Turbulence in the stable atmospheric boundary layer over alpine terrain

An application to katabatic winds on steep slopes

Christophe Brun

LEGI/MEIGE
UGA Grenoble, France

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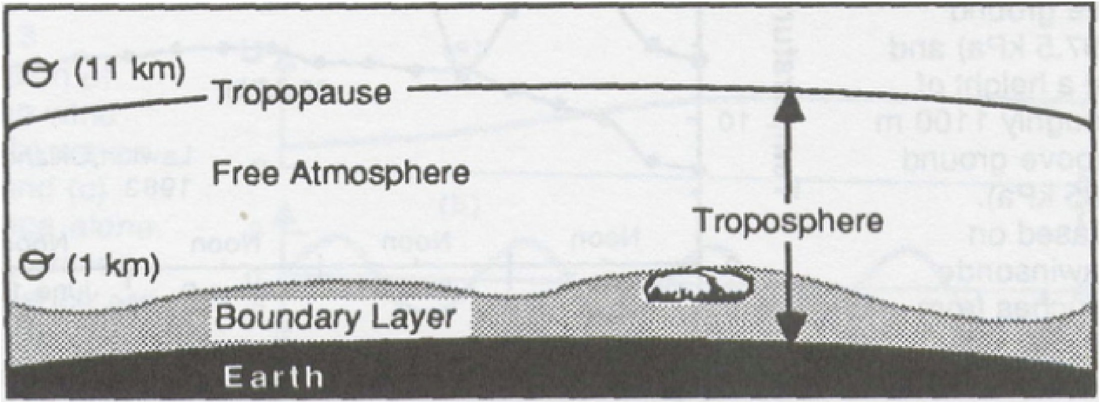
OSUG

Innsbruck valley, February 2024-January 2025



Innsbruck valley, February 2024-January 2025

ABL definition

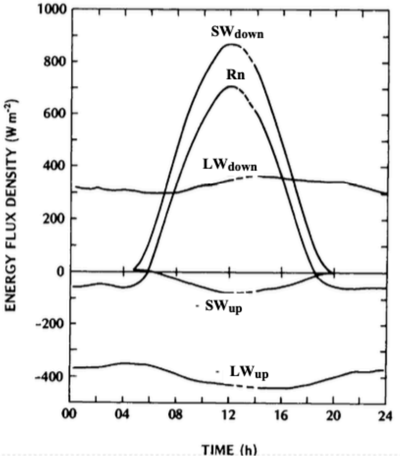


Net radiation budget at the earths surface

Diurnal cycle

$$Rn = SW_{down} - SW_{up} + LW_{down} - LW_{up}$$

- SW Solar radiation
- Earth/Atmosphere radiation $LW = \epsilon \sigma T^4$
- emissivity ϵ
  clear sky: 0.6, clouds : 0.8, snow: 0.99
- Stefan-Boltzmann constant
  $\sigma = 5.67.10^{-8}\ W/m^2/K^4$



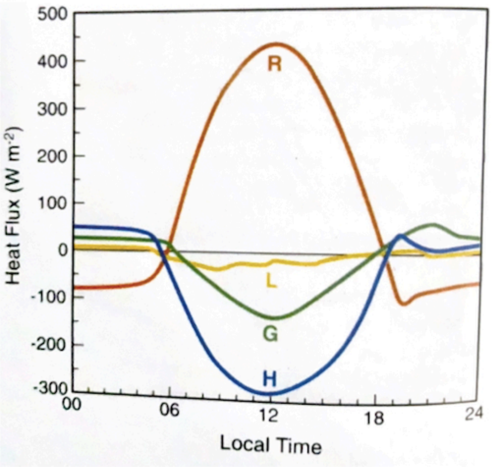
Oke, Boundary layer climate (2002)

ABL energy balance at the ground surface

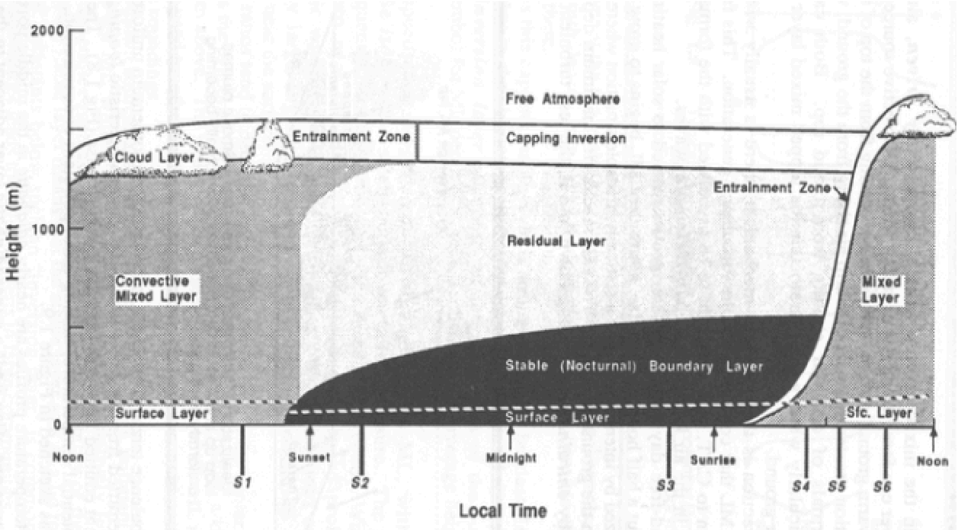
Surface energy redistribution

$$C_s \frac{\partial T_s}{\partial t} = Rn + H_s + LE + G$$

- G Conduction through soil
- LE Turbulent Latent heat flux with moisture
  $LE \approx \rho L_v \overline{w'q'}$
- H_s Turbulent sensible heat flux
  $H_s \approx \rho C_p \overline{w'\theta'}$



ABL energy balance at the ground surface



Thermodynamics

Perfect gas law for dry air

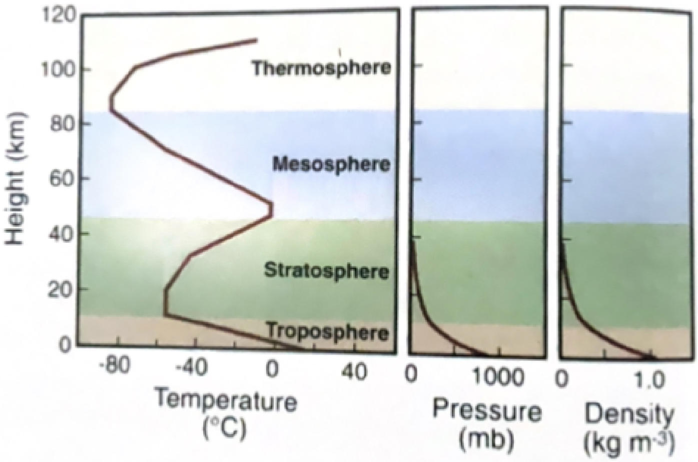
- $P = \rho R_d T$
- with $R_d = C_p - C_v = 287\ J.Kg^{-1}.I$
- and $\gamma = \frac{C_p}{C_v} = 1.4$

Adiabatic conditions

- Entropy $S = \frac{P}{\rho^\gamma} = cst$
- Sonic temperature $C = \sqrt{\gamma R_d T}$

Hydrostatic conditions

- $\frac{\partial P}{\partial z} = -\rho g$



ABL stability

Temperature vertical decay

- $\frac{\partial T}{\partial z} = -\frac{\gamma-1}{\gamma R_d} g \approx -9.8 K.km^{-1}$

Potential temperature

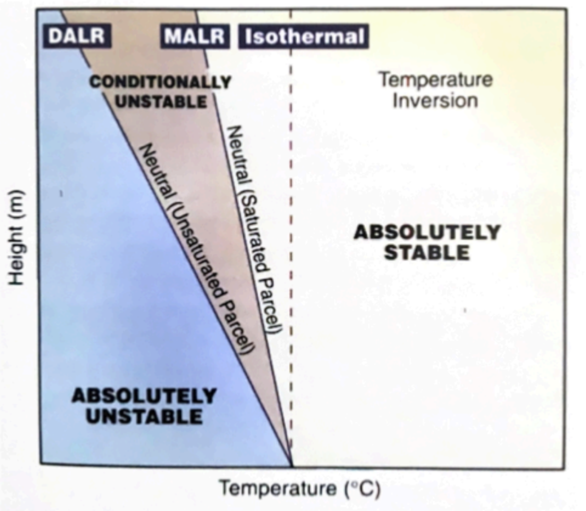
- $\theta(z) = T(z)\left(\frac{P(0)}{P(z)}\right)^{\frac{\gamma-1}{\gamma}}$

Neutral/Unstable/Stable condition

- $\frac{\partial \theta}{\partial z} = 0 \ / \ \frac{\partial \theta}{\partial z} < 0 \ / \ \frac{\partial \theta}{\partial z} > 0$

Background Brunt-Väisälä frequency

- $N = \sqrt{\frac{g}{\theta_s}\frac{\partial \theta}{\partial z}}$



Base state $()_o$

Background isentropic conditions at rest

- $P_o(z) = \rho_o(z) R_d T_o(z)$
- $\frac{\partial P_o(z)}{\partial z} = -\rho_o(z) g$
- $S_o = \frac{P_o(z)}{\rho_o(z)^\gamma} = cst$

Properties

- $\theta_o(z) = Cst$
- $N_o = 0$
- $U_o = 0$

Boussinesq approximation

- $P(\vec{x}) = P_o(z) + \tilde{P}(\vec{x})$
- $\rho(\vec{x}) = \rho_o(z) + \tilde{\rho}(\vec{x})$
- $T(\vec{x}) = T_o(z) + \tilde{T}(\vec{x})$
- $\theta(\vec{x}) = \theta_o + \tilde{\theta}(\vec{x})$

buoyancy terms

- small density and temperature deviations at low Mach number
- $\frac{\tilde{\rho}}{\rho_o(z)} = -\frac{\tilde{T}}{T_o(z)} = -\frac{\tilde{\theta}}{\theta_o}$

Navier-Stokes equations

Mass conservation

- ~~incompressible flow~~
- anelastic approximation

Momentum budget

- no Coriolis effects
- Boussinesq approximation : buoyancy source term

Potential temperature budget

- dry air: no virtual temperature
- non-isentropic nature of the ABL

Navier-Stokes equations

Mass conservation

$$\rho_o \frac{\partial \tilde{u}_i}{\partial x_i} + \tilde{w} \frac{\partial \rho_o}{\partial z} = 0$$

Momentum budget

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{1}{\rho_o} \frac{\partial \tilde{P}}{\partial x_i} - \nu \nabla^2 \tilde{u}_i = g \frac{\tilde{\theta}}{\theta_o} \delta_{i3}$$

Potential temperature budget

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} - \alpha \nabla^2 \tilde{\theta} = 0$$

Navier-Stokes equations

Mass conservation

- incompressible flow
- ~~anelastic approximation~~

Momentum budget

- no Coriolis effects
- Boussinesq approximation : buoyancy source term

Potential temperature budget

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Navier-Stokes equations

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Potential temperature budget

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} - \alpha \nabla^2 \tilde{\theta} = 0$$

Navier-Stokes equations

Reynolds decomposition

- $\tilde{u}_i = \overline{u_i} + u'_i$
- $\tilde{P} = \overline{P} + P'$
- $\tilde{\theta} = \overline{\theta} + \theta'$

Boussinesq Hypothesis (gradient model)

- Reynolds stress tensor $\overline{u'_i u'_j} = -\nu_t \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) + \frac{2}{3} \delta_{ij}\, e$
- Turbulent sensible heat flux $\overline{u'_j \theta'} = -\alpha_t \frac{\partial \overline{\theta}}{\partial x_j}$
- Turbulence Kinetic Energy $e = \frac{1}{2} \overline{u'^2_i}$
- Turbulence Potential Energy $e_p = \frac{1}{2} \frac{g}{\theta_o} \frac{\partial \overline{\theta}}{\partial z}^{-1} \overline{\theta'^2}$

RANS equations for ABL

Mass conservation

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

Momentum budget

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j} + \frac{1}{\rho_o}\frac{\partial \overline{P_t}}{\partial x_i} - \frac{\partial}{\partial x_j}\left((\nu + \nu_t)\frac{\partial \overline{u_i}}{\partial x_j}\right) = g\frac{\overline{\theta}}{\theta_o}\delta_{i3}$$

Potential temperature budget

$$\frac{\partial \overline{\theta}}{\partial t} + \overline{u_j}\frac{\partial \overline{\theta}}{\partial x_j} - \frac{\partial}{\partial x_j}\left((\alpha + \alpha_t)\frac{\partial \overline{\theta}}{\partial x_j}\right) = 0$$

energy budget in the ABL

turbulence kinetic energy budget (TKE)

$$\underbrace{\frac{\partial e}{\partial t}} + \underbrace{\overline{u_j}\frac{\partial e}{\partial x_j}}_{\text{Advection}} = \underbrace{\frac{g}{\theta_o}\overline{w'\theta'}}_{\text{Buoyancy production}} - \underbrace{\overline{u'_i u'_j}\frac{\partial \overline{u_i}}{\partial x_j}}_{\text{Mechanical production}} \underbrace{- \frac{1}{\rho_o}\frac{\partial \overline{p' u'_i}}{\partial x_i} - \frac{1}{2}\frac{\partial \overline{u'^2_i u'_j}}{\partial x_j}}_{\text{Turbulent Transport}} - \underbrace{\epsilon}_{\text{Dissipation}}$$

turbulence potential energy budget (TPE)

$$\underbrace{\frac{\partial e_p}{\partial t}} + \underbrace{\overline{u_j}\frac{\partial e_p}{\partial x_j}}_{\text{Advection}} = \underbrace{-\frac{g}{\theta_o}\overline{w'\theta'}}_{\text{Buoyancy consumption}} \underbrace{- \frac{1}{2}\frac{g}{\theta_o}\frac{\partial \overline{\theta}}{\partial z}^{-1}\frac{\partial \overline{\theta'^2 u'_j}}{\partial x_j}}_{\text{Turbulent Transport}} - \underbrace{\epsilon_p}_{\text{Dissipation}}$$

Turbulent boundary layer on a flat surface (stable conditions)

Turbulent fluxes and turbulent mixing

- $u_*^2 = -\overline{u'w'} > 0$
- $u_*\theta_* = -\overline{w'\theta'} > 0$
- Turbulent flux Richardson number $Ri_f = -\frac{P_B}{P_M} > 0$
- Monin-Obukhov length $L_{MO} = \frac{\theta_o}{g}\frac{\overline{u'w'}^{3/2}}{\overline{w'\theta'}} = \frac{\theta_o}{\kappa g}\frac{{u_*}^2}{\theta_*} = -z\frac{P_M}{P_B} > 0$
- Turbulent mixing $\nu_t = l_m^2\frac{\partial\overline{u}}{\partial z}$
- Prandtl mixing length $l_m = \kappa z\left(1 - \frac{z}{L_{MO}}\right)^{-1}$

Prandtl mixing length model for stable/neutral ABL

$$\underbrace{D_t e}_{\substack{\text{TKE} \\ \text{spatio-temporal} \\ \text{variability}}} = \underbrace{P_M}_{\substack{\text{Mechanical} \\ \text{production}}} + \underbrace{P_B}_{\substack{\text{Buoyancy} \\ \text{production}}} + \underbrace{TT}_{\substack{\text{Turbulent} \\ \text{transport}}} - \underbrace{\epsilon}_{\text{Dissipation}}$$

Prandtl mixing length model for stable/neutral ABL

$$\underbrace{D_t e}_{\text{TKE spatio-temporal variability}} = \underbrace{P_M}_{\text{Mechanical production}} + \underbrace{P_B}_{\text{Buoyancy production}} + \underbrace{TT}_{\text{Turbulent transport}} - \underbrace{\epsilon}_{\text{Dissipation}}$$

Prandtl mixing length model for stable/neutral ABL

$$\underbrace{D_t e}_{\text{TKE spatio-temporal variability}} = \underbrace{\frac{u_*^3}{l_m}}_{\text{Prandtl mixing length}} + \underbrace{\frac{u_*^3}{\kappa L_{MO}}}_{\text{Monin-Oboukhov scale}} + \underbrace{TT}_{\text{Turbulent transport}} - \underbrace{\frac{u_*^3}{\kappa z}}_{\text{Dissipation}}$$

Prandtl mixing length model for stable/neutral ABL

$$\underbrace{D_t e}_{\substack{\text{TKE}\\ \text{spatio-temporal}\\ \text{variability}}} = \underbrace{\frac{u_*^3}{l_m}}_{\substack{\text{Prandtl}\\ \text{mixing length}}} + \underbrace{\frac{u_*^3}{\kappa L_{MO}}}_{\substack{\text{Monin-Oboukhov}\\ \text{scale}}} + \underbrace{TT}_{\substack{\text{Turbulent}\\ \text{transport}}} - \underbrace{\frac{u_*^3}{\kappa z}}_{\text{Dissipation}}$$

Neutral turbulent boundary layer on a flat surface

Monin-Obukhov similarity (stable conditions)

logarithmic law correction

- Momentum ($\alpha_m \approx 5$, z_0 aerodynamic roughness):

$$\overline{u}^+ = \frac{\overline{u}}{u_*} = \frac{1}{\kappa} \ln \frac{z}{z_0} + \frac{\alpha_m}{\kappa} \frac{z}{L_{MO}}$$

- Heat ($\alpha_h \approx 5$, z_h heat roughness):

$$\overline{\theta}^+ = \frac{\overline{\theta}}{\theta_*} = \frac{Pr_t}{\kappa} \ln \frac{z}{z_h} + \frac{\alpha_h}{\kappa} \frac{z}{L_{MO}}$$

- Turbulent Prandtl number $Pr_t = \frac{\nu_t}{\alpha_t} \approx 1$

Zilitinkevich et al. (QJRMS 2000)

Process of katabatic wind formation

Night Anticyclonic conditions

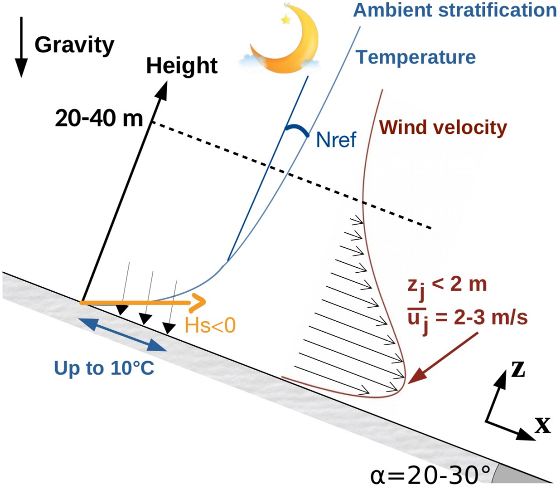
- Negative radiative budget
- $R_n^{night} = LW_{down} - LW_{up} < 0$

Surface temperature cooling $H_s < 0$

- Temperature gradient
- Air cooling / densification

Downslope flow

- Turbulent mixing



Process of katabatic wind formation

Strong radiative cooling

- $H_s \approx \rho C_p \overline{w'\theta'} \in [-50; -100]\ W/m^2$

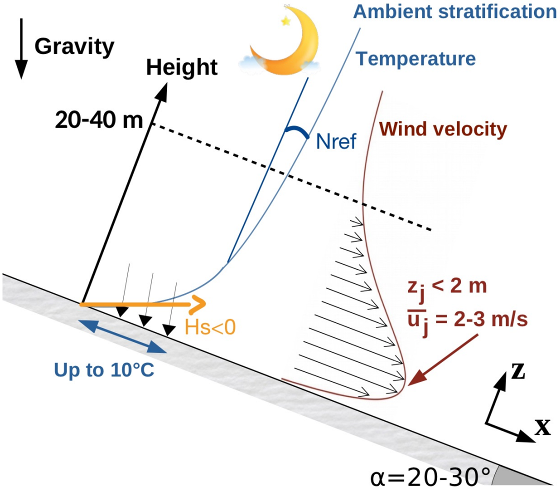
background stratification

- $N = \sqrt{\dfrac{g}{\theta_0}\dfrac{\partial \theta}{\partial z}} \approx 0.01 - 0.02$ Hz

Turbulent flow regime

- $Re = \dfrac{2g}{\nu \theta_s \sin\alpha} \dfrac{H_s}{N^2} \approx 10^5 - 10^6$

Shapiro & Fedorovich (BLM 2014)
Xiao & Senocak (JFM 2019)



RANS equations for katabatic jet

boundary layer on a flat surface

Wyngaard (2010)

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\substack{\text{Turbulent}\\ \text{momentum flux}}} \approx \underbrace{-\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} - \underbrace{g\frac{\overline{\theta}-\theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = 0$$

$$\underbrace{\frac{\partial \tilde{\theta}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{w'\theta'}}{\partial z}}_{\substack{\text{Turbulent sensible}\\ \text{heat flux}}} \approx \underbrace{-\overline{w}\frac{\partial \tilde{\theta}}{\partial z}}_{\text{Advection}} \underbrace{-\overline{u}\frac{\partial \theta_a}{\partial z}\sin\alpha - \overline{w}\frac{\partial \theta_a}{\partial z}\cos\alpha}_{\text{Ambient stratification}} = 0$$

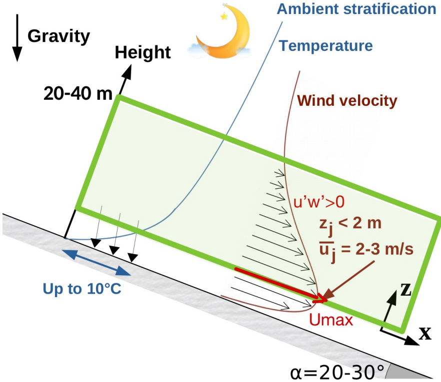
RANS equations for katabatic jet

Prandtl model for Katabatic jet

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Turbulent momentum flux}} \approx \underbrace{-\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} \underbrace{- g\frac{\overline{\theta}-\theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}}$$

$$\underbrace{\frac{\partial \tilde{\theta}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{w'\theta'}}{\partial z}}_{\text{Turbulent sensible heat flux}} \approx \underbrace{-\overline{w}\frac{\partial \tilde{\theta}}{\partial z}}_{\text{Advection}} \underbrace{-\overline{u}\frac{\partial \theta_a}{\partial z}\sin\alpha}_{\text{Ambient stratification}} - \overline{w}\frac{\partial \theta_a}{\partial z}\cos\alpha$$

Prandtl (1942)



RANS equations for katabatic jet

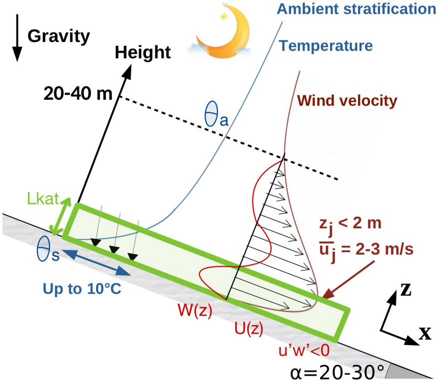
Katabatic boundary layer along a slope

Charrondière et al. (BLM 2022)

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Turbulent momentum flux}} \approx \underbrace{-\bar{w}\frac{\partial \bar{u}}{\partial z}}_{\text{Advection}} - \underbrace{g\frac{\bar{\theta}-\theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}}$$

$$\underbrace{\frac{\partial \tilde{\theta}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{w'\theta'}}{\partial z}}_{\text{Turbulent sensible heat flux}} \approx \underbrace{-\bar{w}\frac{\partial \tilde{\theta}}{\partial z}}_{\text{Advection}} \underbrace{-\bar{u}\frac{\partial \theta_a}{\partial z}\sin\alpha - \bar{w}\frac{\partial \theta_a}{\partial z}\cos\alpha}_{\text{Ambient stratification}}$$

$$\underbrace{\frac{\partial \bar{w}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial \overline{w'^2}}{\partial z}}_{\text{Turbulent velocity variance}} \approx \underbrace{-\bar{w}\frac{\partial \bar{w}}{\partial z}}_{\text{Advection}} + \underbrace{g\frac{\bar{\theta}-\theta_a}{\theta_a}\cos\alpha}_{\text{Katabatic forcing}}$$



1. Atmospheric Boundary Layer (ABL)
2. Turbulent ABL on a flat surface
3. Turbulent ABL on a steep slope
4. *in situ* turbulence measurements

in situ turbulence measurements in the Alps

November 2012

- Blein phD 2016
- Brun et al. JAS 2017
- Charrondière et al. BLM 2020

April 2015

- Unpublished results

February 2019

- Charrondière et al. BLM 2022
- Charrondière et al. JFM 2022
- Charrondière et al. POF 2024

in situ turbulence measurements in the Alps

February 2019

- Zenodo repository 2022
  DOI: 10.5281/zenodo.6546702

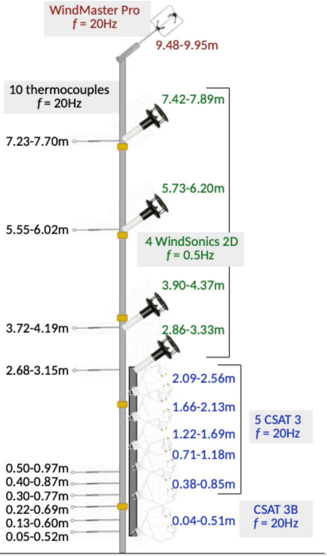
February 2023

- French Alps, Grenoble

January-February 2024-2025

- Austrian Alps, Innsbruck (TEAMx project)

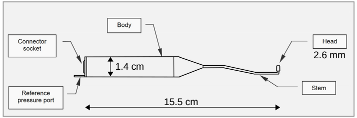
in situ turbulence measurements in the Alps



Winter 2019, Grenoble, 13-28 February

- 4× 2D sonic anemometers
- LW & SW radiation sensor (CNR4)
- 6× 3D sonic anemometers (CSAT/CSAT3B)
- 10 Thermocouples (FW3)
- 3D Pitot sensor (TFI): $z = 2mm - 900mm$
  pressure transducers: $f = 1250$ Hz

in situ turbulence measurements in the Alps

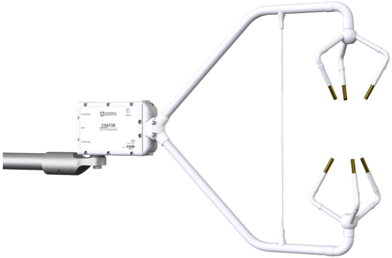


Winter 2023, Grenoble, 3-15 February

- 2D sonic anemometer (Waissala): $z = 3.5m$
- LW radiation sensor (IR120): H_s, T_s
- 3D sonic anemometer (CSAT3B): $z = 1m$
- 4 Thermocouples (FW3):
  $z = 0.7m$, $z = 1.2m$, $z = 2.0m$, $z = 2.9m$
- 3D Pitot sensor (TFI): $z = 2mm - 900mm$
  pressure transducers: f = 1250 Hz
- Micrometric displacement system (Rosier)

in situ turbulence measurements in the Alps

3D Sonic anemometry (acoustic time flight)



$$t_1 = \frac{d}{c + V_d}$$

$$t_2 = \frac{d}{c - V_d}$$

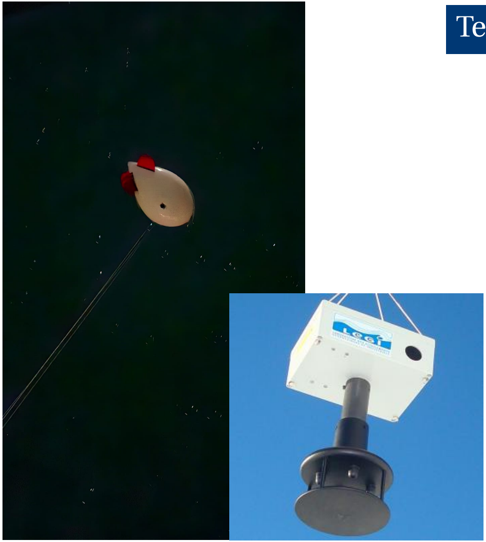
$$V_d = \frac{d}{2}\left(\frac{1}{t1} - \frac{1}{t2}\right)$$

$$c = \sqrt{\gamma R_d T} = \frac{d}{2}\left(\frac{1}{t1} + \frac{1}{t2}\right)$$

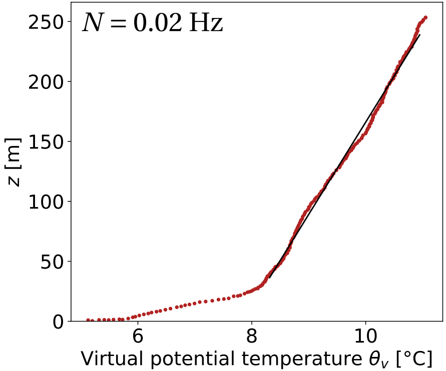
sampling frequency: 20 Hz

measuring volume: $d = 12$ cm

in situ turbulence measurements in the Alps



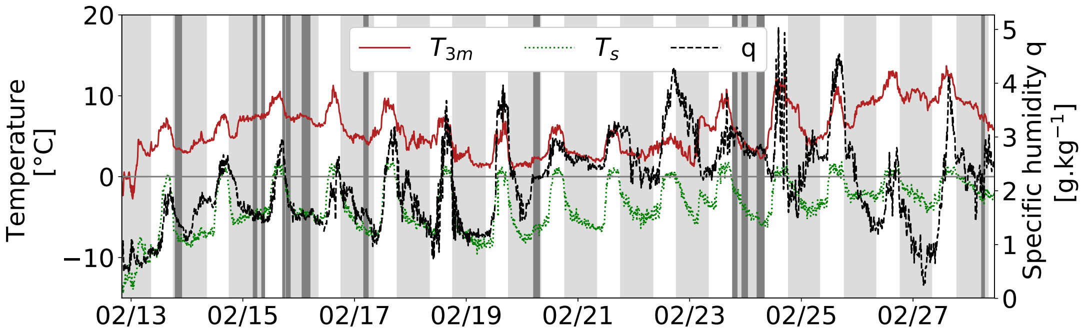
Tethered balloon: $T(z), P(z), q(z)$



15 fev. 2019, 9h00

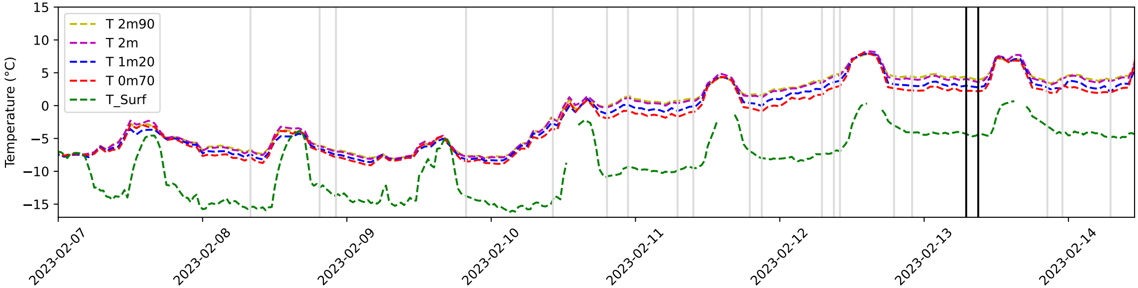
Experimental results

13 katabatic profiles (Grenoble February 2019)



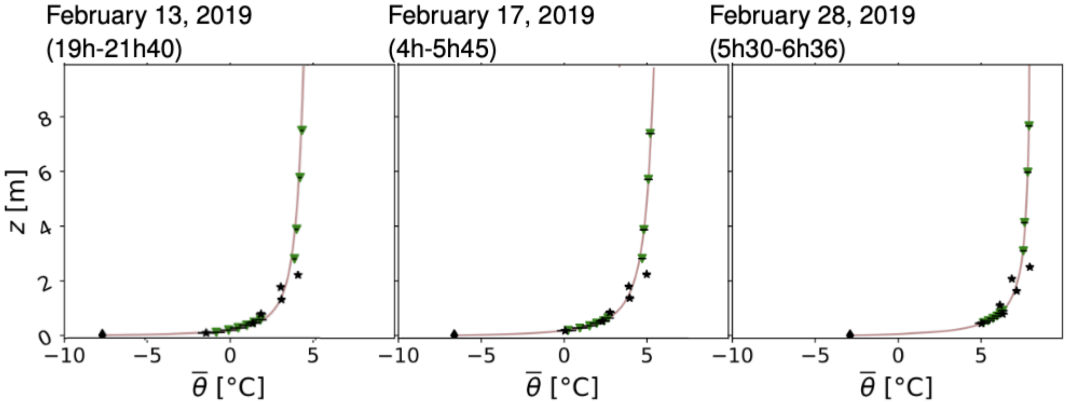
Experimental results

23 katabatic profiles (Grenoble February 2023)



Experimental results

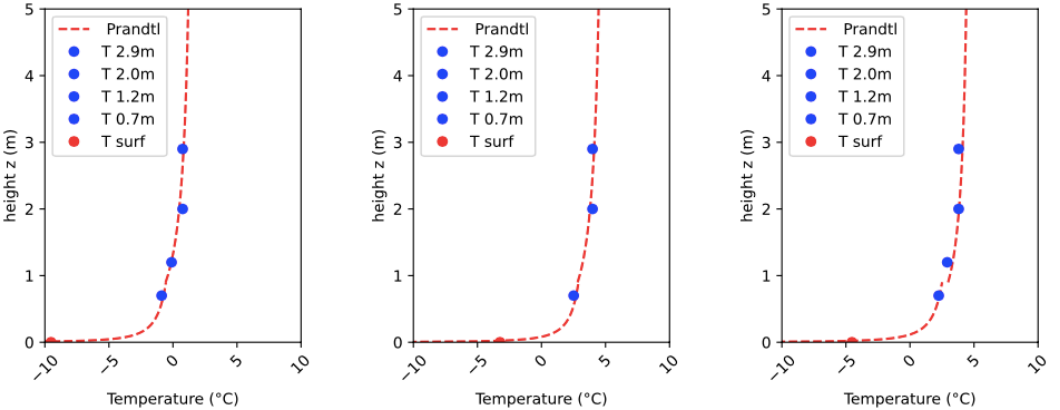
Potential temperature 2019



Experimental results

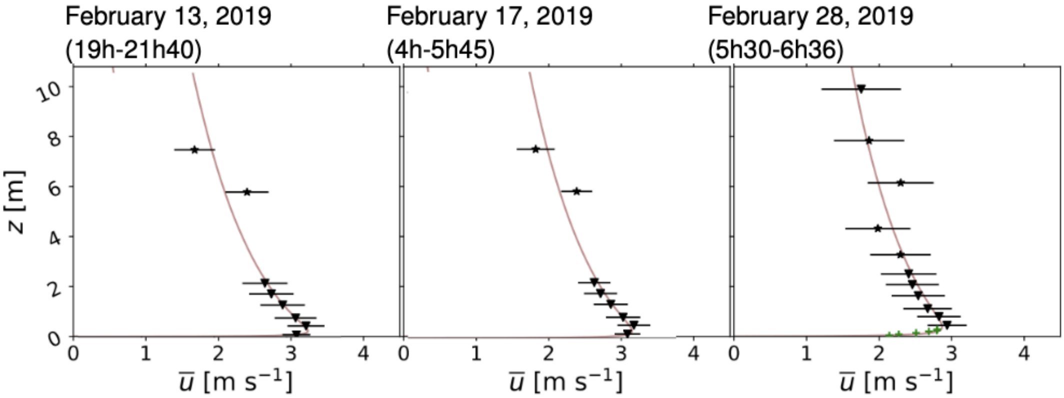
Potential temperature 2023

February 11, 2023 (9h37-10h06) | February 12, 2023 (19h18-19h52) | February 13, 2023 (7h13-7h46)



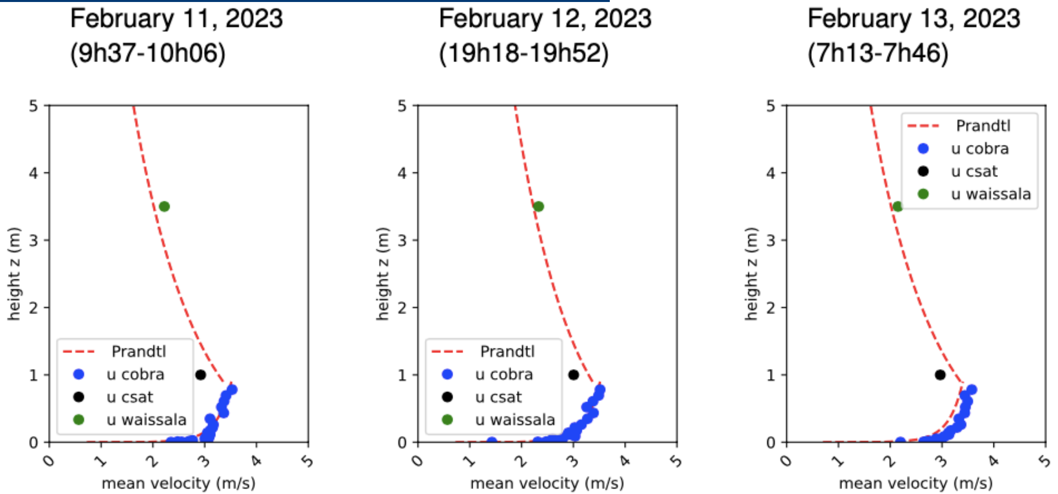
Experimental results

Downslope velocity 2019



Experimental results

Downslope velocity 2023



Experimental results

LES along an ideal curved slope

Brun et al. (JAS 2017)

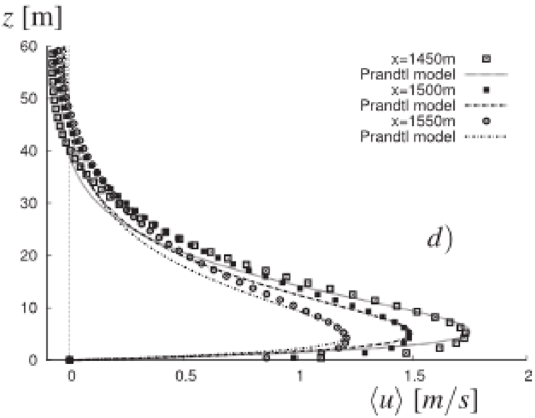
LES Case	$-H_s$ (W m^{-2})	N_{ref} (s^{-1})	α (°)	z_0 (mm)	u_*^{max}(m s^{-1})	θ_*^{max}(°C)
A0 (present study)	10	0.011	13–35.5	35	0.19	0.05
A1 (present study)	30	0.013	13–35.5	35	0.24	0.11
A2 (present study)	10	0.013	13–35.5	35	0.18	0.04
Skyllingstad (2003)	30	0	20	100	—	—
Axelsen and van Dop (2009b)	35–70	0.010–0.014	3–6	200	—	—
Smith and Porté-Agel (2014)	20	0.10	6–18	50	—	—

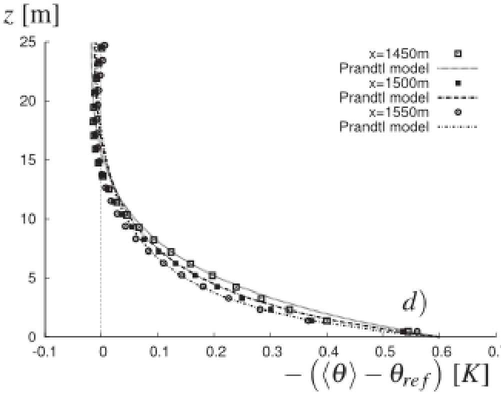
$$u_p(z_n) = V_0 \sin(z_n/L_0) e^{-z_n/L_0}$$

$$\theta_p(z_n) - \theta_{ref}(z_n) = \Theta_0 \cos(z_n/L_0) e^{-z_n/L_0}$$

Experimental results

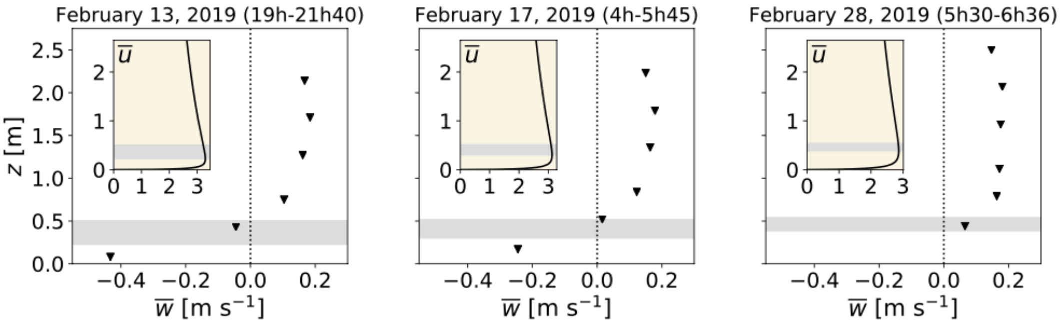
LES vs Prandtl model





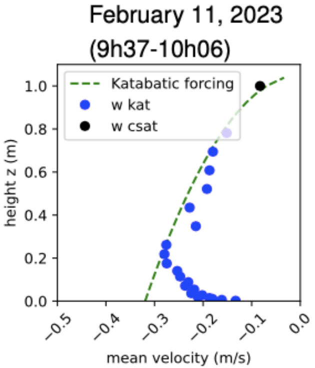
Experimental results

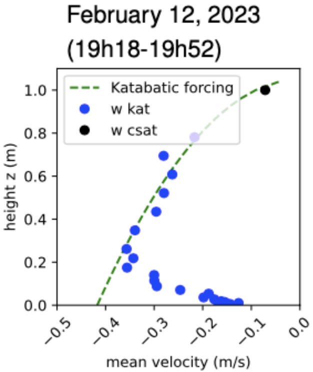
Normal to the slope velocity above jet max

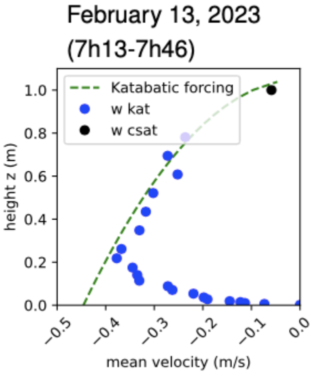


Experimental results

Normal to the slope velocity below jet max







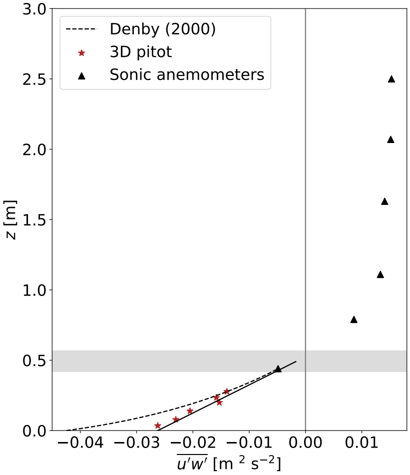
Momentum & heat budget

Non constant flux layer

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Divergence of the turbulent momentum flux}} \approx \underbrace{-g\frac{\overline{\theta_S} - \theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}$$

$$-\overline{u'w'} = u_*^2\left(1 - \frac{z}{L_{Kat}}\right) = \left(\kappa z \frac{\partial \overline{u}}{\partial z}\right)^2$$

February 28, 2019 (5h00-6h36)



Momentum & heat budget

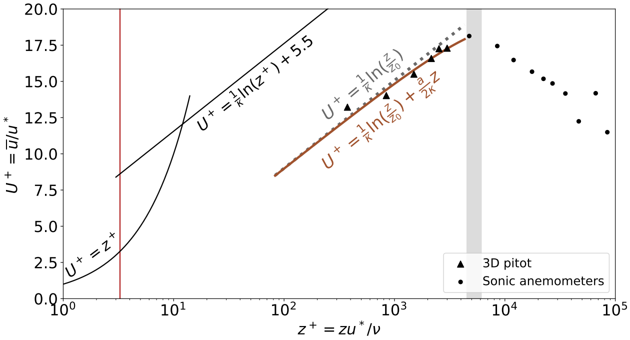
Turbulent velocity profile

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Divergence of the turbulent momentum flux}} \approx \underbrace{-g\frac{\overline{\theta_S} - \theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}$$

$$\overline{u}^+ = \frac{1}{\kappa}\ln\frac{z}{z_0} - \frac{1}{2\kappa}\frac{z}{L_{Kat}} + \beta\frac{z}{L_{MO}}$$

- $L_{Kat} \approx 0.5m$
- $L_{MO} \approx 200m$

February 28, 2019 (5h00-6h36)



Momentum & heat budget

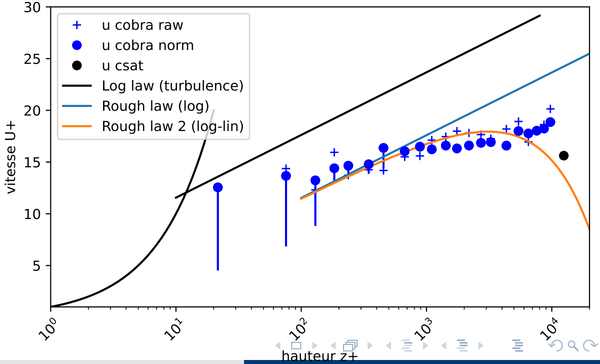
Turbulent velocity profile

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Divergence of the turbulent momentum flux}} \approx \underbrace{-g\frac{\overline{\theta_S} - \theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}$$

$$\overline{u}^+ = \frac{1}{\kappa}\ln\frac{z}{z_0} - \frac{1}{2\kappa}\frac{z}{L_{Kat}} + \beta\frac{z}{L_{MO}}$$

- $L_{Kat} \approx 0.5m$
- $L_{MO} \approx 200m$

February 11, 2023 (9h37-9h56)



Momentum & heat budget

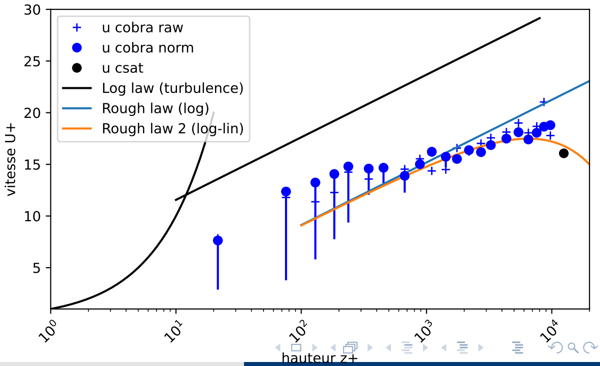
Turbulent velocity profile

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Divergence of the turbulent momentum flux}} \approx \underbrace{-g\frac{\overline{\theta_S}-\theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}$$

$$\overline{u}^+ = \frac{1}{\kappa}\ln\frac{z}{z_0} - \frac{1}{2\kappa}\frac{z}{L_{Kat}} + \beta\frac{z}{L_{MO}}$$

- $L_{Kat} \approx 0.5m$
- $L_{MO} \approx 200m$

February 12, 2023 (19h18-19h52)



Momentum & heat budget

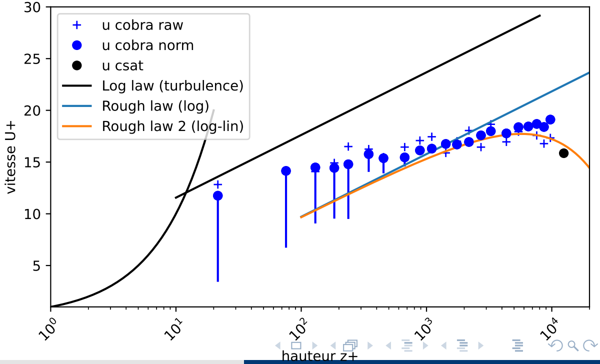
Turbulent velocity profile

$$\underbrace{\frac{\partial \overline{u}}{\partial t}}_{\text{Inertia}} + \underbrace{\overline{w}\frac{\partial \overline{u}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{\partial \overline{u'w'}}{\partial z}}_{\text{Divergence of the turbulent momentum flux}} \approx \underbrace{-g\frac{\overline{\theta_S}-\theta_a}{\theta_a}\sin\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}$$

$$\overline{u}^+ = \frac{1}{\kappa}\ln\frac{z}{z_0} - \frac{1}{2\kappa}\frac{z}{L_{Kat}} + \beta\frac{z}{L_{MO}}$$

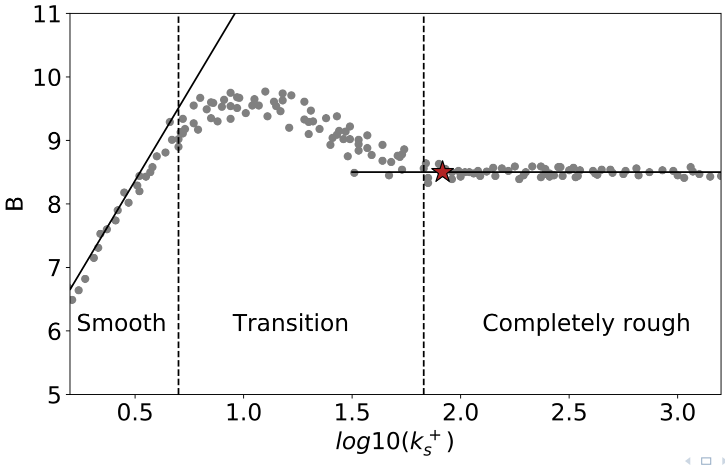
- $L_{Kat} \approx 0.5m$
- $L_{MO} \approx 200m$

February 13, 2023 (07h13-07h47)



Momentum & heat budget

Surface Roughness (Schlichting)



Momentum & heat budget

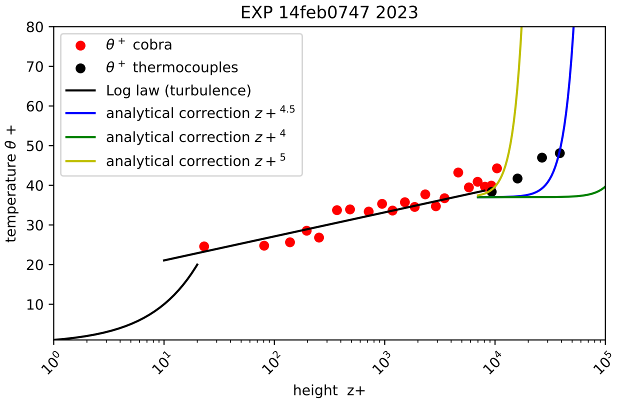
Non constant flux layer

$$\underbrace{\frac{\partial \overline{w'\theta'}}{\partial z}}_{\text{Turbulent heat flux}} \approx -\underbrace{\overline{w}\frac{\partial \overline{\theta}}{\partial z}}_{\text{Advection}}$$

Turbulent temperature profile

$$\overline{\theta}^{+} - \theta_s^{+} = \frac{Pr_t}{\kappa} \ln \frac{z}{z_T}$$

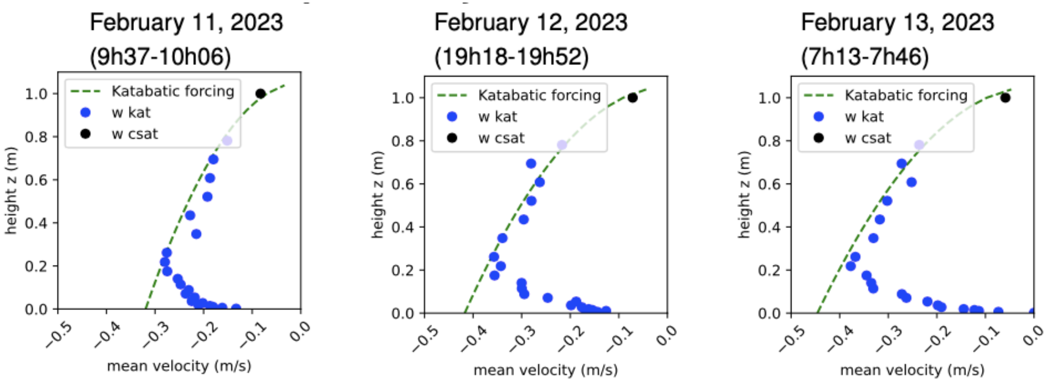
February 14, 2023 (7h47-8h15)



Momentum & heat budget

Normal to the slope velocity $\overline{w}$

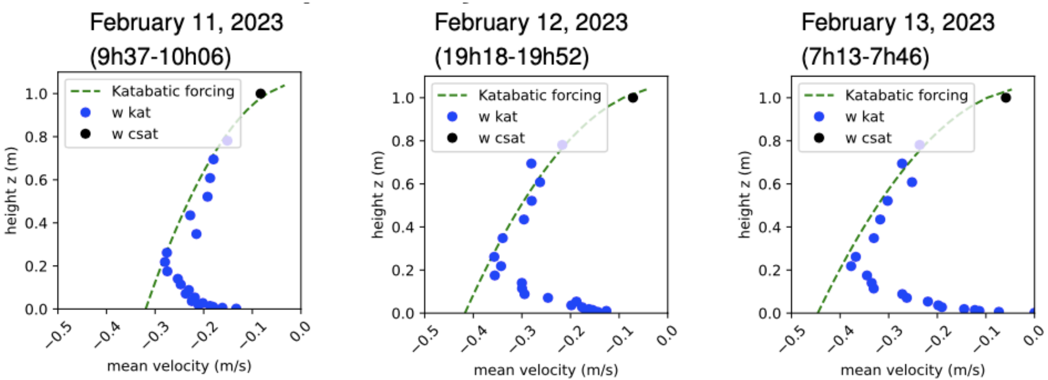
$$\underbrace{\frac{\partial \overline{w}}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{1}{2}\frac{\partial \overline{w'^2}}{\partial z}}_{\text{Advection}} + \underbrace{\frac{1}{2}\frac{\partial \overline{w}^2}{\partial z}}_{} \approx \underbrace{g\frac{\overline{\theta_S} - \theta_a}{\theta_a}\cos\alpha}_{\text{Katabatic forcing}} = \frac{u_*^2}{L_{Kat}}\frac{1}{\tan\alpha}$$



Momentum & heat budget

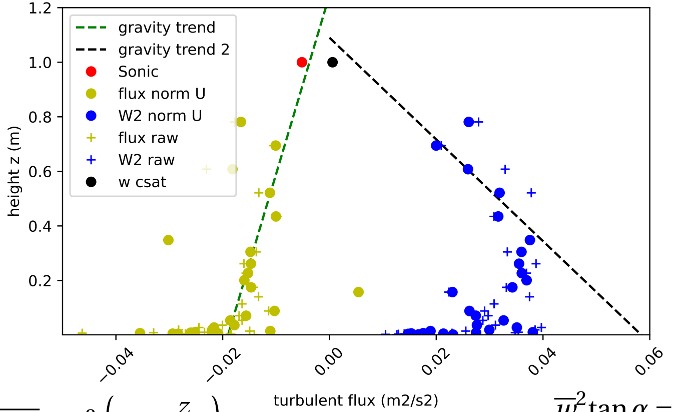
Normal to the slope velocity $\overline{w}$

$$\overline{w}^2 = w_o^2\left(1 - \frac{z}{L_{Kat}}\right)\frac{2}{\tan\alpha} = u_*^2\left(1 - \frac{z}{L_{Kat}}\right)$$



Momentum & heat budget

Momentum flux $\overline{u'w'}$

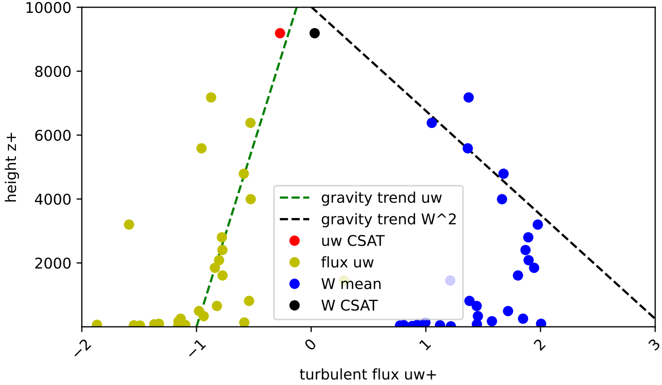


$$-\overline{u'w'} = u_*^2\left(1 - \frac{z}{L_{Kat}}\right)$$

$$\overline{w}^2 \tan\alpha = 2u_*^2\left(1 - \frac{z}{L_{Kat}}\right)$$

Momentum & heat budget

Momentum flux $\overline{u'w'}$ vs $\overline{w}^2$



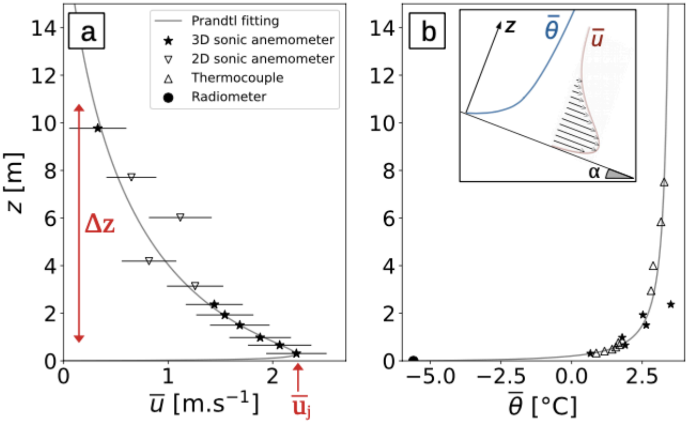
$$-\overline{u'w'}^{+} = \left(1 - \frac{z}{L_{Kat}}\right)$$

$$\overline{w}^{+2} \tan\alpha = 2\left(1 - \frac{z}{L_{Kat}}\right)$$

Energy spectra

outer layer of the katabatic jet

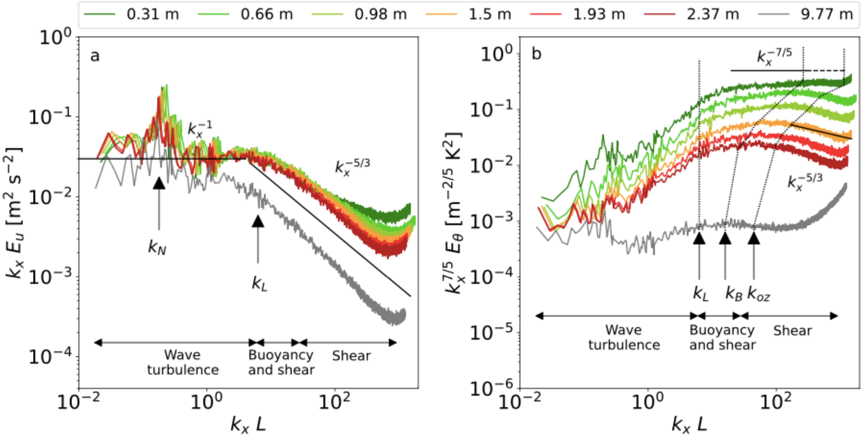
Charrondière et al (POF 2024)



Energy spectra

Strong wave turbulence and Bolgiano spectra

Charrondière et al (POF 2024)



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