New Challenges in Turbulence Research VII, February 14th, 2025

Measuring turbulence with particle imaging: from common practical use to advanced methods

Benjamin Leclaire, ONERA/DAAA

Metrology, Data assimilation and Flow Physics unit

+

F. Champagnat, A. Cheminet, P. Cornic, S. Davoust, M. Hebey, C. Illoul, L. Jacquin, J. Kantharaju, G. Le Besnerais, J. Le Bris, Y. Le Sant, G. Losfeld, O. Marquet, V. Mons, A. Plyer, R. Yegavian, M. Zauner…

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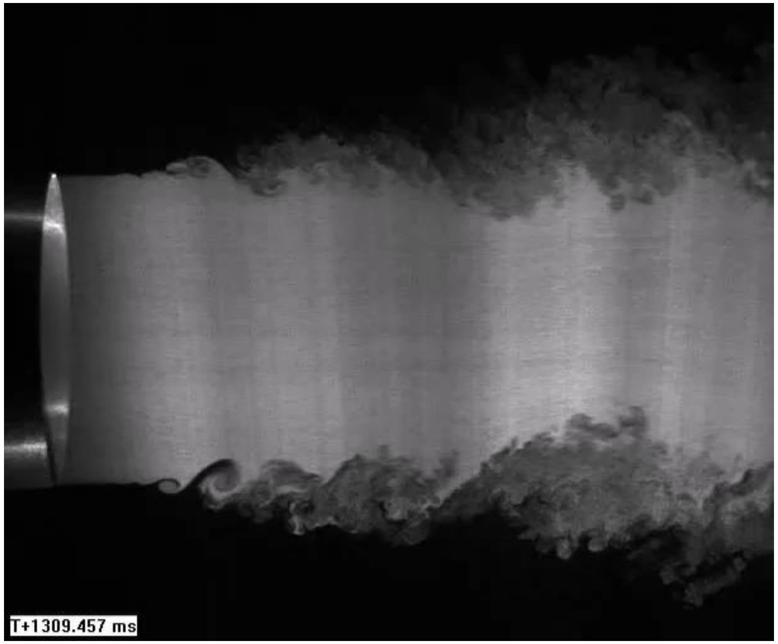
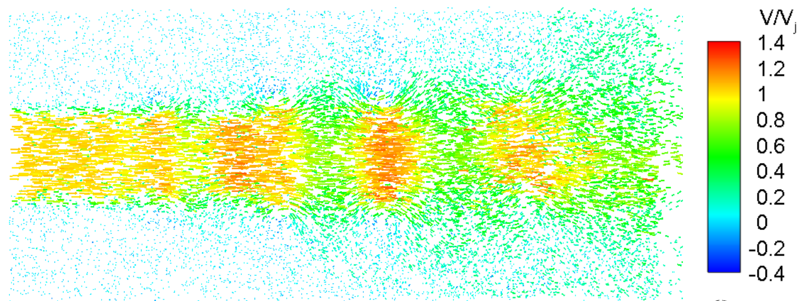
Introduction

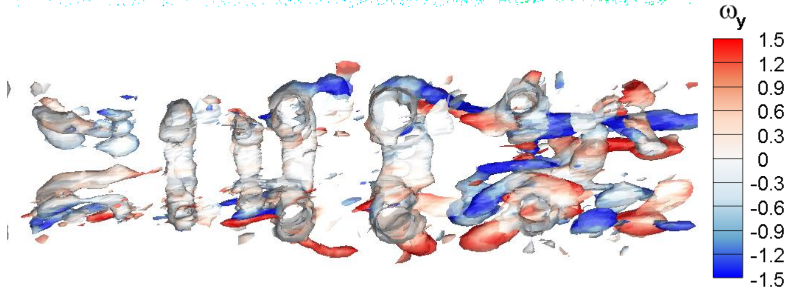


From smoke visualization…

… to volumetric vector fields

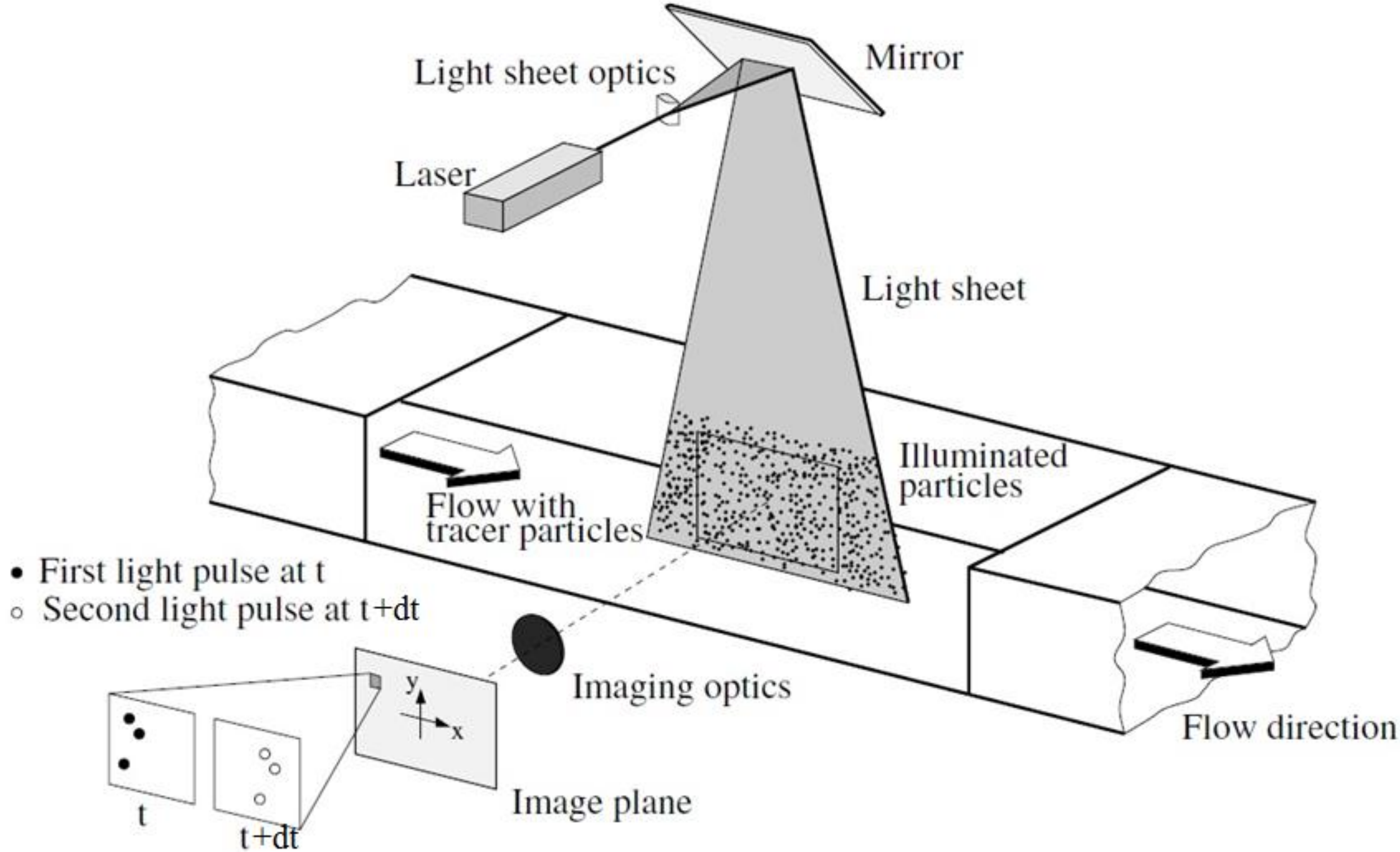


… then full flow fields!



Introduction

General principle

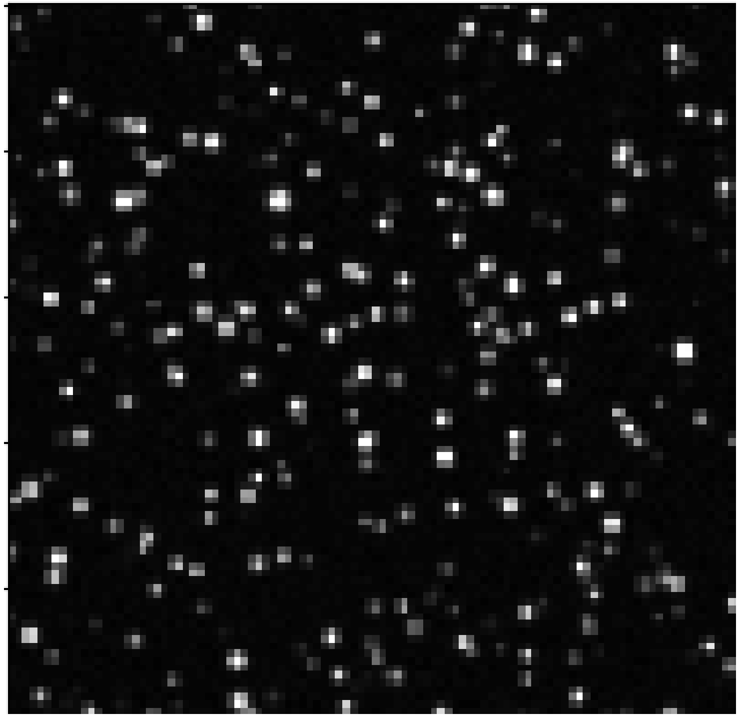


Raffel et al., PIV: a practical guide, 2018

Introduction

Whatever the variant (2D, 3D, PIV/PTV, etc…), data should look like this:

t

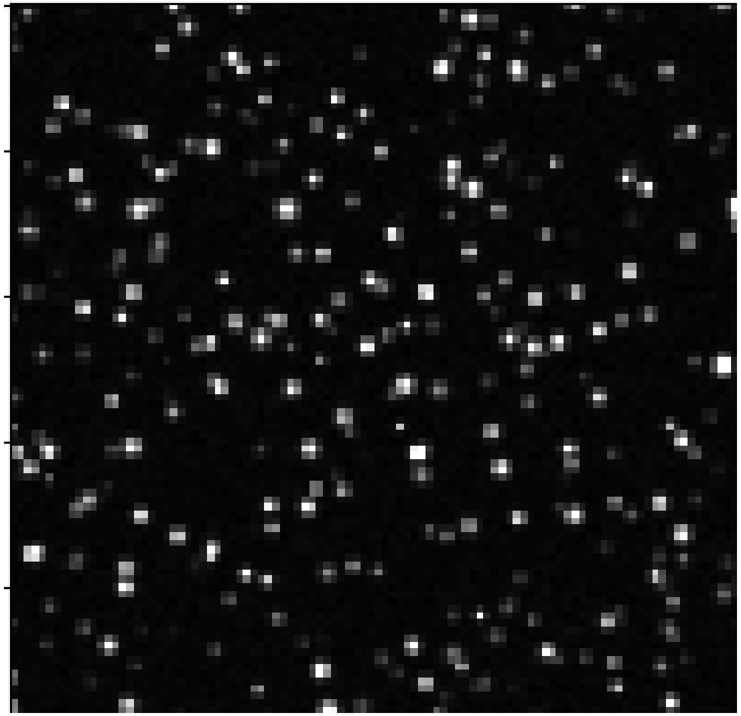


3rd PIV Challenge (2005), case B
Stanislas et al., Exp. Fluids 2008

Introduction

Whatever the variant (2D, 3D, PIV/PTV, etc…), data should look like this:

$t + dt$



3rd PIV Challenge (2005), case B
Stanislas et al., Exp. Fluids 2008

Synthetic images: mimic experimental conditions, with known particle intensities, positions, displacement…

Introduction

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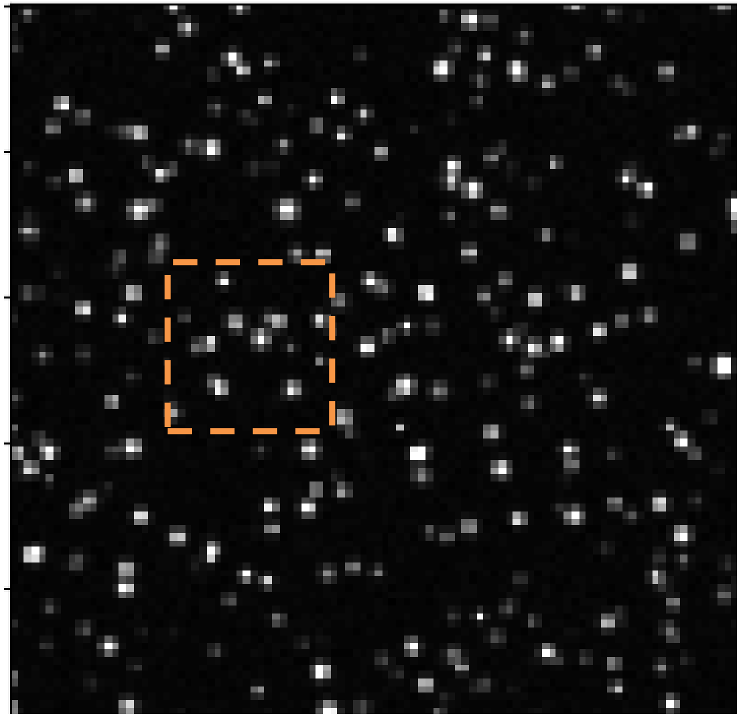
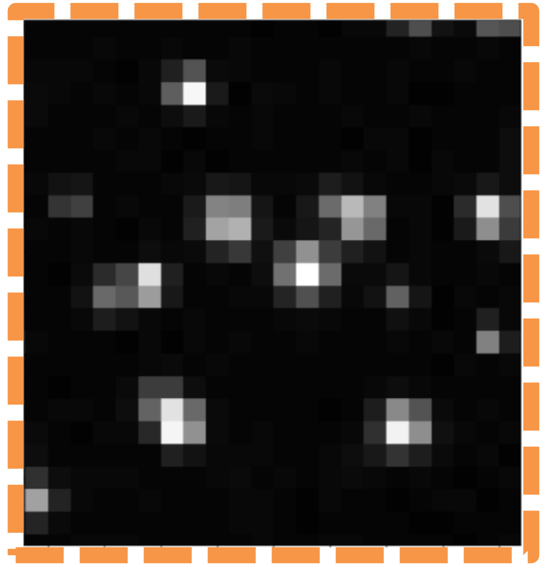




3rd PIV Challenge (2005), case B
Stanislas et al., Exp. Fluids 2008

Bright particle images, each of size $\approx 2 - 3$ pixels (we'll see why)

Outline

I. Seeding and image formation

II. Basics: 2D, two-component PIV

III. Towards more complexity: Stereo PIV, Time-Resolved PIV

IV. Volumetric and Tracking approaches, and beyond

- A subjective selection:
  - Data processing > hardware
  - 2D PIV: quick account on basics, and then:
    - examples of use for turbulent flow analysis
    - … and of precautions that should be taken

    from the speaker's experience!
  - More emphasis on 3D methods and related (especially with two-pulse acquisition: more versatile, more of interest for ONERA research!)

I. **Seeding and image formation**

II. Basics: 2D, two-component PIV

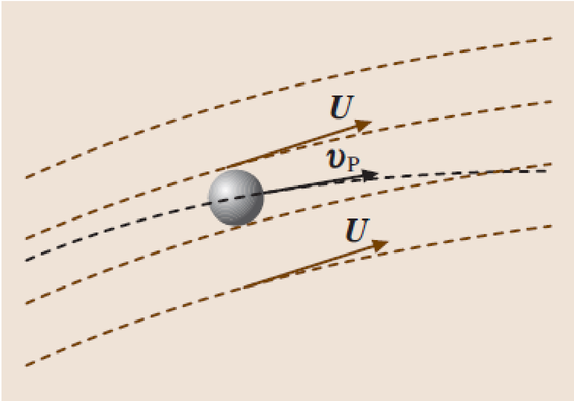
III. Towards more complexity: Stereo PIV, Time-Resolved PIV

IV. Volumetric and Tracking approaches, and beyond

Seeding and image formation

Passively entrained tracers?...

Tropea et al., Springer handbook of experimental fluid mechanics, 2007



Stokes regime: Response time for a particle (d_p, ρ_p) to a change in flow (ρ_f, μ_f) velocity (estimate based on settling velocity):

$$\tau_s = \frac{(\rho_p - \rho_f) d_p^2}{18 \mu_f}$$

τ_η being the smallest flow time scale, the **Stokes number**

$$\boxed{St = \frac{\tau_s}{\tau_\eta}}$$

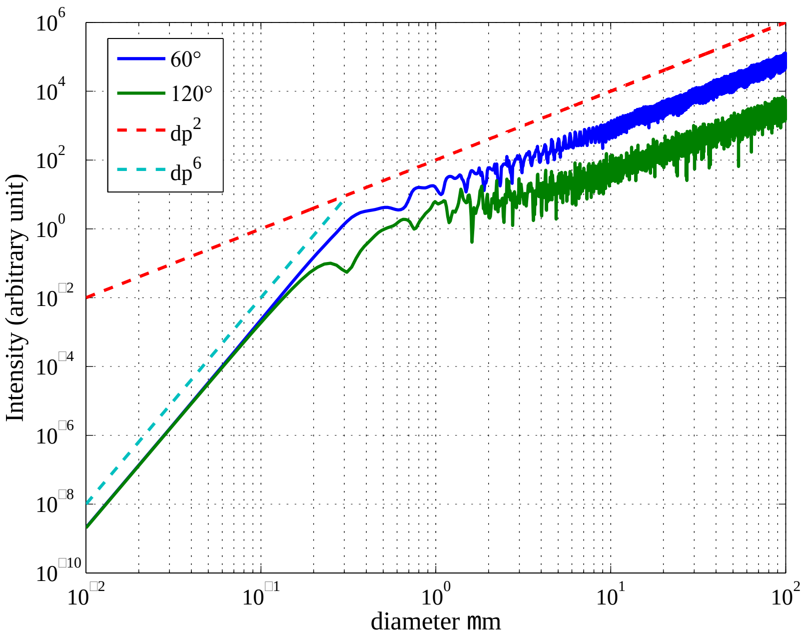
must be minimal

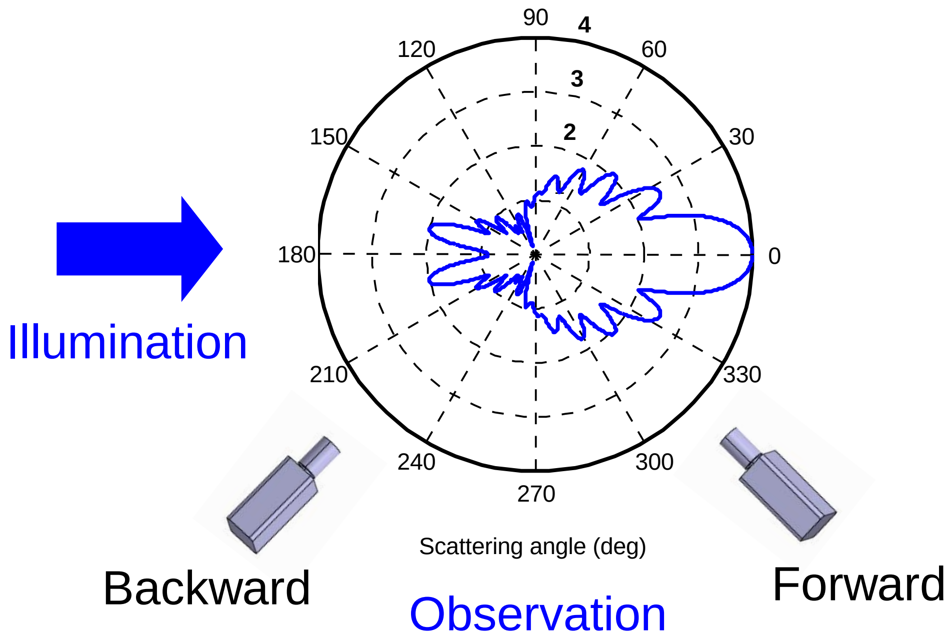
$\Rightarrow$ target **either $\rho_p \sim \rho_f$, and/or minimal d_p**!

Seeding and image formation

$d_p \sim \lambda$ or $d_p > \lambda$: Mie scattering

… But emitted intensity roughly evolves as d_p^2!… **Trade-off good tracer / brightness**





Cheminet, PhD Univ. Paris-Scalay, 2016

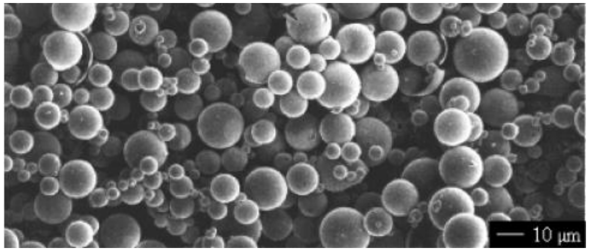
(… and is very irregular depending on viewing angle
- to keep in mind for Stereo and 3D experiments!)

Seeding and image formation

Liquid droplets or solid particles

Raffel et al., 2018

Silver coated hollow spheres



Al_2O_3 (reactive flows)

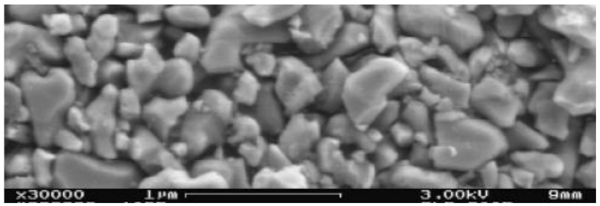


Table 2.1. Seeding materials for liquid flows.

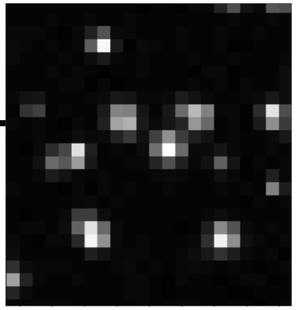
Type	Material	Mean diameter in μm
Solid	Polystyrene	10 – 100
	Aluminum flakes	2 – 7
	Hollow glass spheres	10 – 100
	Granules for synthetic coatings	10 – 500
Liquid	Different oils	50 – 500
Gaseous	Oxygen bubbles	50 – 1000

Table 2.2. Seeding materials for gas flows.

Type	Material	Mean diameter in μm
Solid	Polystyrene	0.5 – 10
	Alumina Al_2O_3	0.2 – 5
	Titania TiO_2	0.1 – 5
	Glass micro-spheres	0.2 – 3
	Glass micro-balloons	30 – 100
	Granules for synthetic coatings	10 – 50
	Dioctylphathalate	1 – 10
	Smoke	< 1
Liquid	Different oils	0.5 – 10
	Di-ethyl-hexyl-sebacate (DEHS)	0.5 – 1.5
	Helium-filled soap bubbles	1000 – 3000

Large volumes (and low speeds): Helium-Filled Soap Bubbles ($\sim$300 μm): see later!

Seeding and image formation



Tropea et al., Springer handbook of experimental fluid mechanics, 2007

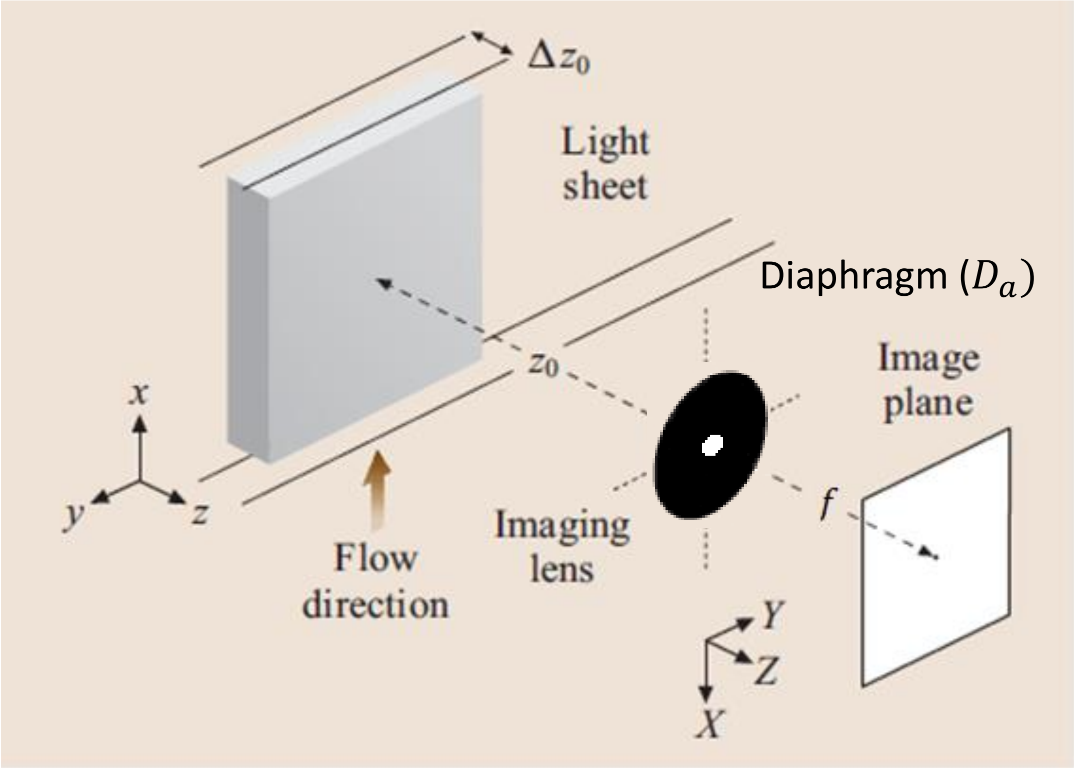


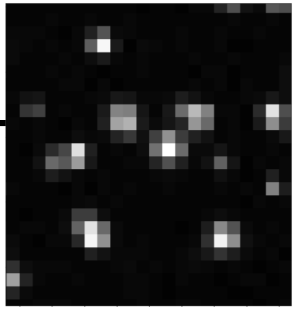
Image size on the sensor d_τ of a particle of diameter d_p:

$$d_\tau = \sqrt{\left(M d_p\right)^2 + d_{diff}^2} \approx d_{diff} \text{ (air)}$$

$$d_{diff} = 2.44 \frac{f}{D_a}(M+1)\lambda$$

f focal length, D_a diaphragm aperture
$M = z_0/f$ Magnification
λ light wavelength

Seeding and image formation



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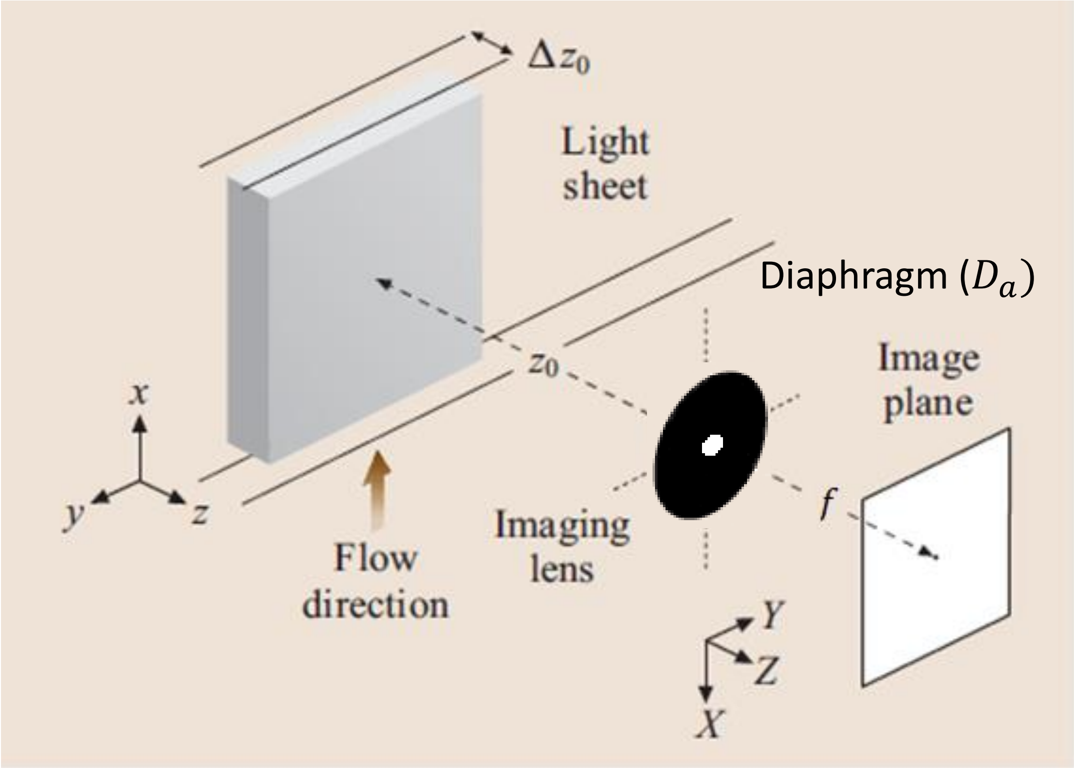

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$$d_{diff} = 2.44\frac{f}{D_a}(M+1)\lambda$$

Small aperture favorable for **subpixel information**, but detrimental to **SNR**
⇒ **trade-off!**
→ *Slight defocus can come to the rescue*

I. Seeding and image formation

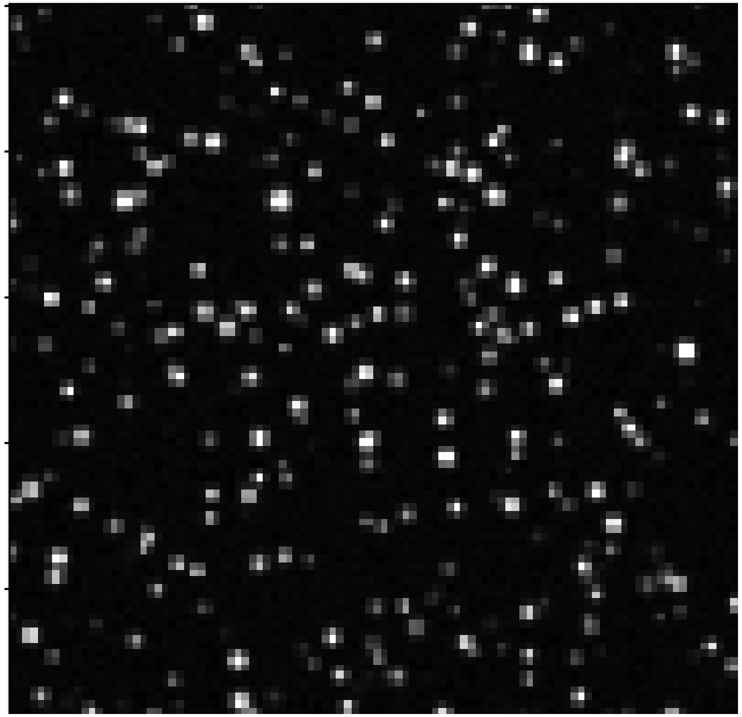
II. Basics: 2D, two-component (2D2C) PIV

III. Towards more complexity: Stereo PIV, Time-Resolved PIV

IV. Volumetric and Tracking approaches, and beyond

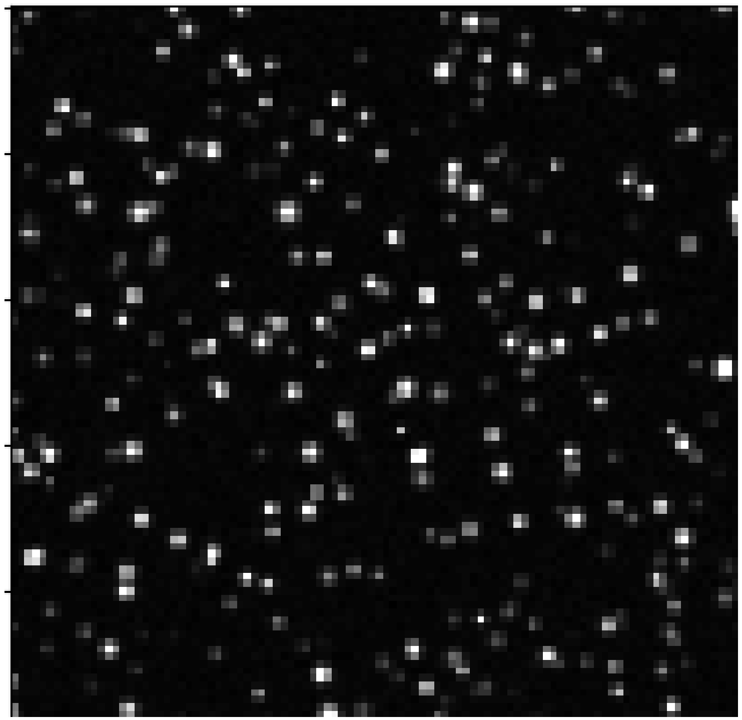
2D2C PIV: displacement estimation

t



2D2C PIV: displacement estimation

$t + dt$



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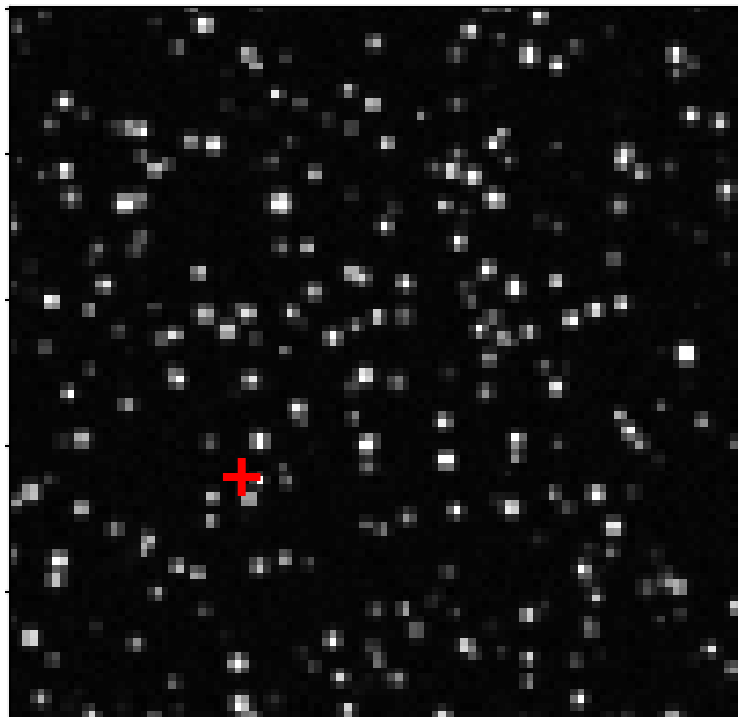
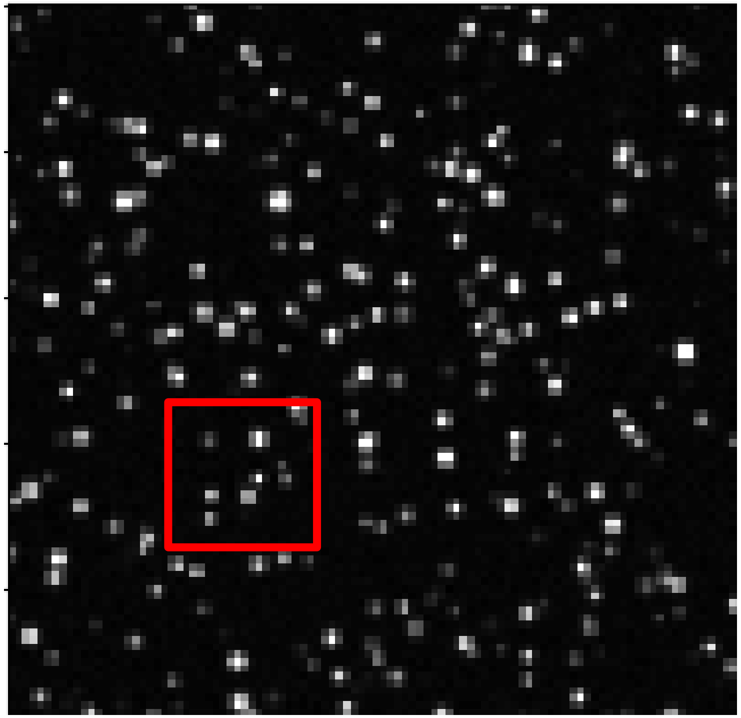


Objective: find displacement at pixel k

2D2C PIV: displacement estimation

t

Interrogation window at pixel k: $W(k)$



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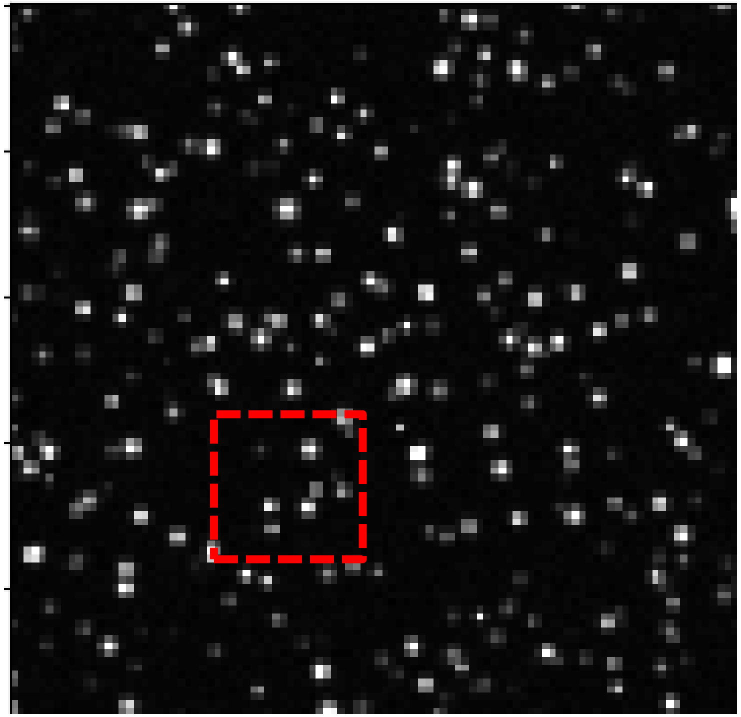

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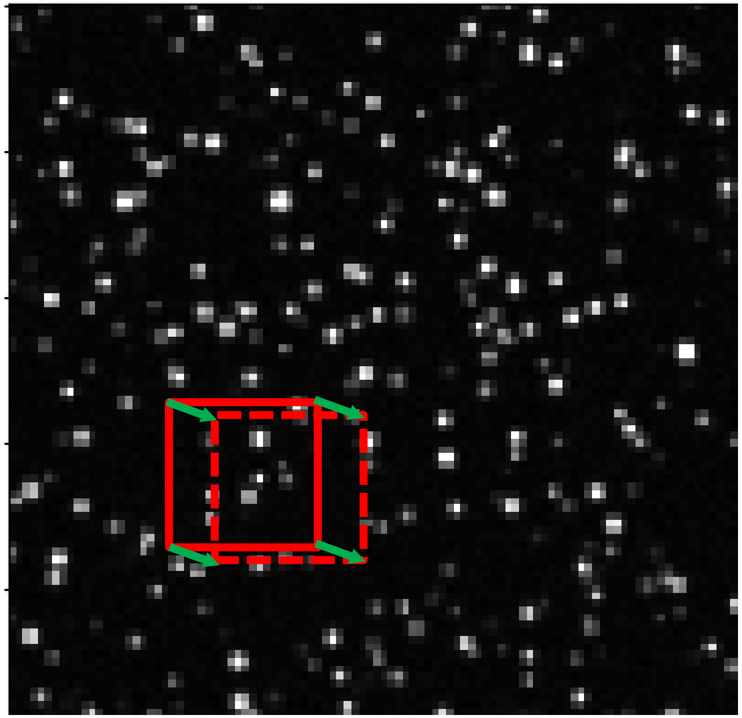

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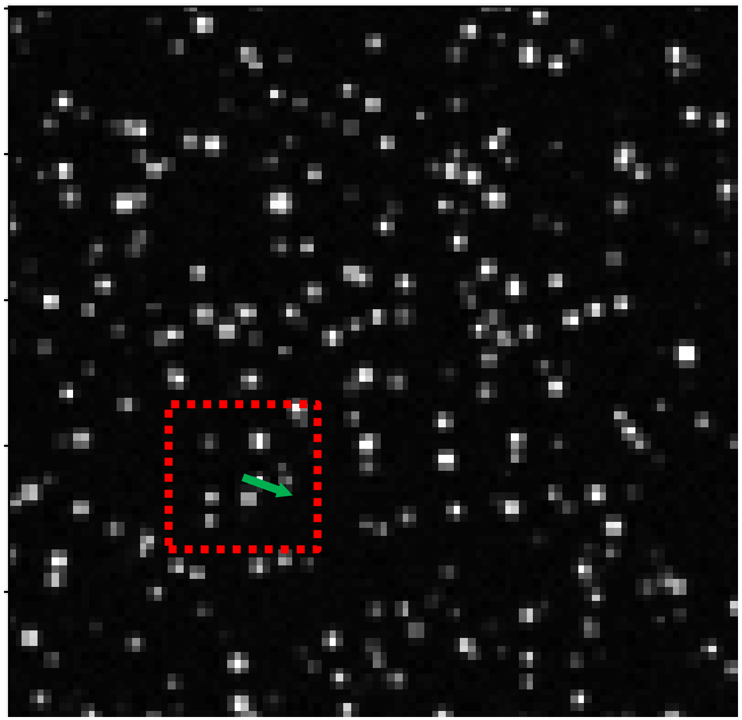
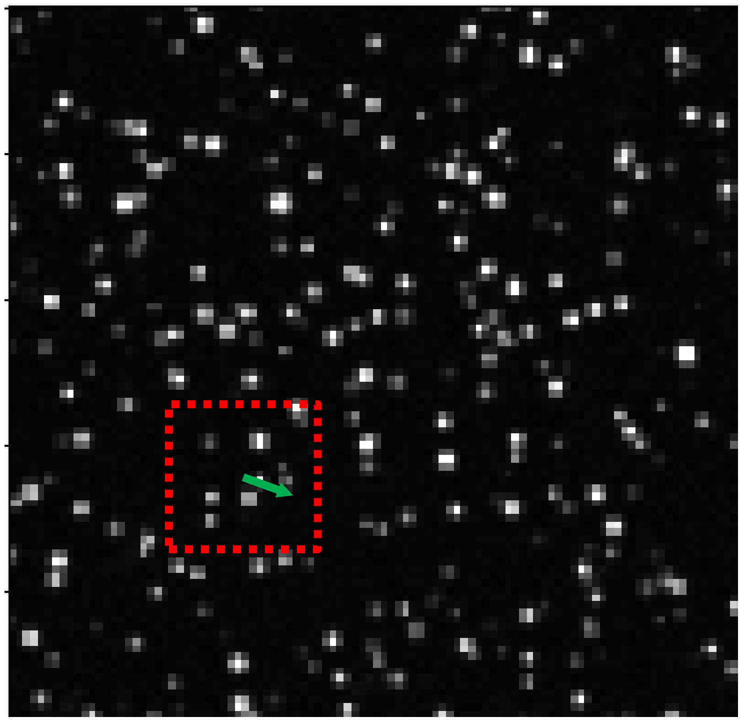


Objective: find displacement at pixel k

PIV tracks the *particle pattern* in an interrogation window

2D2C PIV: displacement estimation

t



Interrogation window at pixel k: $W(k)$

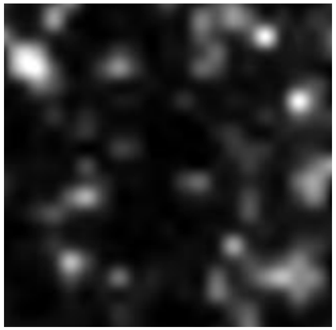
Objective: find displacement at pixel k

One vector for a group of particles: what if finer spatial scales than size of W?...
→ **more later**

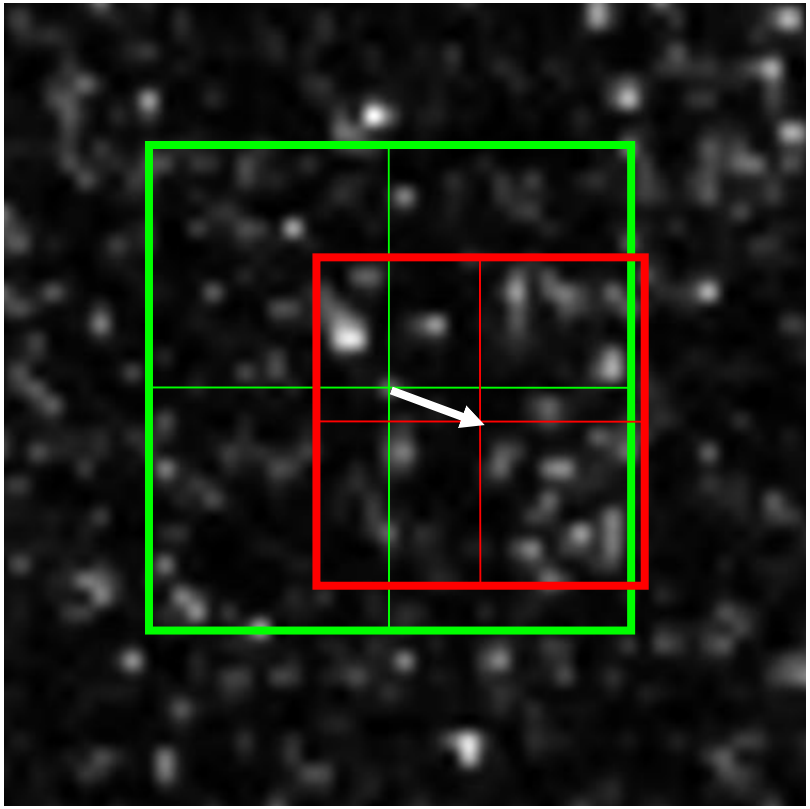
2D2C PIV: displacement estimation

Automating this: cross-correlation (CC)

$W(k)$

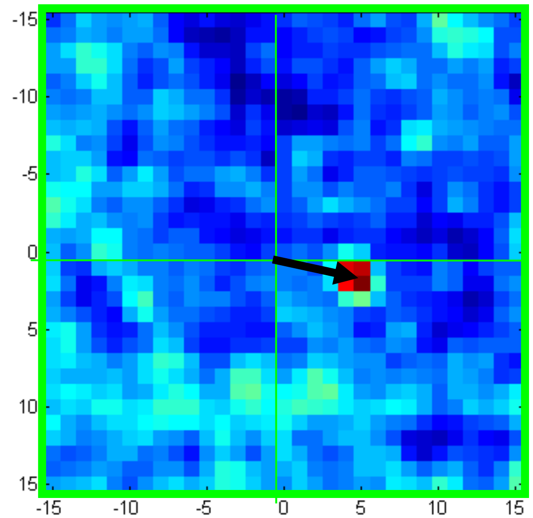


$I_1(t)$



$I_2(t + dt)$

Correlation map



$$CC\big(\Delta X(k)\big) = \sum_{m \in W(k)} I_1(m) I_2(m - \Delta X(k))$$

Displacement $\Delta X(k)$ at pixel k found as **maximum** of cross-correlation $CC(\Delta X(k))$

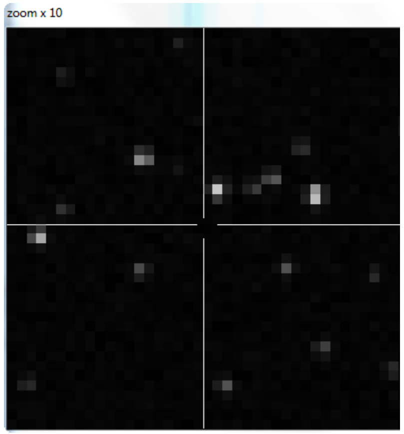
PIV is an optimization problem for each vector!

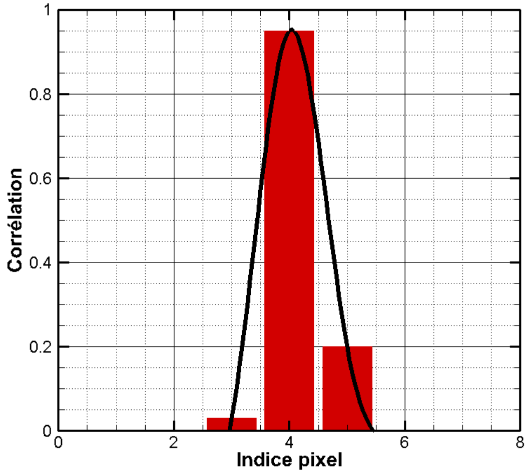
PIV: ideal particle image size

Width of $CC(\Delta X(k))$ peak ≈ particle image diameter d_τ

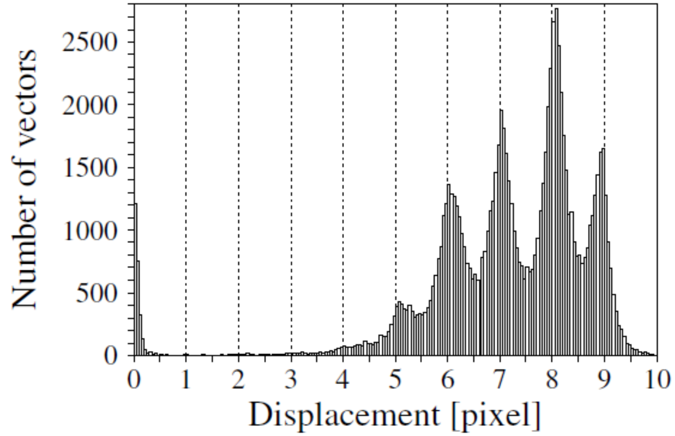
⇒ **peak-locking bias** (= interpolation error!) unless $d_\tau \geq 2 - 3$ pixels

d_τ < 1.5 px



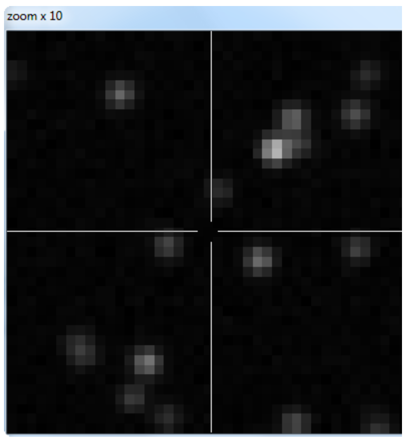


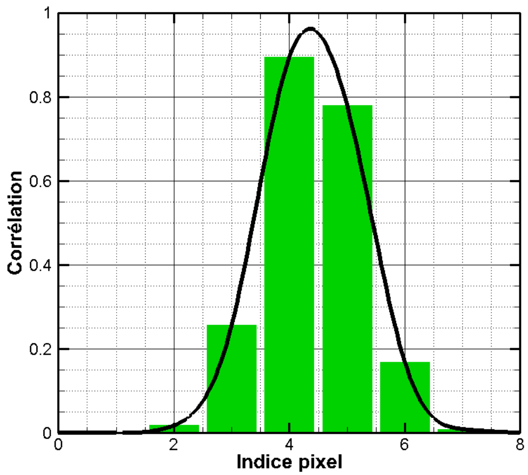
Peak locking: bias towards integer values



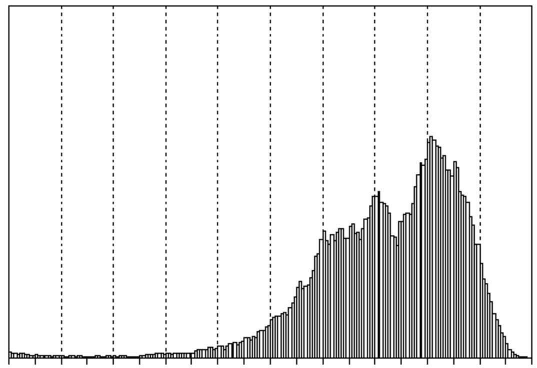
True displacement: 4.4 px

d_τ ≈ 2-3 px





OK: subpixel accuracy preserved



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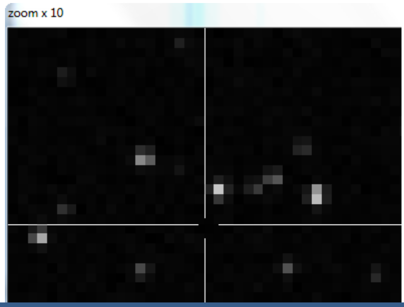

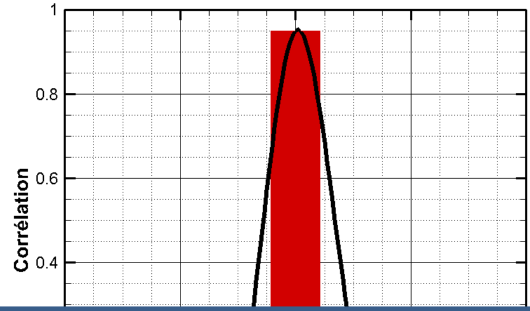

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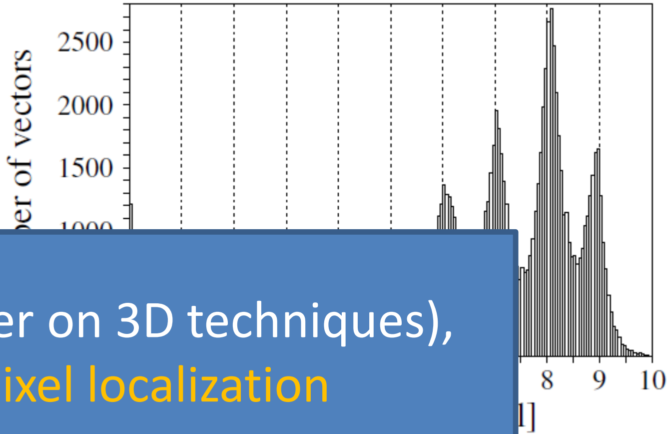
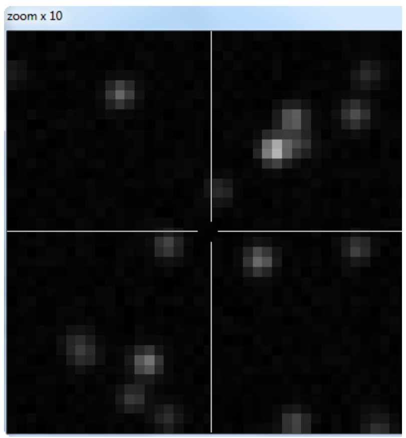
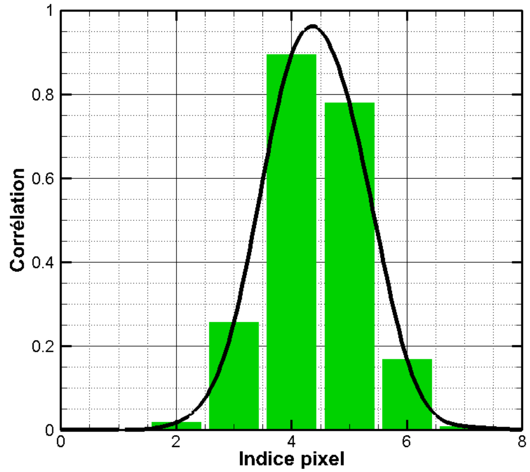


- In particle tracking methods (PTV – see later on 3D techniques), $d_\tau \approx$ 2-3 pixel ensures accurate subpixel localization

$d_\tau \approx$ 2-3 px





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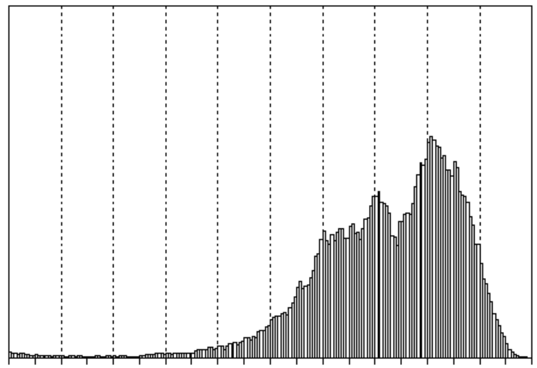
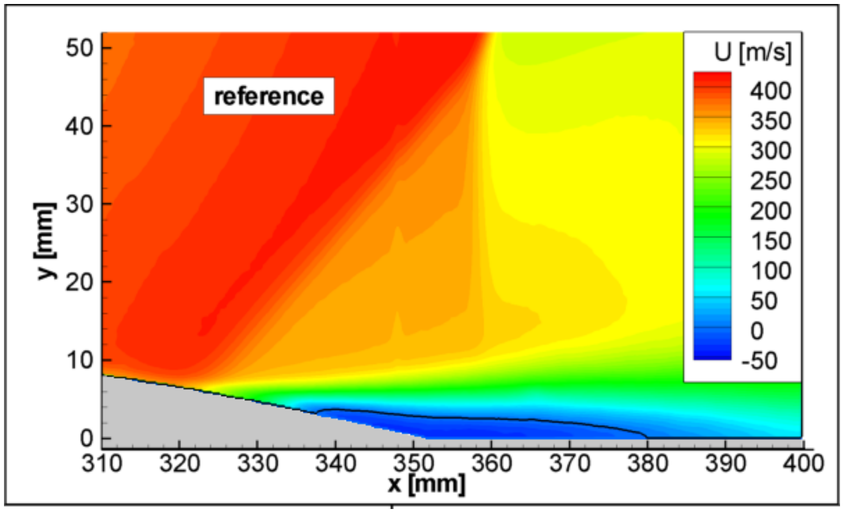


PIV: noise/resolution trade-off

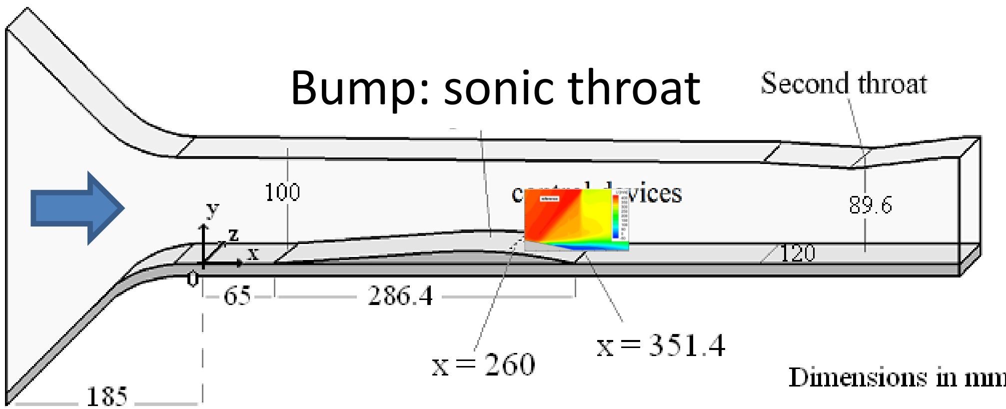
Example: shock-wave boundary layer interaction

Flow accelerated to supersonic, until Mach M ≈ 1.4

Lambda shock on bump downstream side, induces separation



Time-averaged horizontal velocity



ONERA S8Ch wind-tunnel (Sartor et al., Exp. Fluids 2012)

PIV: noise/resolution trade-off

A simple mathematical model

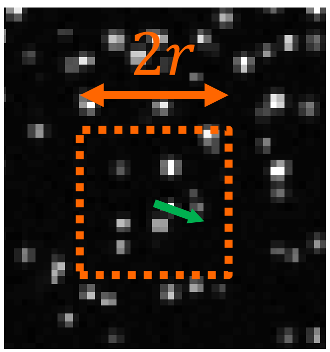
$$u_{PIV}(x) = F(u(x)) + \varepsilon_{noise}$$

- $u(x)$ true displacement value
- F spatial transfer function (only spatial filtering part here) = bias
- ε_{noise} : measurement noise (**random by definition ≠ bias**)

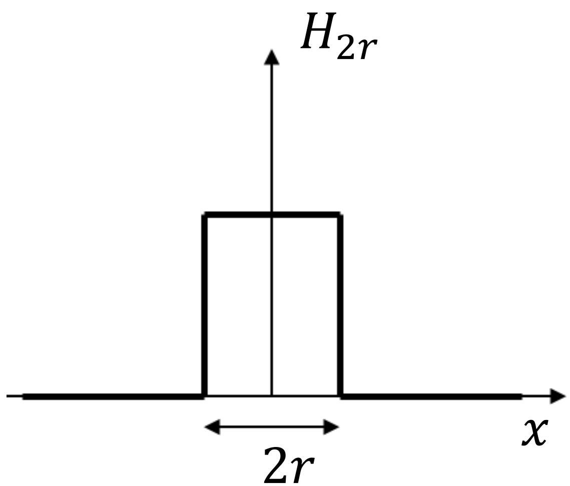
$$\Rightarrow \langle u_{PIV}(x) \rangle = F(u(x))$$

- F should model the effect of the interrogation window:
  - ✓ A priori, top-hat filter of same width as the interrogation window ($2r$)
  - ✓ If yes, then $F\big(u(x)\big)$ provided by a *convolution*:

$$F(u(x)) = (H_{2r} * u)(\mathrm{x}) = \int H_{2r}(x - \xi)u(\xi)d\xi$$



$\Longleftrightarrow$



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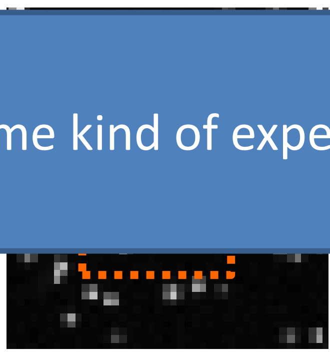
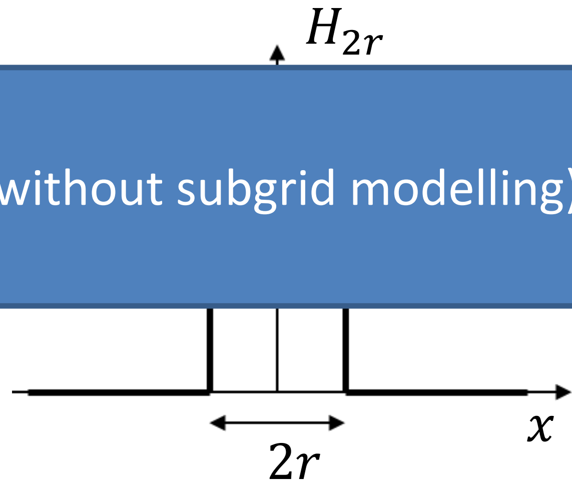




PIV: some kind of experimental LES (without subgrid modelling) ?

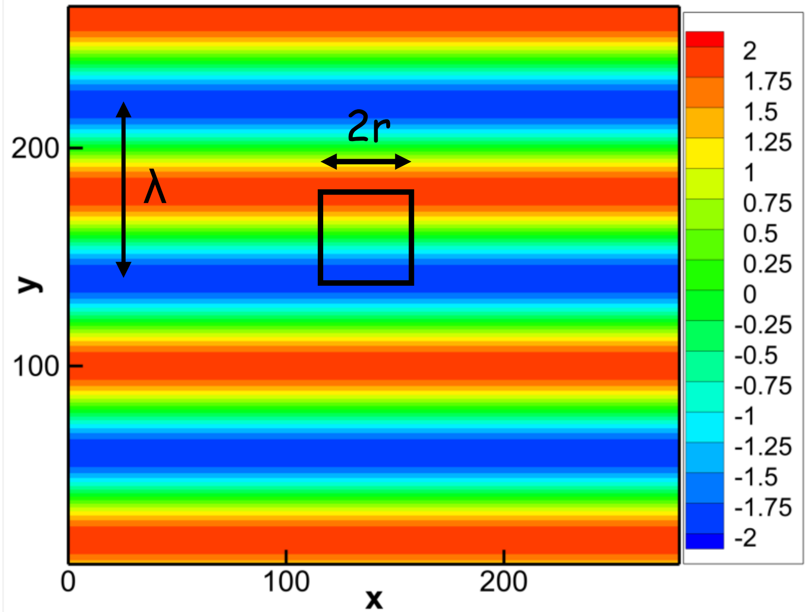
PIV: noise/resolution trade-off

Spatial filtering: Velocities

- Synthetic images with sinusoidal displacement

$$\begin{pmatrix} U \\ V \end{pmatrix}(x,y) = \begin{pmatrix} A\sin(2\pi\frac{y}{\lambda}) \\ 0 \end{pmatrix}$$

- Process with different window sizes $2r$ and compare A_{PIV} with A



Scarano & Riethmuller, Exp. Fluids 2000

PIV: noise/resolution trade-off

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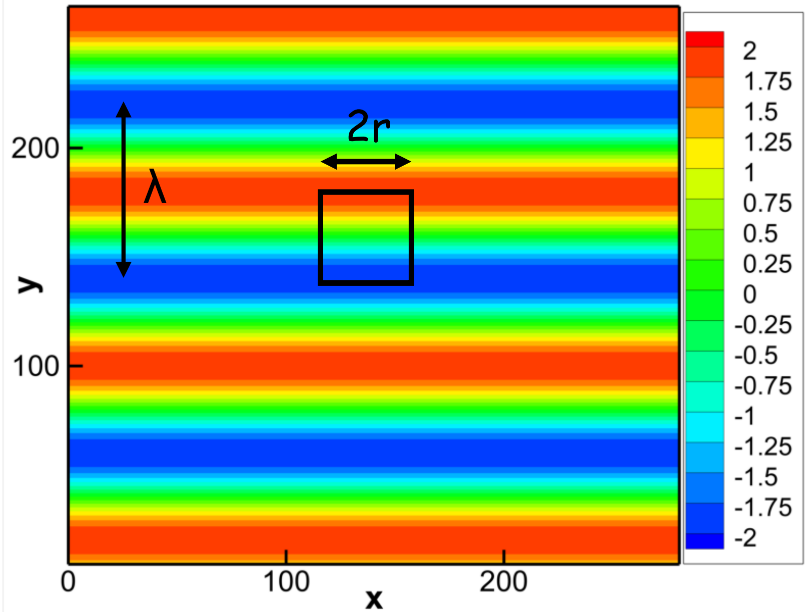
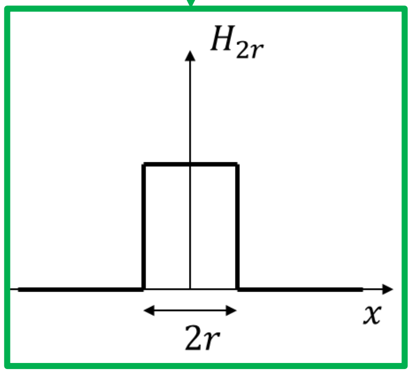
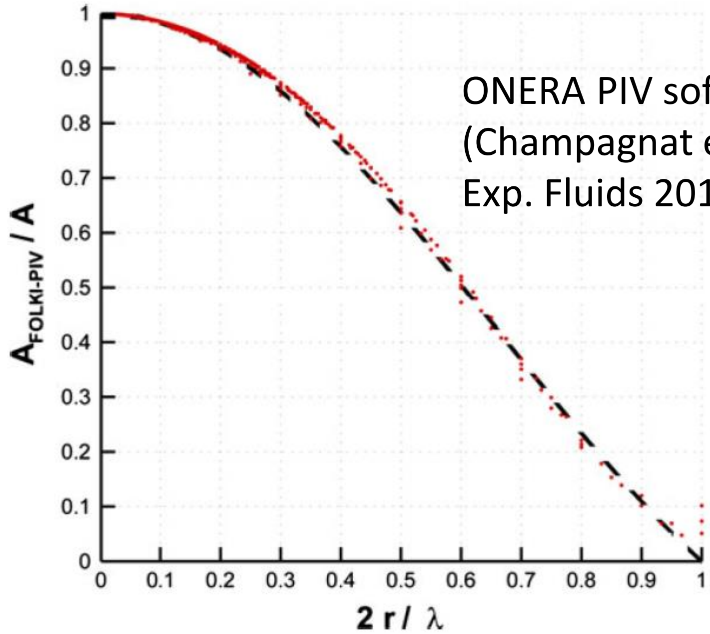


One should have $A_{PIV} \approx \underbrace{\frac{\sin(2\pi r/\lambda)}{(2\pi r/\lambda)}} A$

Fourier transform



$2r/\lambda$:
effective window size



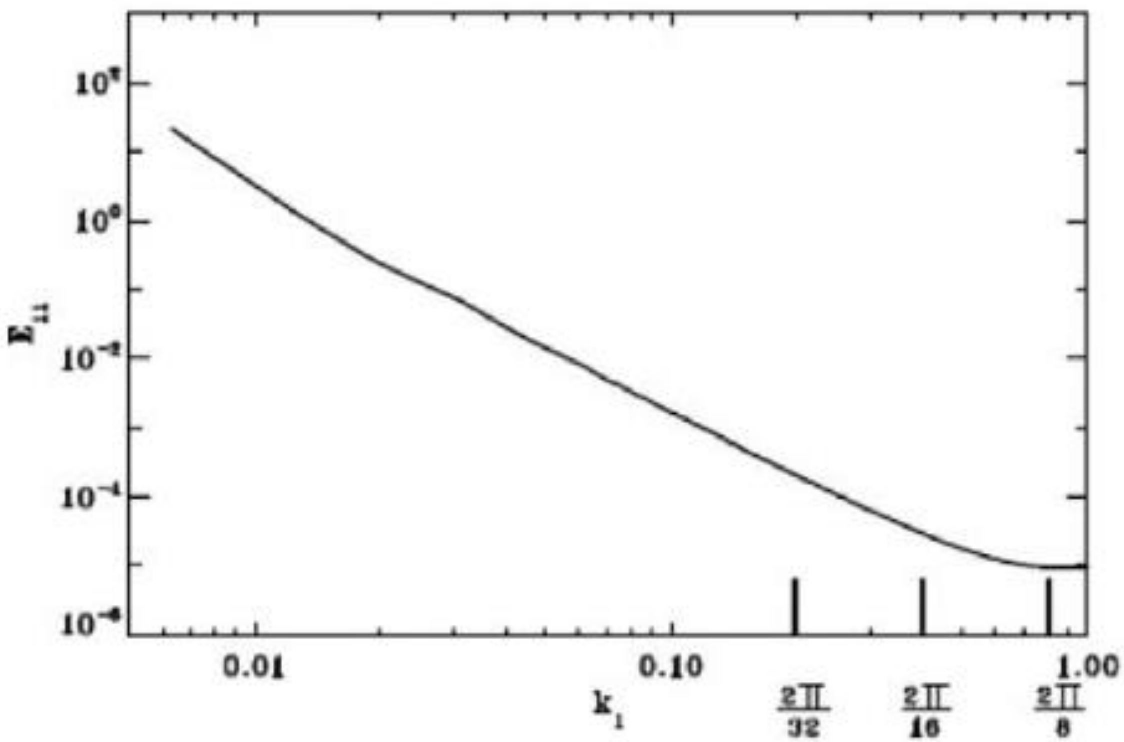
ONERA PIV software (Champagnat et al., Exp. Fluids 2011)

PIV: noise/resolution trade-off

Spatial filtering: fluctuations / spectra

- A2 test case from the 3rd international PIV Challenge (Stanislas et al., Exp. Fluids, 2008): DNS of 2D turbulence (k^{-3} spectrum)

<u>Qualitatively</u>: how can we expect the PIV spectra to compare with the ground truth?

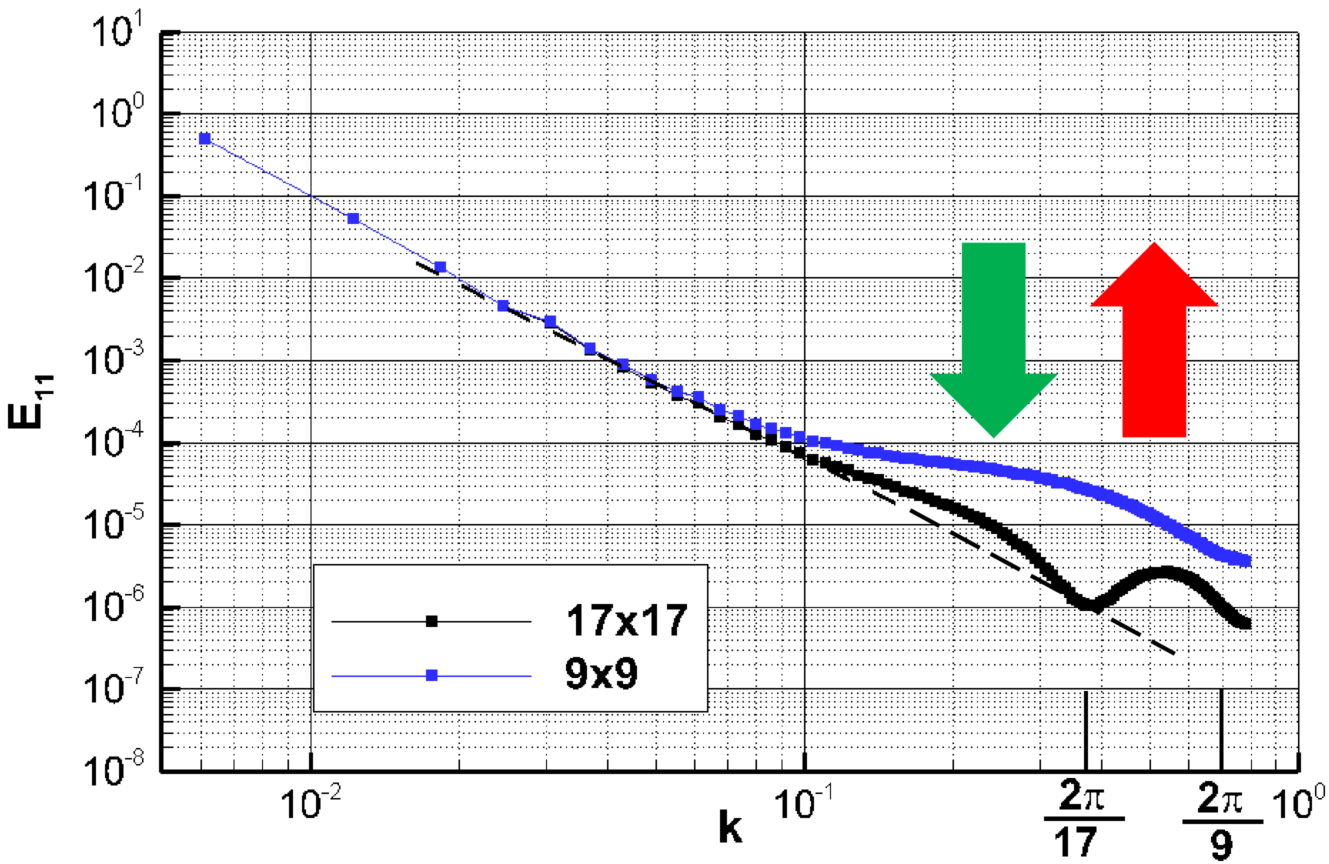


PIV: noise/resolution trade-off

Spatial filtering: fluctuations / spectra

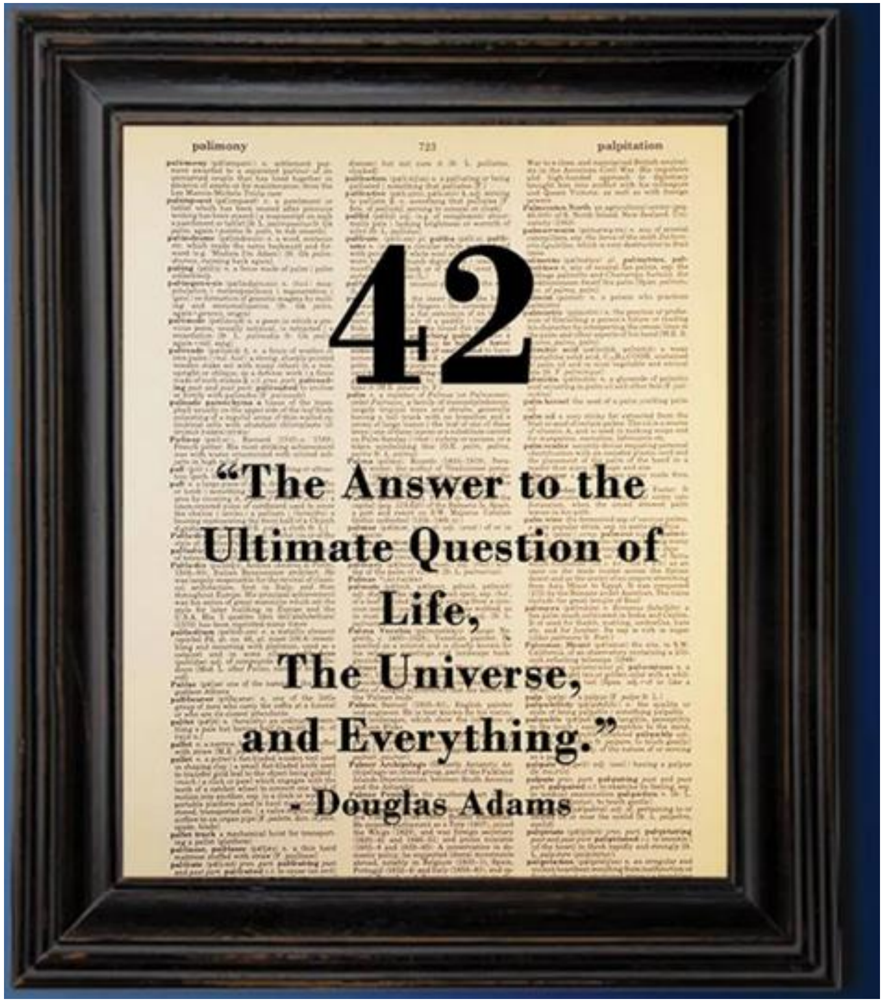
$$u_{PIV}(\underline{x}) = (H_{2r} * u)(\underline{x}) + \varepsilon_{noise}$$

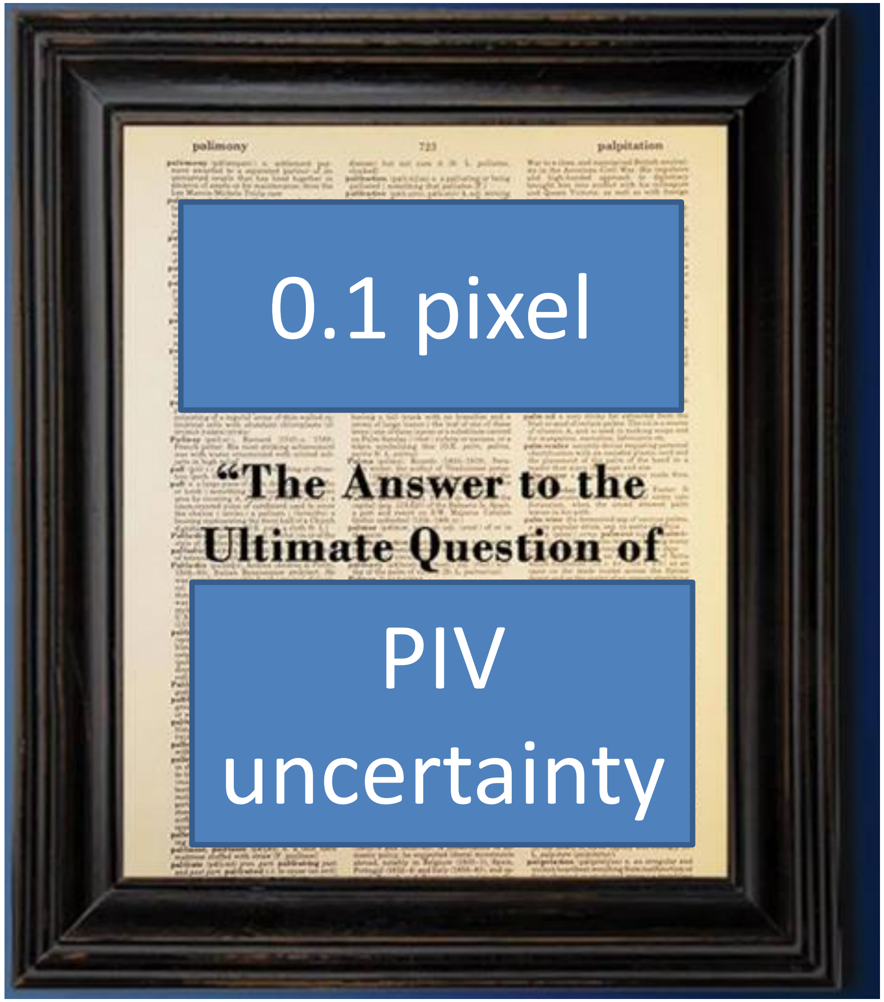
$$\Rightarrow \qquad \left\langle \left| \hat{u}_{PIV}(k) \right|^2 \right\rangle = \left\langle \left| \boxed{\frac{\sin(kr)}{(kr)} \hat{u}(k)} + \boxed{\hat{\varepsilon}_{noise}(k)} \right|^2 \right\rangle$$



PIV: uncertainty quantification?...

Some time ago...





To refine the 0.1 pixel view: Sciacchitano, Meas. Sci. Technol. 2019 (topical review)

PIV: uncertainty quantification

1. **Instantaneous velocity vector**
2. **Statistical estimates**

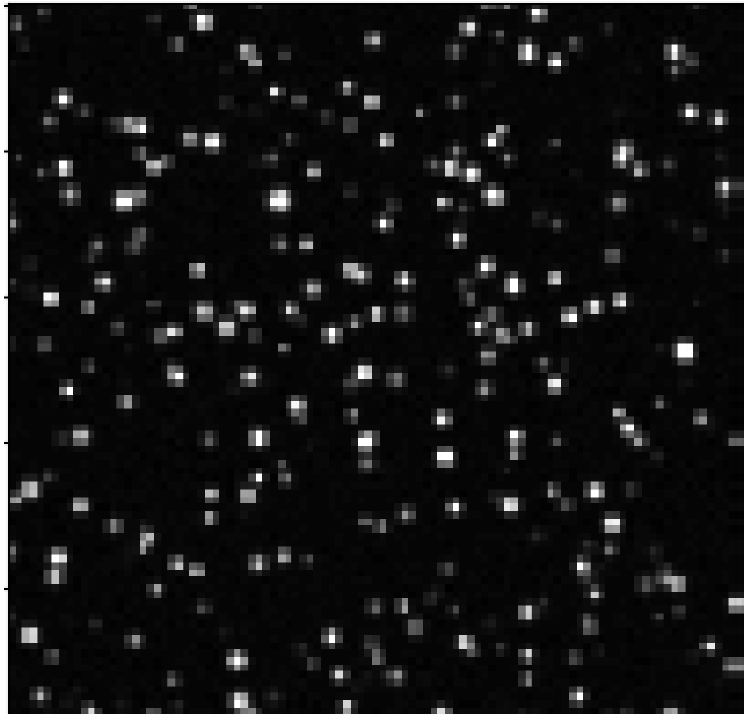
PIV: uncertainty quantification

1. **Instantaneous velocity vector**

PIV: uncertainty quantification

1. Instantaneous velocity vector

Way #1 (« a priori »): Quantify effect of individual parameters on the measurement error, either theoretically or using synthetic images



Fundamental to understand **parameterwise effects**, but: synthetic images always **« too perfect »** + **error sources add** within the images, and their relative amplitude **can vary locally within the images!**

PIV: uncertainty quantification

1. Instantaneous velocity vector

Way #1 (« a priori »): Quantify effect of individual parameters on the measurement error, either theoretically or using synthetic images

…we just did that in the case of spatial filtering!

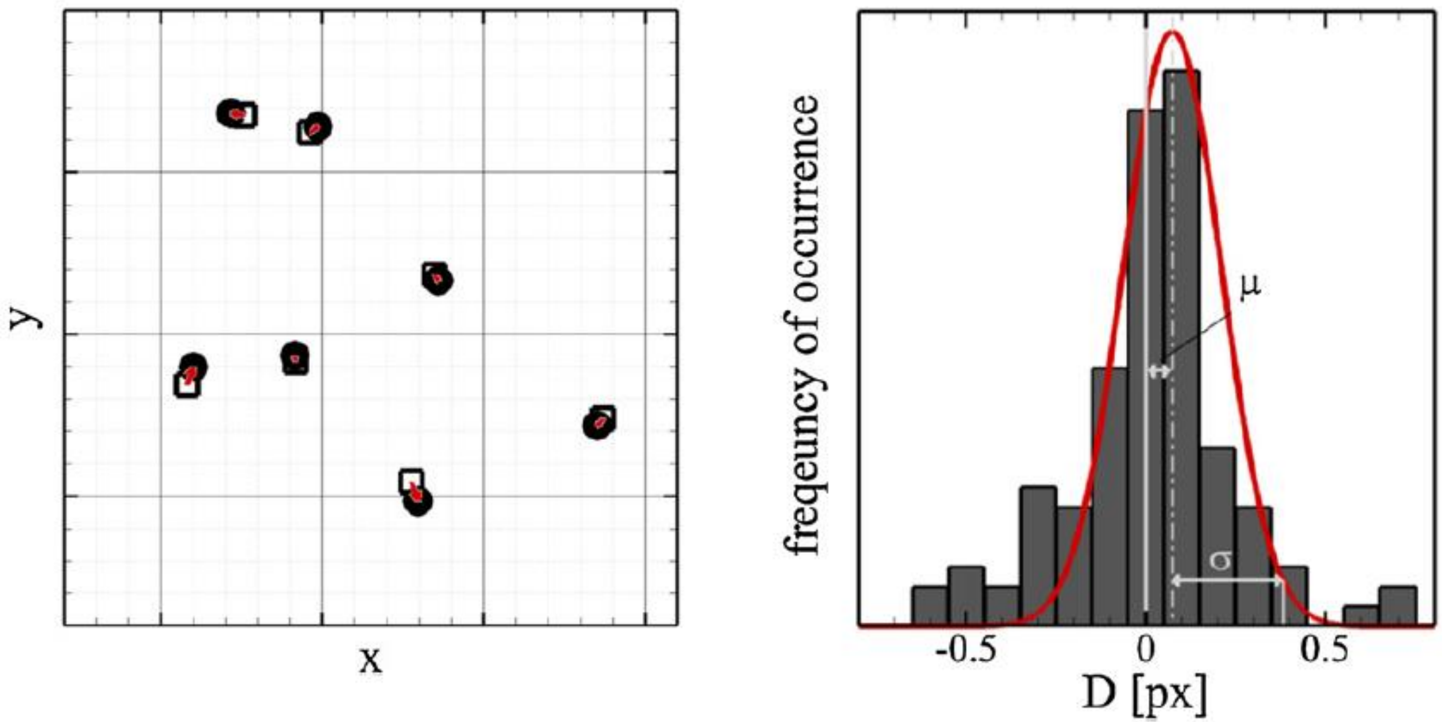
Fundamental to understand **parameterwise effects**, but: synthetic images always **« too perfect »** + **error sources add** within the images, and their relative amplitude **can vary locally within the images!**

PIV: uncertainty quantification

1. Instantaneous velocity vector

<u>Way #2 (« a posteriori »)</u>: Derive formula/algorithm estimating UQ **of each individual vector** in the PIV result *given the image pair*

Ex.: particle disparity method



Sciacchitano et al., Meas. Sci. Technol. 2013

Individual vector uncertainty depending on local image characteristics, but potential **variability** and **account for part of error sources present in the images**

(+ only consider error sources contained in the images, as a priori methods!)

PIV: uncertainty quantification

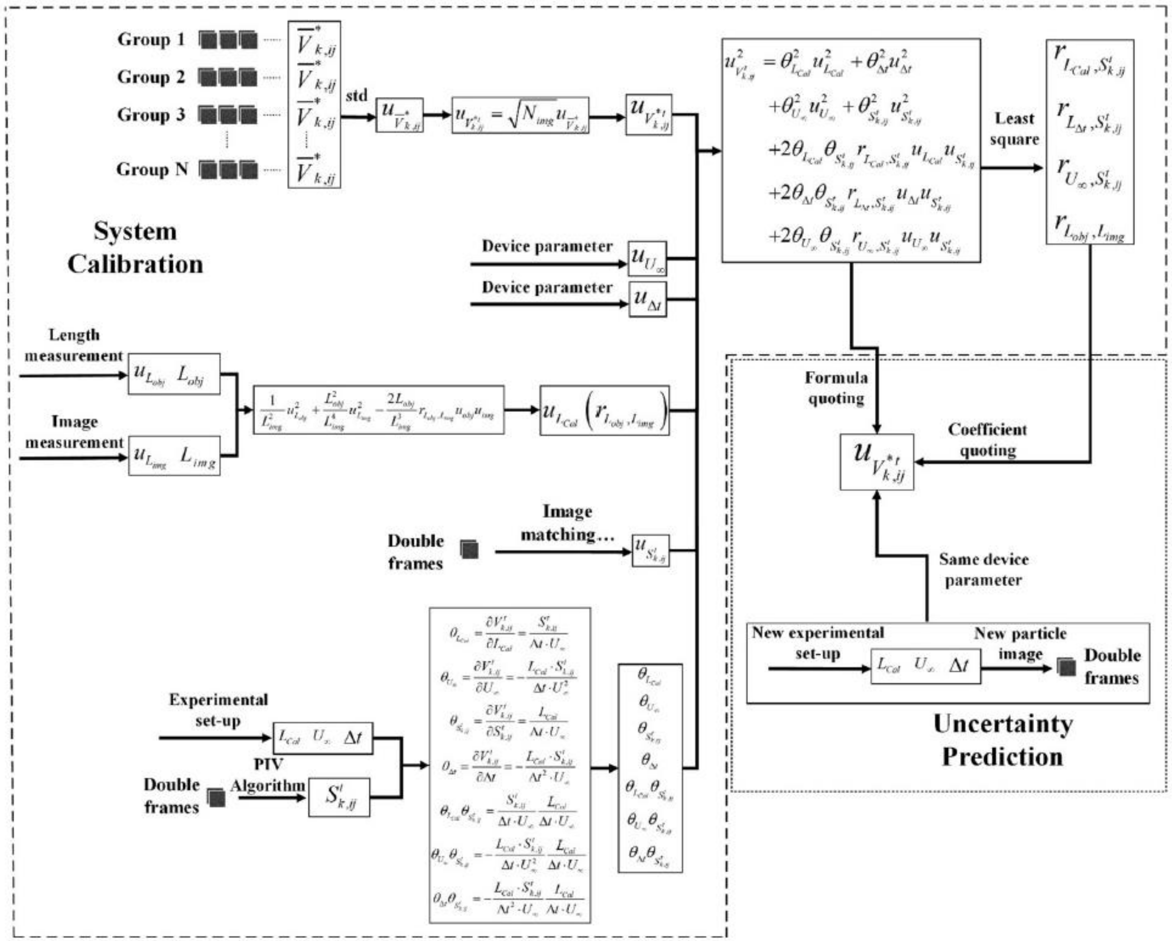
1. **Instantaneous velocity vector – so what?...**

PIV: uncertainty quantification

1. Instantaneous velocity vector – so what?...

... well, research again!...

Theoretical model of the whole chain! The solution?...



Fu, , Meas. Sci. Technol. 2024

PIV: uncertainty quantification

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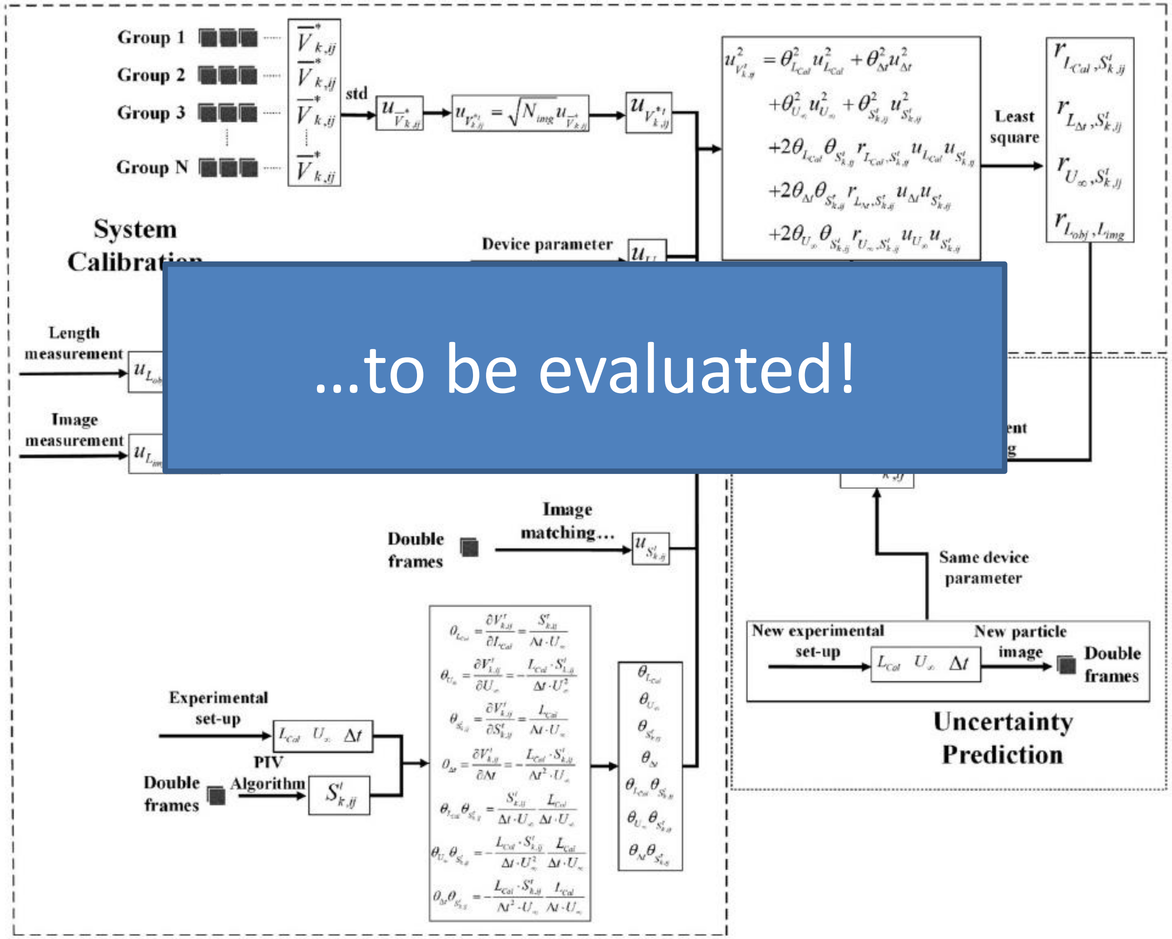


Fu, , Meas. Sci. Technol. 2024

PIV: uncertainty quantification

2. Statistical estimates – random part only

Table 1. Estimator variances multiplied by N

Benedict & Gould, Exp. Fluids 1996

- Estimator? e.g.: estimator of time-averaged velocity: $\overline{U} = \frac{1}{N}\sum_{i=1}^{N} u_i$
- Variance of estimator: how far we are from true value (e.g. true mean)

PIV: uncertainty quantification

2. Statistical estimates – random part only

Table 1. Estimator variances multiplied by N

Statistic	Valid for any distribution	Normal assumption
$\bar{U}$	$\overline{u^2}$	$\overline{u^2}$
$\sqrt{\overline{u^2}}$	$[\overline{u^4}-(\overline{u^2})^2]/4\overline{u^2}$	$\overline{u^2}/2$
$\overline{uv}$	$\overline{u^2v^2}-(\overline{uv})^2$	$(1+R_{uv}^2)(\overline{u^2})(\overline{v^2})$
$R_{uv}=\dfrac{\overline{uv}}{(\overline{u^2})^{1/2}(\overline{v^2})^{1/2}}$	$R_{uv}^2\left\{\dfrac{\overline{u^2v^2}}{(\overline{uv})^2}+\dfrac{1}{4}\left(\dfrac{\overline{u^4}}{(\overline{u^2})^2}+\dfrac{\overline{v^4}}{(\overline{v^2})^2}+\dfrac{2\overline{u^2v^2}}{(\overline{u^2})(\overline{v^2})}\right)-\left(\dfrac{\overline{u^3v}}{(\overline{uv})(\overline{u^2})}+\dfrac{\overline{uv^3}}{(\overline{uv})(\overline{v^2})}\right)\right\}$	$(1-R_{uv}^2)^2$
$\overline{u^2}$	$\overline{u^4}-(\overline{u^2})^2$	$2(\overline{u^2})^2$
$\overline{u^3}$	$\overline{u^6}-(\overline{u^3})^2-6(\overline{u^4})(\overline{u^2})+9(\overline{u^2})^3$	$6(\overline{u^2})^3$

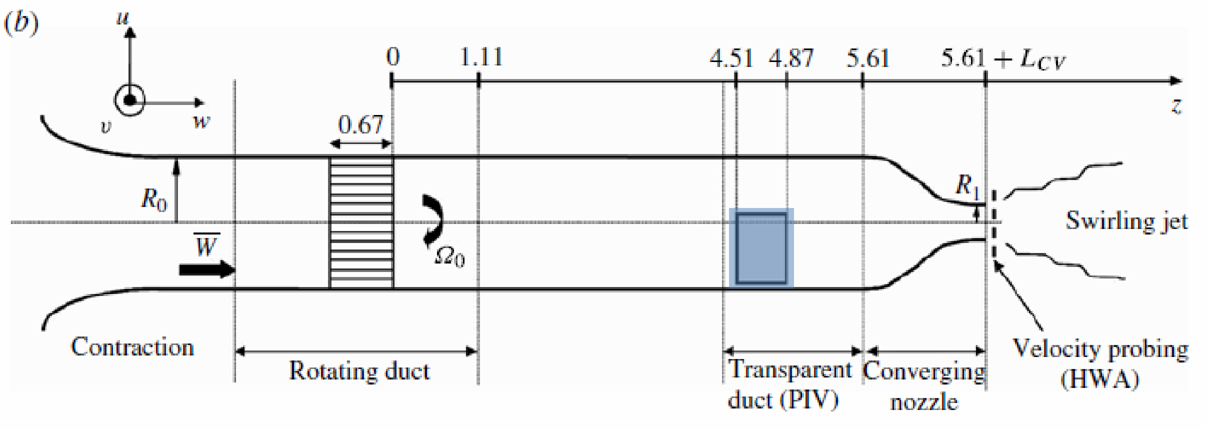
Benedict & Gould, Exp. Fluids 1996

- Estimator? e.g.: estimator of time-averaged velocity: $\overline{U}=\frac{1}{N}\sum_{i=1}^{N}u_i$
- Variance of estimator: how far we are from true value (e.g. true mean)
- Table above: case of **independent samples**
- If **correlated samples** (e.g. high-speed PIV): replace N by $N_{eff}=T/(2T_{int})$ (T measurement duration for acquiring the N samples, T_{int} integral time)

Turbulence (resp. measurement) often non-gaussian (resp. noisy)
⇒ **what if high-order moments not reliable… ?**

PIV: uncertainty quantification

2. Statistical estimates (random only): example



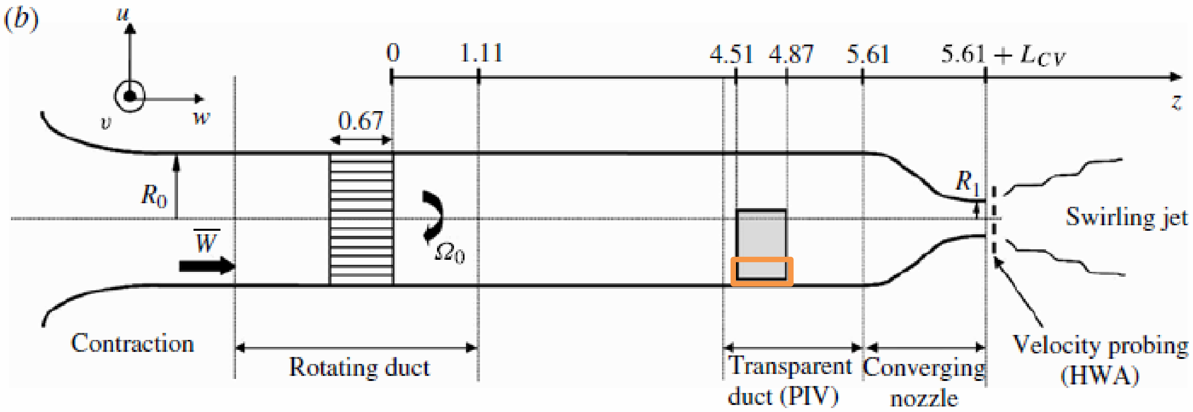
Plug axial flow + solid-body rotation

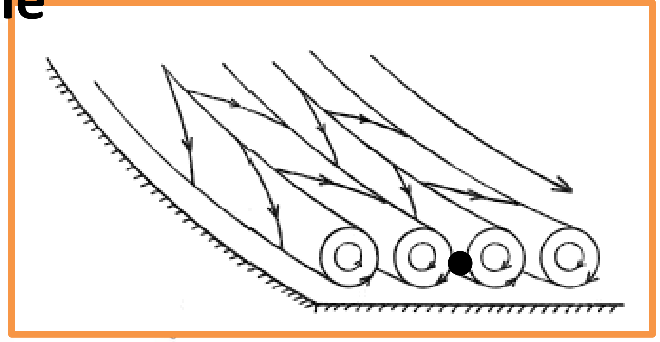
(Stereo) PIV in a longitudinal plane

Leclaire & Jacquin, J. Fluid Mech., 2012

PIV: uncertainty quantification

2. Statistical estimates (random only): example



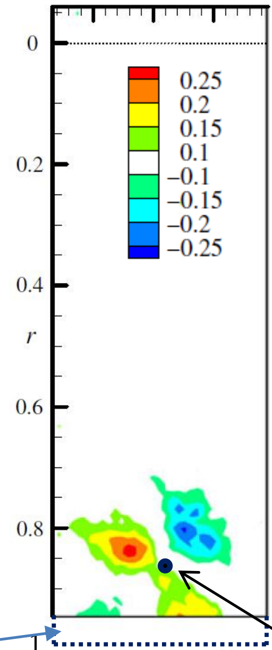


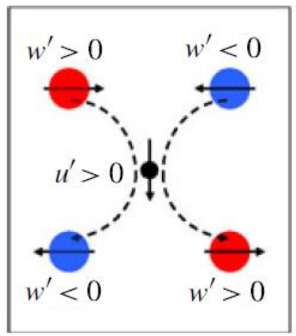
Plug axial flow + solid-body rotation

(Stereo) PIV in a longitudinal plane

Presence of (intermittent) **Görtler vortices at the wall?...**

$$R_{uw}(\underline{x}_0, \underline{x}) = \langle u'(\underline{x}_0) w'(\underline{x}) \rangle$$



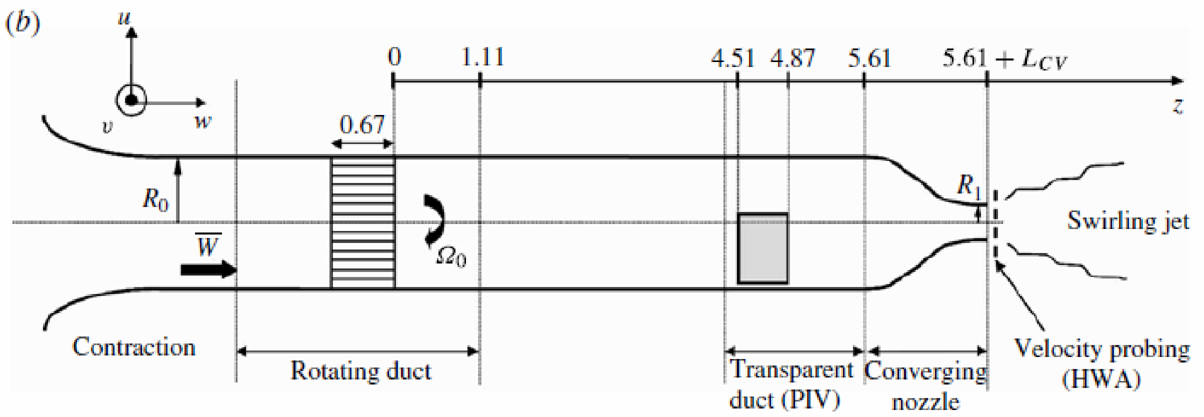


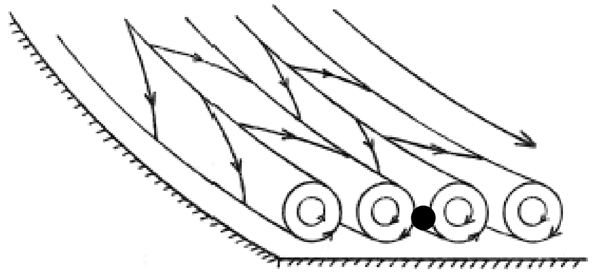
No info here: **impact of laser on wall induces image saturation!**
A few workarounds: use fluorescence, or automatic masking algorithms…

Leclaire & Jacquin, J. Fluid Mech., 2012

PIV: uncertainty quantification

2. Statistical estimates (random only): example





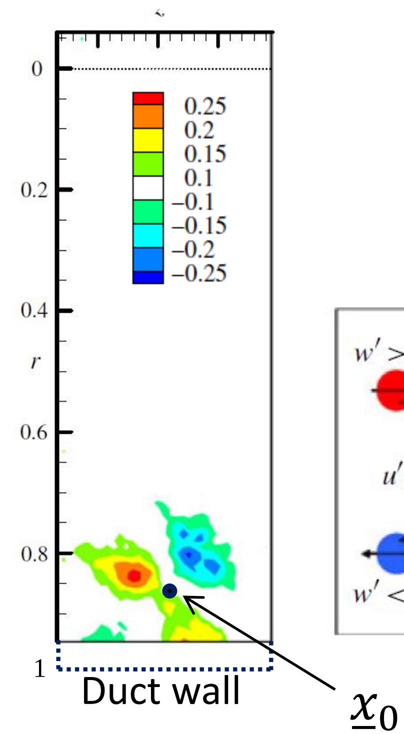
Plug axial flow + solid-body rotation

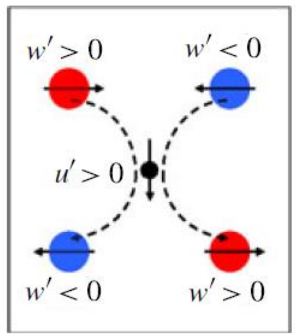
(Stereo) PIV in a longitudinal plane

Presence of (intermittent) **Görtler vortices at the wall?...**

$$R_{uw}(\underline{x}_0, \underline{x}) = \langle u'(\underline{x}_0) w'(\underline{x}) \rangle$$

But low levels: uncertainty?...





	$S_0 = 2.01$		$S_0 = 2.77$		$S_0 = 3.35$	
	ϕ	$\sigma(\phi)$	ϕ	$\sigma(\phi)$	ϕ	$\sigma(\phi)$
$R_{uu}(4.70, 0; 4.70, 0.2)$	0.570	0.055	0.367	0.046	0.652	0.039
$R_{uw}(4.70, 0.85; 4.75, 0.80)$	−0.197	0.052	−0.209	0.048	−0.213	0.052

TABLE 2. Values and standard deviations σ of correlation coefficient components R_{uu} and R_{uw}, probed at selected locations in the flow, $S_0 = 0$, 2.01 and 3.35, $\chi_{CV} = 18.4$.

<u>Jackknife</u>: resampling-based estimation of statistical uncertainty (e.g. Benedict & Gould Exp. Fluids 1996)

Leclaire & Jacquin, J. Fluid Mech., 2012

PIV: uncertainty quantification

2. Statistical estimates – bias *and* random errors

Design Of Experiments (DOE) for PIV UQ – but not only!

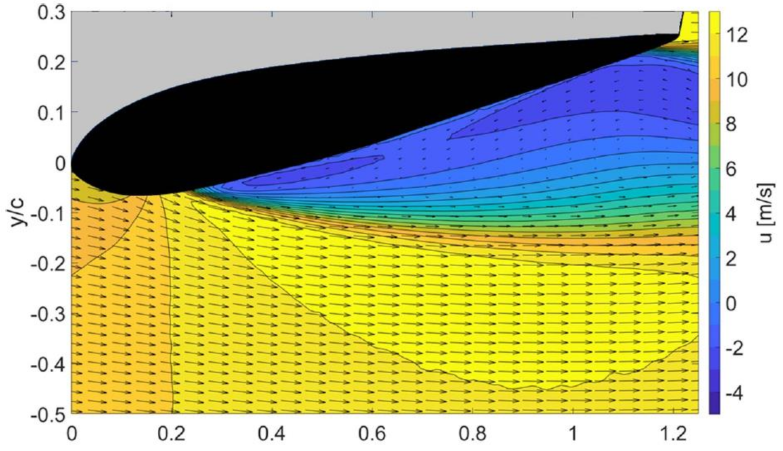
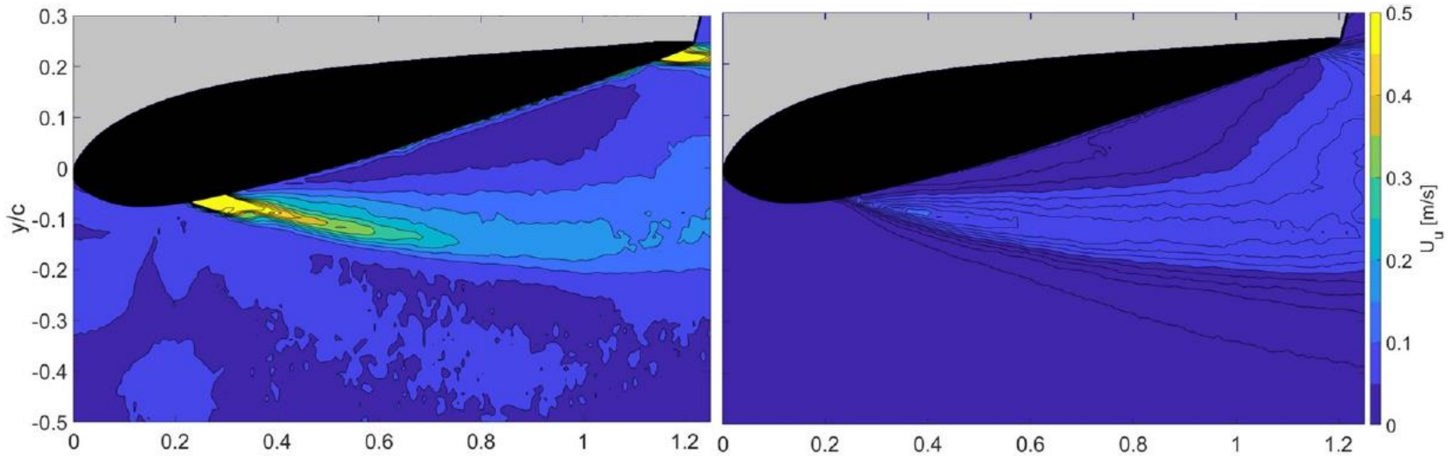


Table 2. Factors and their levels in the planar PIV measurements of the flow over a NACA0012 airfoil.

Factor	Parameter	Levels
A	$f\#$	4, 8
B	Δt	50, 70 μs
C	D_{I}	16 × 16, 64 × 64 pixels (0.95 × 0.95, 3.78 × 3.78 mm)
D	Δz	1, 3 mm
Block	Seeding density	0.01–0.02, 0.08–0.09 ppp (mean particle distances of 0.2 and 0.5 mm)



(a) total uncertainty by proposed methodology based on DOE

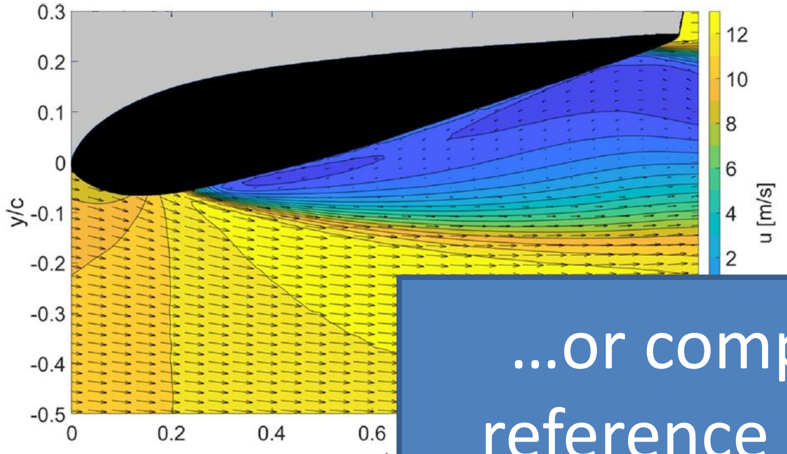
(b) random uncertainty from data statistics $(\sigma/\sqrt{N_s})$

Adatrao et al., Meas. Sci. Technol., 2022

PIV: uncertainty quantification

2. Statistical estimates – bias *and* random errors

Design Of Experiments (DOE) for PIV UQ – but not only!



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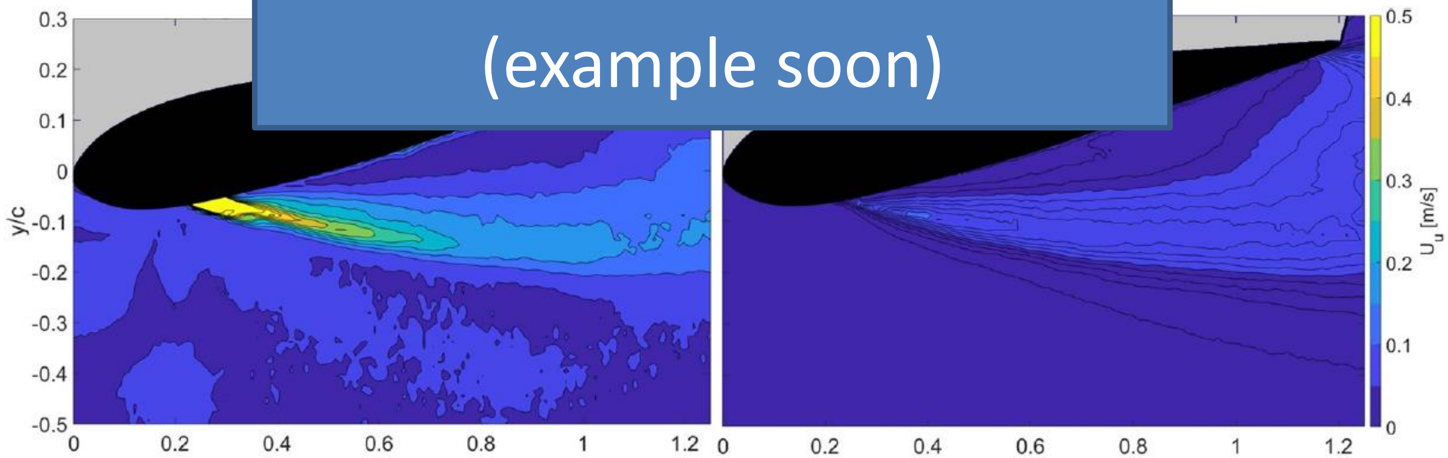
C	D_I	16 × 16, 64 × 64 pixels (0.95 × 0.95, 3.78 × 3.78 mm)
		…n
		…02, 0.08–0.09 ppp
		…article distances of 0.2 and
		…)

…or comparison with a reference measurement! (example soon)



(a) total uncertainty by proposed methodology based on DOE

(b) random uncertainty from data statistics ($\sigma/\sqrt{N_s}$)

Adatrao et al., Meas. Sci. Technol., 2022

I. Seeding and image formation

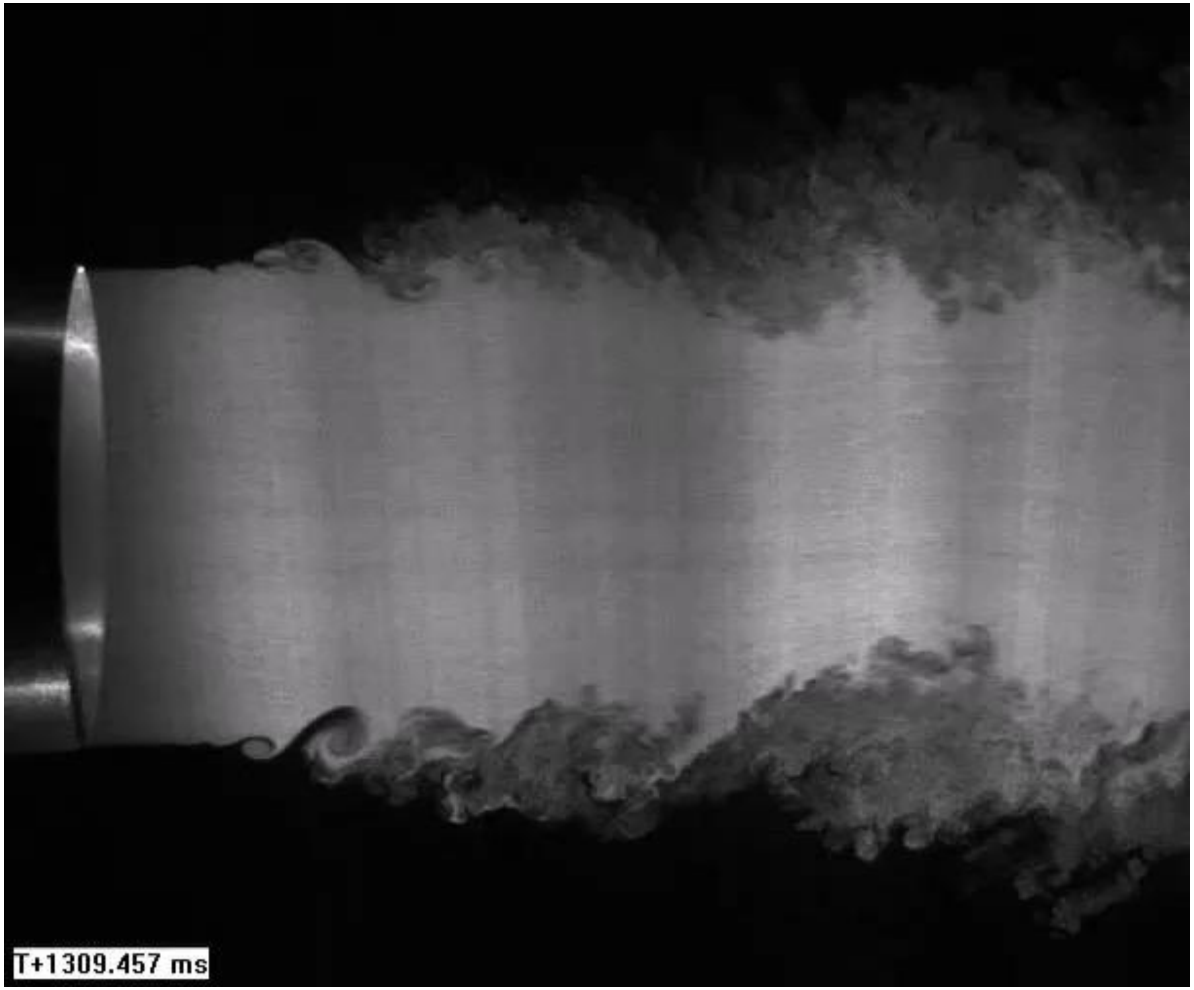
II. Basics: 2D, two-component (2D2C) PIV

III. **Towards more complexity: Stereo PIV, Time-Resolved PIV**

IV. Volumetric and Tracking approaches, and beyond

Stereo (2D3C) PIV

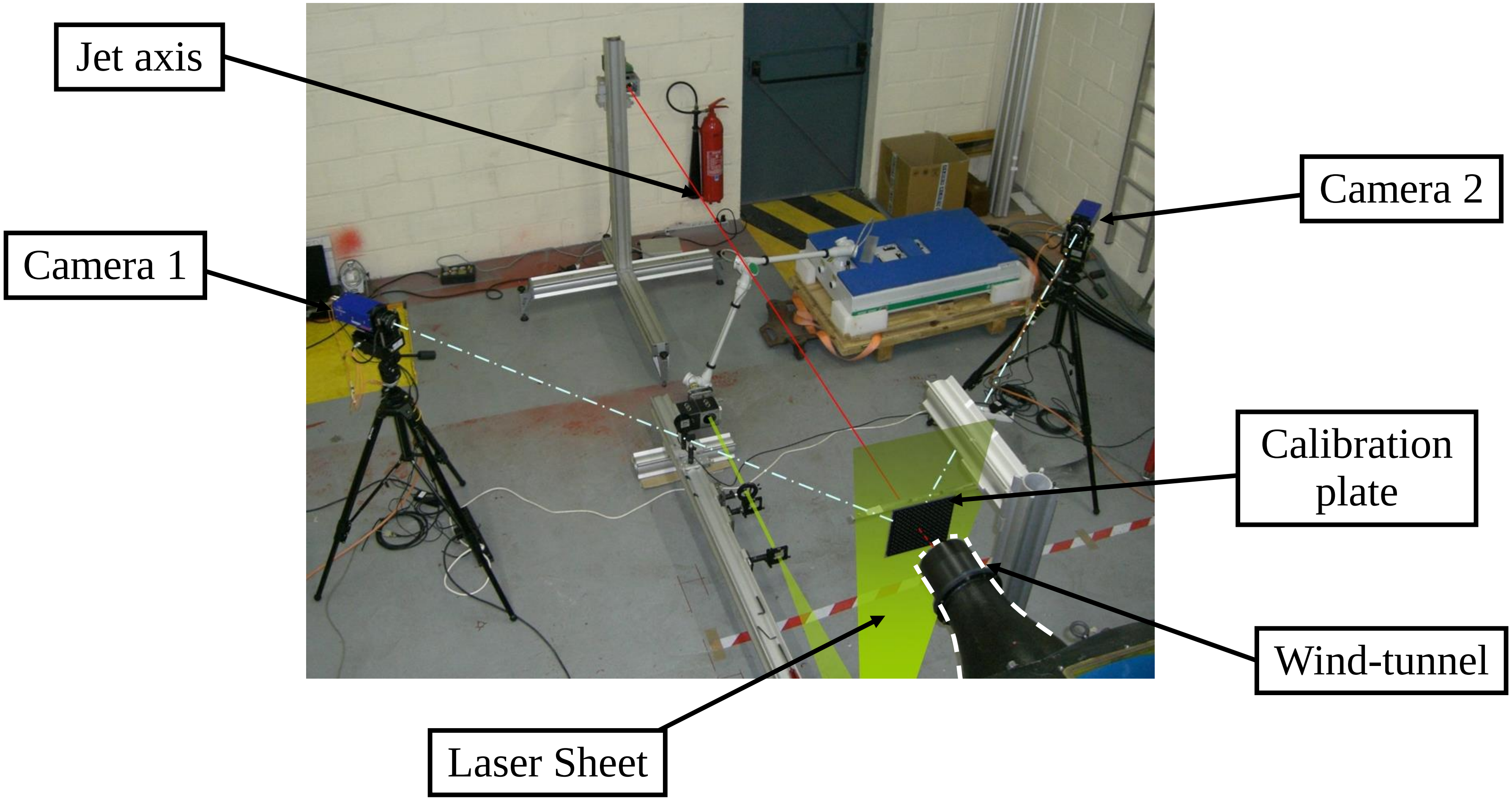
Example: a cylindrical air jet



ONERA R4Ch wind-tunnel, PhD S. Davoust (2011)

Stereo (2D3C) PIV

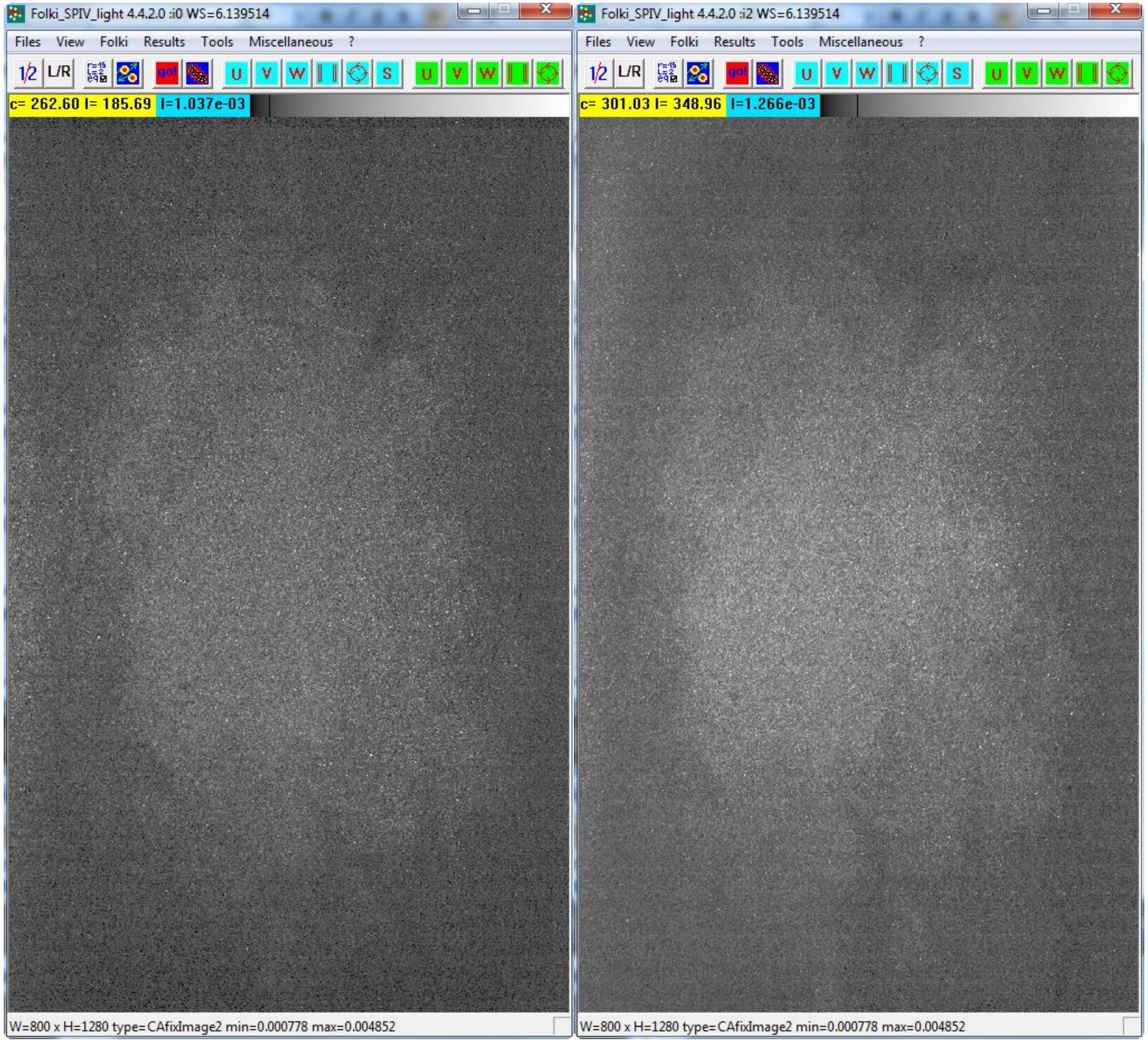
Setup



Stereo (2D3C) PIV

Sample images

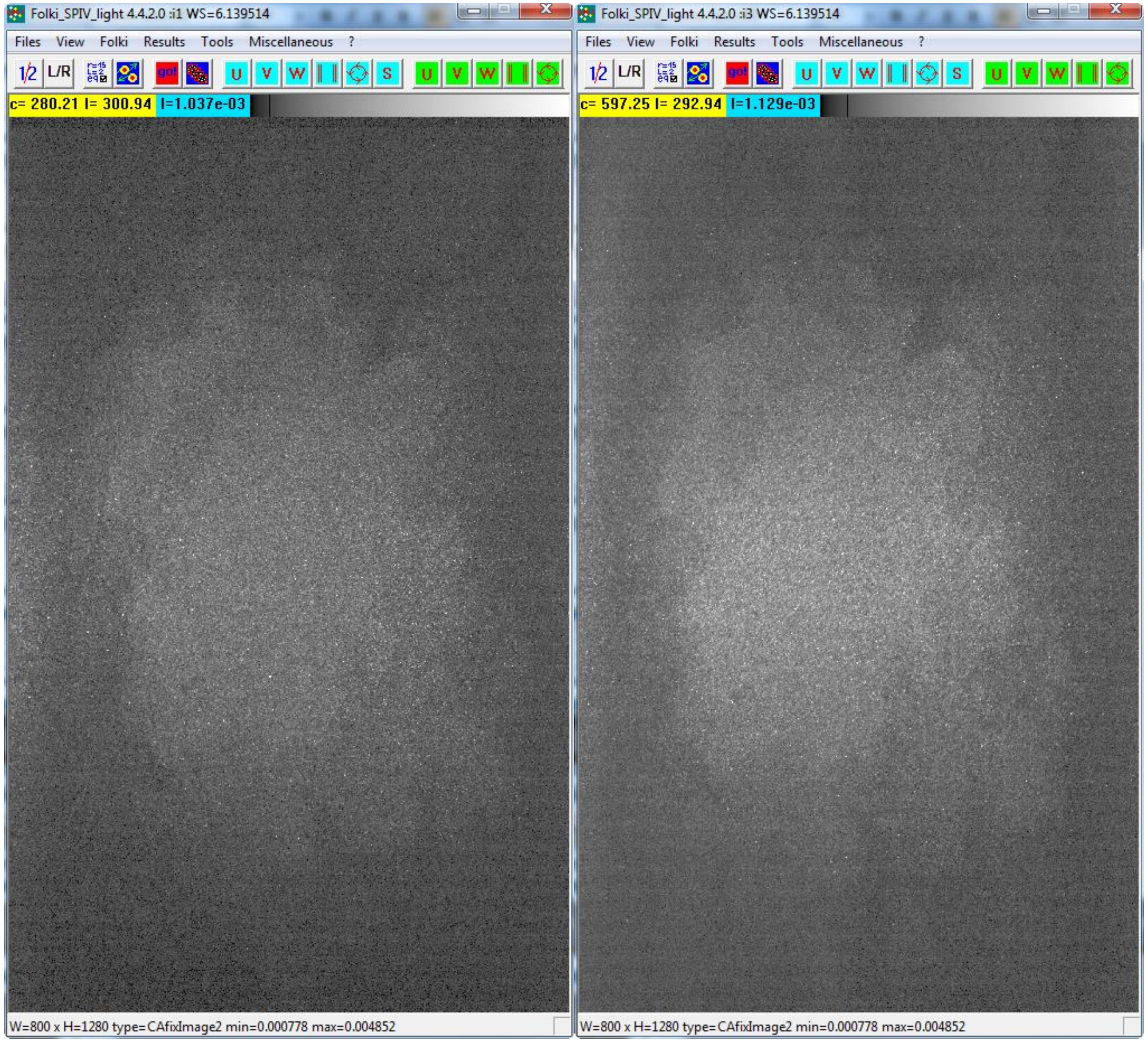
t



Stereo (2D3C) PIV

Sample images

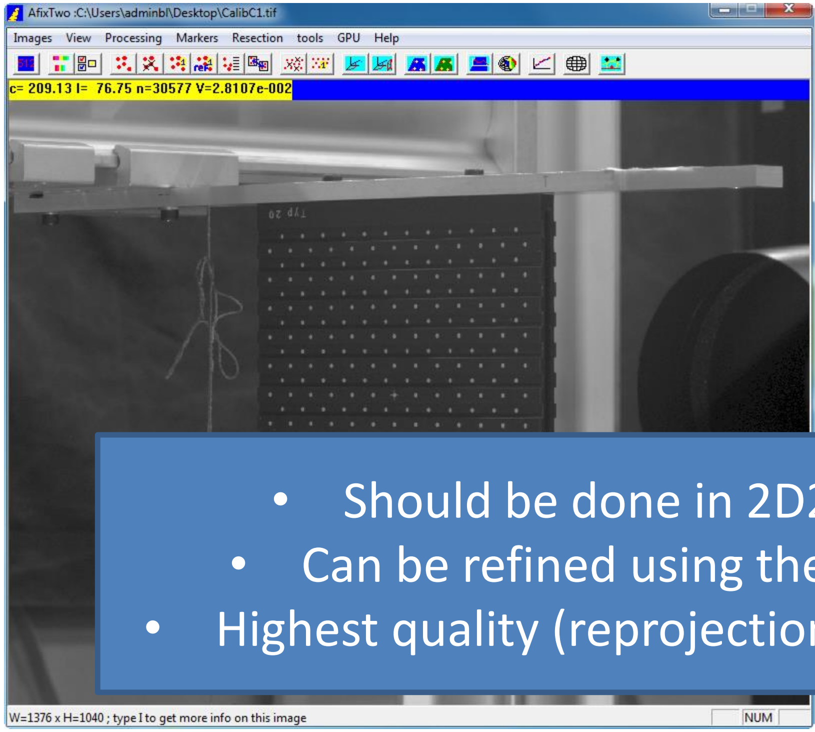
$t + dt$

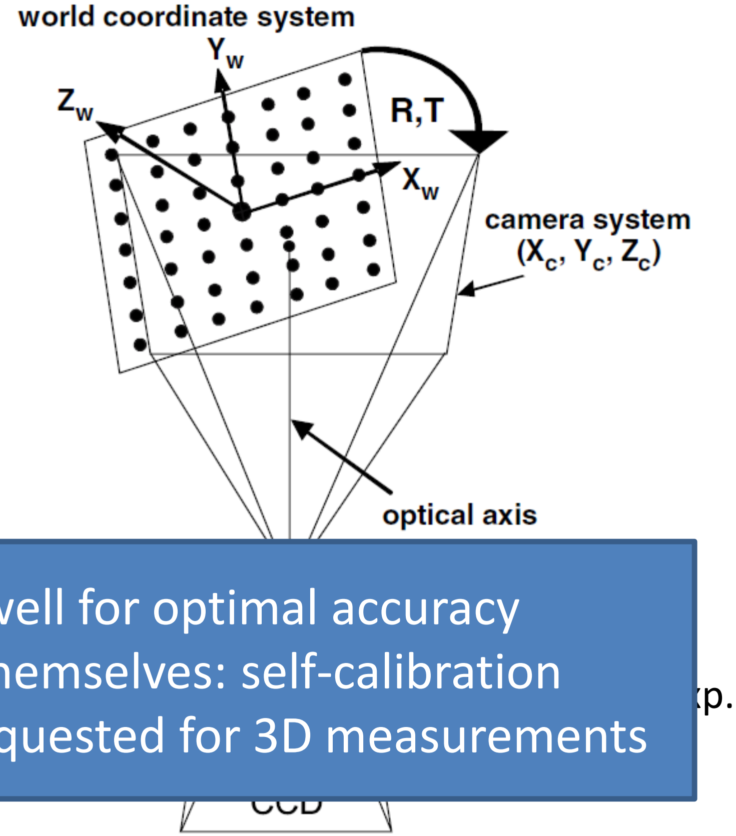


How to handle perspective viewing and obtain the final 3C displacement?

Stereo (2D3C) PIV

Calibration





- Should be done in 2D2C PIV as well for optimal accuracy
- Can be refined using the images themselves: self-calibration
- Highest quality (reprojection error) requested for 3D measurements

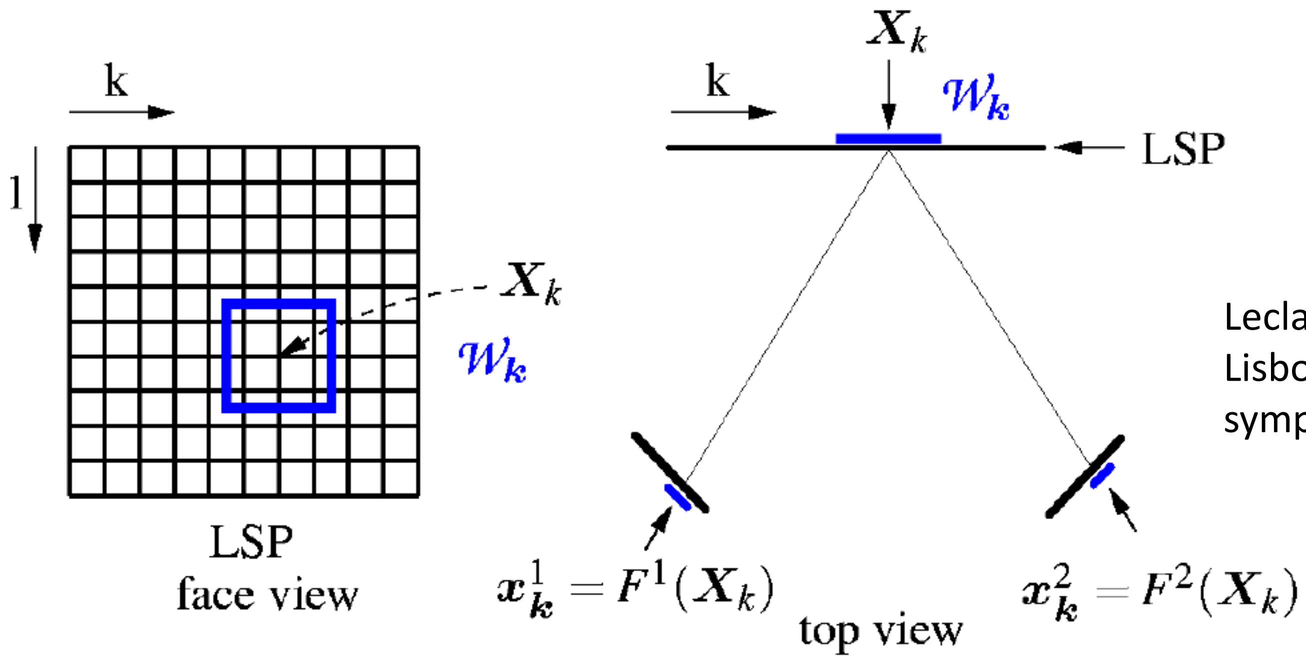
$\underline{X}$ point in 3D space, $\underline{x}$ 2D position on camera sensor

Calibration = determine parameters of camera **projection functions** $\underline{\boldsymbol{x}} = \boldsymbol{F}(\underline{\boldsymbol{X}})$
→ this is in fact stereovision / computer vision! (robotics, etc…)

Common projection models: pinhole (physical), polynomial (e.g. distorsions)

Stereo (2D3C) PIV

From 2D correlation to 2D3C displacement



Leclaire et al., Lisbon laser symposium 2012

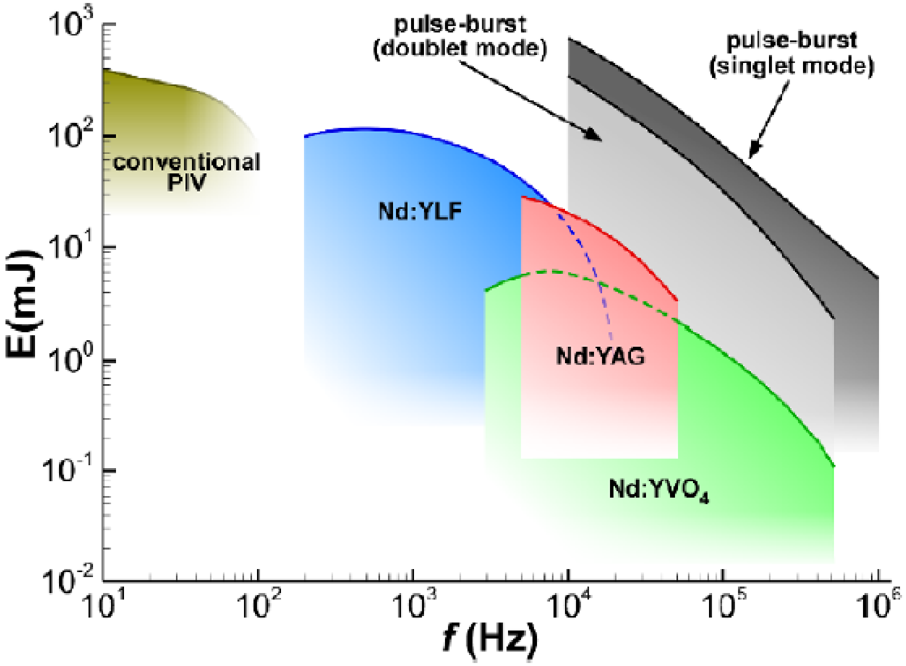
- At a given pixel k, find displacements $\underline{\Delta x}^i$ on each camera i
- 3 unknowns: components of 3C displacement $\underline{\Delta X}$, 4 data: $\underline{\Delta x} = (\underline{\Delta x}^1, \underline{\Delta x}^2)$

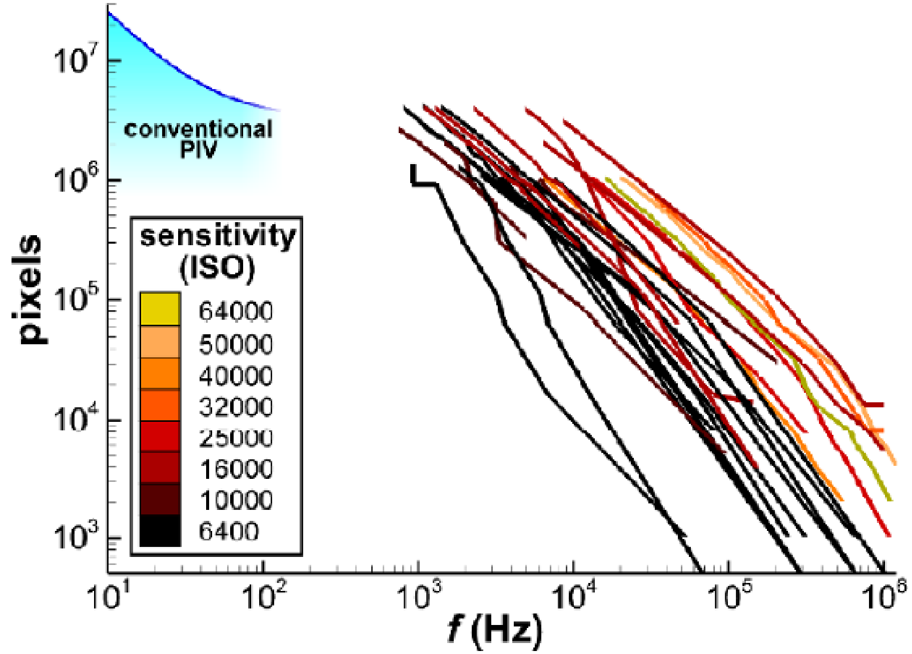
$\Rightarrow$ $\underline{\Delta X}$ found by least-squares inversion: minimization of

$$\varepsilon = \left\| \underline{\underline{\nabla F}} \cdot \underline{\Delta X} - \underline{\Delta x} \right\|$$

« Time-resolved » PIV

Beresh, Meas. Sci. Technol., 2021 (topical review on TR-PIV)





<u>Standard PIV</u>:

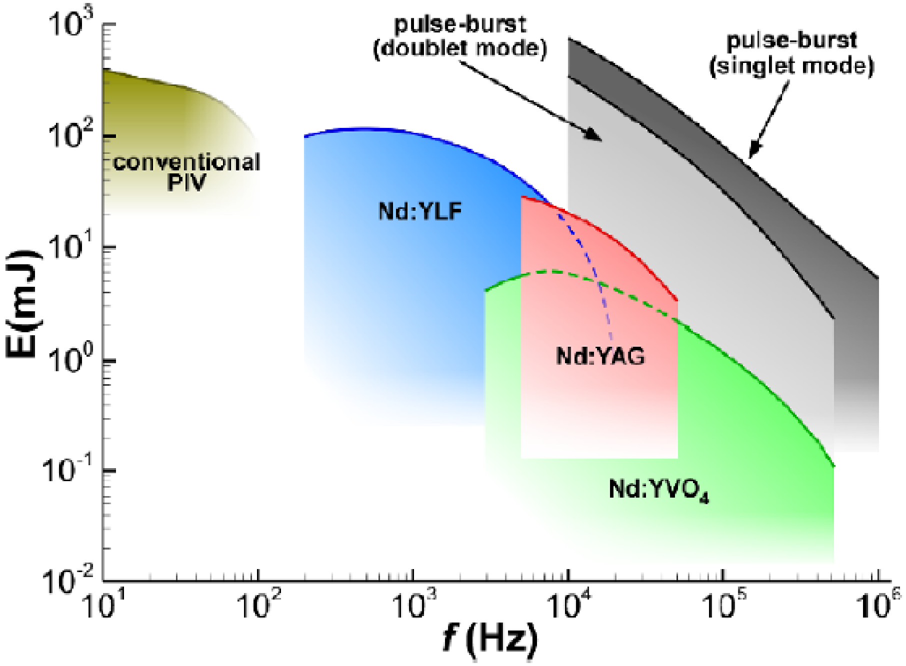
- Flow snapshots every $1 - 10\ Hz$
- Max light per pulse $\sim 400\ mJ$
- Max cam sensor size $\sim 40\ Mpix$
- Typical pixel pitch: $\sim 5 - 10\ \mu m$

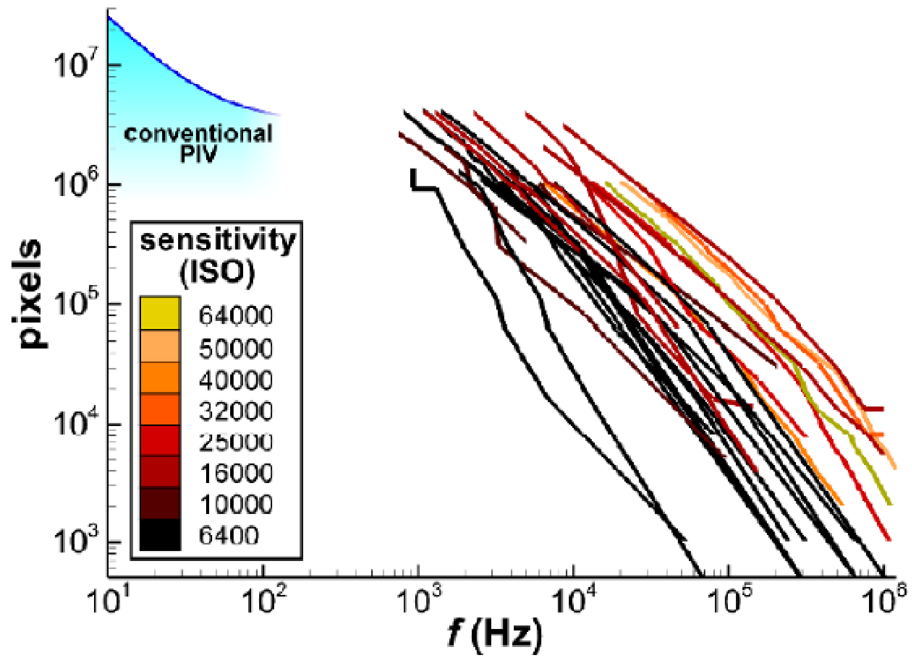
<u>High Speed (HS) PIV</u>:

- Flow snapshots every $1 - 10\ kHz$
- Max light per pulse $\sim 40\ mJ$ (decreases if frequency increases)
- Max cam sensor size $\sim 4\ Mpix$ (decreases if frequency increases)
- Typical pixel pitch: $\sim 10 - 20\ \mu m$

« Time-resolved » PIV

Beresh, Meas. Sci. Technol., 2021 (topical review on TR-PIV)





Specificities of HS-PIV to be expected:

- Lower SNR*
- Poorer spatial resolution
- More prone to peak-locking*
- Aliasing of temporal spectra

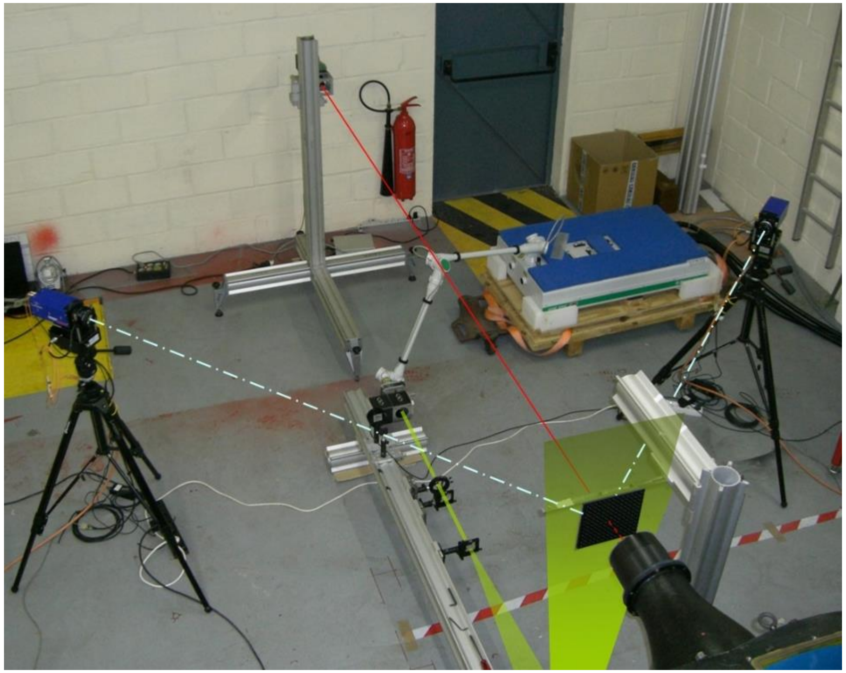
*in practice for air flows, peak-locking rather minimized through ***defocus blur*** than with diaphragm opening!

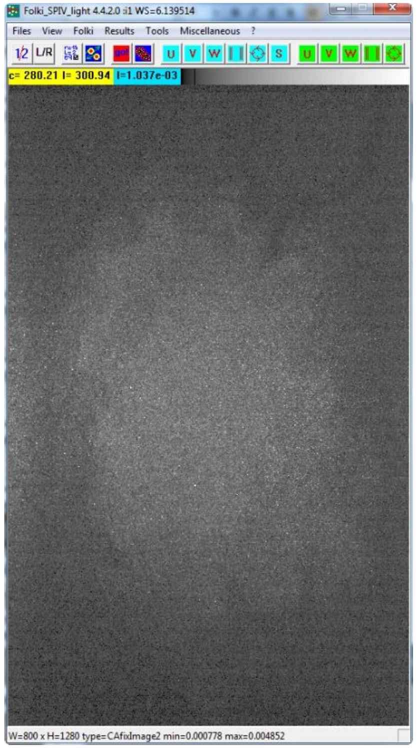
<u>High Speed (HS) PIV</u>:

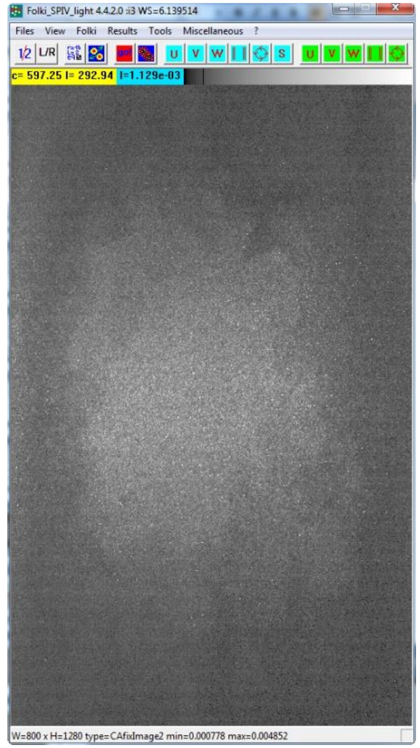
- Flow snapshots every $1-10\ kHz$
- Max light per pulse $\sim 40\ mJ$ (decreases if frequency increases)
- Max cam sensor size $\sim 4\ Mpix$ (decreases if frequency increases)
- Typical pixel pitch: $\sim 10-20\ \mu m$

High-Speed Stereo PIV

Quasi-3D turbulence characterization





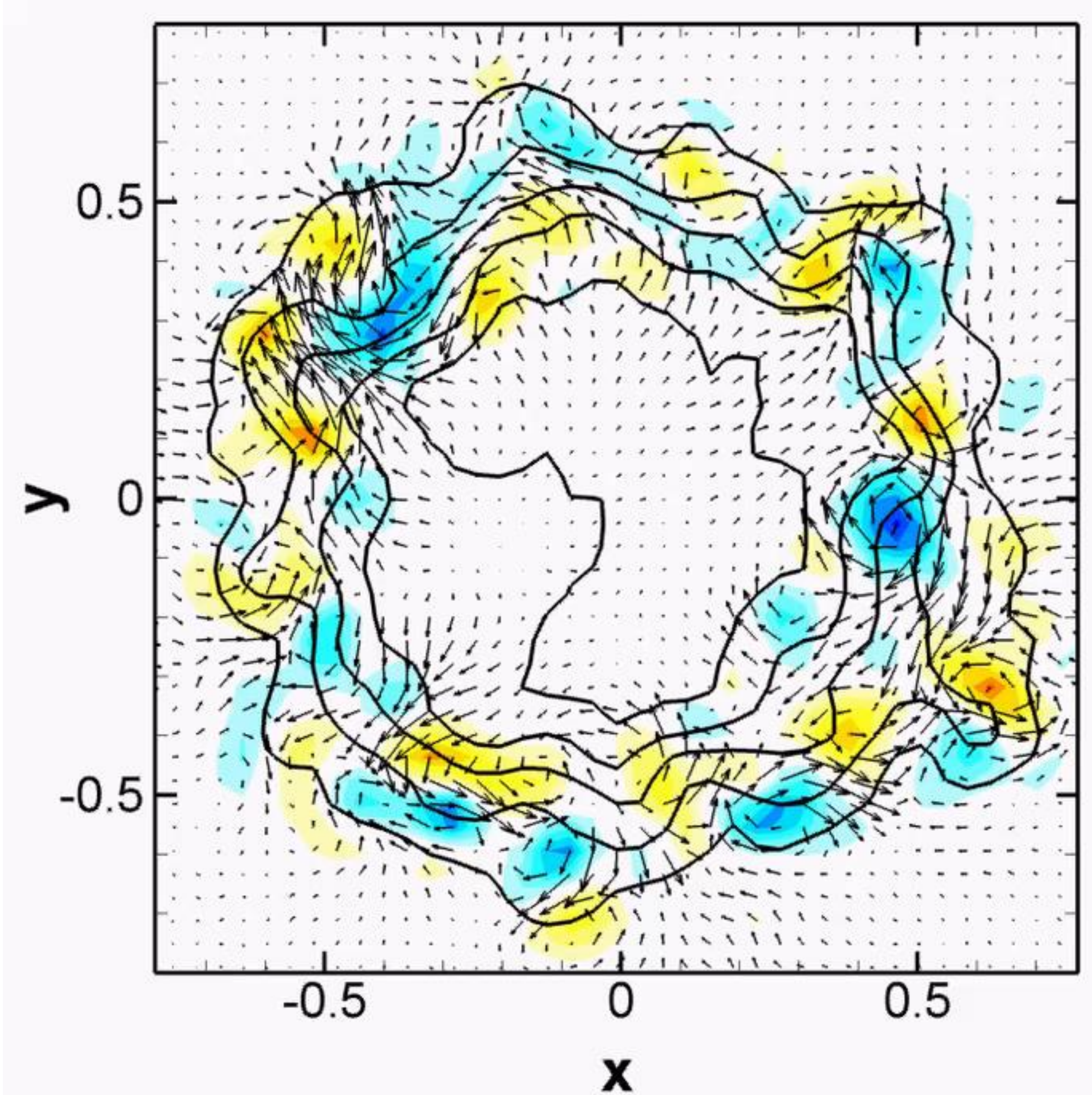


Cylindrical air jet, $Re = 2.10^5$

Davoust et al., J. Fluid Mech. 2012

High-Speed Stereo PIV

Quasi-3D turbulence characterization



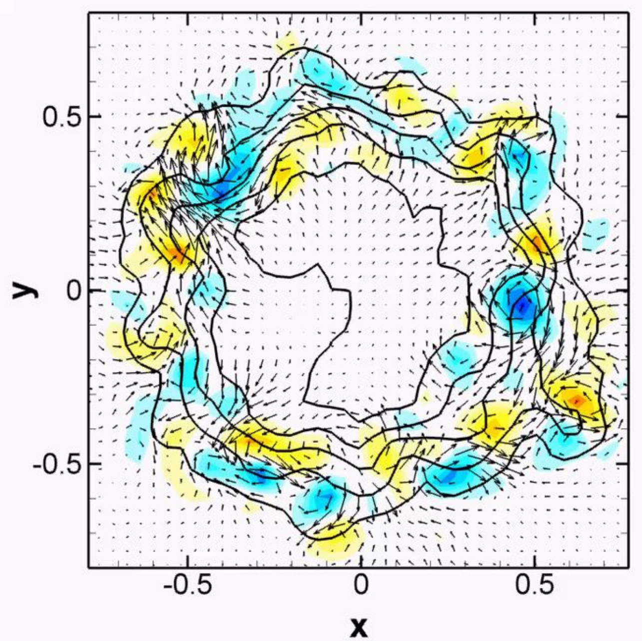
- Colored contours: axial vorticity fluctuations of opposite signs
- Arrows: fluctuation velocity vector
- Black lines: contours of (full) axial velocity

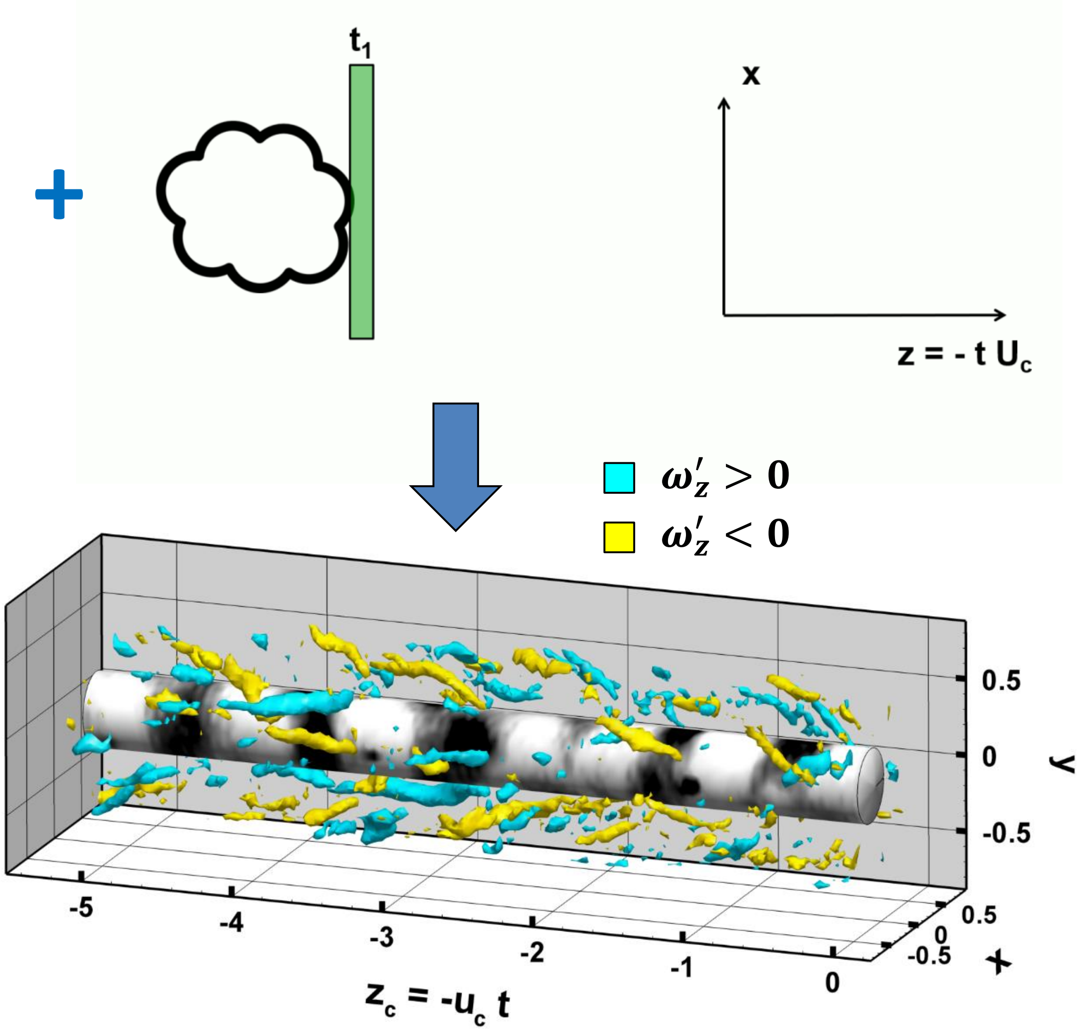
Interest in their structures due to their potential for mixing

Davoust et al., J. Fluid Mech 2012

High-Speed Stereo PIV + Taylor's hypothesis

Quasi-3D turbulence characterization





Pseudo-spatial reconstruction of streamwise vortices, and interplay with Kelvin-Helmholtz rollers

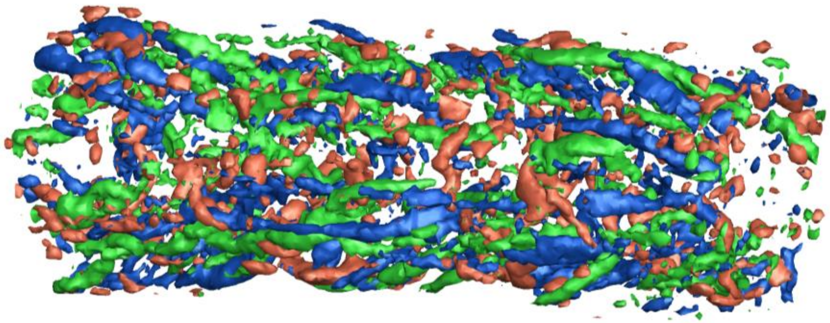
Davoust et al., J. Fluid Mech 2012

High-Speed Stereo PIV + Taylor's hypothesis

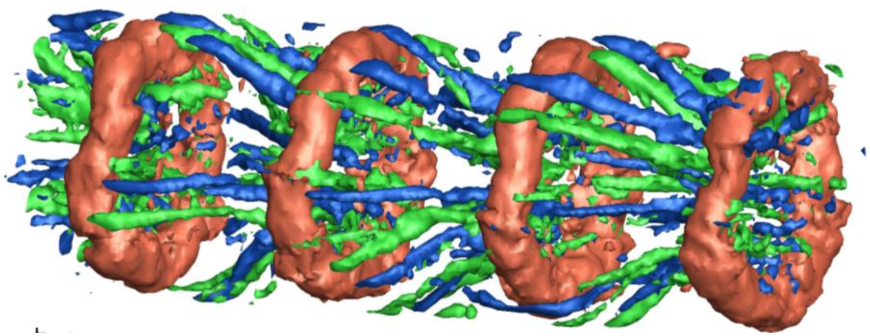
Quasi-3D turbulence characterization

- $\omega_z' > 0$
- $\omega_z' < 0$
- $\omega_\theta' > 0$

Unforced



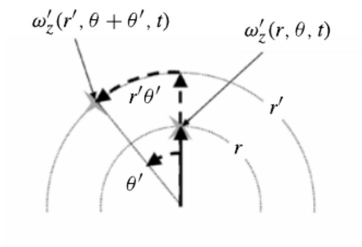
Forced



Effect of acoustic forcing with loudspeaker in wind-tunnel settling chamber (excites axisymmetric perturbation = Kelvin-Helmholtz rollers!)

Kantharaju et al., J. Fluid Mech 2020

High-Speed Stereo PIV + Taylor's hypothes



Quasi-3D turbulence characterization

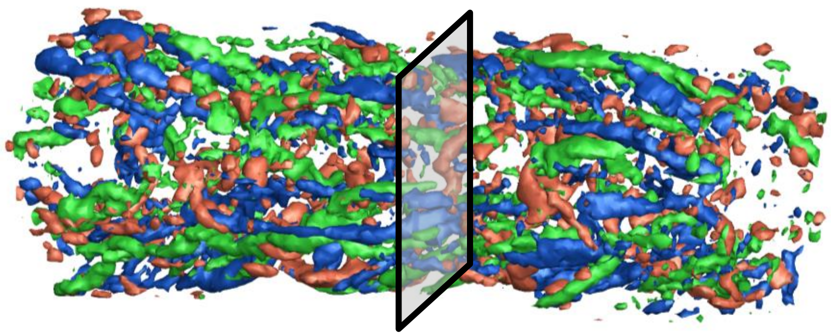
$\omega_z' > 0$

$\omega_z' < 0$

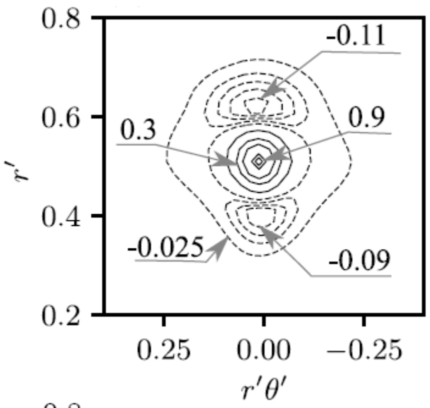
$\omega_\theta' > 0$

$$C_{\omega_z\omega_z}(r,r',\theta',t') = \frac{\langle \omega_z'(r,\theta,t)\,\omega_z'(r',\theta+\theta',t+t')\rangle_\theta}{\langle \omega_z'^2(r,\theta,t)\rangle_\theta^{1/2}\,\langle \omega_z'^2(r',\theta,t)\rangle_\theta^{1/2}}$$

Unforced



Auto-correlation of fluctuating axial vorticity

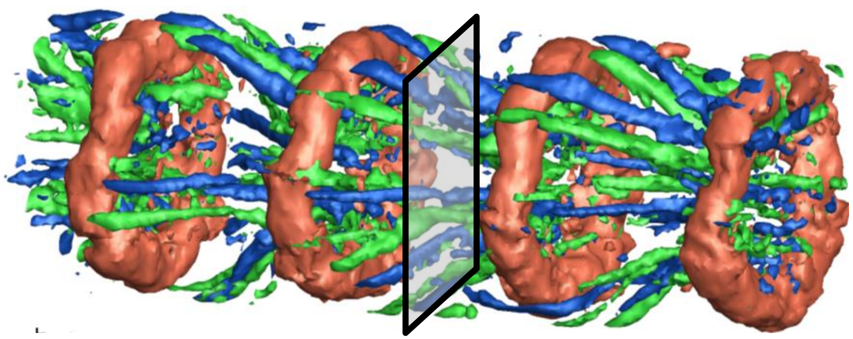


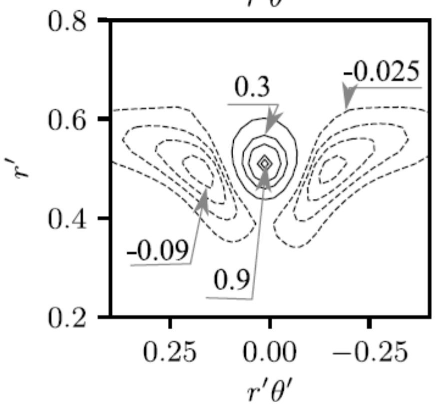
Radial stracking

$Re \sim 2.10^5$

Davoust et al. J. Fluid Mech 2012, Kantharaju et al. J. Fluid Mech. 2020

Forced





Azimuthal stacking

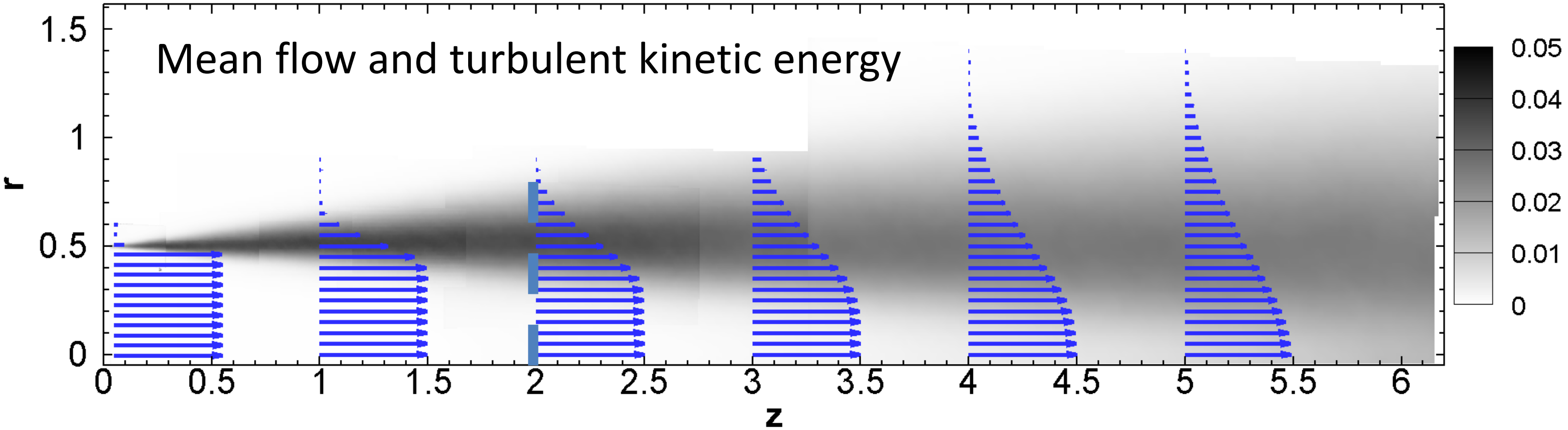
$Re \leq 0.8\ 10^4$

Previous literature
(e.g. Citriniti & George, 2000)

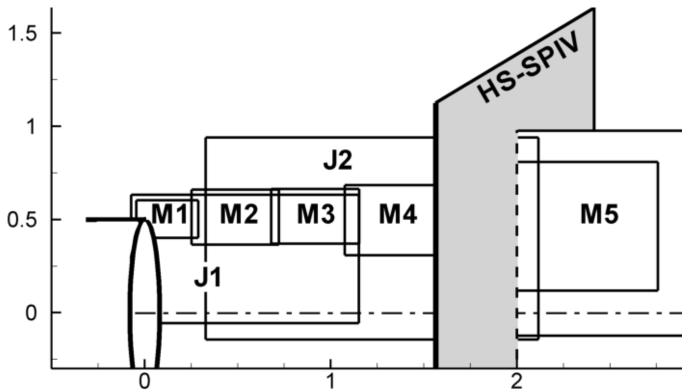
Kantharaju et al., J. Fluid Mech 2020

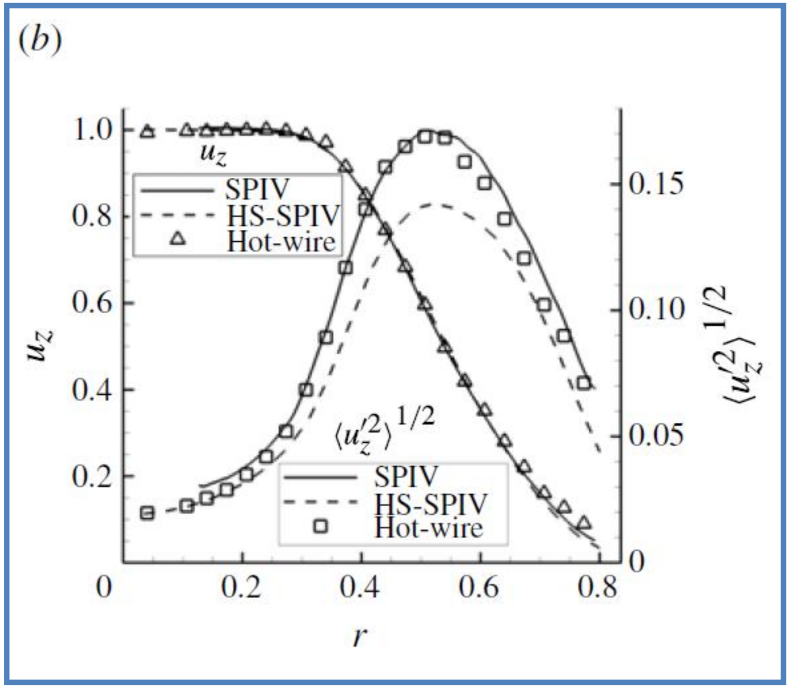
(High Speed) PIV: spatial filtering in practice

Mean and fluctuating velocities



SPIV (white)
HS-SPIV (grey): lower resolution



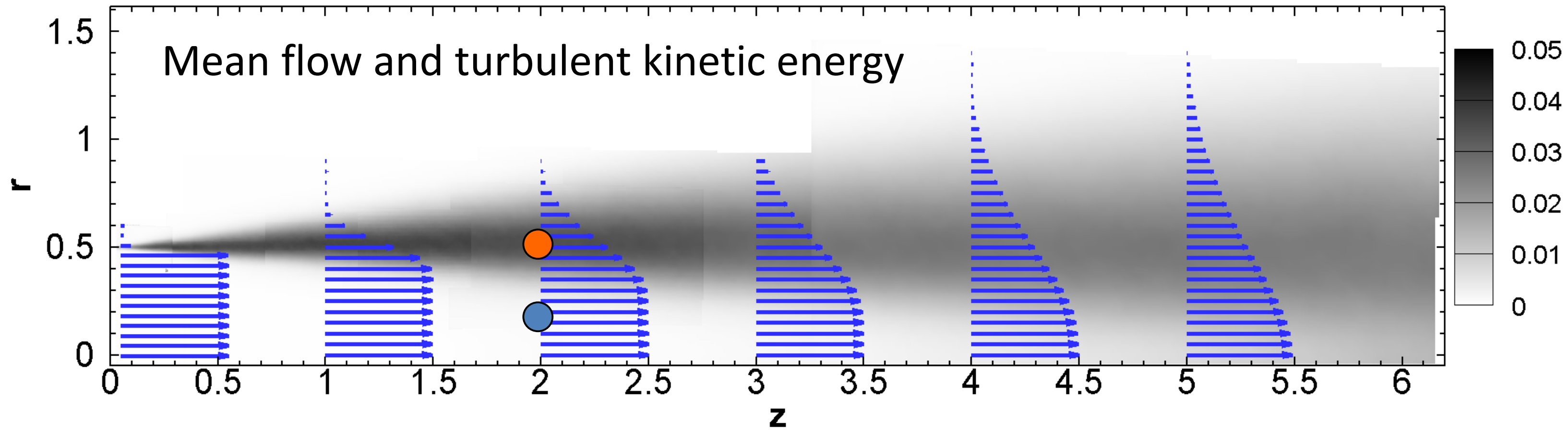


+ Hot-wire to the rescue!

Davoust et al., J. Fluid Mech 2012

High Speed PIV vs. Time-Resolved PIV

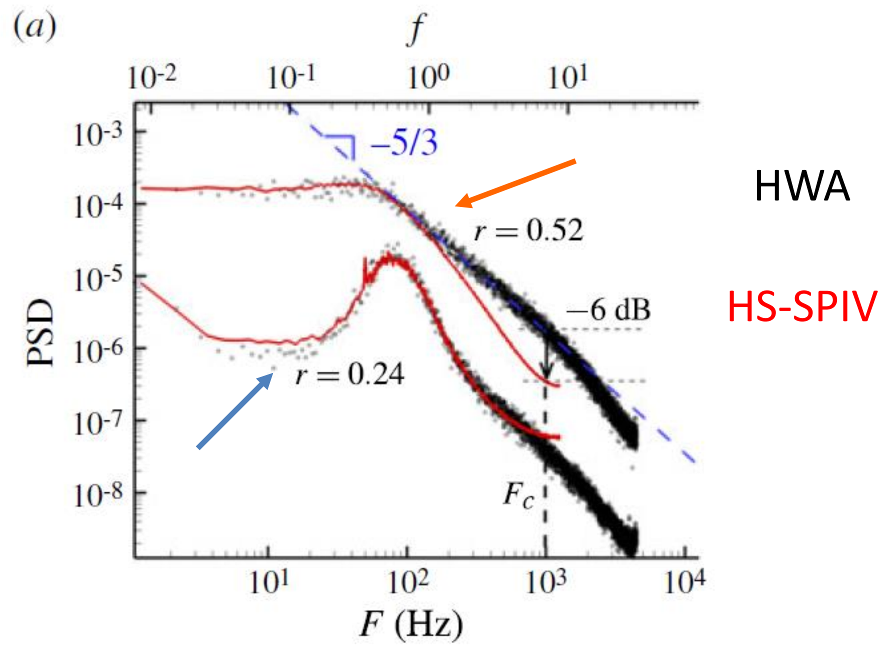
Temporal spectra: aliasing?



Acquisition at 2.5 kHz while spectral content beyond: aliasing was expected…

…but here: spatial **filtering acted as a temporal filter as well** (thanks to **turbulence**)!

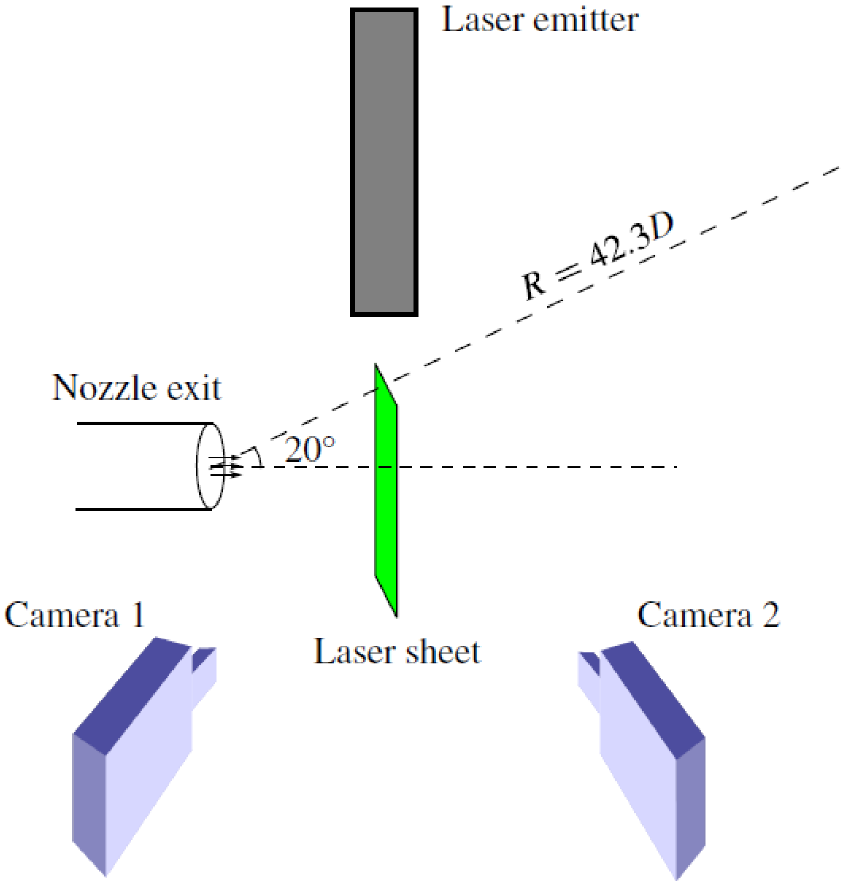
Calibration of frequency cut-off of HS-PIV (in this experiment) thanks to HWA

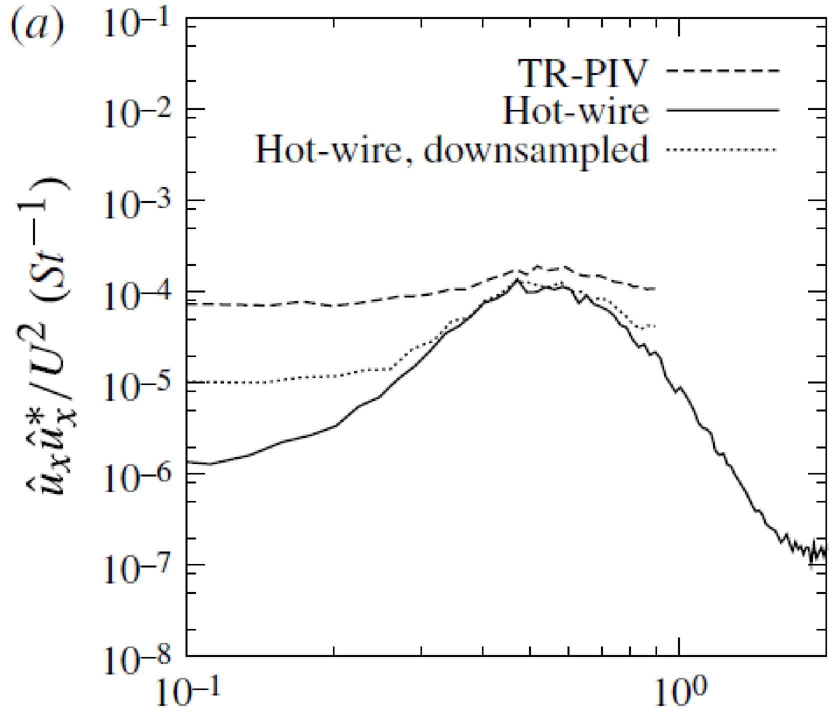


Davoust et al., J. Fluid Mech 2012

High Speed PIV vs. Time-Resolved PIV

Temporal spectra: aliasing





Axial velocity spectra @2D, jet centreline

Cylindrical air jet, higher Mach number

Cavalieri et al., J. Fluid Mech. 2013

I. Seeding and image formation

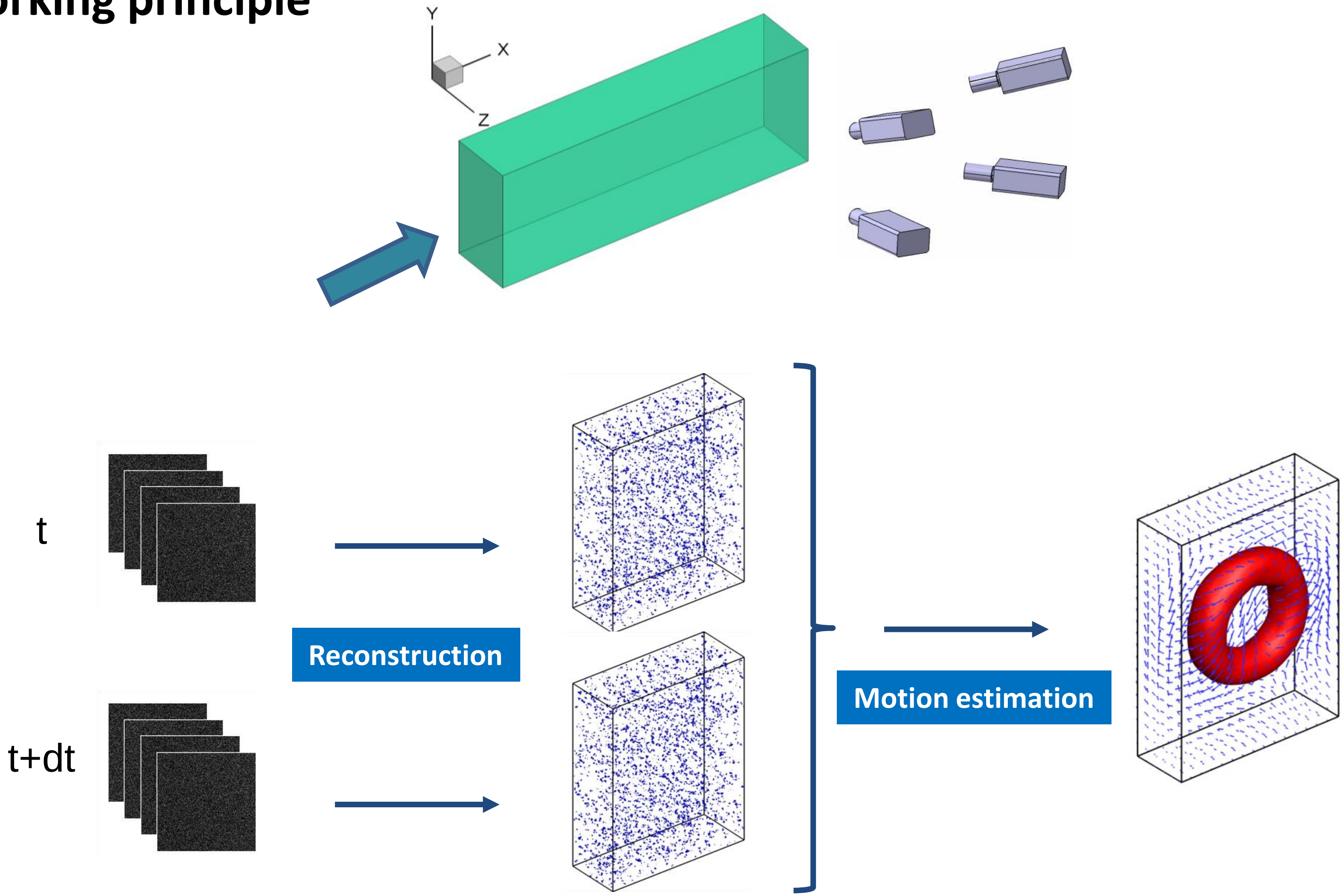
II. Basics: 2D, two-component (2D2C) PIV

III. Towards more complexity: Stereo PIV, Time-Resolved PIV

IV. Volumetric and Tracking approaches, and beyond

Volumetric methods

Working principle



Volumetric methods

New difficulties #1

Thicker light sheet in 3D
(1-2 cm, vs. 1-2 mm in 2D), with:

- Same hardware

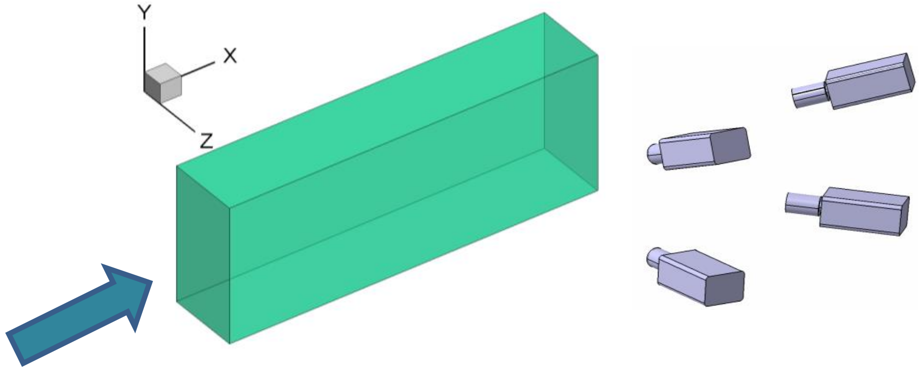
⇒ **lower SNR**

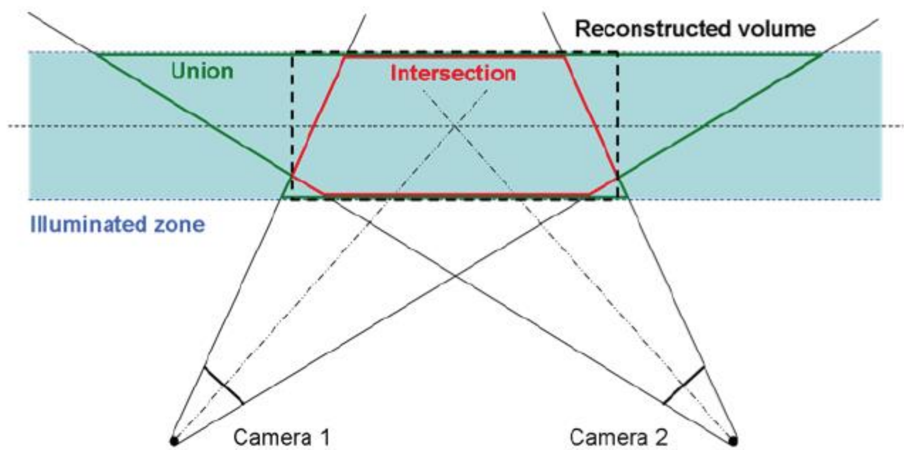
- Comparable / slightly inferior image seeding density

⇒ **lower volumetric particle concentration**

- A multi-camera system (minimum of 4 advised):

⇒ **more geometric constraints: some illuminated zones not viewed by all cams!**



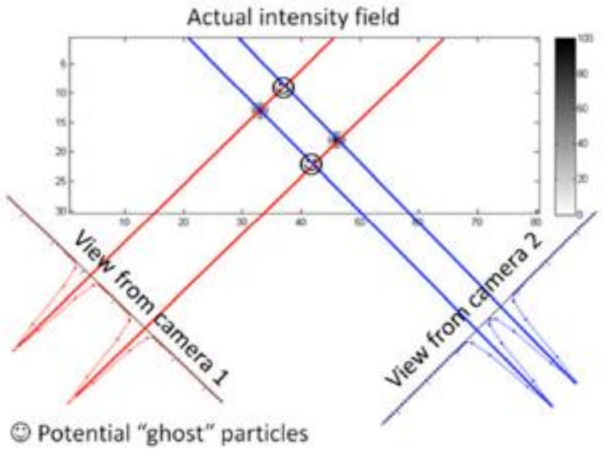


Cheminet, PhD Univ. Paris-Scalay, 2016

Volumetric methods

New difficulties #2: ghost particles

Scarano, Meas. Sci. Technol. 2013

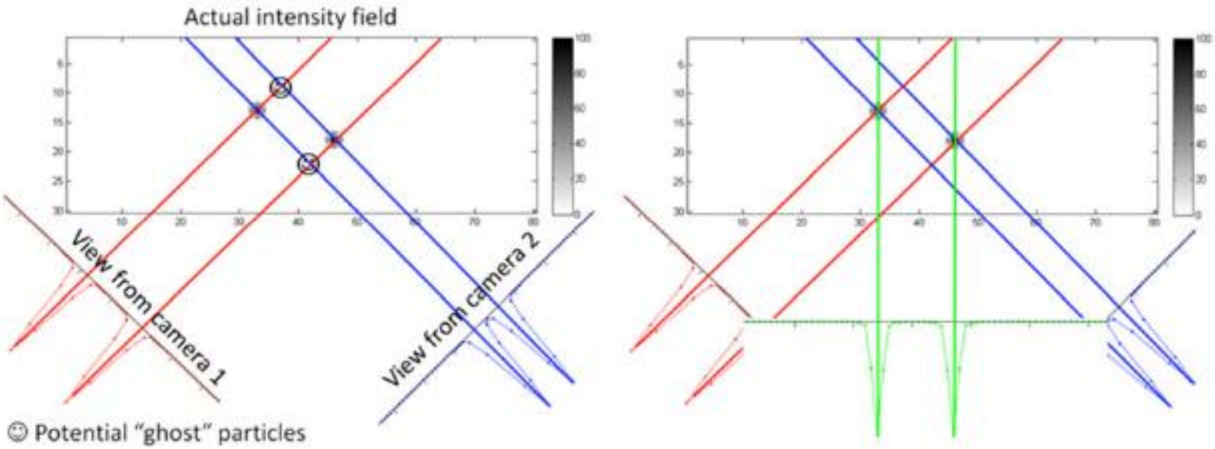


<u>Number of ghosts</u>:

Volumetric methods

New difficulties #2: ghost particles

Scarano, Meas. Sci. Technol. 2013



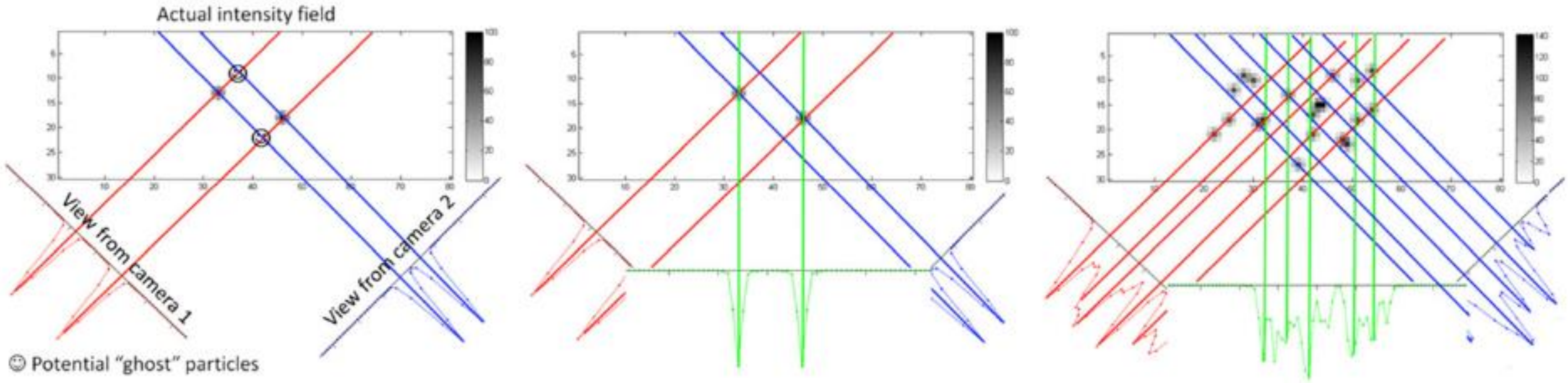
Number of ghosts:

- decreases with number of cameras (but they are expensive! **Trade-off**: 4 cams)

Volumetric methods

New difficulties #2: ghost particles

Scarano, Meas. Sci. Technol. 2013



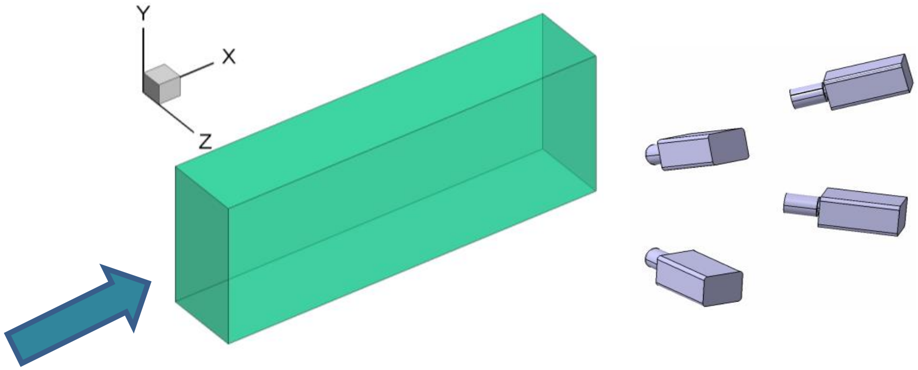
Number of ghosts:

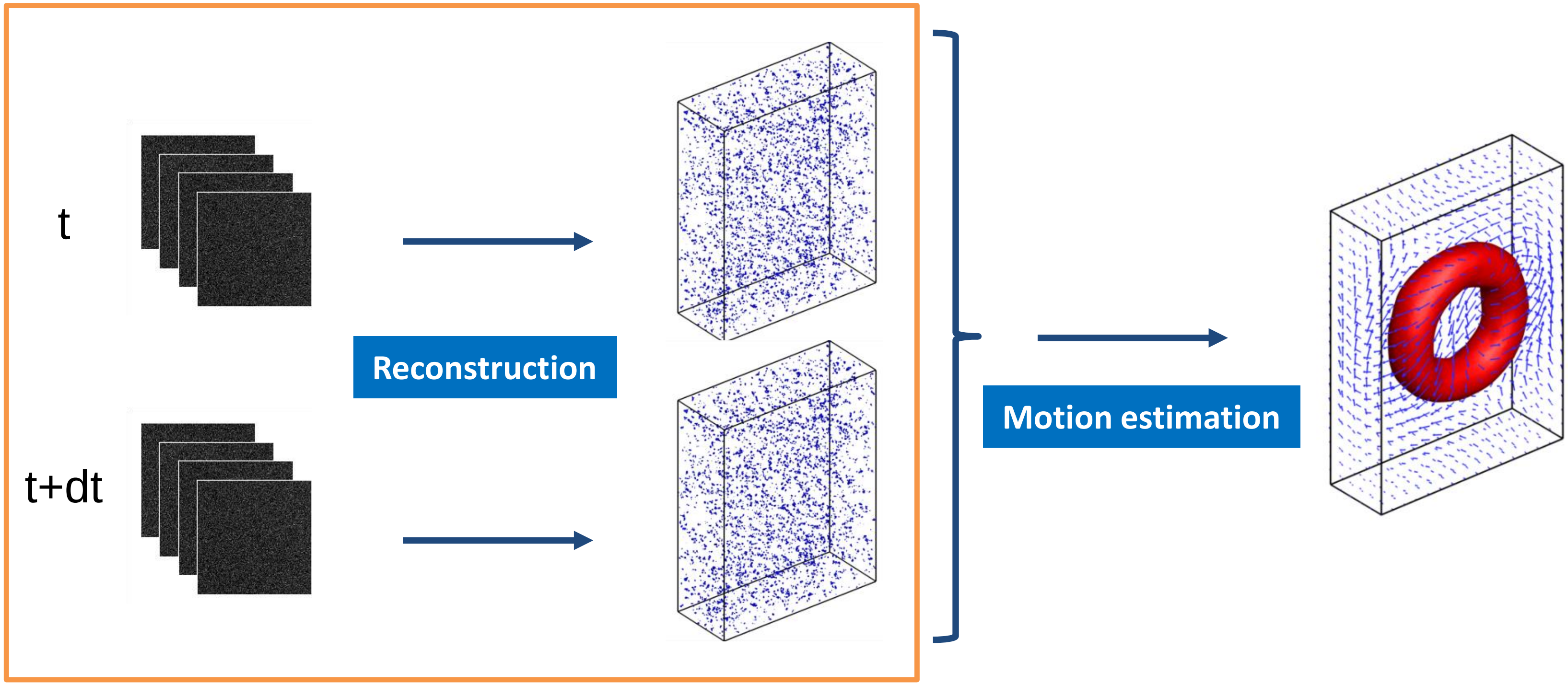
- decreases with number of cameras (but they are expensive! Trade-off: 4 cams)
- increases with particle concentration (**a problem for turbulent flows!**)

Strategies to limit their number:

- At each instant separately: exploit their differences wrt true particles (e.g. intensity, usually inferior)
- Exploit temporal context: t and $t + dt$ (or beyond: Lagrangian Particle Tracking, see later)

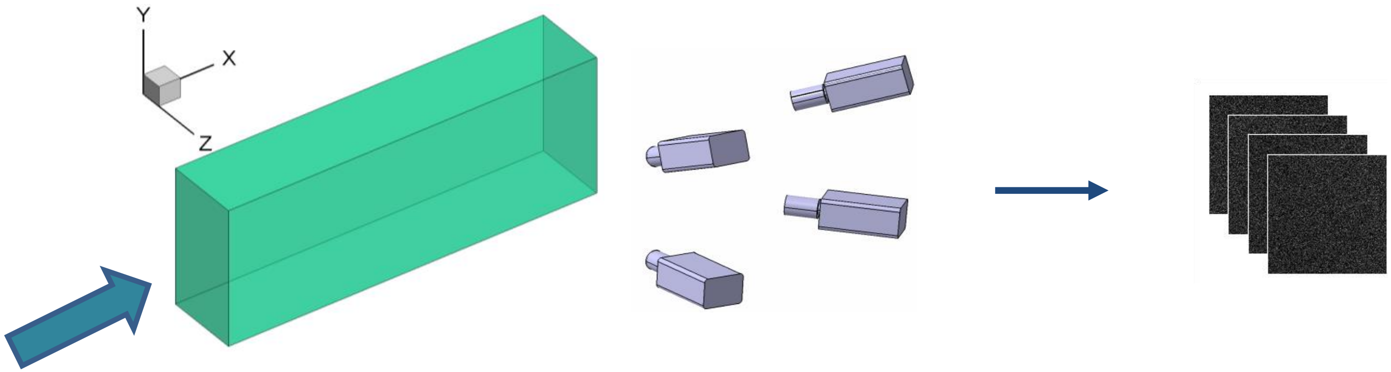
Volumetric methods





Volumetric methods

Reconstruction step = *invert image formation* (= direct problem)

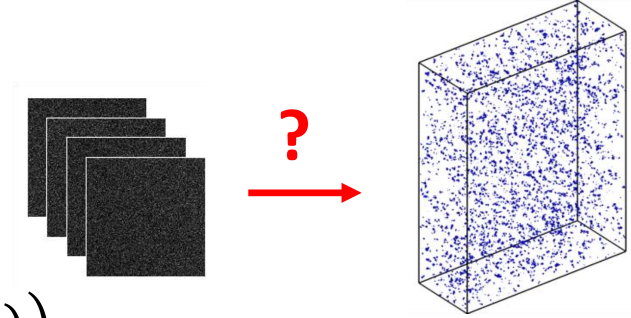


P particles, of intensities E_p, located at $\underline{X}_p$
Grey level I at pixel position $\underline{x}$ on a camera (projection function F):

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

$h(\underline{x})$: Point Spread Function / Optical Transfer Function: *models diffraction-limited imaging (Gaussian integrated over the pixel)*

Volumetric methods



$\underline{X}_p, E_p$?

Reconstruction: particles in 3D from multi-view images

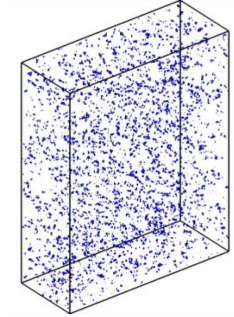
$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F(\underline{X}_p)\right)$$

2 strategies: 3D / Tomo-PIV, and 3D PTV

3D PIV

Reconstruction: particles in 3D from multi-view images

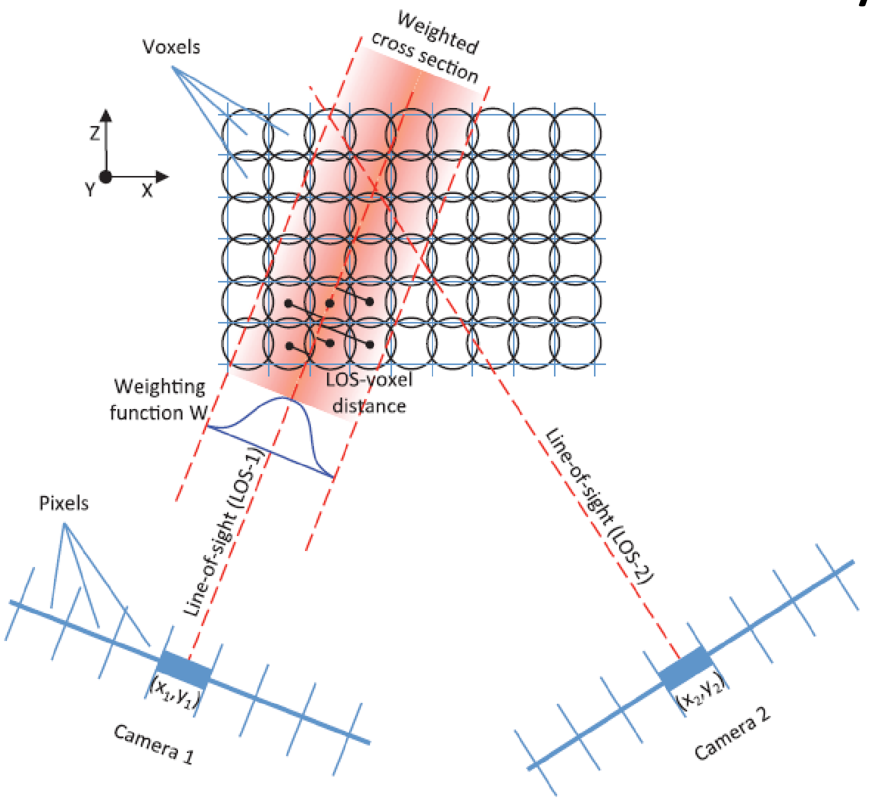
?



$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

$\underline{X}_p\,,\, E_p$?

3D PIV / Tomo-PIV

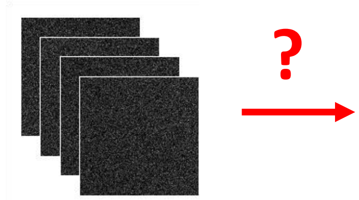


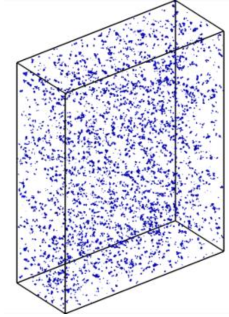
Scarano, Meas. Sci. Technol. 2013

- 3D space discretized in voxels, size ~ back-projected pixel ⇒ $\boldsymbol{I = WE}$

3D PIV

Reconstruction: particles in 3D from multi-view images

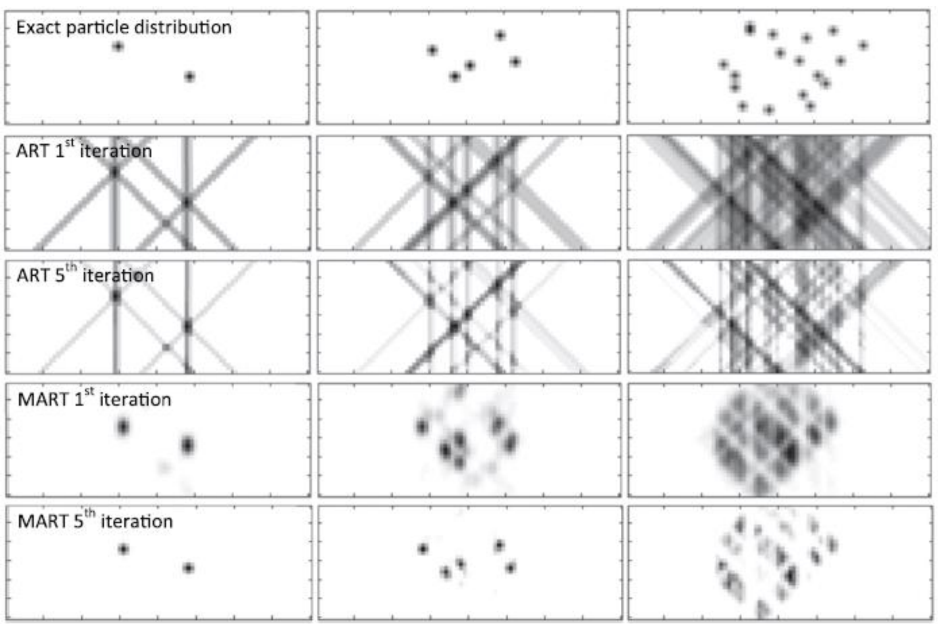




$\underline{X}_p$, E_p ?

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

3D PIV / Tomo-PIV

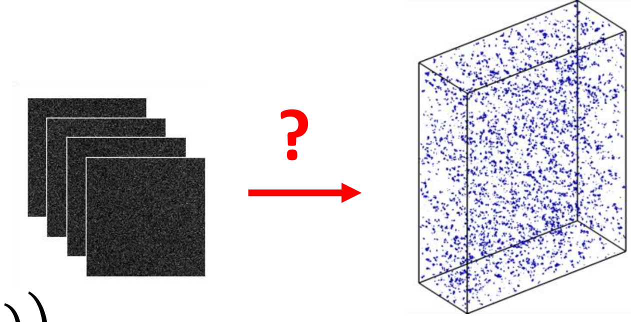


- 3D space discretized in voxels, size ~ back-projected pixel ⇒ $\boldsymbol{I = WE}$
- **Tomographic reconstruction = iteratively solving this underdetermined linear system**

Scarano, Meas. Sci. Technol. 2013

3D PIV

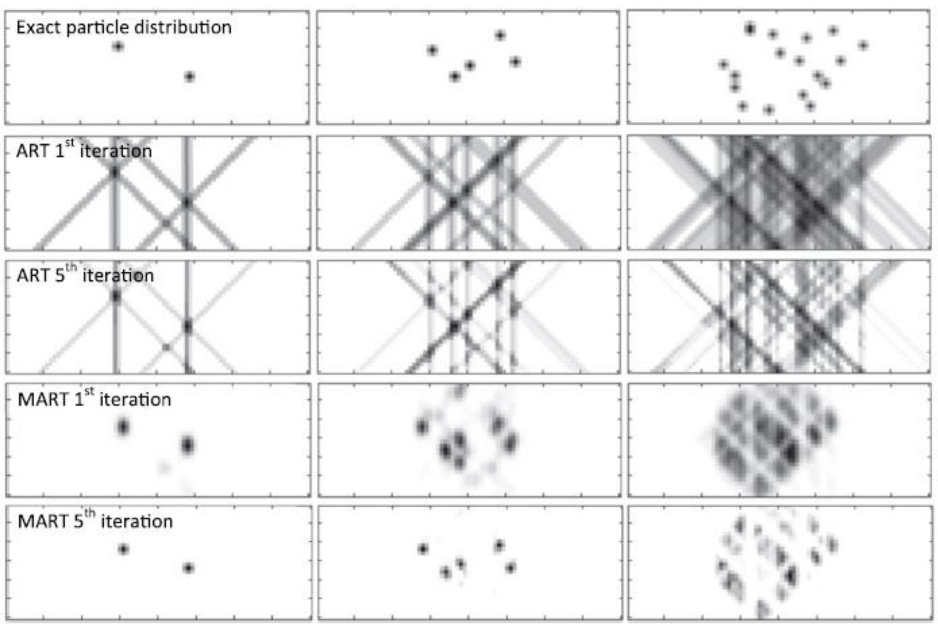
Reconstruction: particles in 3D from multi-view images



$\underline{X}_p\,,\,E_p$?

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

3D PIV / Tomo-PIV

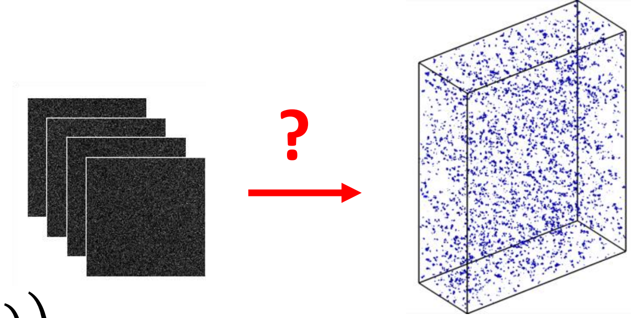


- 3D space discretized in voxels, size ~ back-projected pixel ⇒ $\boldsymbol{I = WE}$
- **Tomographic reconstruction = iteratively solving this underdetermined linear system**
- Particles represented as **intensity blobs** on a 3D grid, (« blobs »: because spread over several neighboring voxels)

Scarano, Meas. Sci. Technol. 2013

3D PIV

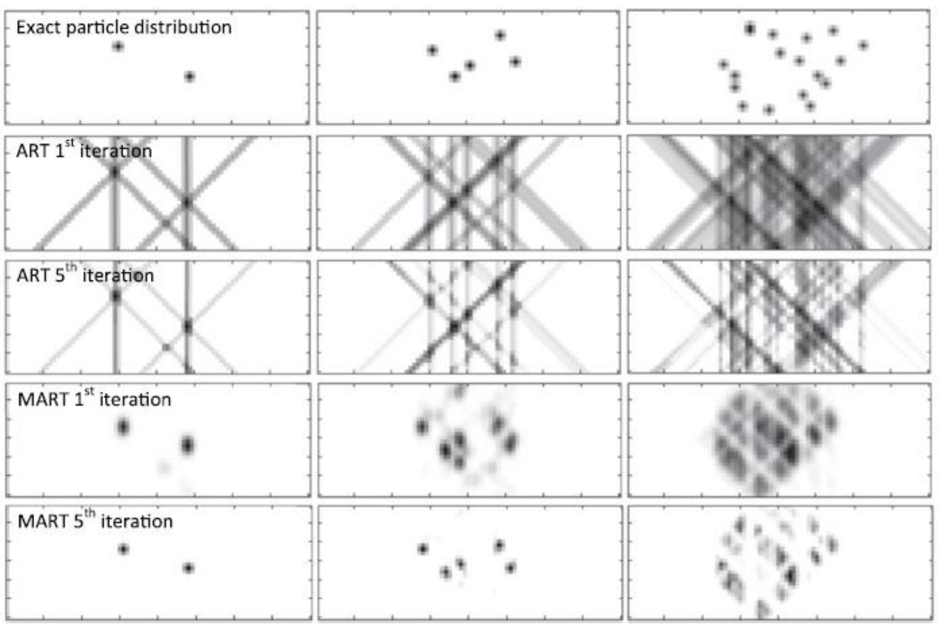
Reconstruction + motion estimation



$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

$\underline{X}_p\,,\, E_p$?

3D PIV / Tomo-PIV



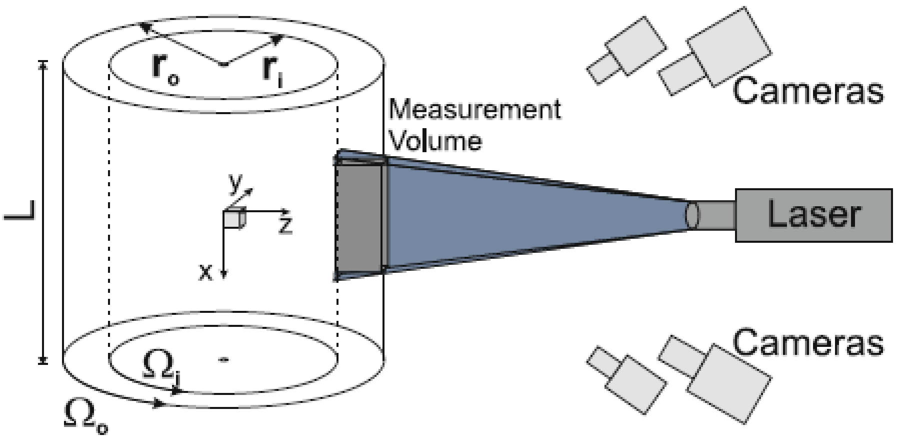
Scarano, Meas. Sci. Technol. 2013

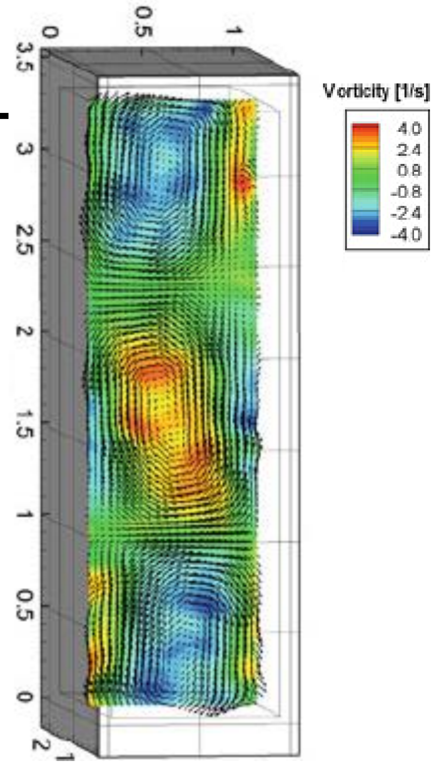
- 3D space discretized in voxels, size ~ back-projected pixel ⇒ $\boldsymbol{I = WE}$
- **Tomographic reconstruction = iteratively solving this underdetermined linear system**
- Particles represented as **intensity blobs** on a 3D grid, (« blobs »: because spread over several neighboring voxels)

⇒ displacement estimation can be done by **3D correlation** (Interrogation Volumes, instead of Windows)

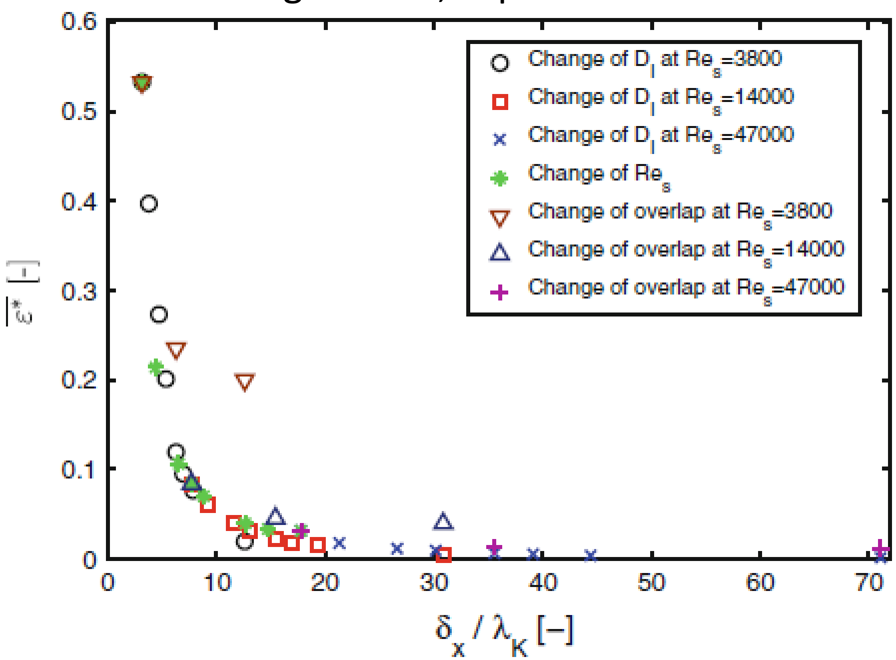
3D PIV

An example in turbulence





Tokgoz et al., Exp. Fluids 2012

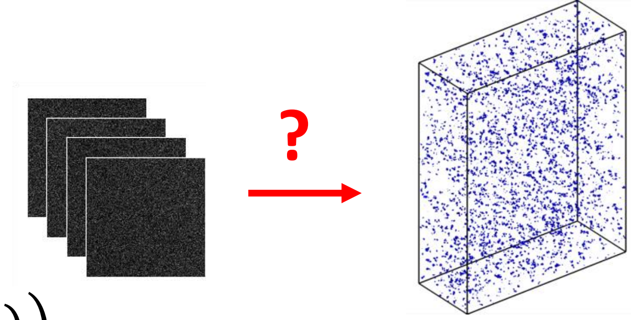


- λ_K Kolmorogov scale
- δ_X ~ interrogation volume size
- $\overline{\varepsilon}^*$ dissipation rate normalized by the actual value (torque measurement)

3D correlation: **(very) significant filtering of spatial scales!**
→ Due to lower concentration of particles than in 2D, whereas still a minimum of particles in the interrogation volume needed!

3D PTV

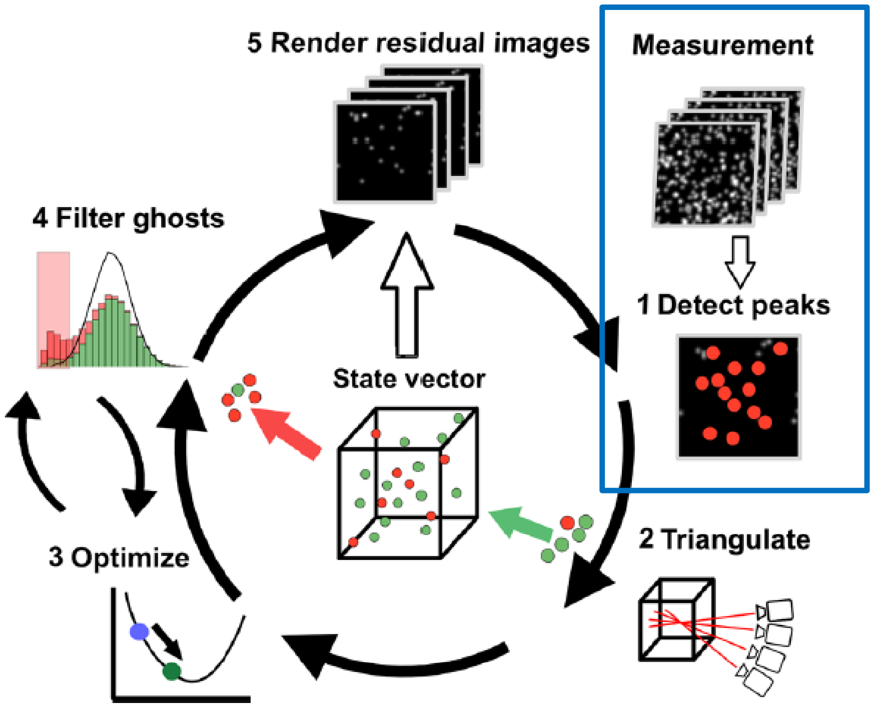
Reconstruction: particles in 3D from multi-view images



$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

$\underline{X}_p, E_p$?

3D PTV



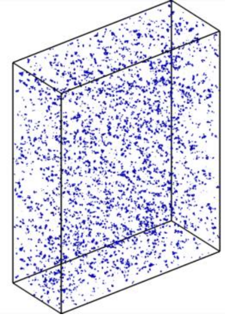
Enhanced IPR, Jahn et al., Exp. Fluids 2021

- **Locate** particle positions in the images

Subpixel accuracy guaranteed by the 2 – 3 pixel image size!

3D PTV

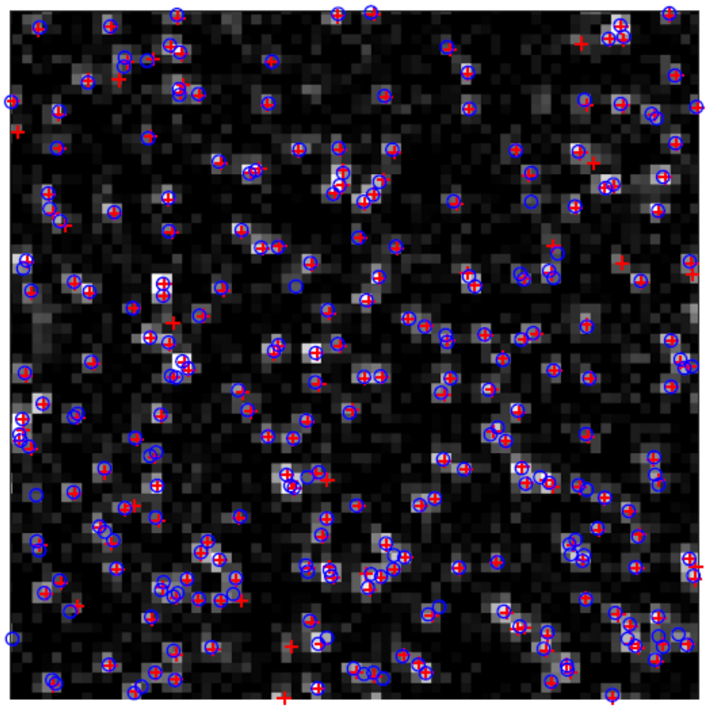
Reconstruction: particles in 3D from multi-view images



$\underline{X}_p, E_p$?

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

3D PTV



PIR_{NNLS}, Cheminet et al., Meas. Sci. Technol. 2018

- **Locate** particle positions in the images

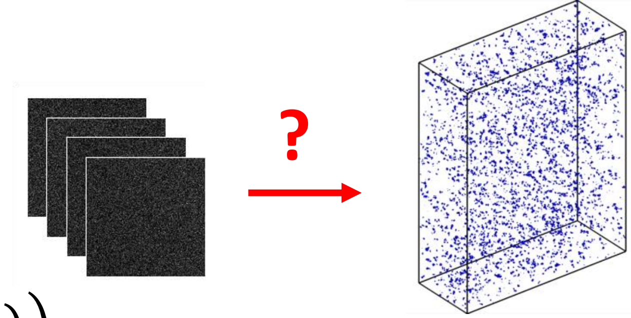
Necessary to handle **large image densities / important image overlap**, otherwise max volumetric density limited!

⇒ *Advanced methods using:*

Image formation physics

3D PTV

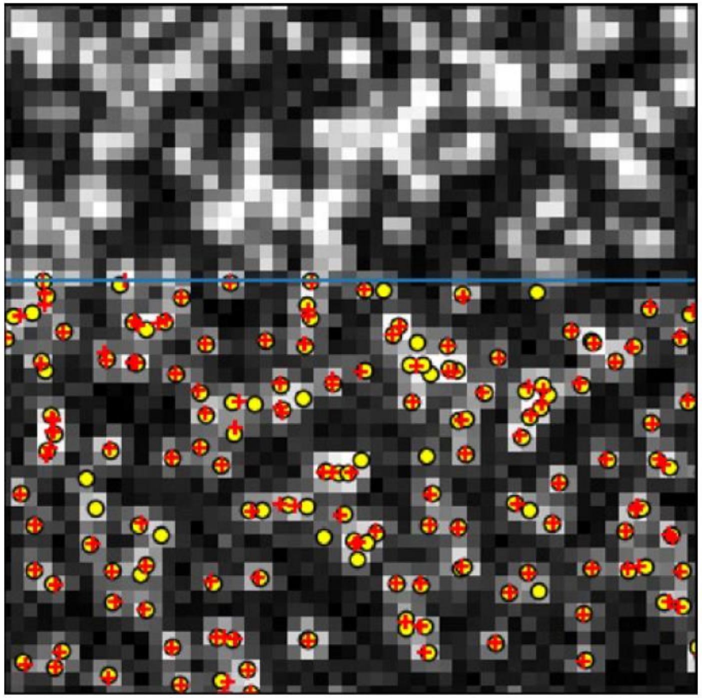
Reconstruction: particles in 3D from multi-view images



$\underline{X}_p\,, E_p$?

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

3D PTV



$Peak - CNN$, Godbersen et al., Exp. Fluids 2024

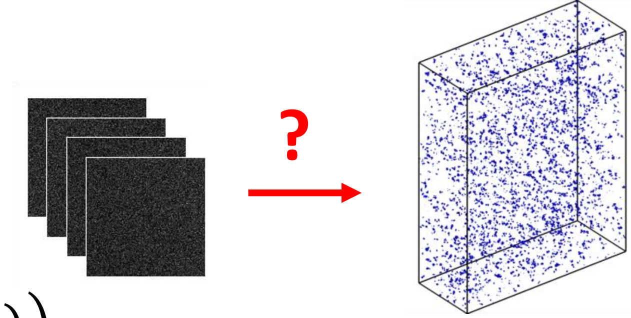
- **Locate** particle positions in the images

Necessary to handle **large image densities / important image overlap**, otherwise max volumetric density limited!

⇒ *Advanced methods using:*

Learned image formation

3D PTV

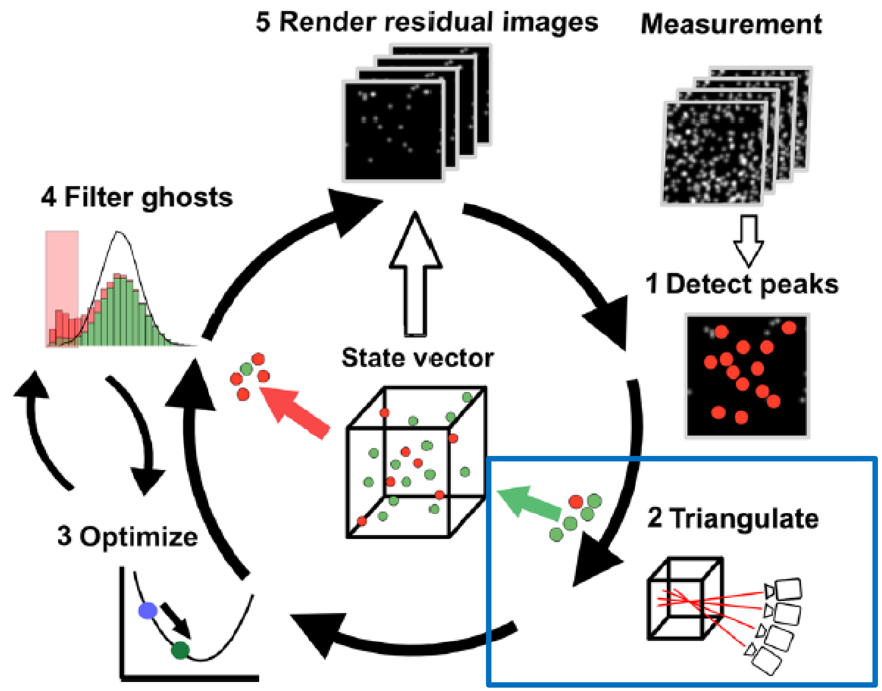


$\underline{X}_p$, E_p ?

Reconstruction: particles in 3D from multi-view images

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$

3D PTV



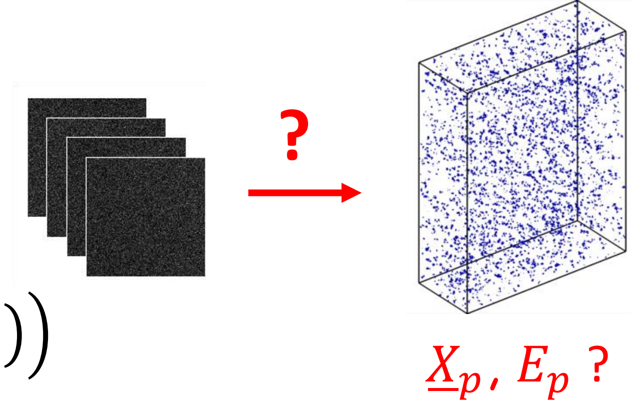
Enhanced IPR, Jahn et al., Exp. Fluids 2021

- **Locate** particle positions in the images
- **Triangulate**:

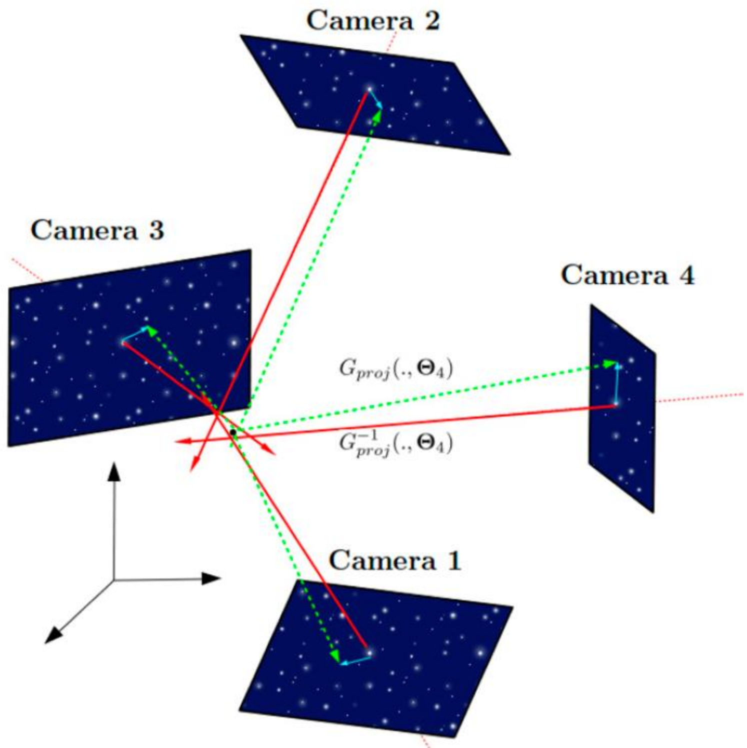
3D PTV

Reconstruction: particles in 3D from multi-view images

$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F\left(\underline{X}_p\right)\right)$$



3D PTV

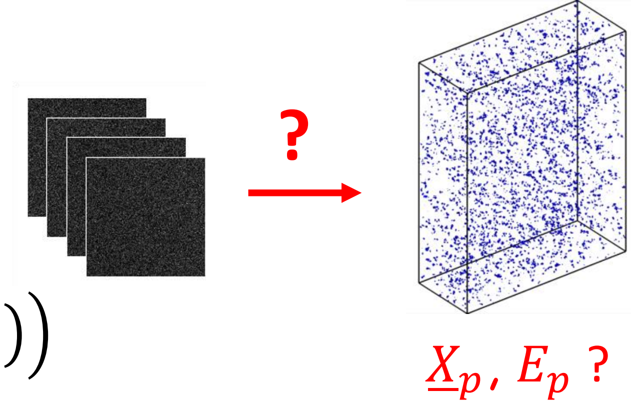


Cornic et al., Meas. Sci. Technol. 2016

- **Locate** particle positions in the images
- **Triangulate**:
  - Back-project (ray tracing) particle positions to volume
  - 3D particle positions are in the centre of zones where 4 rays are close to crossing (never cross exactly: **residual calibration errors!**)

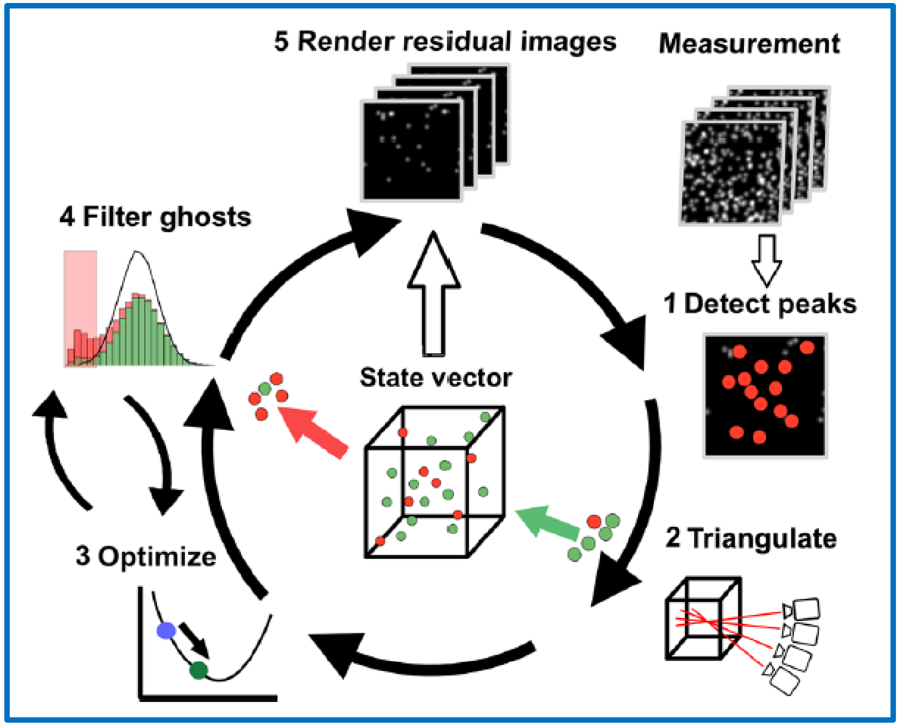
3D PTV

Reconstruction: particles in 3D from multi-view images



$$I(\underline{x}) = \sum_{p=1}^{P} E_p h\left(\underline{x} - F(\underline{X}_p)\right)$$

3D PTV

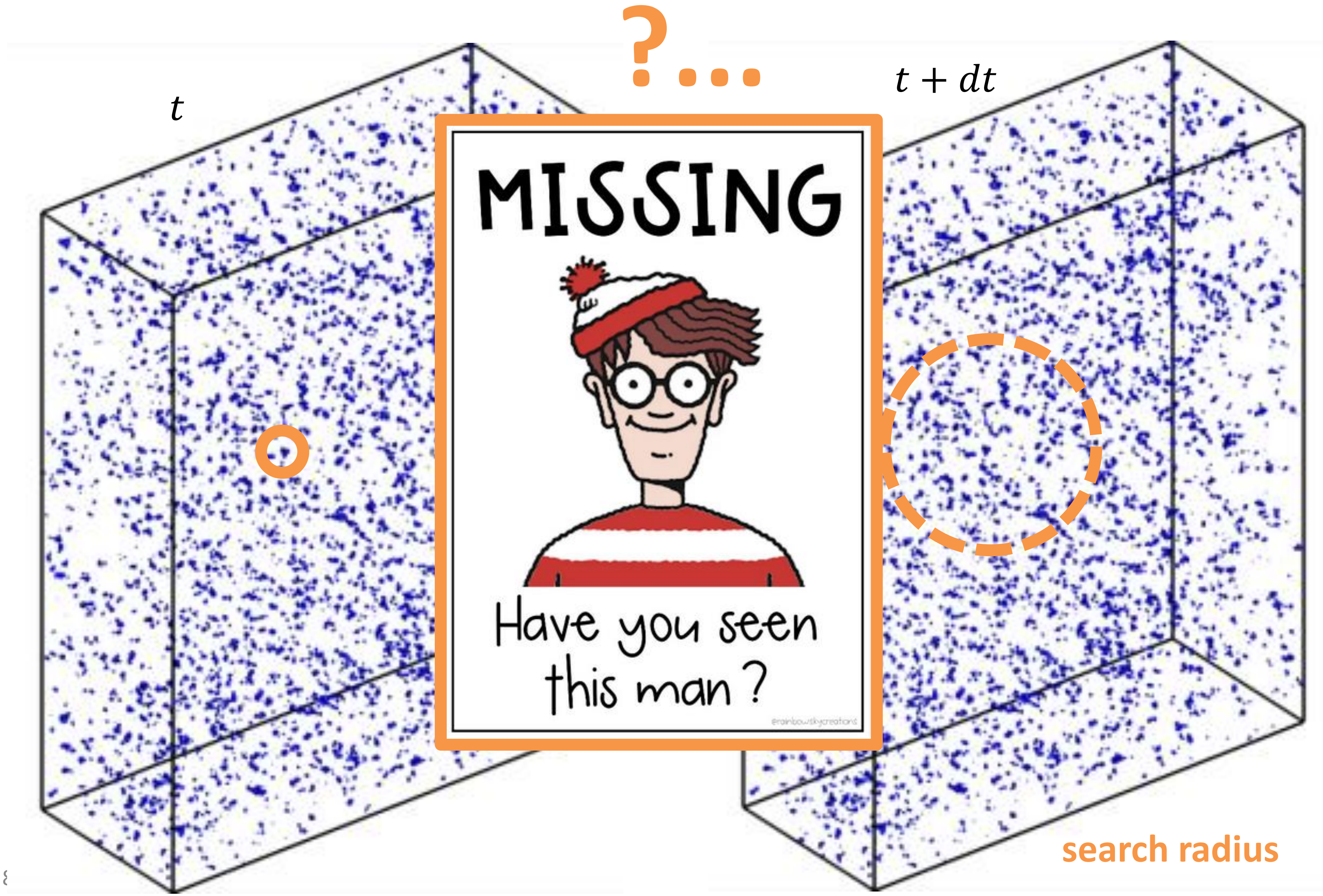


Enhanced IPR, Jahn et al., Exp. Fluids 2021

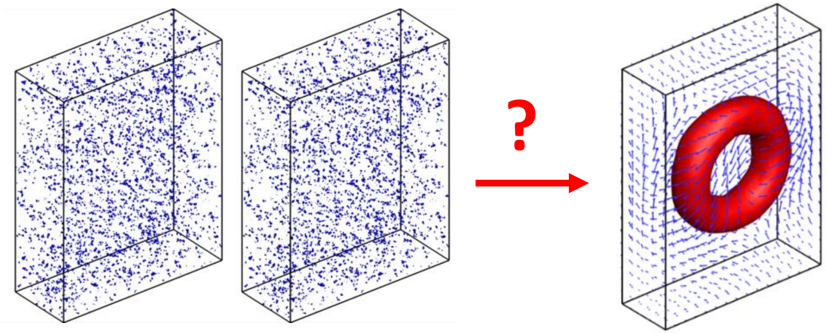
- **Locate** particle positions in the images
- **Triangulate**:
  - Back-project (ray tracing) particle positions to volume
  - 3D particle positions are in the centre of zones where 4 rays are close to crossing (never cross exactly: residual calibration errors!)
- **Iterative approach** aiming at minimizing **residual images:** discrepancy between actual images and projection from 3D particle estimate (= **synthetic image!**)

3D PTV

Motion estimation: Matching

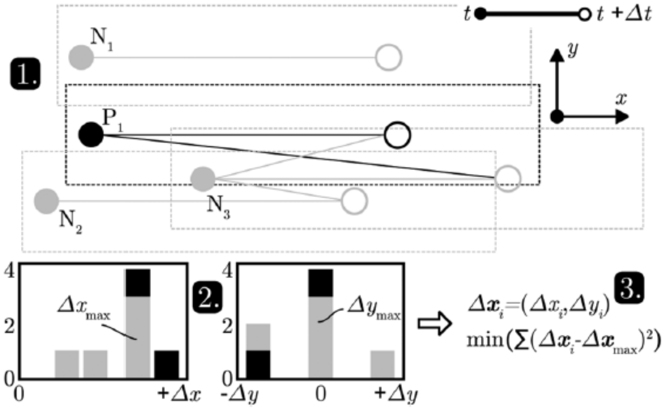


3D PTV



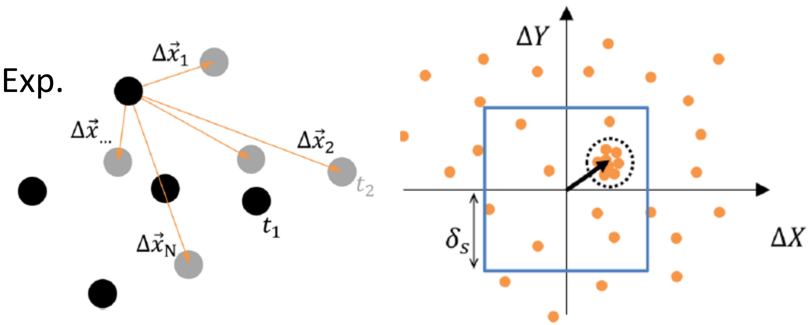
Motion estimation: Matching

Fuchs et al., Exp. Fluids 2017



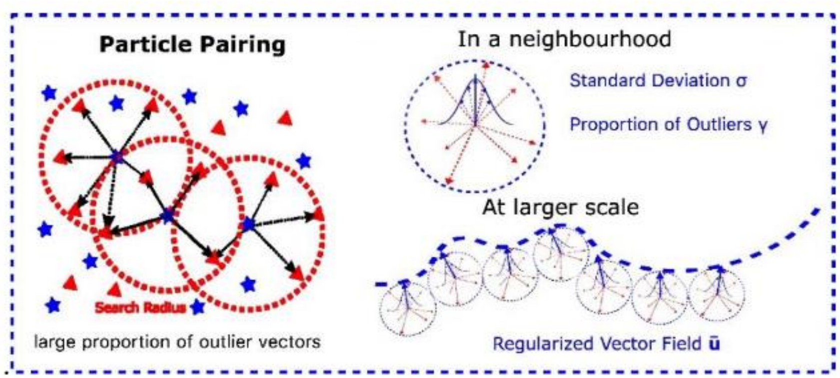
- **Nearest-neighbor** accounting for average (in space) displacement: *remains accurate if displacement larger than inter-particle distance*

Novara et al., Exp. Fluids 2023



- **3D Correlation-based predictor**:
  - 3D correlation on a coarse grid (Cornic et al., Exp. Fluids 2020)
  - Particle Space Correlation (Novara et al., Exp. Fluids 2023)

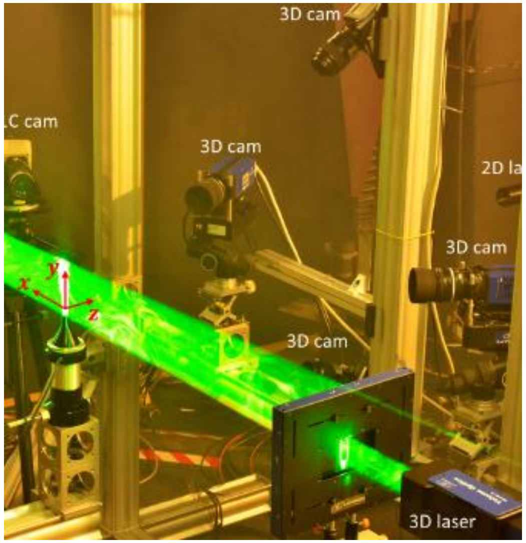
Le Bris et al., 2025



- The full package: matching by **predictor estimation with embedded ghost rejection**: Vector Field Consensus (Le Bris et al., let's hope accepted at ISPIV 2025!)

3D PTV

An example: Double-Frame Tomo-PTV

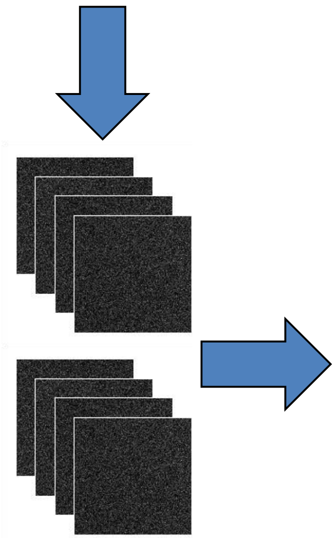


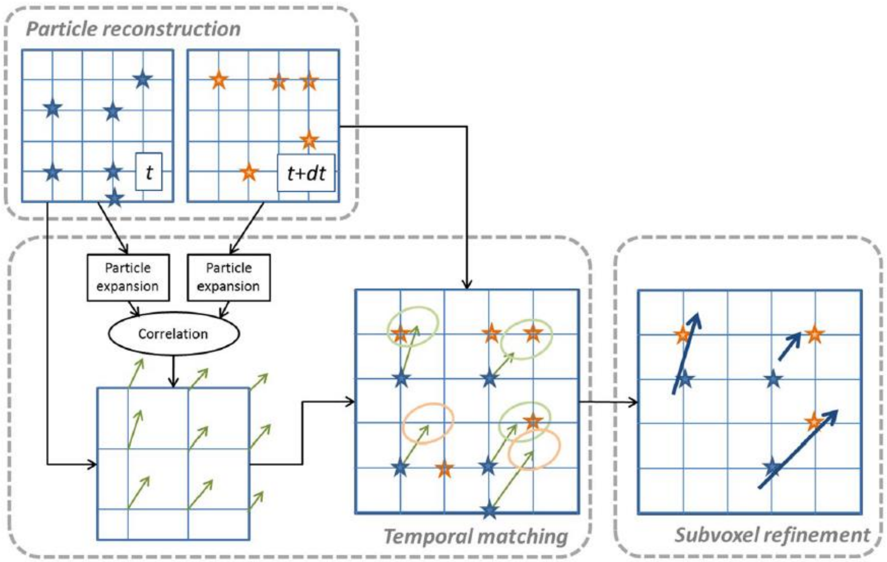
$Re = 4600$

« Tomo-PTV »: goal is PTV but at some stages we exploit principles from Tomo-PIV!

t

t+dt

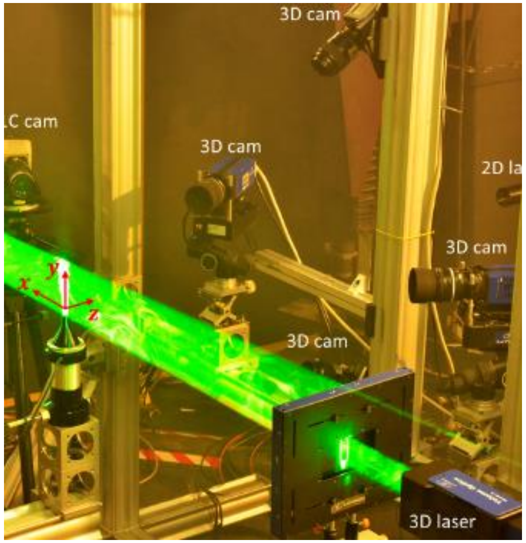




Cornic et al.
Exp. Fluids 2020

3D PTV

An example: Double-Frame Tomo-PTV



After matching (or prediction of position at next time – LPT), **refinement of positions (X_p) and intensities (E_p) is** necessary:

- Obtained by minimization of residual images:

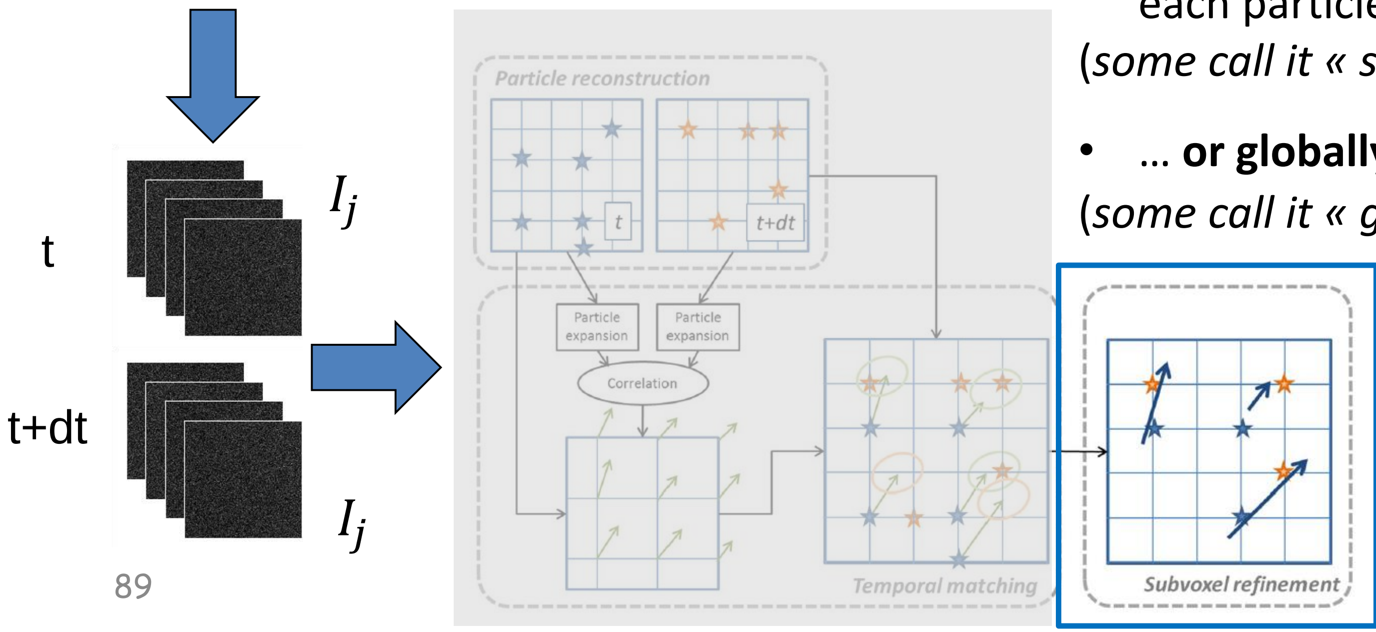
$$\sum_j \sum_{\underline{x}} \left\| I_j(\underline{x}) - \sum_p E_p h\left(\underline{x} - F_j(\underline{X}_p)\right) \right\|^2$$

- Optimization **either individually** for each particle

(*some call it « shaking »*)

- … **or globally**, all $\underline{X}_p$ and E_p at once

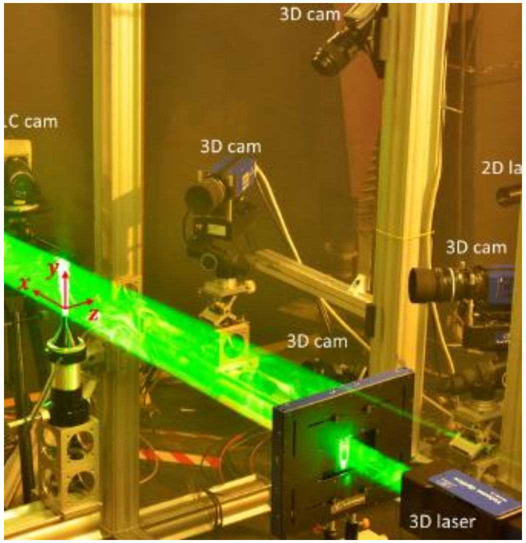
(*some call it « global shake »*)



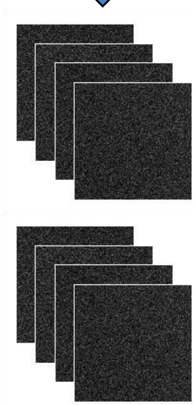
Cornic et al.
Exp. Fluids 2020

3D PTV

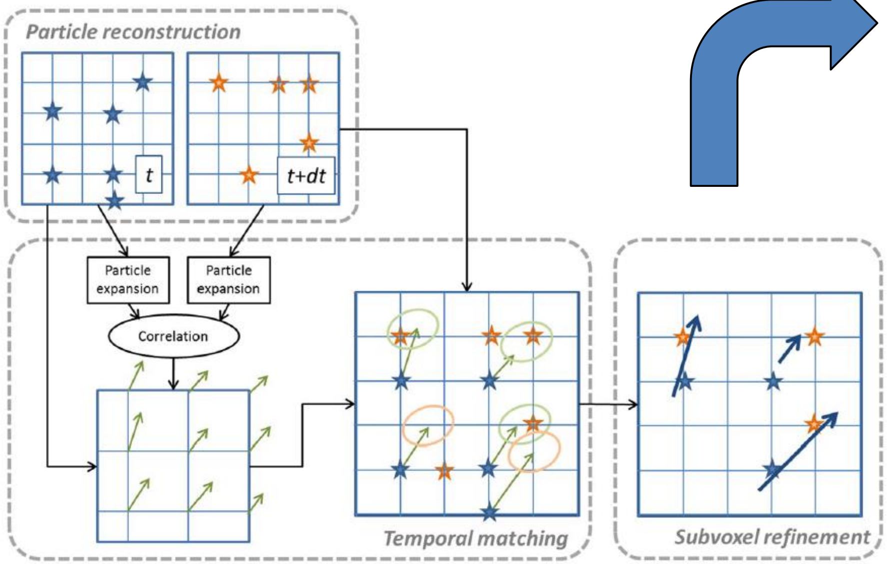
An example: Double-Frame Tomo-PTV

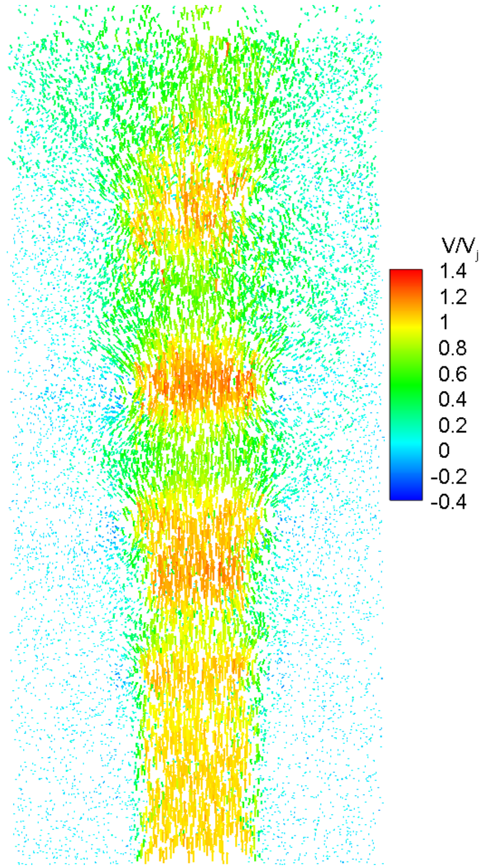


t



t+dt

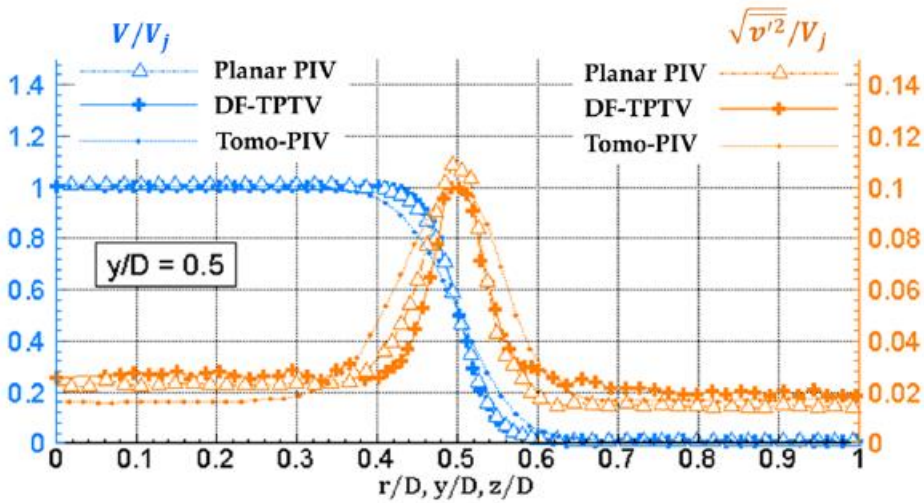


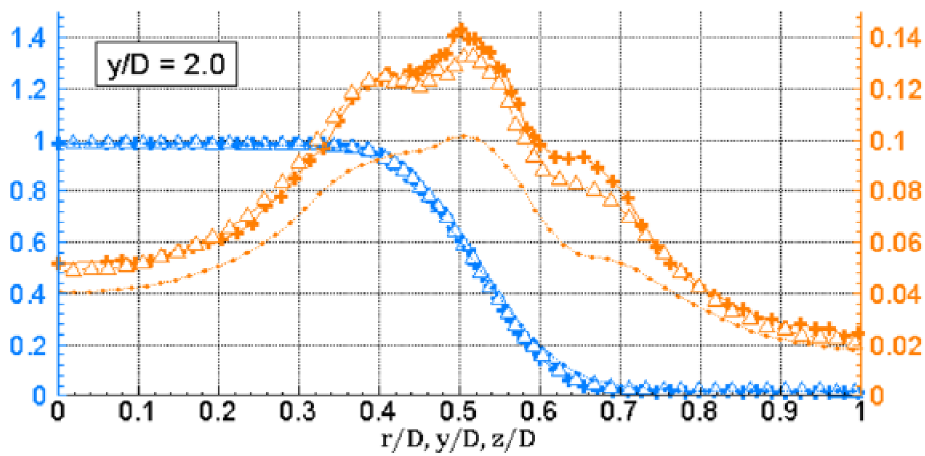


Cornic et al.
Exp. Fluids 2020

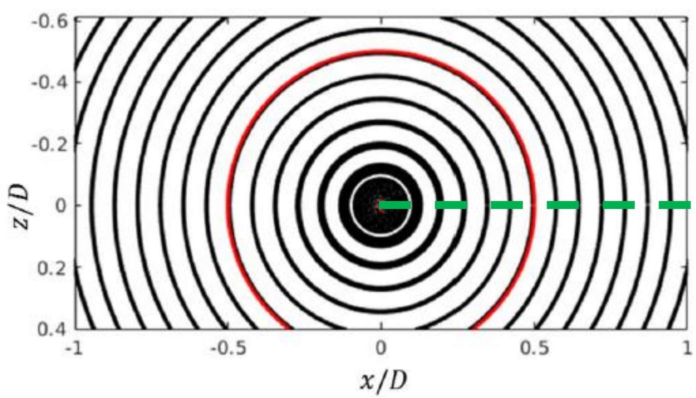
3D PTV

Statistics: bin-averaging





- Bin averaging: discretize space in small volumic cells and perform statistics on all vectors that were once in the cell
- Small bins require high seeding density and/or large number of snapshots (if no spatial invariance)
- **Bin-averaged statistics of 3D PTV of higher quality than standard statistics of TomoPIV (confirmed!)**

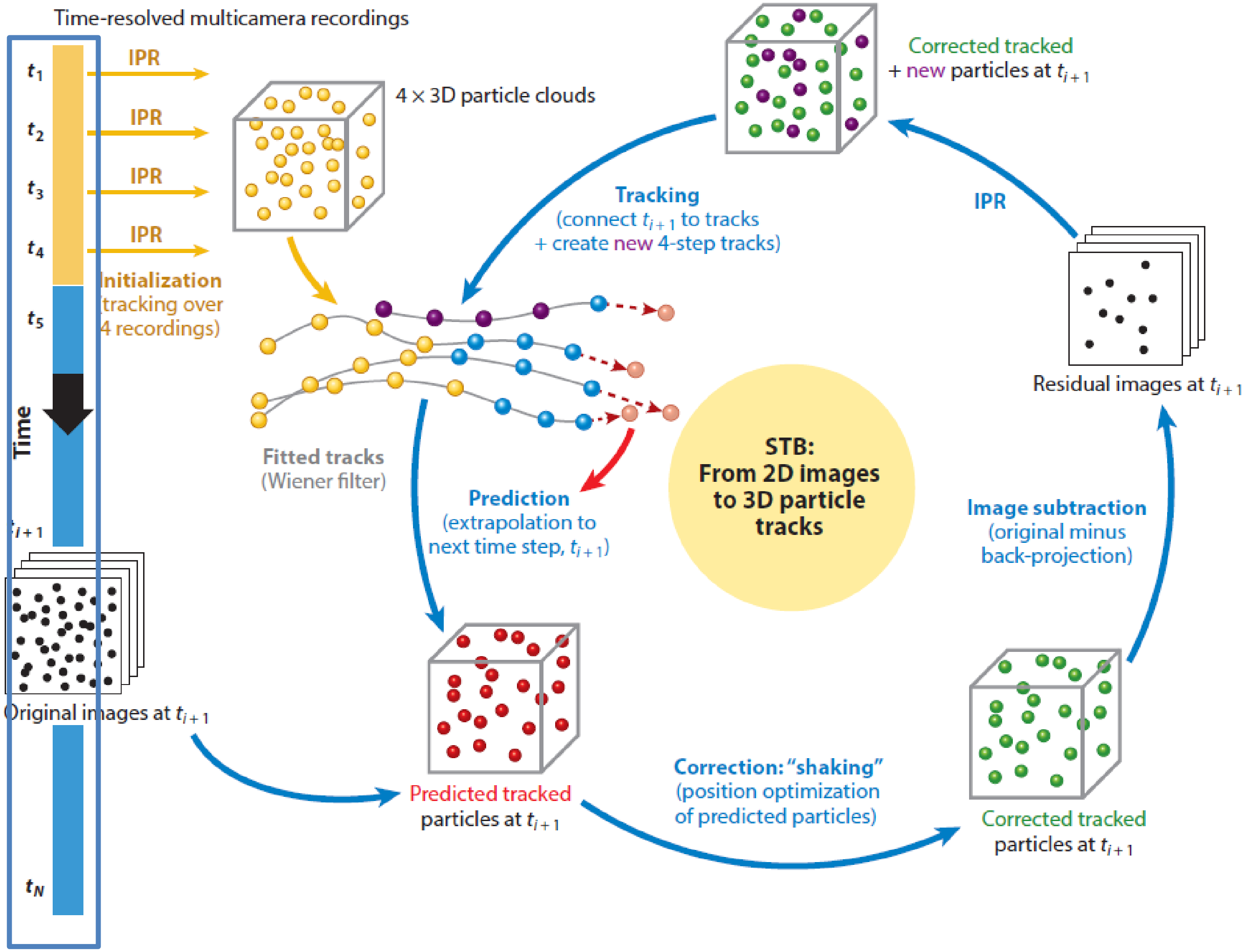


Cornic et al. Exp. Fluids 2020

3D PTV / LPT

Exploiting temporal consistency: Lagrangian Particle Tracking

Schröder et al., Ann. Rev. Mech. 2023



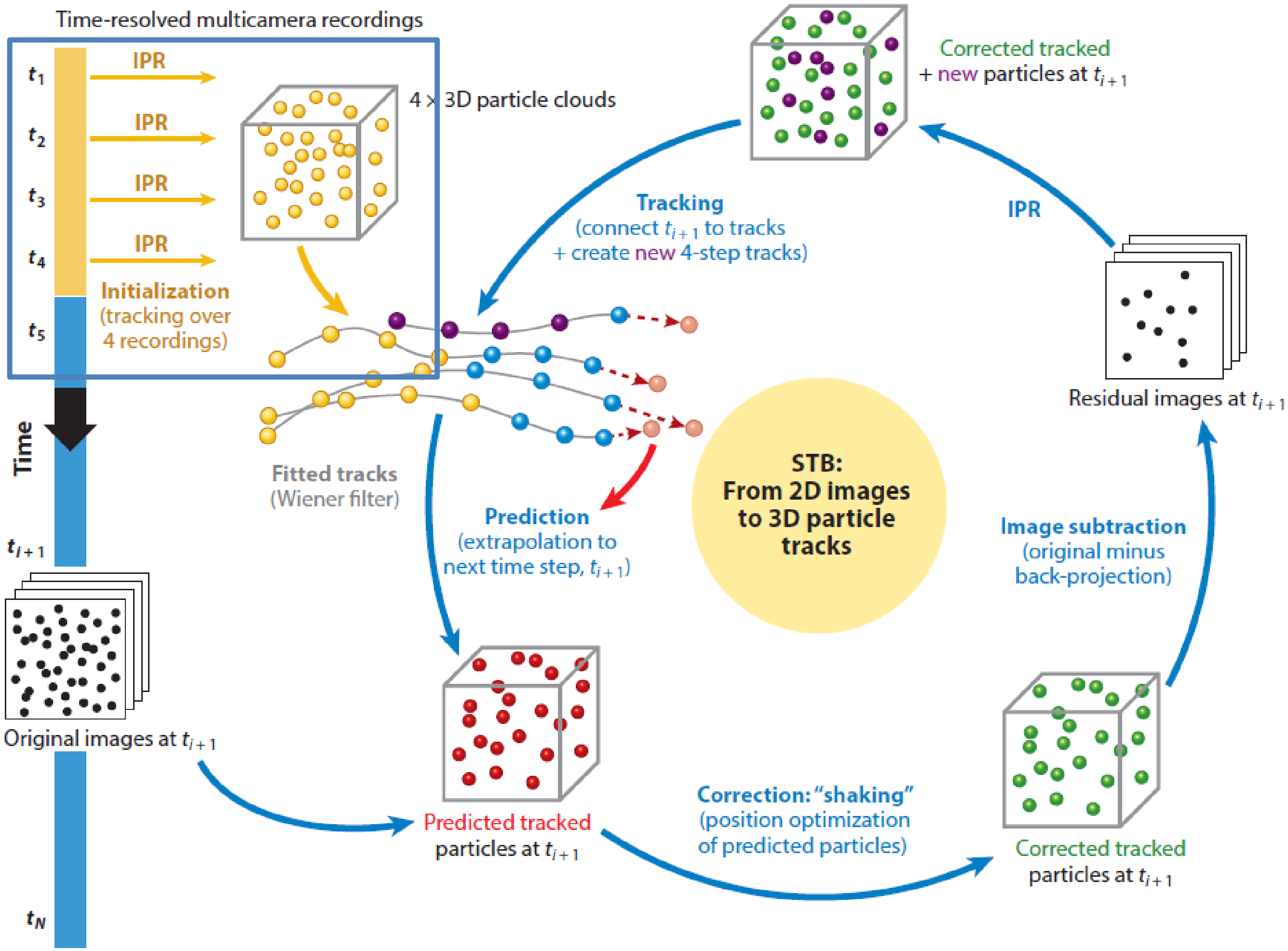
Acquisition in single-frame mode

⇒ high frame rate lasers and cams
⇒ dt given by acquisition freq.
⇒ upper bound on max flow speed

3D PTV / LPT

Exploiting temporal consistency: Lagrangian Particle Tracking

Schröder et al., Ann. Rev. Mech. 2023



Acquisition in *single-frame mode*

⇒ upper bound on max flow speed

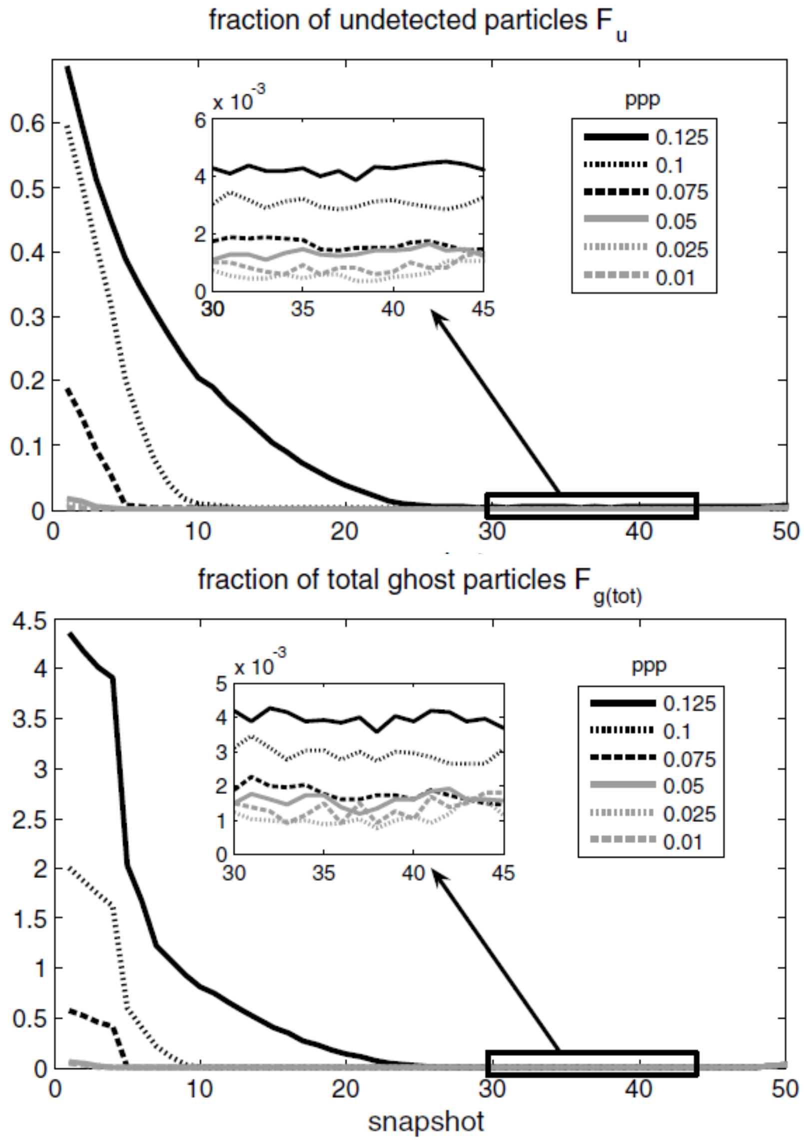
If ok:

- **cost-effective:** Particle Reconstruction only performed at initial instants (mostly)

3D PTV / LPT

Exploiting temporal consistency: Lagrangian Particle Tracking

Schanz et al., Exp. Fluids 2016



Acquisition in *single-frame mode*

⇒ upper bound on max flow speed

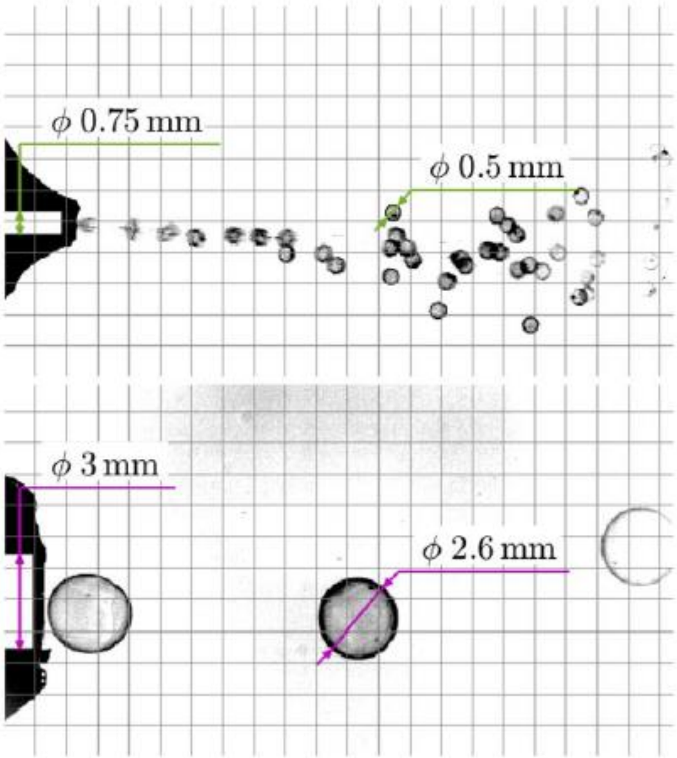
If ok:

- **cost-effective:** Particle Reconstruction only performed at initial instants (mostly)
- and **the most accurate option!**

Large-scale 3D PIV / LPT

Helium Filled Soap Bubbles

Grille Guerra et al., Exp. Fluids 2024



- (Very) large particle sizes ($\sim 300\ \mu m$ most common)

⇒ **much brighter**
⇒ **much larger volumes** (or planes!), using multi-LED systems

- Neutral buoyancy thanks to Helium
- Depending on optical setup: form similar images to other tracers, or images with glare points

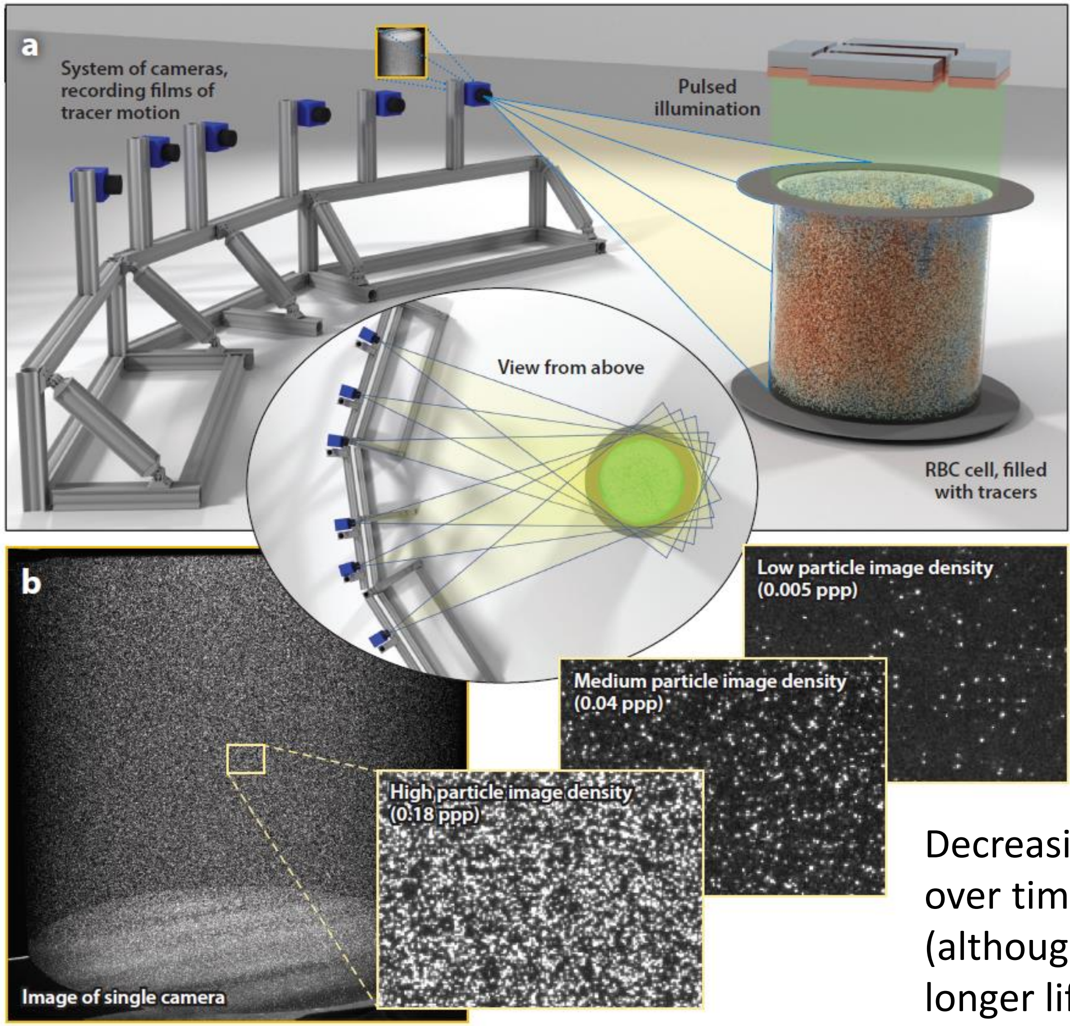
Limits:

- Turbulent flow: their size! (could be of order of turbulent sizes)
- Short lifetime / fragility:
  - Break-up due to shear (near-wall)
  - Must be injected quite close to test section ⇒ possible disturbance of flow by injection devices

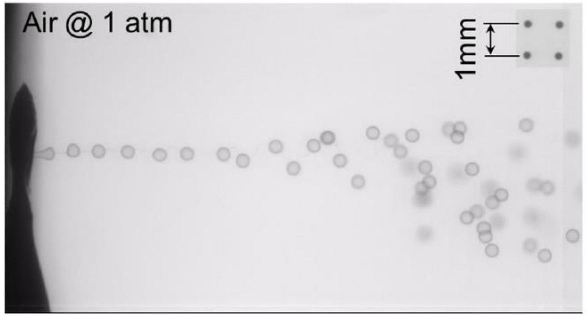
Large-scale 3D PIV / LPT

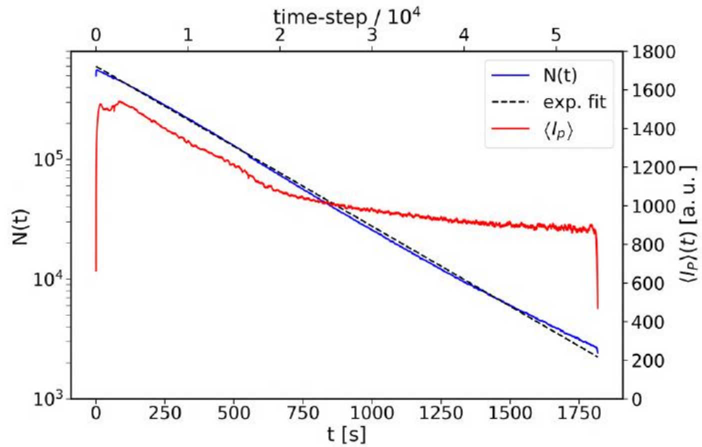
Rayleigh-Bénard Convection with HFSB

Schröder et al., Ann. Rev. Mech. 2023



Bosbach et al., 14th Int. Symp. on PIV, 2021





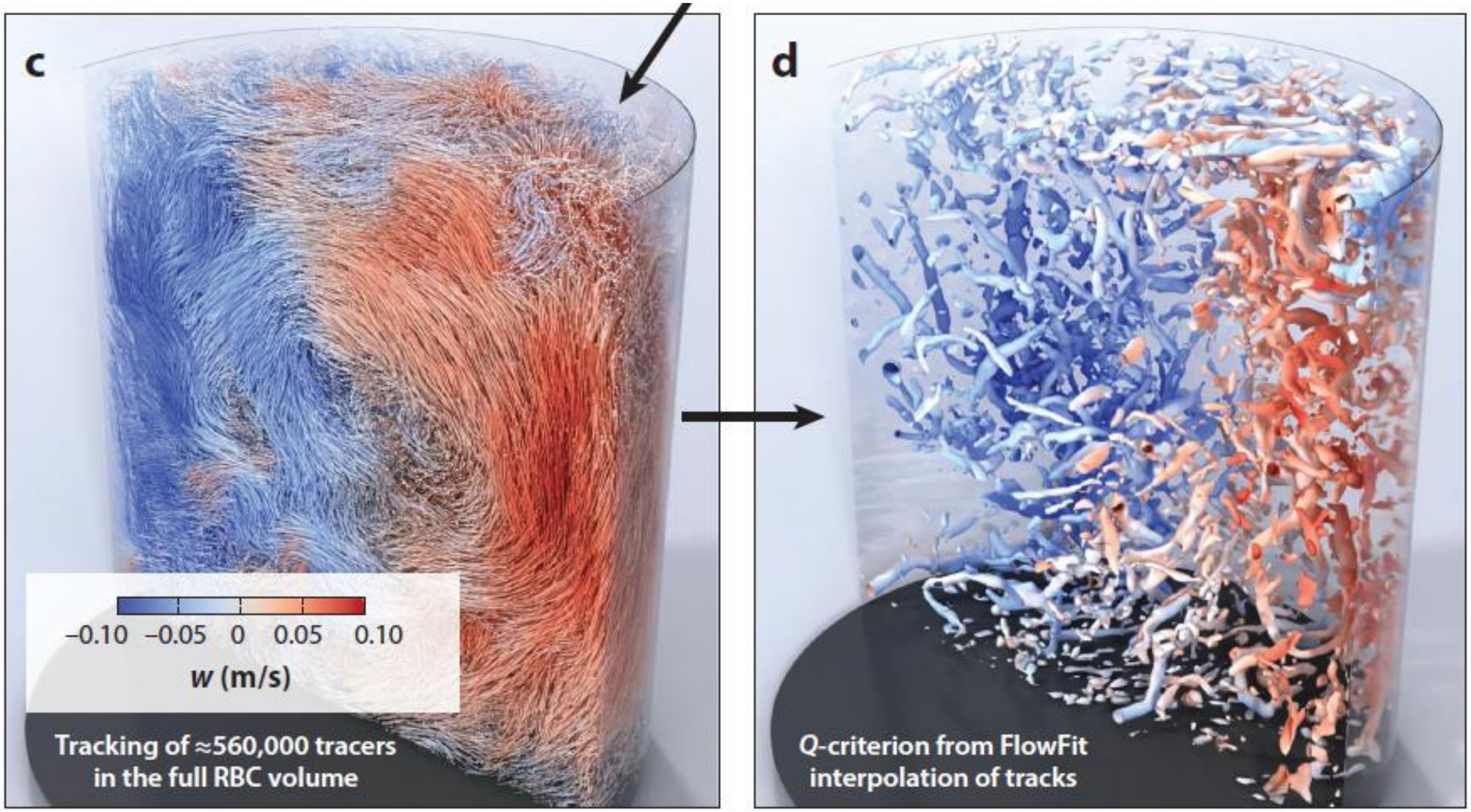
Decreasing concentration over time: **lifetime of HFSB!** (although dedicated ∼3 × longer lifetime system)

https://gfm.aps.org/meetings/dfd-2020/5f5fe77f199e4c091e67bfe8

Large-scale 3D PIV / LPT + Data assimilation

Rayleigh-Bénard Convection with HFSB

Schröder et al.,
Ann. Rev. Mech.
2023

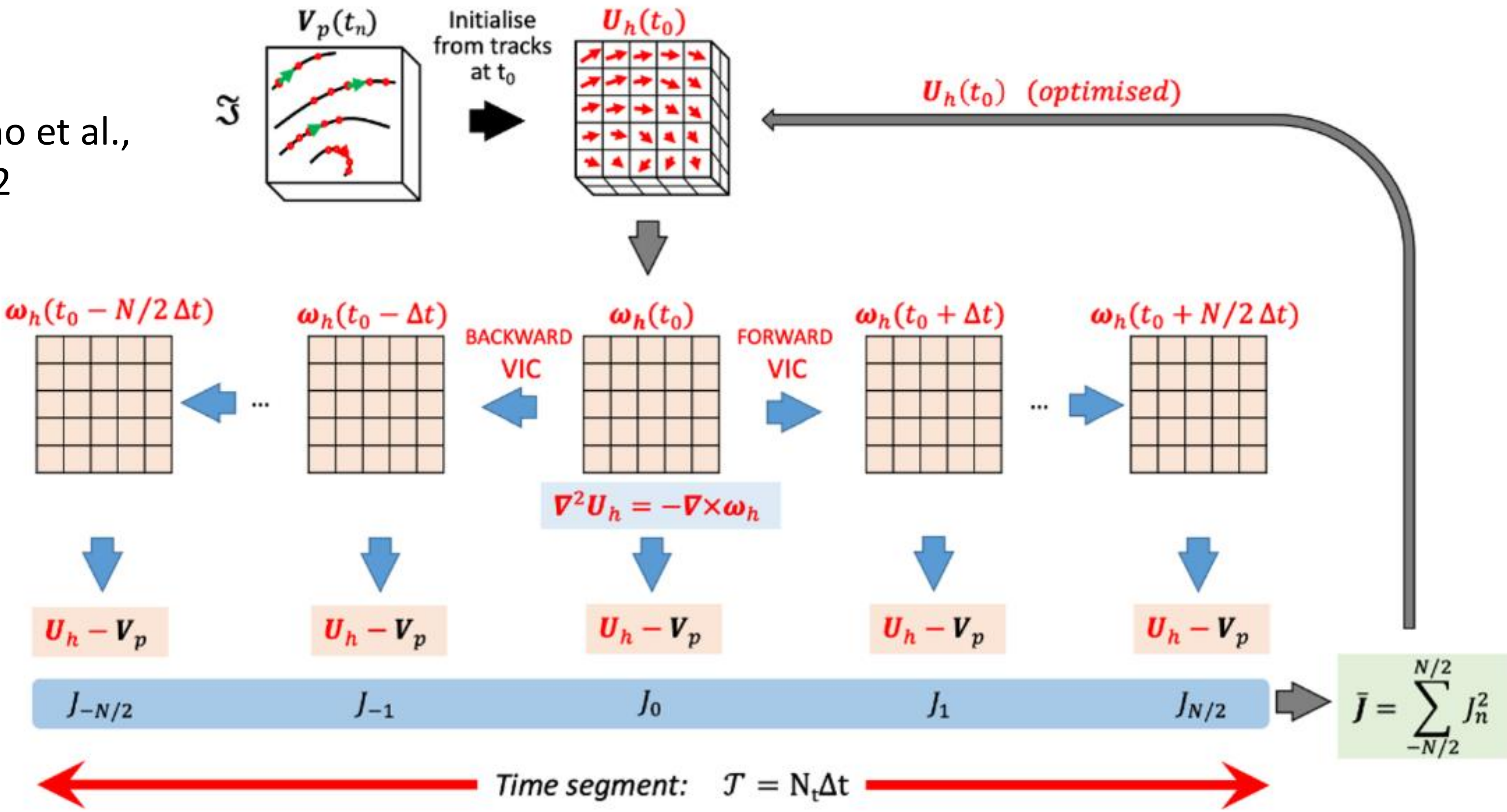


https://gfm.aps.org/meetings/dfd-2020/5f5fe77f199e4c091e67bfe8

Data assimilation

Filling gaps using physics… mostly from time-resolved tracks (LPT)…

VIC-TSA, Scarano et al., Exp. Fluids 2022

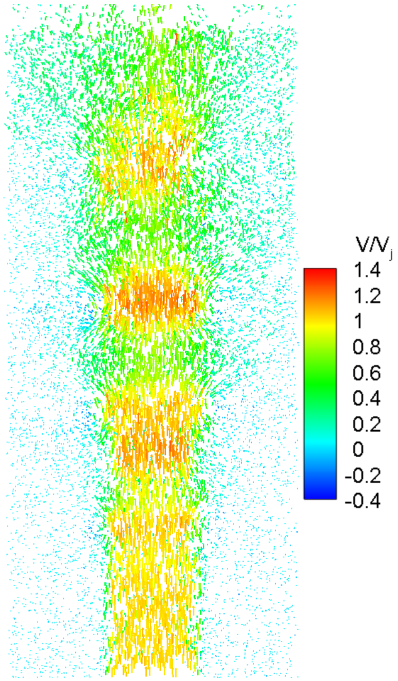


- Numerical velocity field on a 3D grid field sought as an ensemble of base functions (**vortices, B-splines**) located at mesh nodes
- **Coefs optimized so that numerical flow close to measurement and (incompressible) Navier-Stokes equations** (mostly penalization: no hard constraint, or only $\nabla \cdot u = 0$)
- Variants: input = one snapshot (**with acceleration**) / a sequence of instants

Data assimilation

… or the harder way (from a single velocity snapshot, no acceleration!)

measured data: $\boldsymbol{m}$



Available: $\boldsymbol{u}$
at particles' positions only

Objective: minimize

$$\min_{\boldsymbol{f}} \left\{ J = \frac{1}{2} \|\boldsymbol{m} - \boldsymbol{h}(\boldsymbol{u})\|^2 \right\}$$

h : measurement operator: mimics PTV

under incompressible Navier-Stokes constraint:

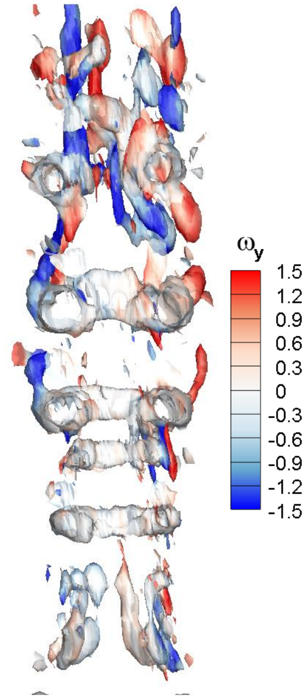
$$\nabla \cdot \boldsymbol{u} = 0$$

$$(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - Re^{-1}\Delta \boldsymbol{u} + \nabla p = \boldsymbol{f}$$

… when minimum is reached, control parameter f yields $-\partial \boldsymbol{u}/\partial t$!

Nice, but variational optimization + DNS
⇒ **very expensive!**

simulation : $\boldsymbol{u}$

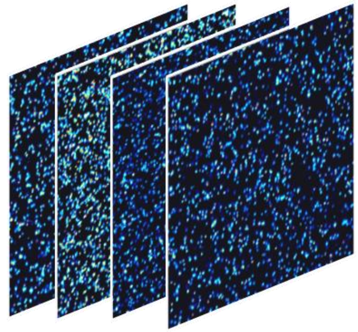


Available: $\boldsymbol{u}$, p, $\partial \boldsymbol{u}/\partial t$
on a regular grid
⇒ ∇u, eddies…

Mons et al., Meas. Sci. Technol. 2022

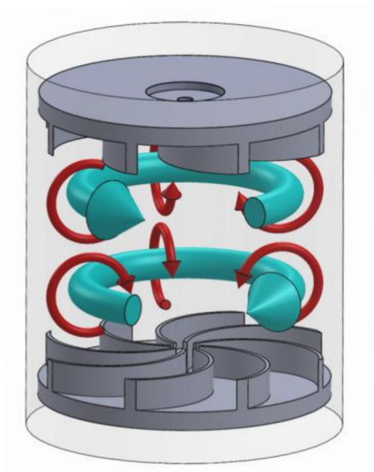
What should we do next?...

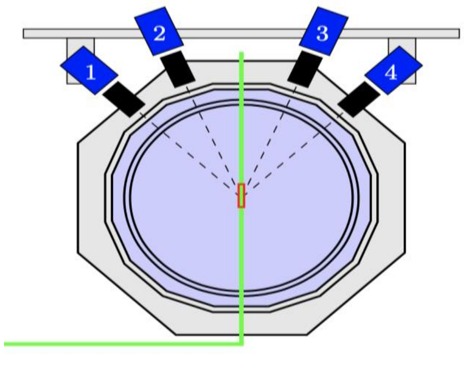
Improve our tools to investigate singularities!...

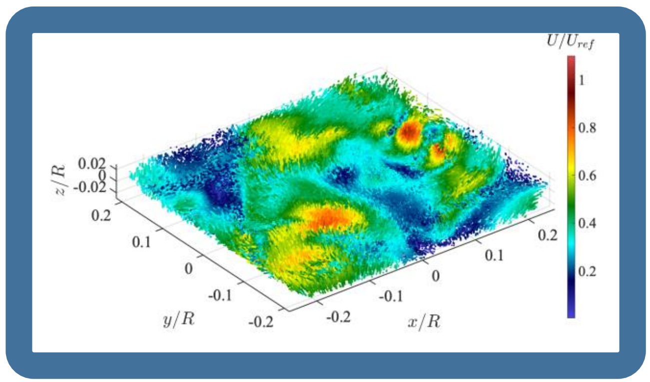


4D–PTV

~~(Shake-The-Box)~~



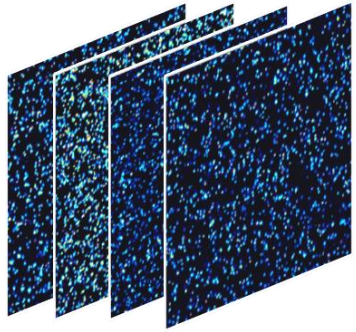




French National Research Agency funded project BANG: CEA/SPEC (**B. Dubrulle**, F. Daviaud, A. Cheminet, J. Le Bris, et al.), LMFL (N. Tawdi, et al.), ONERA (B. Leclaire, M. Hebey et al.)

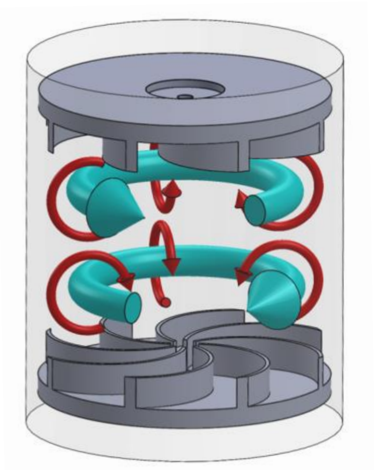
What should we do next?...

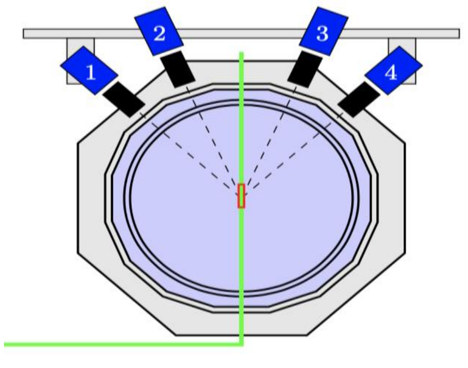
Improve our tools to investigate singularities!...

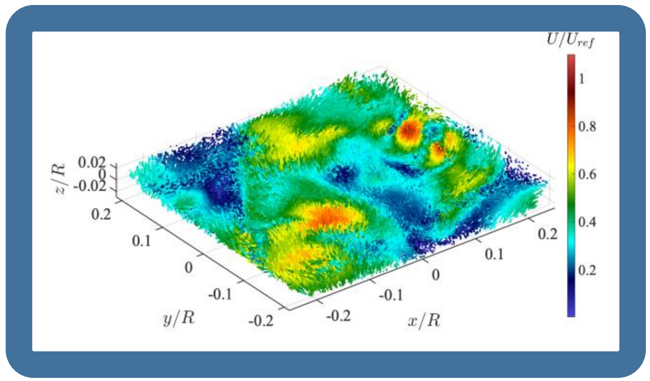


4D–PTV
(XXXXX*)

* fancy name to be found by the end of BANG, if we are happy with the result!







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What should we do next?...

Challenges might also tell!...