition

monodisperse: $P(R,0) = \delta(R_0, 0)$

DV, Watanabe, Ray, Picardo, *J. Fluid Mech.* (2021)

Breakup: Decay of the number of unbroken polymers

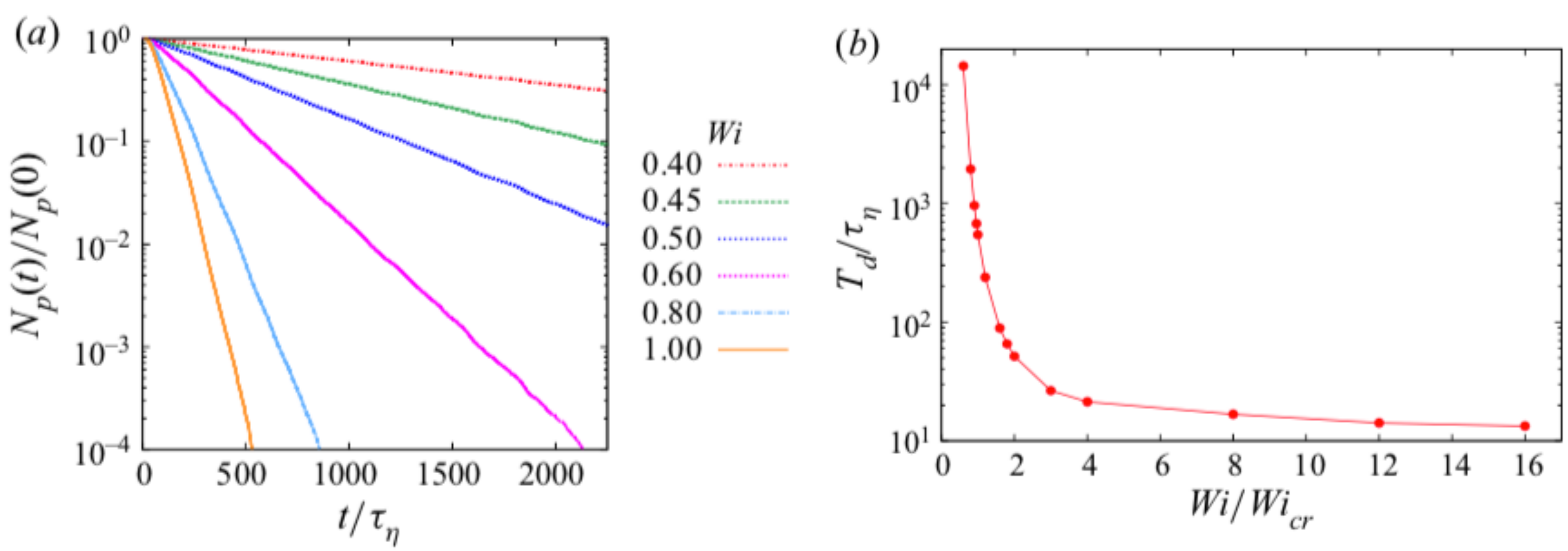
Fraction of polymers surviving at time t

$$\frac{N_p(t)}{N_p(0)} = \int_0^{R_{\text{break}}} P(R,t)\,\mathrm{d}t \sim \mathrm{e}^{-t/T_d}$$

T_d is the inverse of the smallest positive eigenvalue of the FP operator with the above b.c.

Lagrangian simulations
in a periodic cube at $R_\lambda \approx 110$

$N_p(0) = 9 \times 10^5$ polymeric chains
with $\mathcal{N} = 10$ beads each



DV, Watanabe, Ray, Picardo, *J. Fluid Mech.* (2021)

Breakup: Time integrated PDF of the extension

$$\partial_t P = -\partial_R(D_1 P) + \partial_R^2(D_2 P)$$

Monodisperse i.c. $P(R,0) = \delta(R_0,0)$

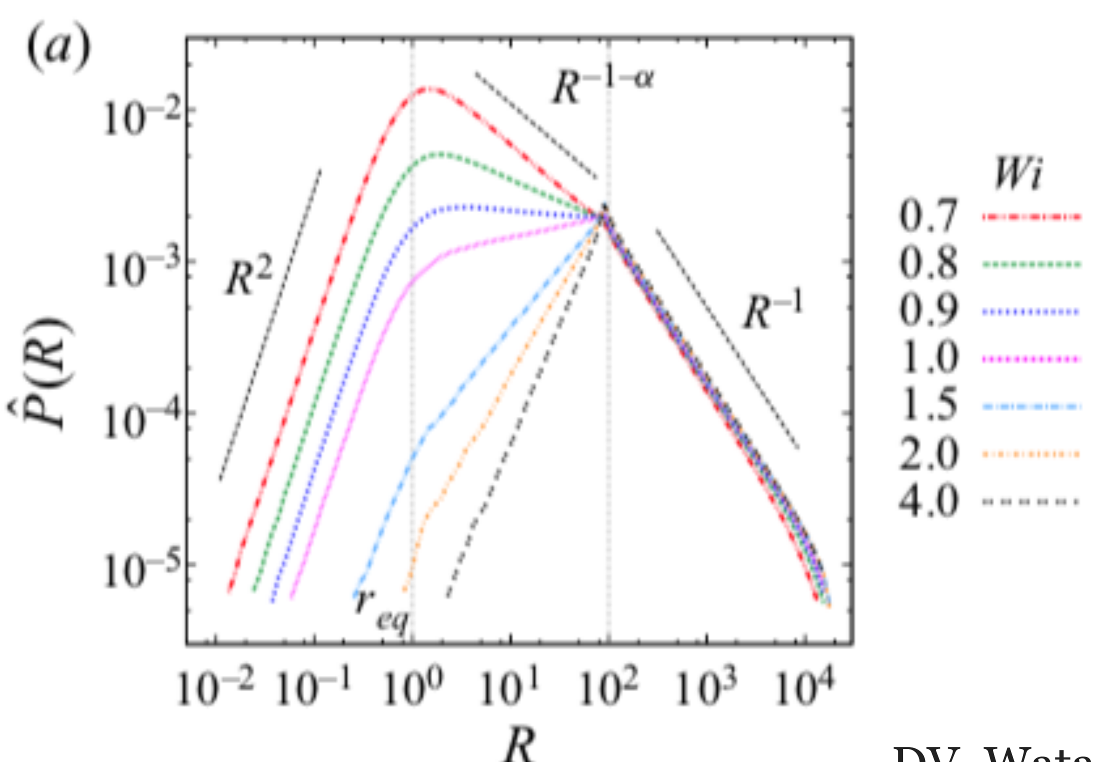
Time integrated PDF of the extension
for unbroken polymers

$$\hat{P}(R) = \int_0^\infty P(R,t)\,\mathrm{d}t$$

$$-D_1\hat{P} + \frac{\mathrm{d}}{\mathrm{d}R}(D_2\hat{P}) = \begin{cases} 1 & \text{if } 0 \leqslant R \leqslant R_0 \\ 0 & \text{if } R_0 \leqslant R \leqslant R_{\text{break}} \end{cases}$$

$$\hat{P}(R) \sim \begin{cases} R^{d-1} & \text{if } 0 \leqslant R \leqslant R_{\text{eq}} \\ R^{-1-\alpha} & \text{if } R_{\text{eq}} \leqslant R \leqslant R_0 \\ R^{-1-\beta} & \text{if } R_0 \leqslant R \leqslant R_{\text{break}} \end{cases}$$

$$\beta = \begin{cases} \alpha & \text{if Wi} \leqslant \text{Wi}_{\text{cr}} \\ 0 & \text{if Wi} > \text{Wi}_{\text{cr}} \end{cases}$$



DV, Watanabe, Ray, Picardo, *J. Fluid Mech.* (2021)

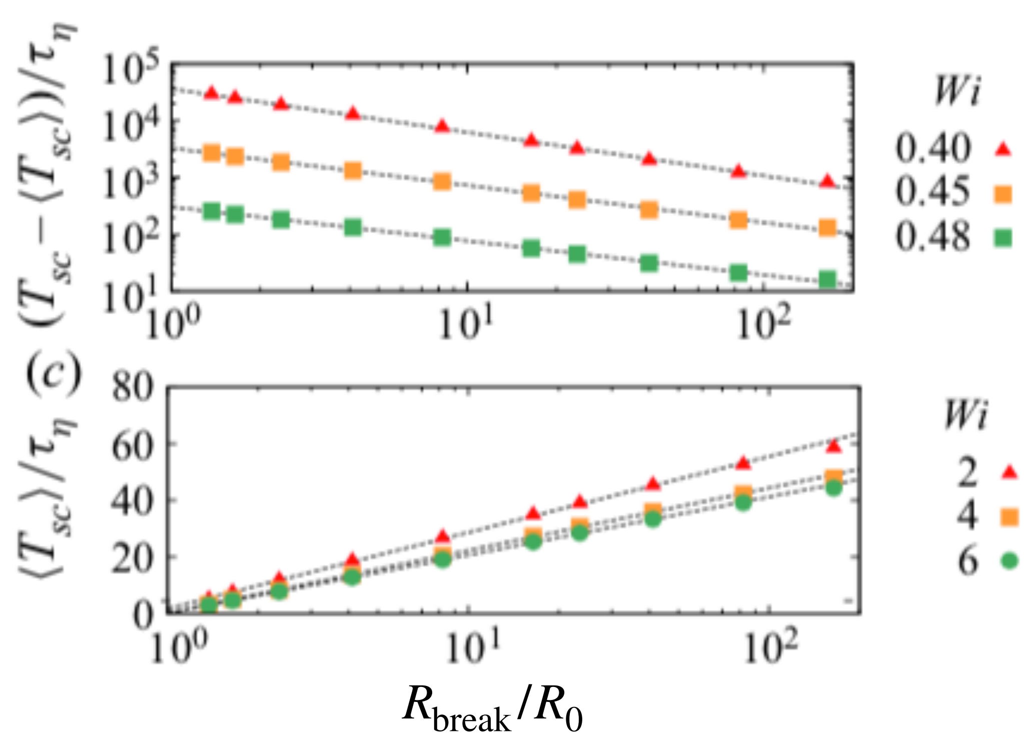
Breakup: Average breakup time

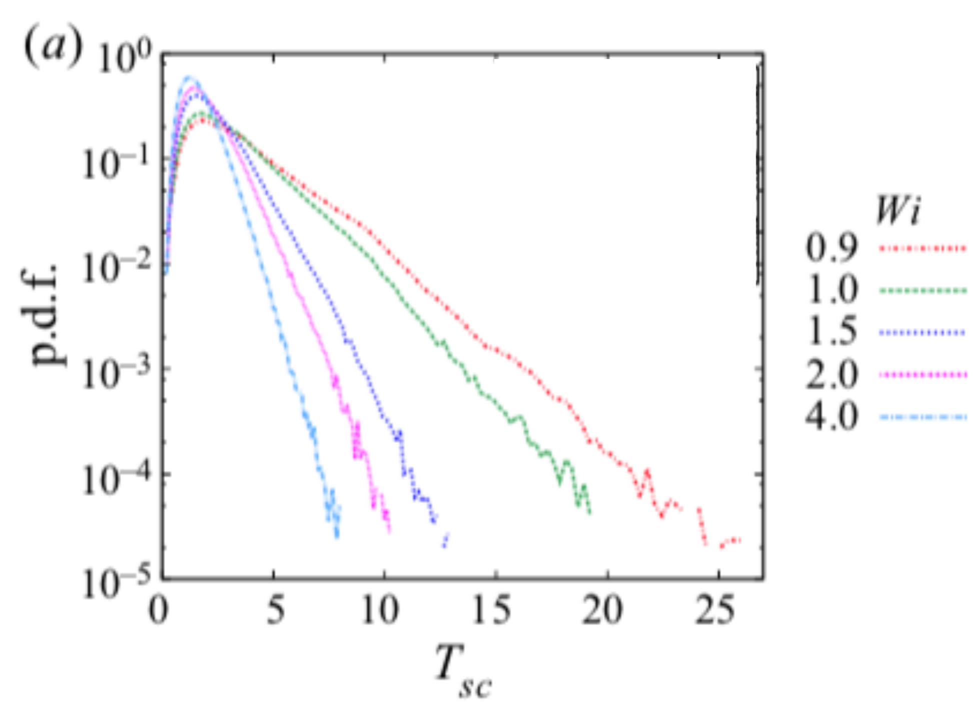
$T_{\rm sc}$ time it takes for a polymer to break in a given realisation of the flow and of thermal noise

$$F(t) = \mathrm{Prob}\{T_{\rm sc} \geqslant t\} = \int_0^{R_{\rm break}} P(R,t)\,\mathrm{d}t$$

$$\langle T_{\rm sc}\rangle = -\int_0^\infty t\,\partial_t F\,\mathrm{d}t = \int_0^\infty F(t)\,\mathrm{d}t = \int_0^\infty \mathrm{d}t \int_0^{R_{\rm break}} \mathrm{d}R\,P(R,t) = \int_0^{R_{\rm break}} \hat{P}(R)\,\mathrm{d}R$$

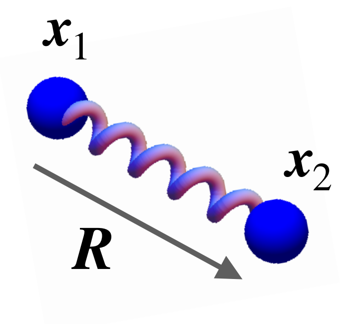
$$\lambda\langle T_{\rm sc}\rangle \sim \begin{cases} \left(\dfrac{R_{\rm break}}{R_0}\right)^\beta & \text{if Wi} \leqslant \text{Wi}_{\rm cr} \\ \ln\left(\dfrac{R_{\rm break}}{R_0}\right) & \text{if Wi} > \text{Wi}_{\rm cr} \end{cases}$$





DV, Watanabe, Ray, Picardo, *J. Fluid Mech.* (2021)

Stretching beyond the Kolmogorov scale



$R_{eq} \ll \eta_K$ and $\eta_K \ll L$

$$\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \nabla \boldsymbol{u} \cdot \boldsymbol{R} - \frac{f(R)}{2\tau}\boldsymbol{R} + \sqrt{\frac{R_{eq}^2}{\tau}}\,\boldsymbol{\xi}(t)$$

$$\downarrow$$

$$\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \boldsymbol{u}(\boldsymbol{x}_2, t) - \boldsymbol{u}(\boldsymbol{x}_1, t) - \frac{f(R)}{2\tau}\boldsymbol{R} + \sqrt{\frac{R_{eq}^2}{\tau}}\,\boldsymbol{\xi}(t)$$

De Lucia, Mazzino & Vulpiani, *EPL* (2002)
Davoudi & Schumacher, *Phys. Fluids* (2006)

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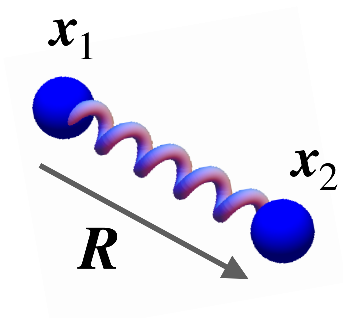

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Davoudi & Schumacher, *Phys. Fluids* (2006)

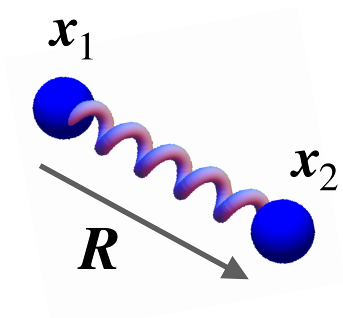
Kraichnan flow

$$\langle u_i(\boldsymbol{x}+\boldsymbol{r}, t)\, u_j(\boldsymbol{x}, t')\rangle = [D_{ij}(0) - d_{ij}(\boldsymbol{r})]\;\delta(t-t')$$

$$d_{ij}(\boldsymbol{r}) = d_{NN}(r)\,\delta_{ij} + [d_{LL}(r) - d_{NN}(r)]\,\hat{r}_i\hat{r}_j$$

$$d_{NN}(r) = d_{LL}(r) + r d'_{LL}(r)/2$$

Stretching beyond the Kolmogorov scale



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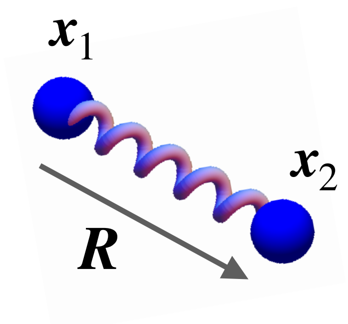
$$\partial_t P = -\,\partial_R(D_1 P) + \partial_R^2(D_2 P)$$

$$\begin{cases} D_1 = \dfrac{4\tau d_{LL}}{R} + 2\tau d'_{LL} - f(R)R + \dfrac{2R_0^2}{R} \\ D_2 = 2\tau d_{LL} + R_0^2 \end{cases}$$

Stretching beyond the Kolmogorov scale



$R_{eq} \ll \eta_K$ and $\eta_K \ll L$

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$$\downarrow$$

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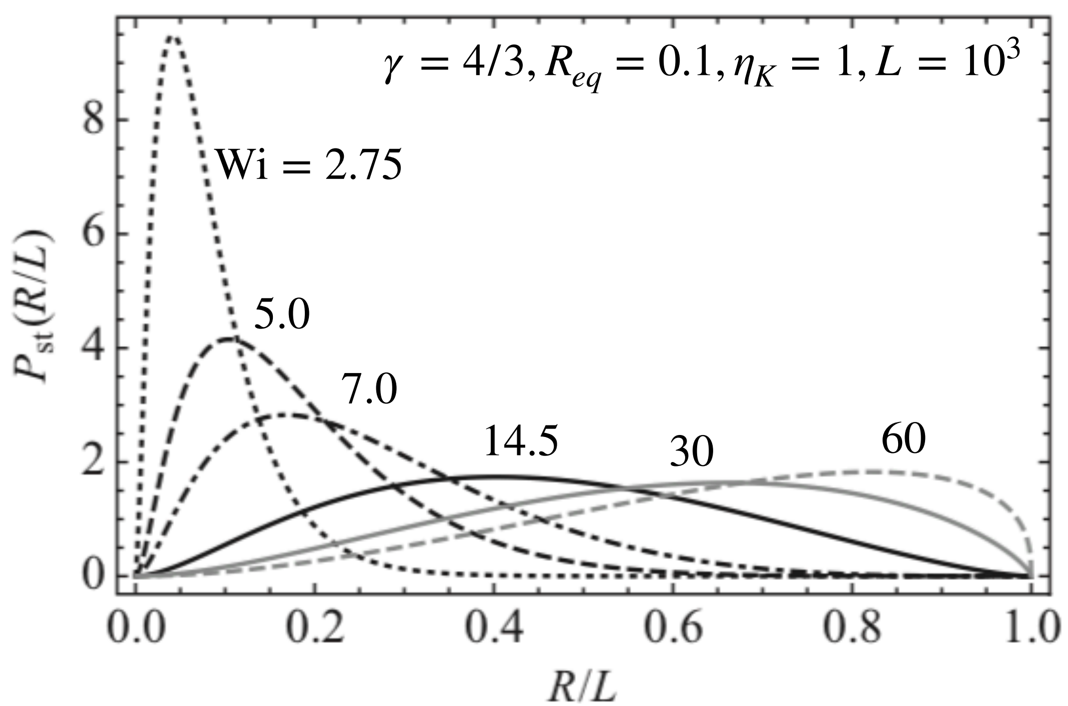
$$E(k) = c\mathrm{e}^{-\eta^2k^2}k^{-1-\gamma} \quad (0<\gamma<2)$$

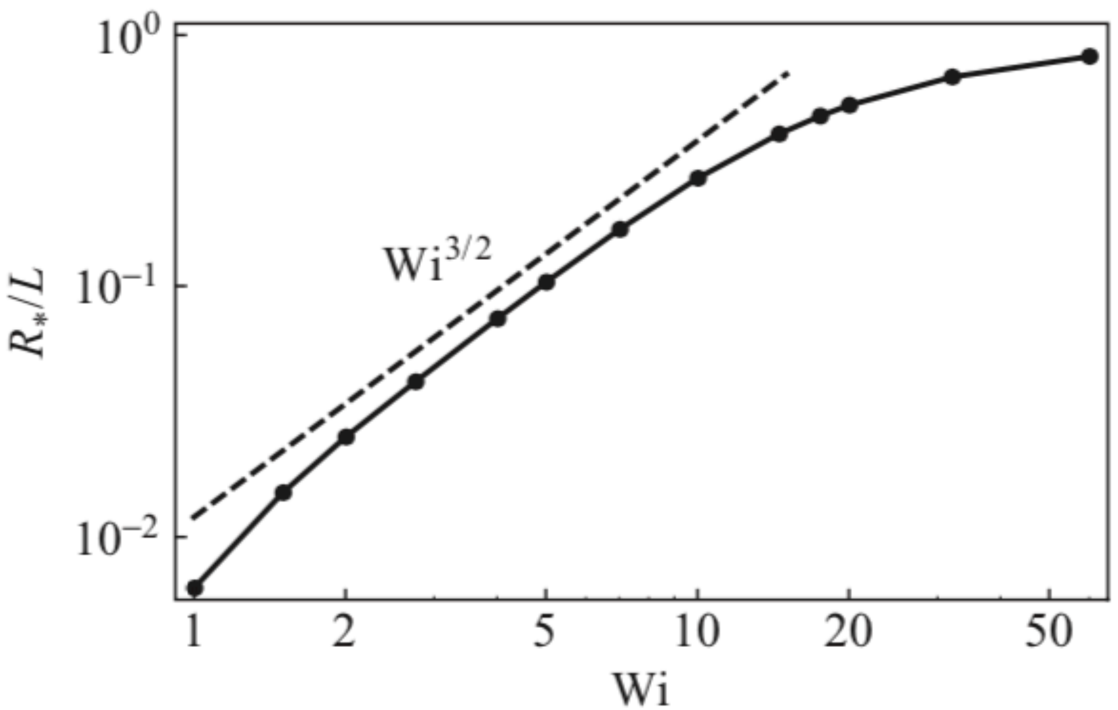
$$\begin{cases} d_{LL} \sim 2D_1 r^2 & (r \ll \eta) \\ d_{LL} \sim 2D_1 a(\gamma)\,\eta^{2-\gamma} r^{\gamma} & (r \gg \eta) \end{cases}$$

Stretching beyond the Kolmogorov scale

$$P(R) \sim \begin{cases} R^{d-1} & 0 \leqslant R \ll R_{eq} \\ R^{-1-\alpha} & R_{eq} \ll R \ll \eta_K \\ R^{d-1} \exp\left\{ -\frac{a_\gamma}{\mathrm{Wi}} \left(\frac{R}{\eta_K}\right)^{2-\gamma} \right\} & \eta_K \ll R \ll L \end{cases}$$

Lumley scale $R_\star \sim \eta_K \, \mathrm{Wi}^{1/(2-\gamma)}$

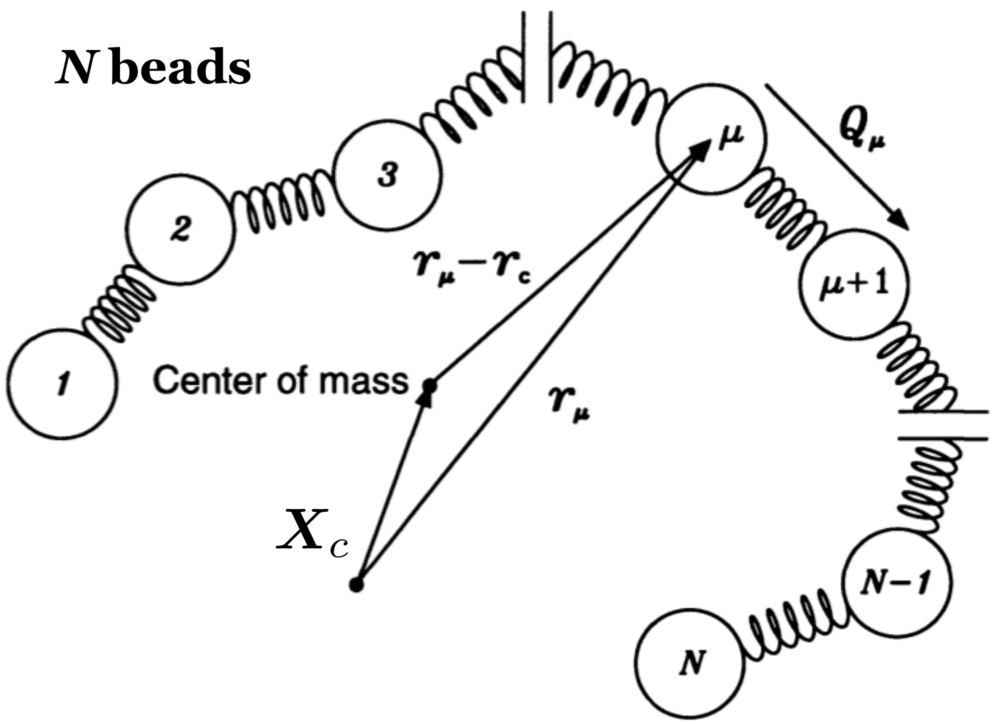




- The stretched state emerges through the shift of the maximum of the PDF, i.e. $R_\star$
- There is no coil—stretch transition in this case

Ahmad & DV, *Phys. Rev.* E (2016)

Elastic filaments in 2D turbulence



equilibrium length $\ll \ell_\mathrm{f}$

$$L_{\max} = (N-1)Q_{\max} > \ell_\mathrm{f}$$

ℓ_f scale of the forcing

$$\frac{\mathrm{d}\boldsymbol{X}_c}{\mathrm{d}t} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{u}(\boldsymbol{x}_i, t) + \frac{1}{N}\sqrt{\frac{R_{eq}^2}{6\tau}}\sum_{i=1}^{N}\boldsymbol{\xi}_i(t)$$

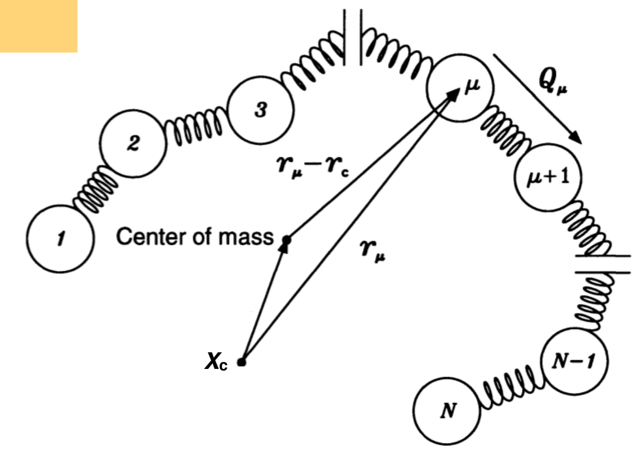
$$\frac{\mathrm{d}\boldsymbol{Q}_i}{\mathrm{d}t} = \boldsymbol{u}(\boldsymbol{x}_{i+1}, t) - \boldsymbol{u}(\boldsymbol{x}_i, t) - \frac{1}{4\tau}(2f_i\boldsymbol{Q}_i - f_{i+1}\boldsymbol{Q}_{i+1} - f_{i-1}\boldsymbol{Q}_{i-1}) + \sqrt{\frac{R_{eq}^2}{6\tau}}[\boldsymbol{\xi}_{i+1}(t) - \boldsymbol{\xi}_i(t)], \qquad \tau = 4k/\zeta$$

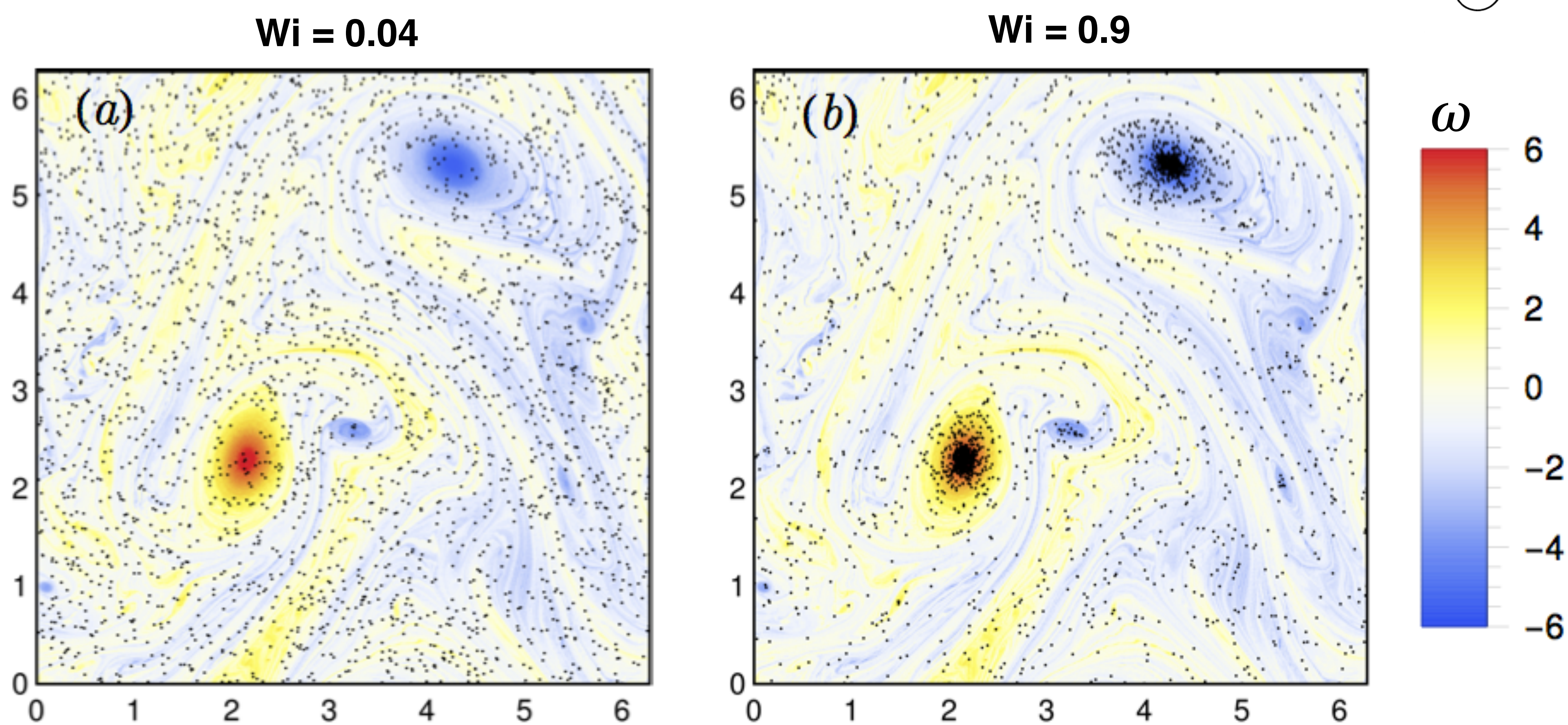
Preferential sampling of vortices

number of beads $N = 10$

chain length $L_{\max} = 4$

forcing scale $\ell_f = 2\pi/k_f = 2$





Weissenberg number: $\mathrm{Wi} = \tau_{\mathrm{chain}}/t_f$ with $t_f = \ell_f/\sqrt{2E}$ (E mean kinetic energy)

2D Navier–Stokes DNS on $[0,2\pi]^2$, grid resolution 1024^2, forcing $f = -F_0\, k_f \cos(k_f\, x)$

Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)

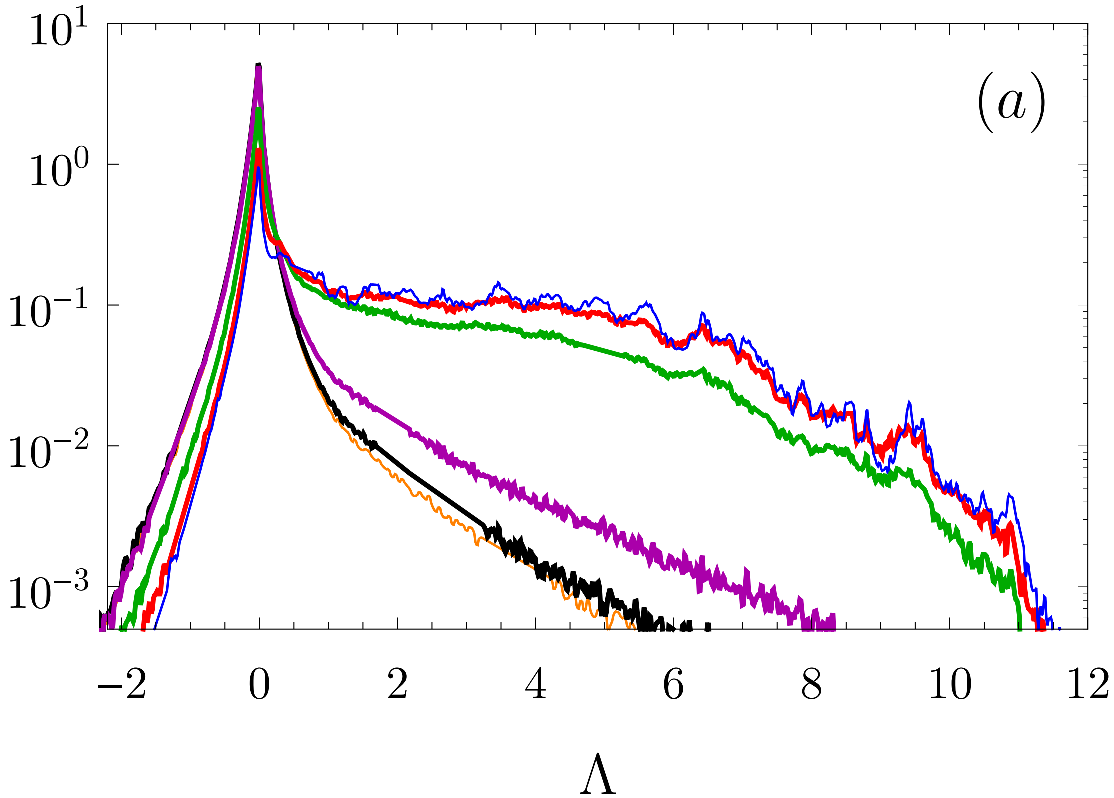
Okubo—Weiss parameter

$$\Lambda = \frac{\omega^2 - \sigma^2}{4\langle \omega^2 \rangle}$$

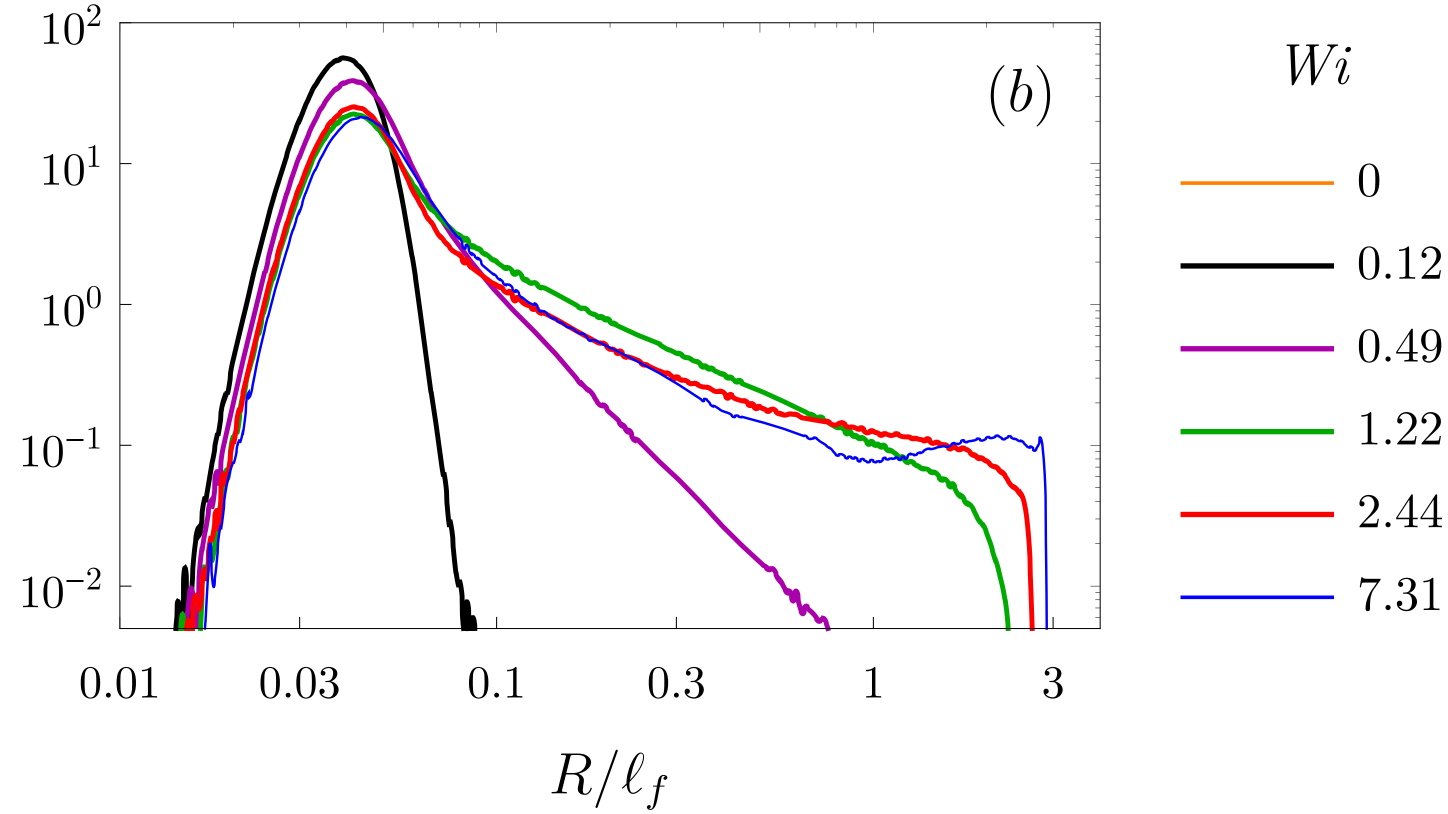
$\sigma^2 = 2\mathsf{S}_{ij}\mathsf{S}_{ij}$ strain rate

$\mathbf{S} = [\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^\top]/2$

PDF of Λ

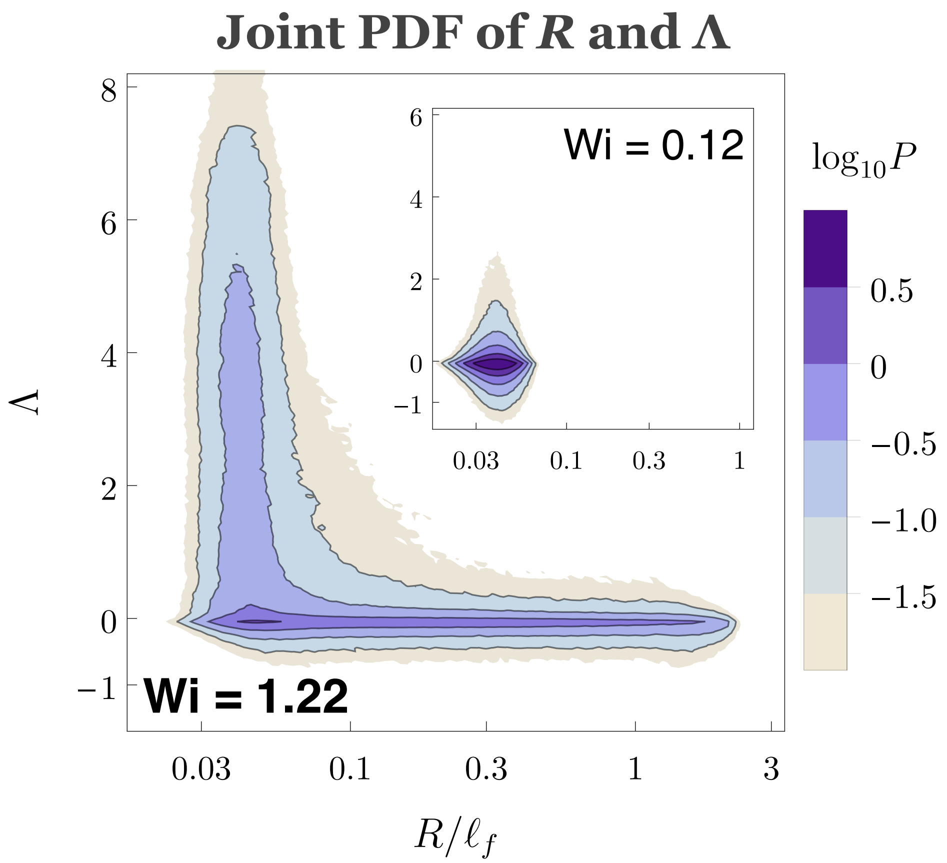


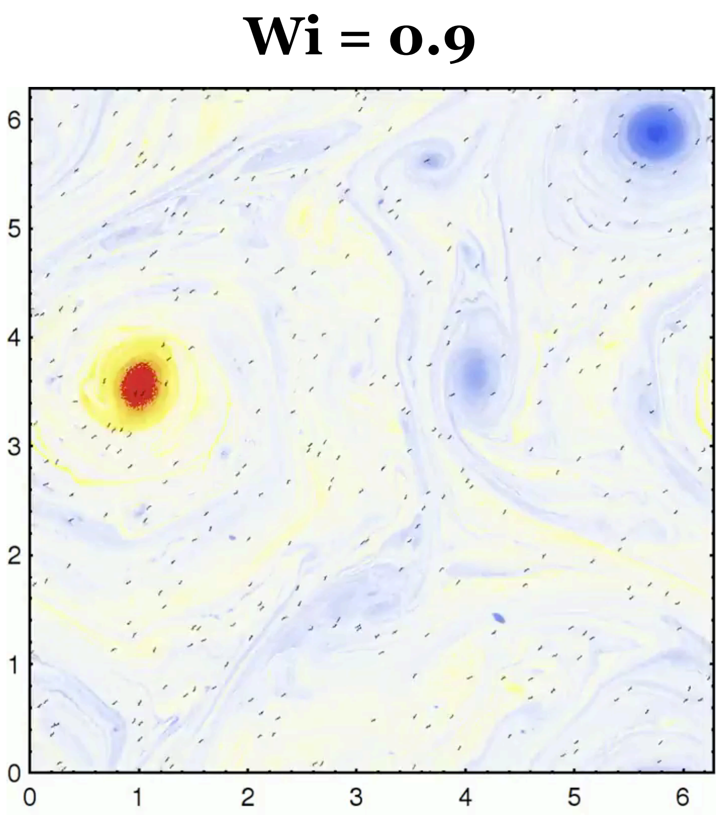
PDF of chain extension



Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)

Selection of vortical regions

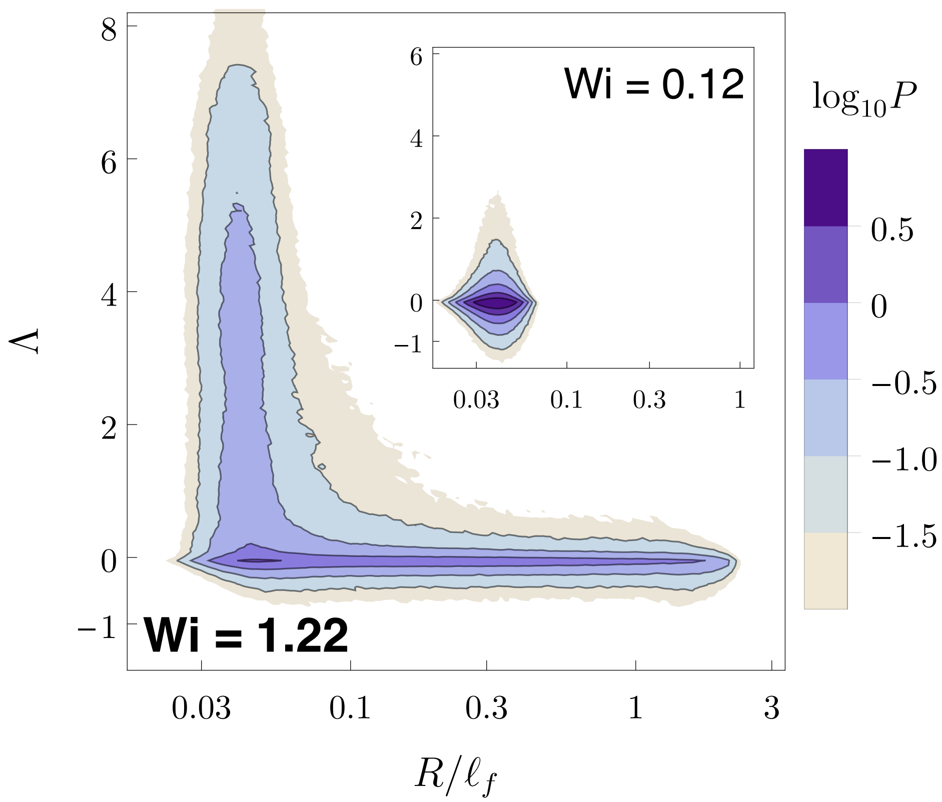




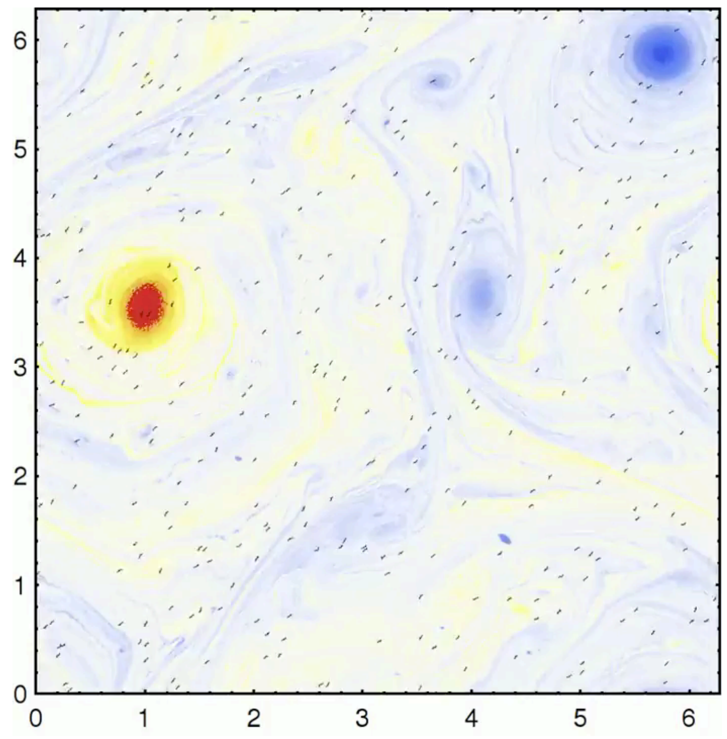
Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)
Singh, Gupta, Picardo, Dv & Ray, *Phys. Rev.* E (2020)
Picardo, Singh, Ray & DV, *Phil. Trans. R. Soc.* A (2020)

Selection of vortical regions

Joint PDF of R and Λ



Wi = 0.9



Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)
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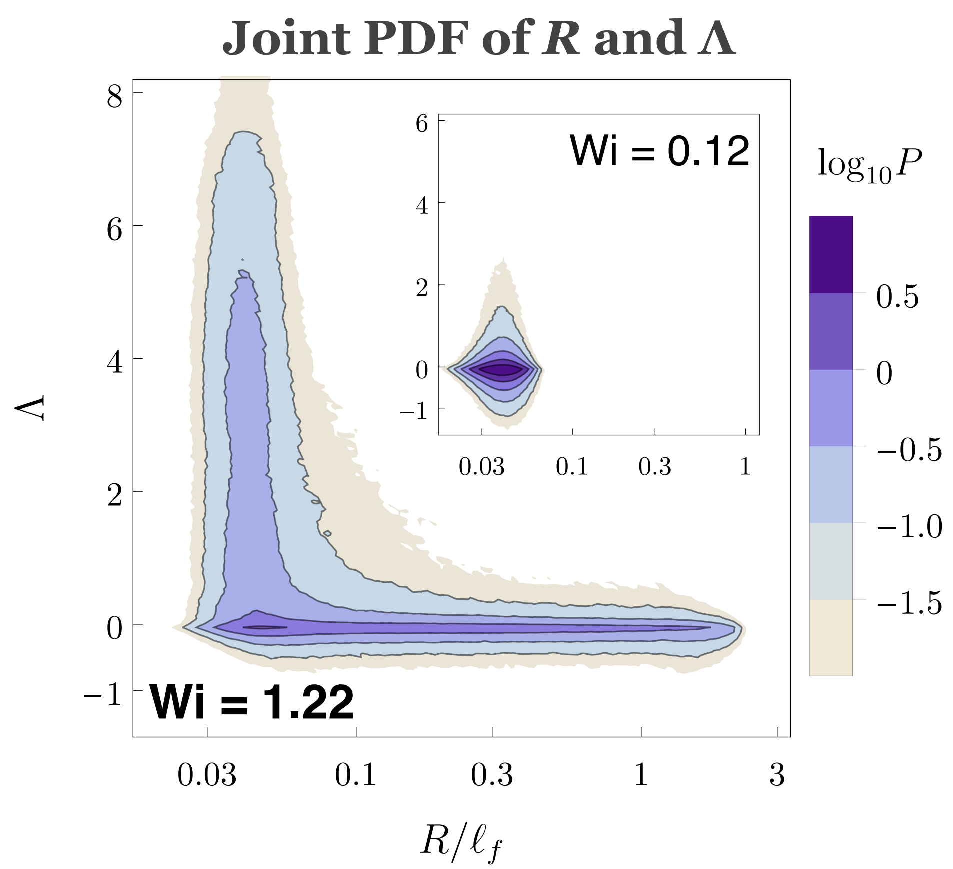
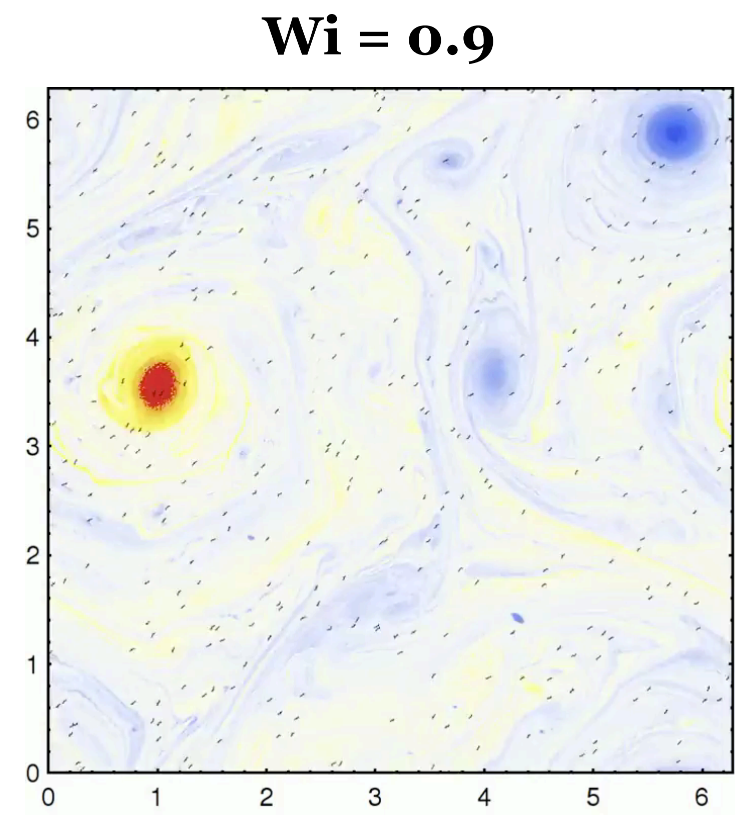
Selection of vortical regions



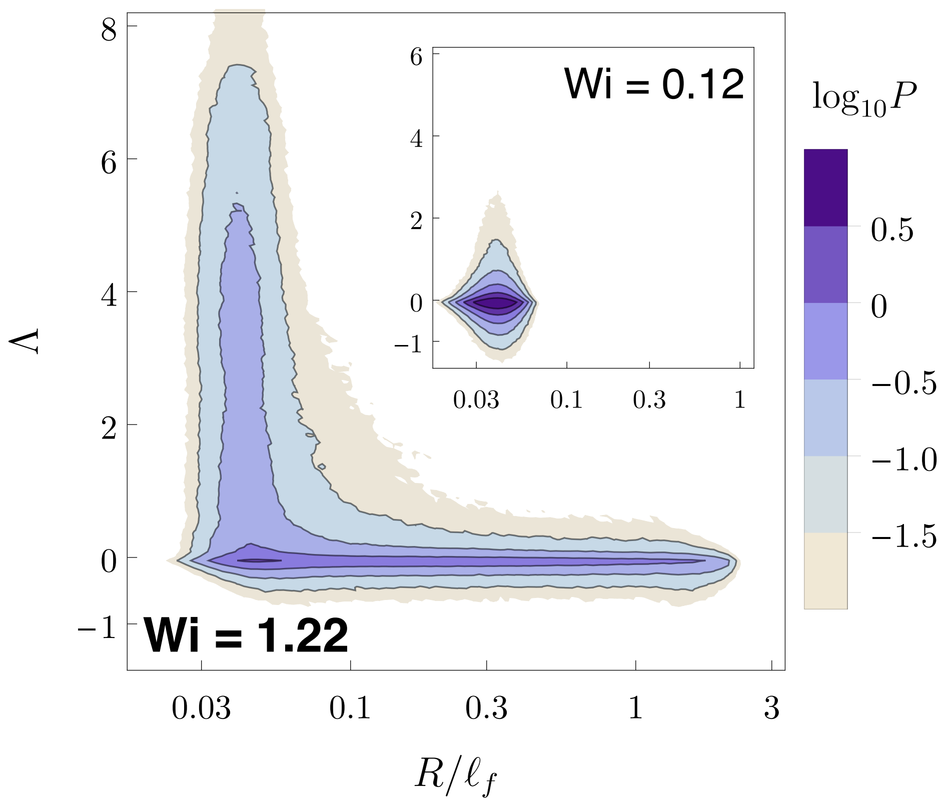
Inextensible filaments



Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)
Singh, Gupta, Picardo, Dv & Ray, *Phys. Rev.* E (2020)
Picardo, Singh, Ray & DV, *Phil. Trans. R. Soc.* A (2020)

Selection of vortical regions

Joint PDF of R and Λ



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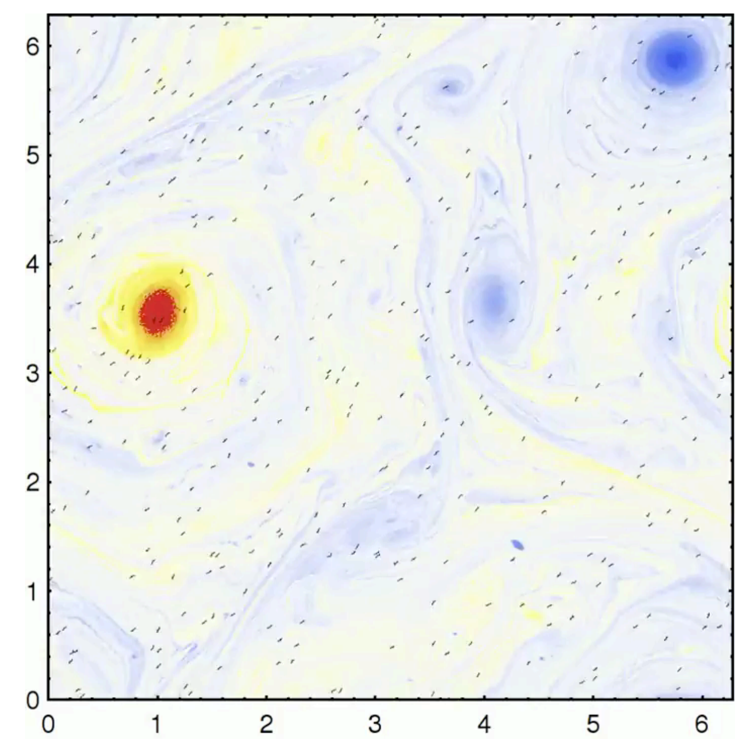
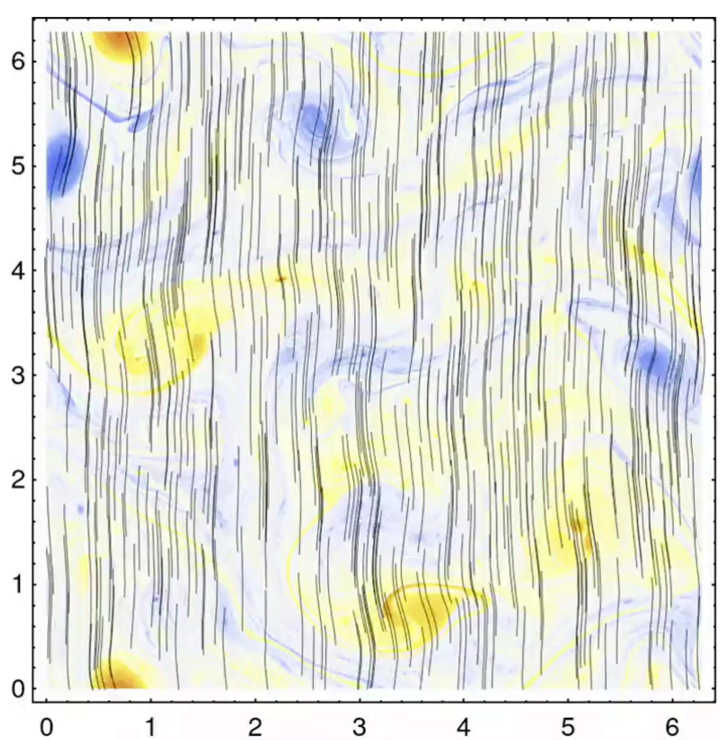


Inextensible filaments



Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)
Singh, Gupta, Picardo, Dv & Ray, *Phys. Rev.* E (2020)
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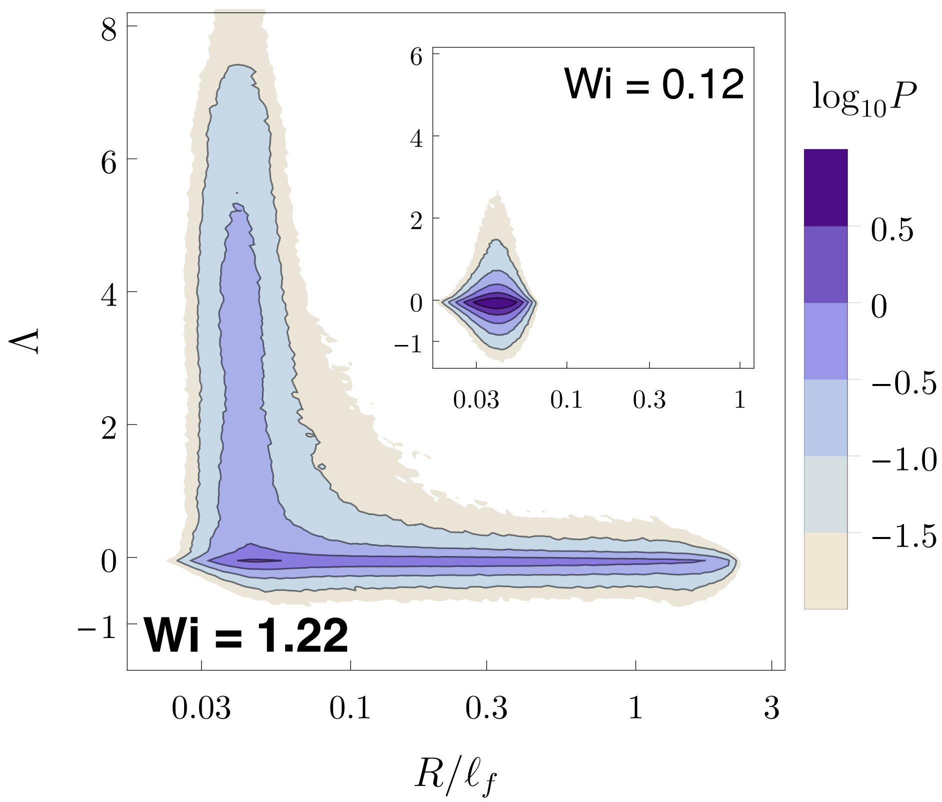

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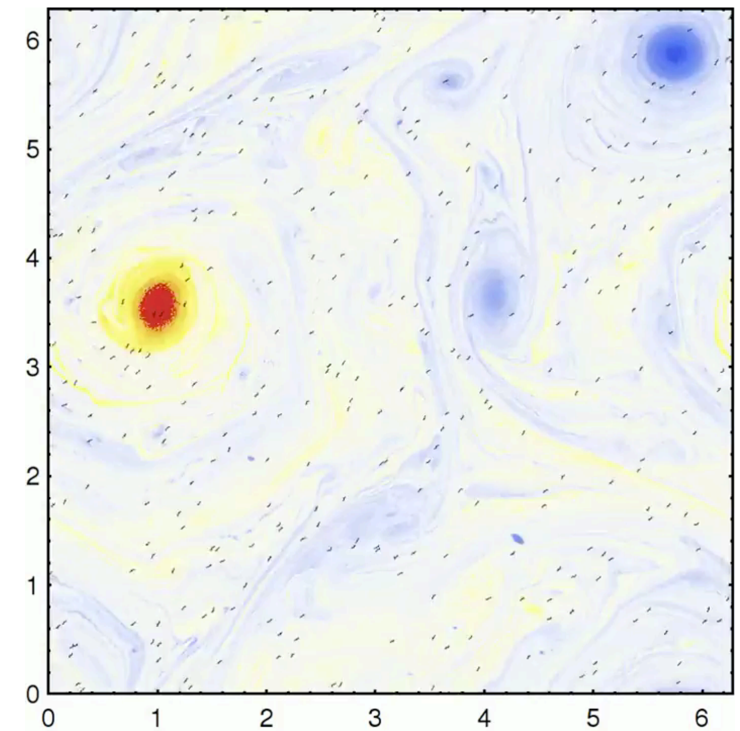
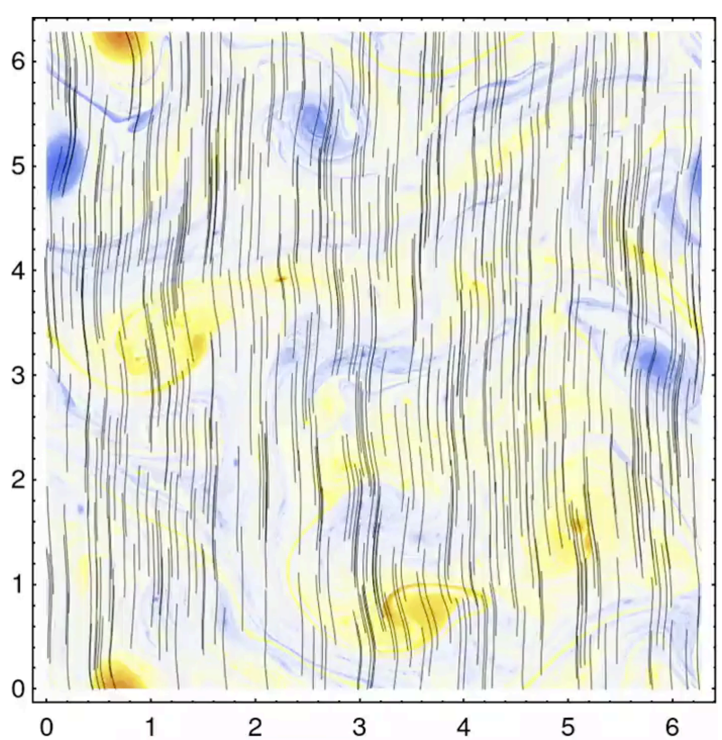


Inextensible filaments



3D turbulence

- The preferential sampling of vortices persists, but is much weaker
- The mechanism is the alignment of the filament the with axis of the vortices

Picardo, DV, Pal & Ray, *Phys. Rev. Lett* (2018)
Singh, Gupta, Picardo, Dv & Ray, *Phys. Rev.* E (2020)
Picardo, Singh, Ray & DV, *Phil. Trans. R. Soc.* A (2020)

Perspectives

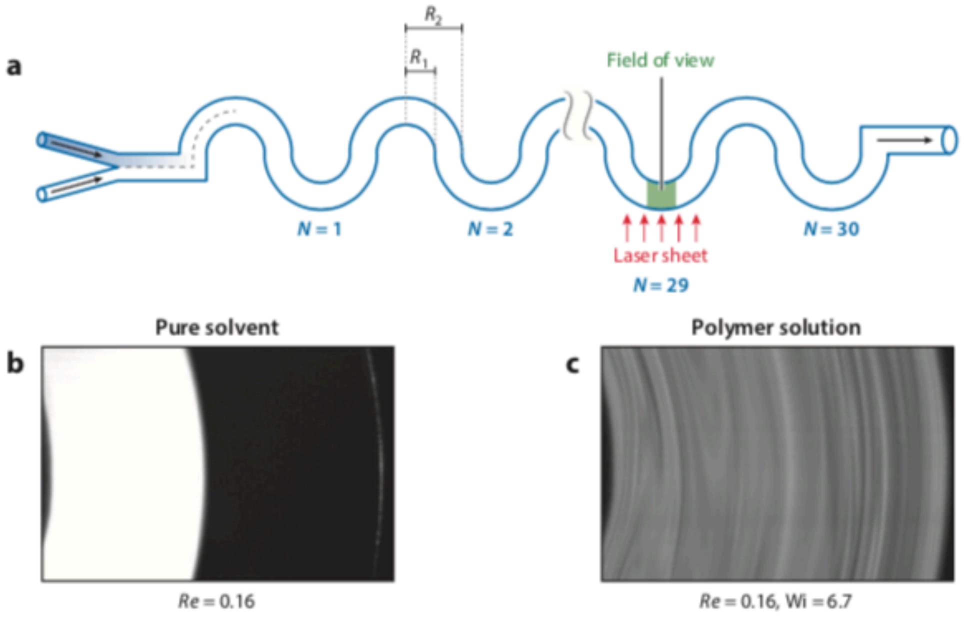
From *mesoscopic* models of polymer molecules
to *macroscopic* models of polymer solutions:

How to include the knowledge on the coil—
stretch transition into continuum models
of polymer solutions?

Turbulent drag reduction (high *Re*)

White & Mungal, *Annu. Rev. Fluid Mech.* (2008)
Graham, *Phys. Fluids* (2014)

Elastic turbulence (small *Re*)



Groisman & Steinberg, *Nature* (2000, 2001)
Steinberg, *Annu. Rev. Fluid Mech.* (2021)