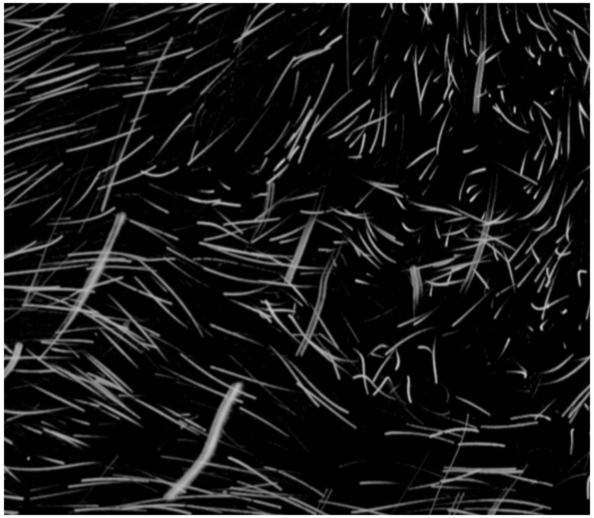
Dynamics of inertial particles and finite size particles in turbulence



Romain Volk

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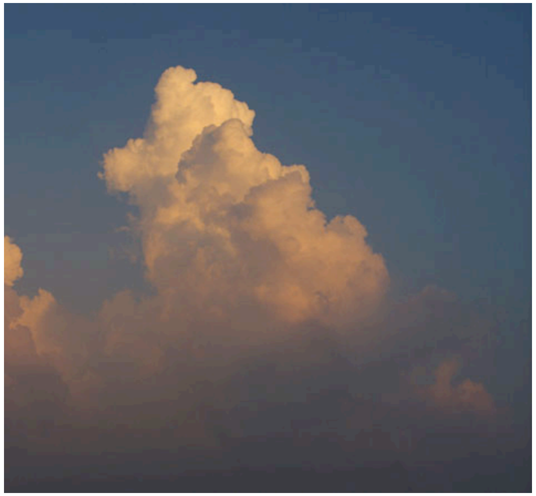
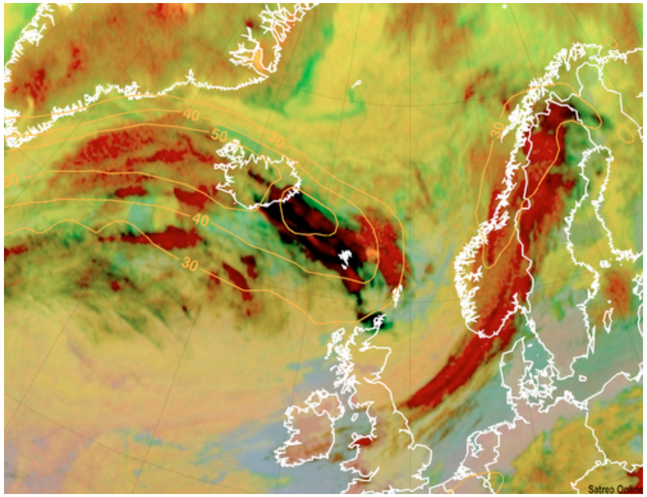
R. Zimmermann, N. Machicoane, L. Fiabane

E. Calzavarini, M. Bourgoin, N. Mordant, J.-F. Pinton

Inertial and material particles





Situations involving heavy particles

How far are heavy particles transported ?

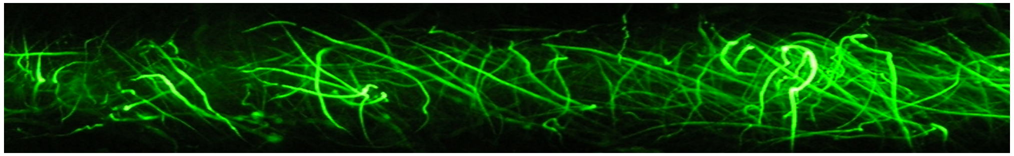
Inertial and material particles

Sand

Plastic

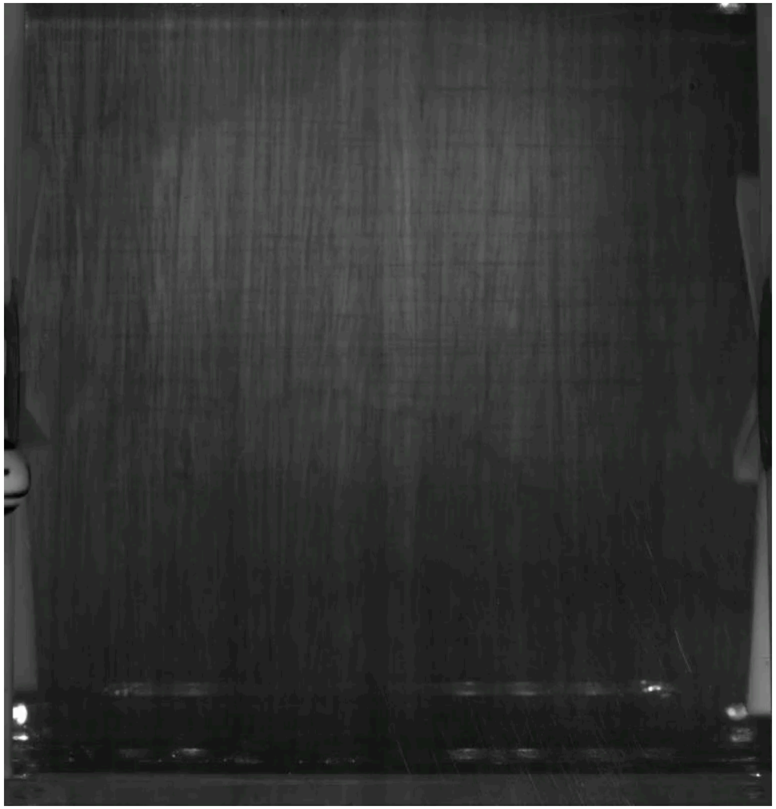
Iceberg

What is the inertia of those objets ?

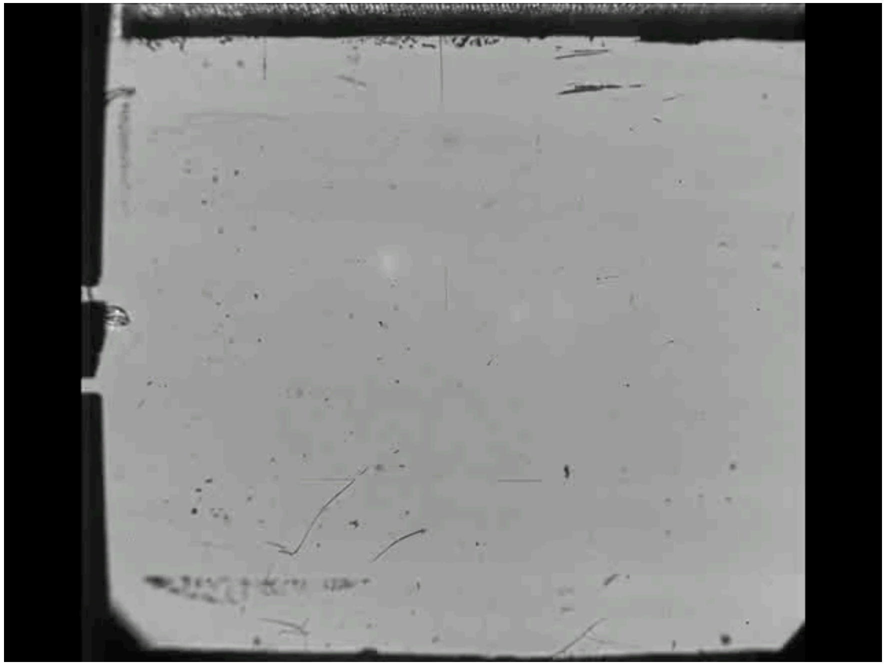


Bubbles

Inertial and material particles



R. Zimmermann



N. Machicoane

How large spheres move in turbulence ?

How fast does a large sphere melts in turbulence ?

An ambitious outline

From Euler to Lagrange

Dynamics of fluid tracers and inertial particles

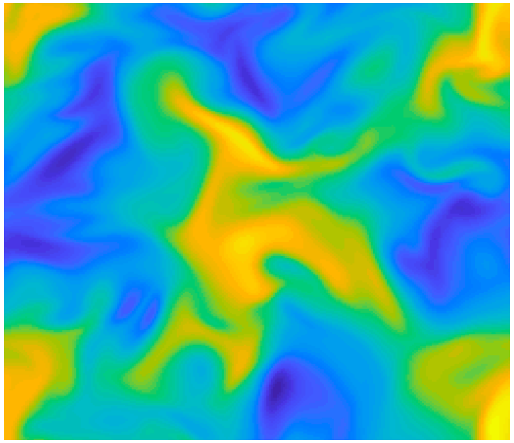
Particles larger than the dissipative scale

Dynamics of very large particles

Melting of freely transported ice balls

From Euler to Lagrange

Eulerian point of view : fixed probe in space

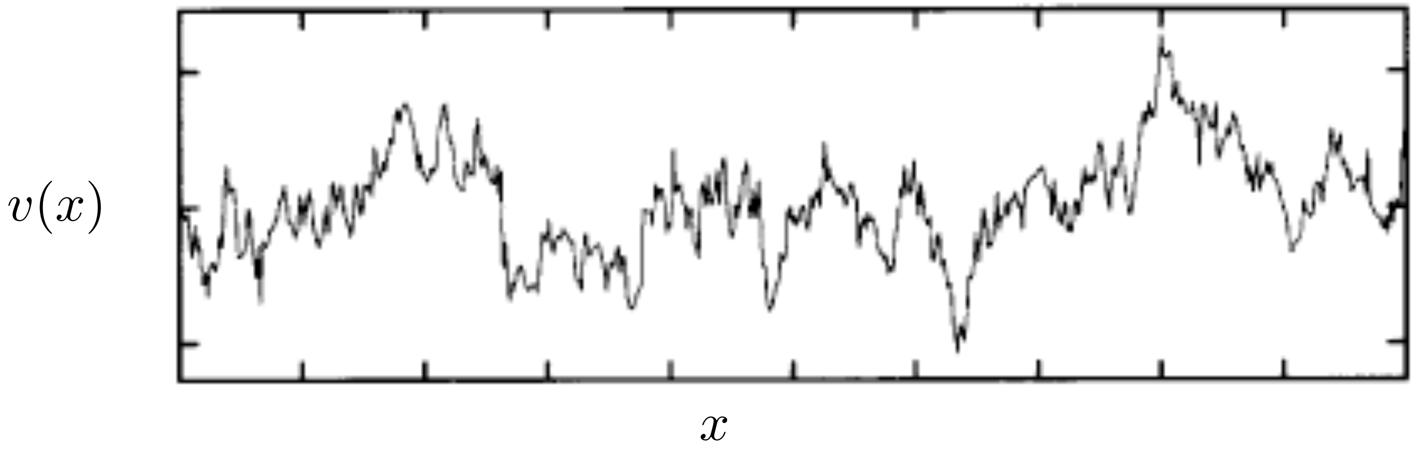


Vector field or one point measurement

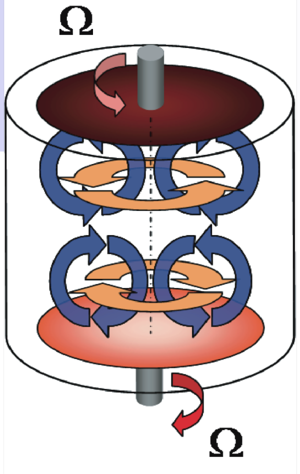
Look at spatial increments

$$S_{2,\parallel}(\ell) = \left\langle \left(\left(\vec{v}(\vec{x}) - \vec{v}(\vec{x}+\vec{\ell}) \right) \cdot \frac{\vec{\ell}}{\ell} \right)^2 \right\rangle$$

Hot wire measurement transformed into v(x) (energy at all scales)



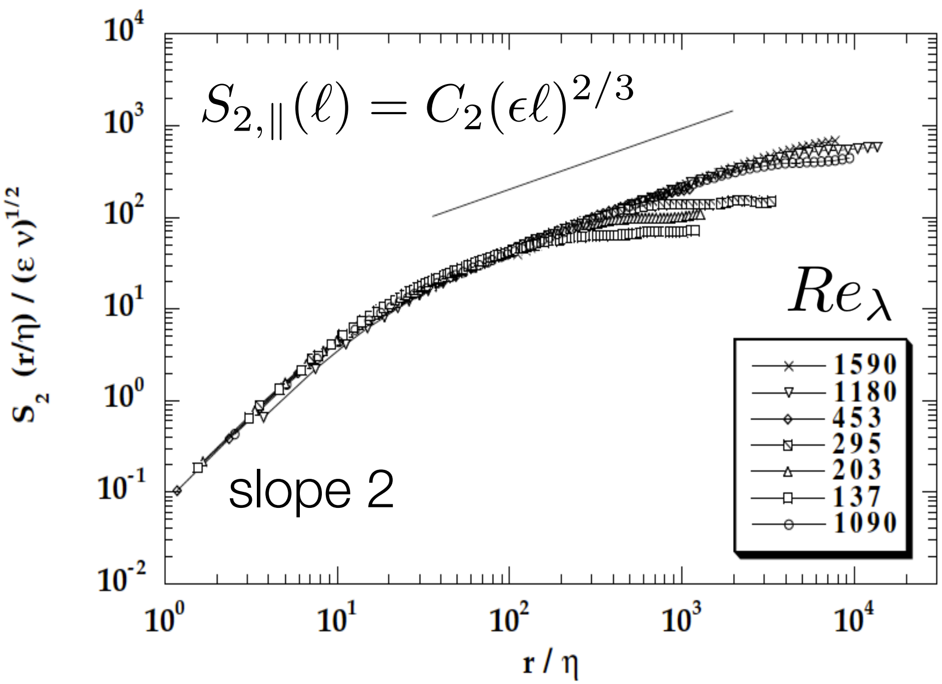
From Euler to Lagrange



Eulerian point of view : fixed probe in space

$$S_{2,\parallel}(\ell) = \left\langle \left(\left(\vec{v}(\vec{x}) - \vec{v}(\vec{x}+\vec{\ell}) \right) \cdot \frac{\vec{\ell}}{\ell} \right)^2 \right\rangle$$

F. Moisy (hot wire in He, von Karman flow)



$$v' = \sqrt{\langle v^2 \rangle - \langle v \rangle^2}$$

Dissipation

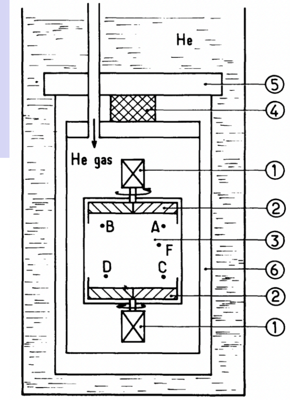
$$\epsilon \simeq 15\nu \left\langle \left(\frac{\partial v_x}{\partial x} \right)^2 \right\rangle = 15\nu \frac{v'^2}{\lambda^2}$$

Taylor based Reynolds number

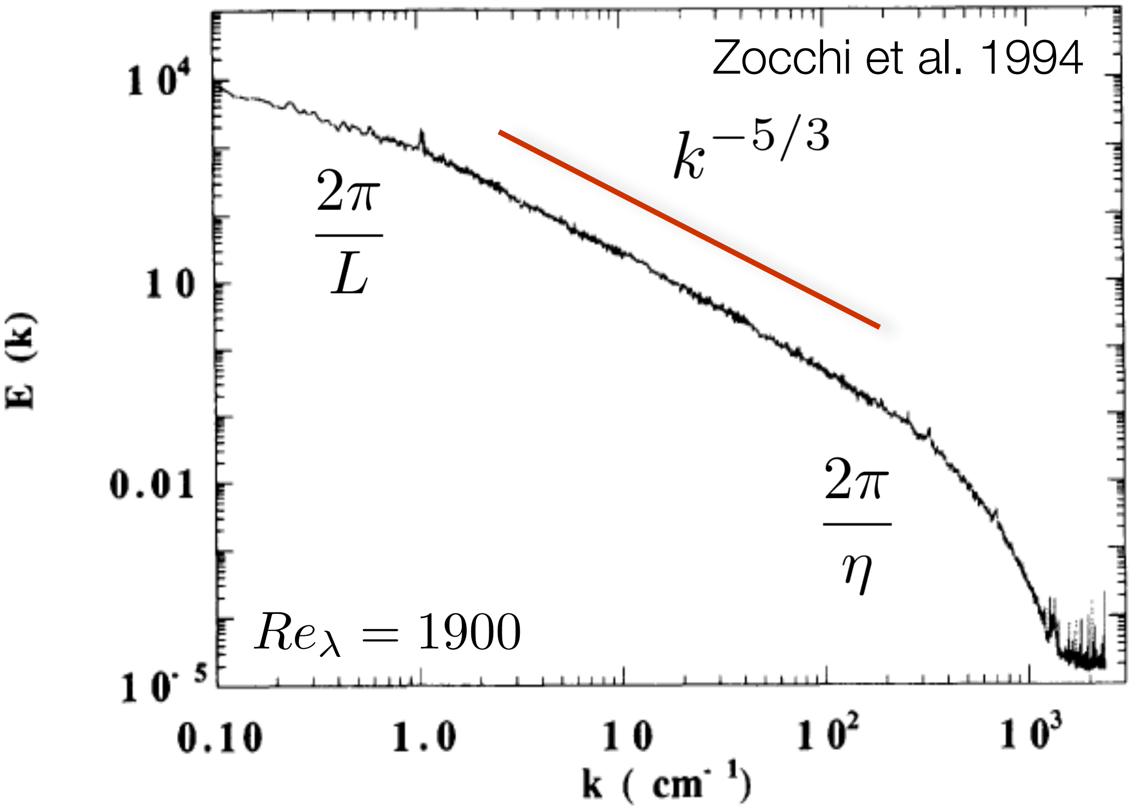
$$Re_\lambda = \frac{v'\lambda}{\nu}$$

Fully turbulent regime : $v' \propto R\Omega$ $\qquad \epsilon = \dfrac{v'^3}{L}$

From Euler to Lagrange



Power spectrum in Eulerian framework : fully turbulent case



Integral scale L

$$\epsilon = \frac{v'^3}{L}$$

Dissipative scale

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

$$Re_L = \frac{v' L}{\nu} \propto Re_\lambda^2$$

$$\frac{L}{\eta} = (Re_L)^{3/4}$$

From Euler to Lagrange

A typical Lagrangian problem

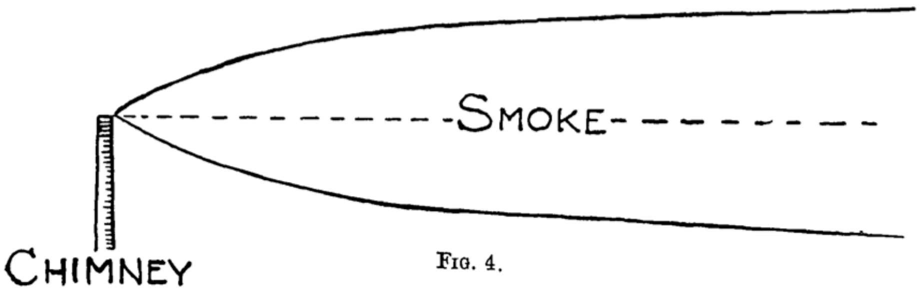


FIG. 4.

Average concentration with a continuous source ? [Euler]

$$\partial_t \langle c \rangle + \vec{\nabla} \cdot \langle \vec{v} c \rangle = D_c \triangle \langle c \rangle$$

Probability of finding the particles at some location ? [Lagrange]

$$d\vec{X} = \vec{v}(\vec{X}(t), t)dt + \sqrt{2D_c} d\vec{W}$$

From Euler to Lagrange

A typical Lagrangian problem

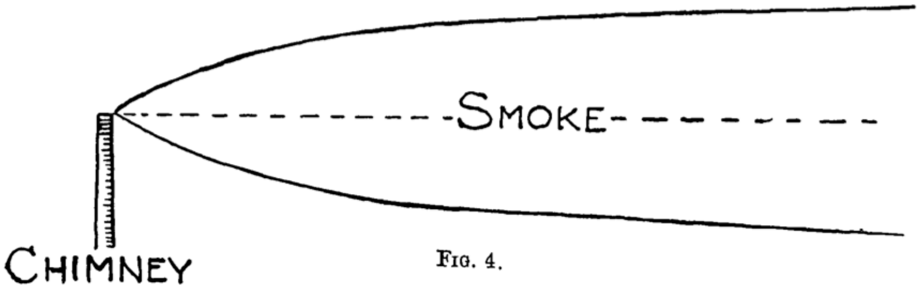


FIG. 4.

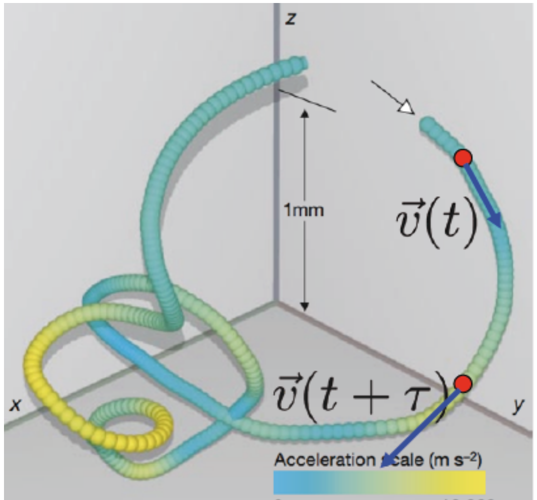
Lagrangian point of view : tracking objects

Trajectory

$$\vec{X}(t_0) = \vec{X}_0 \rightarrow \vec{X}(t)$$

$$\vec{V}(t) = \dot{\vec{X}}(t)$$

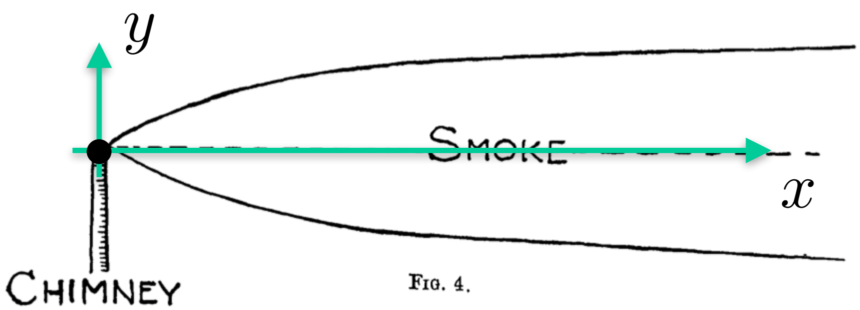
$$\vec{X}(t) = \vec{X}_0 + \int_0^t \vec{V}(t')dt'$$



La Porta et al, Nature, **409**, 1017 (2001)

From Euler to Lagrange

Let's find the relevant quantities on a simple case



Forget about molecular diffusivity (tracer) $$\vec{X}(t) = \vec{X}_0 + \int_0^t \vec{V}(t')dt'$$

Assume HIT with constant mean flow $$\vec{V} = U\vec{e}_x + \vec{V}' \qquad \langle \vec{V}' \rangle = \vec{0}$$

The mean square displacement is a key quantity (in HIT)

$$\langle X \rangle(t) = X_0 + Ut$$

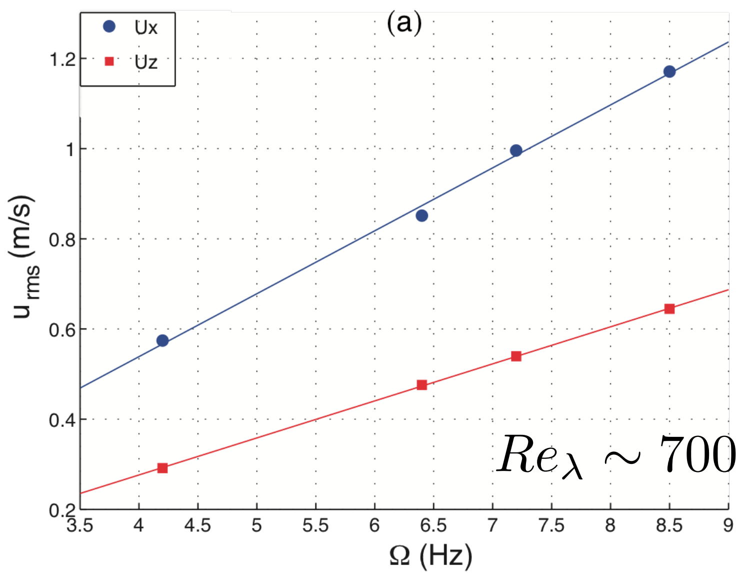
$$\frac{d\langle (Y(t) - Y(0))^2 \rangle}{dt} = 2\int_0^t \langle V_y'(t) V_y'(t') \rangle d\tau$$

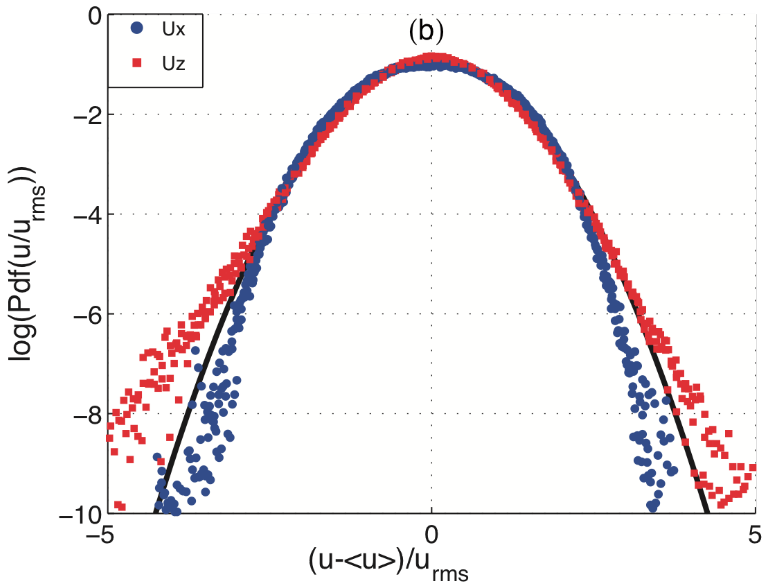
$$\langle Y \rangle(t) = Y_0$$

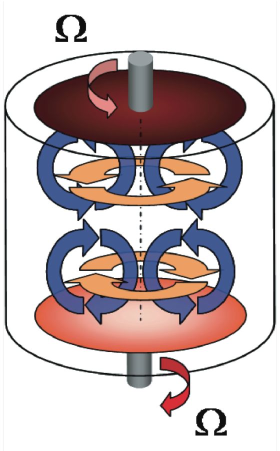
We need the velocity correlation function

Dynamics of fluid tracers

First key quantity : Lagrangian velocity fluctuations







$$V_{rms} \propto 2\pi R\Omega$$

Developed turbulence

$$\epsilon = \frac{V_{rms}^3}{L} \simeq 10 \text{ W/kg}$$

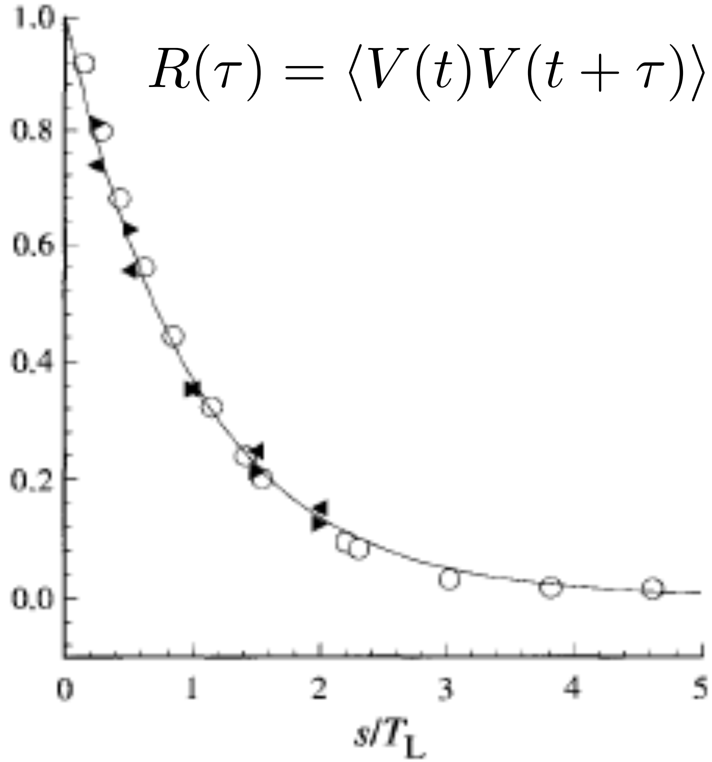
PDF(V/V_{rms})
Quasi Gaussian

Agrees with Eulerian measurement

Dynamics of fluid tracers

Velocity correlations

$$R(\tau) = \langle V(t)V(t+\tau)\rangle \,/\, \langle V^2\rangle$$

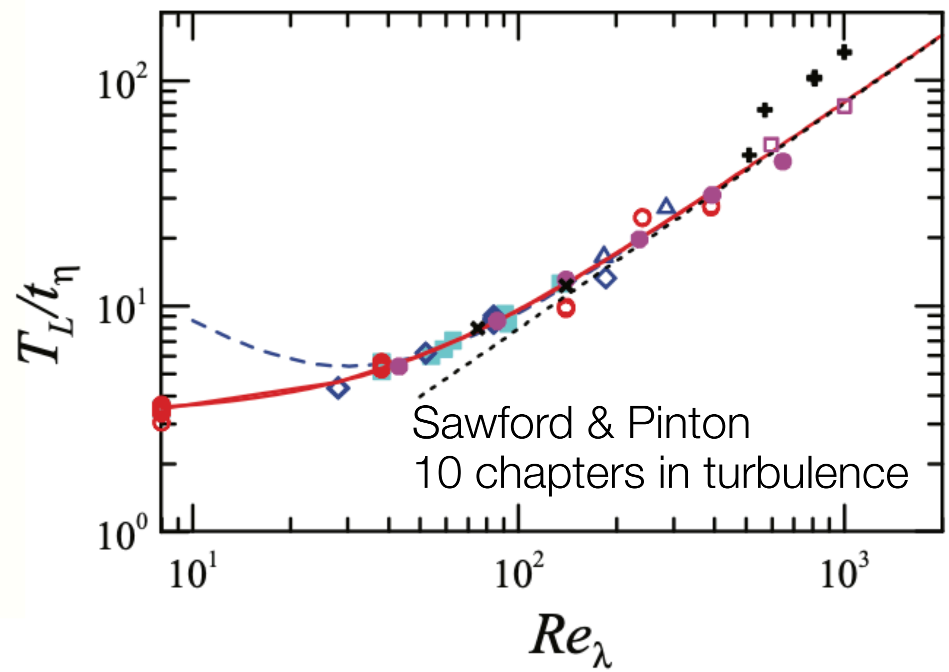


Sato, 1987 – Yeung 1989

Mordant et al, 2001

$$\langle V_i(t)V_j(t+\tau)\rangle \propto \exp\left(-\frac{\tau}{T_L}\right)\delta_{ij}$$

Integral time



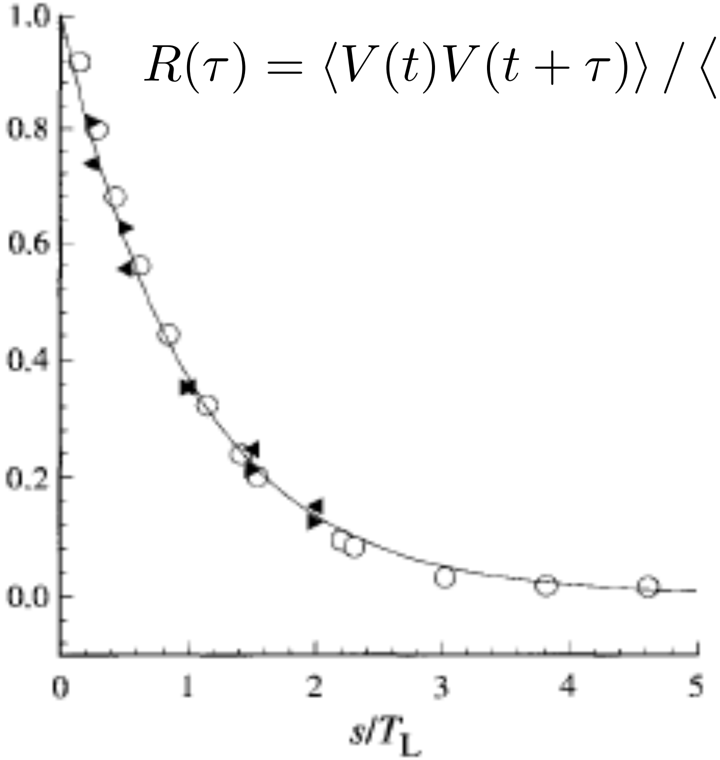
$$T_E = \frac{L}{v'} = \frac{v'^2}{\epsilon} \qquad \tau_\eta = \sqrt{\frac{\nu}{\epsilon}}$$

$$\frac{T_\mathrm{L}}{t_\eta} = \frac{T_\mathrm{L}}{T_\mathrm{E}} Re^{1/2} = \frac{T_\mathrm{L}}{T_\mathrm{E}}\frac{R_\lambda}{\sqrt{15}}$$

Dynamics of fluid tracers

Velocity correlations

$$R(\tau) = \langle V(t)V(t+\tau)\rangle \,/\, \langle V^2 \rangle$$



Consequence on dispersion

$$\frac{d\langle (Y(t)-Y(0))^2\rangle}{dt} = 2v'^2 \int_0^t R(\tau)d\tau$$

Dispersion at short time

$$\langle (Y(t)-Y(0))^2\rangle = v'^2 t^2$$

Ballistic regime

Long time dispersion

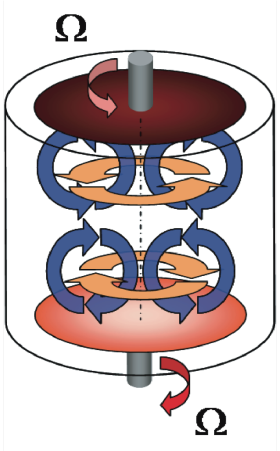
$$\langle (Y(t)-Y(0))^2\rangle \sim 2v'^2 T_L t + C$$

Turbulent diffusion

$$D_{\mathrm{turb}} = v'^2 T_L$$

Dynamics of fluid tracers

Dynamics in the dissipative range

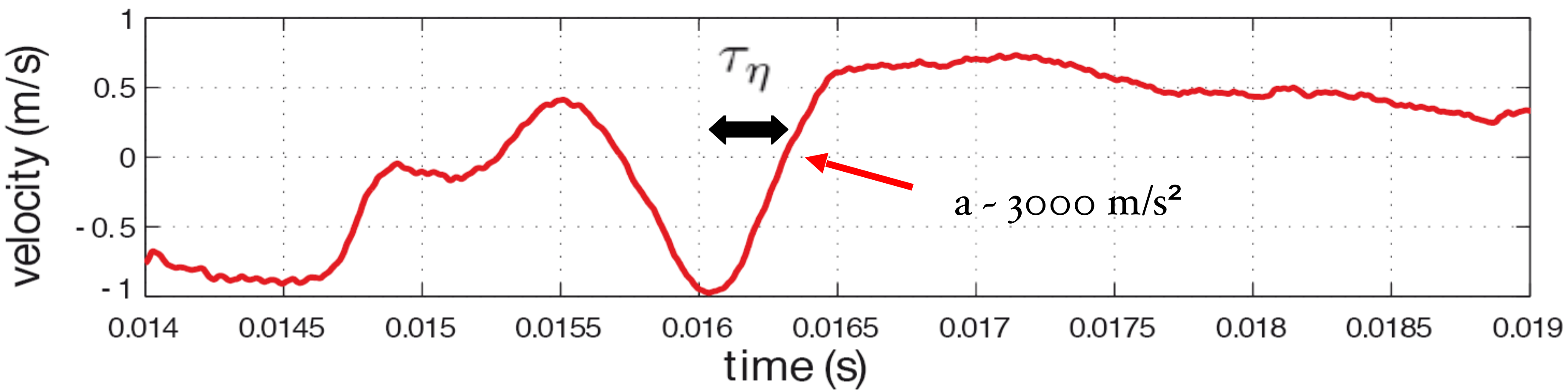


$$D_2^L(\tau) = \langle (V'(t+\tau) - V'(t))^2 \rangle = 2v'^2(1 - R(\tau))$$

If exponential correlation $D_2^L(\tau) = 2\frac{v'^2}{T_L}t \qquad t \ll T_L$

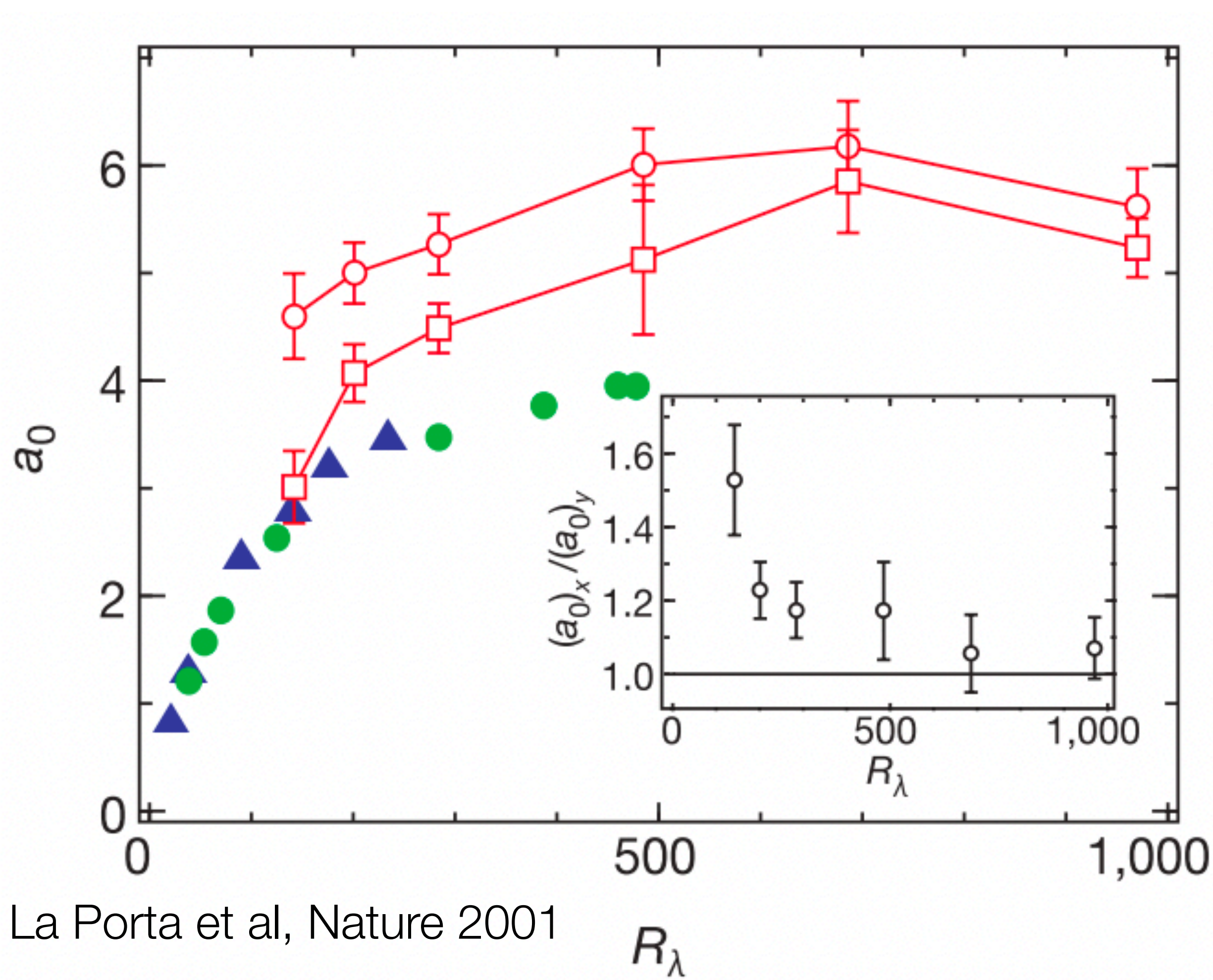
But velocity is smooth $D_2^L \simeq \tau^2 \langle a^2 \rangle \qquad t \ll \tau_\eta$

Temporal evolution of the Lagrangian acceleration $\qquad Re_\lambda = \frac{u_{rms}\lambda}{\nu} \sim 700$

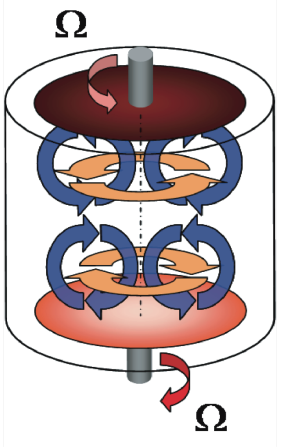


Dimensional analysis : $\langle a^2 \rangle = f(\epsilon, \nu, L) = a_0(Re_\lambda)\epsilon^{3/2}\nu^{-1/2}$

Dynamics of fluid tracers



La Porta et al, Nature 2001



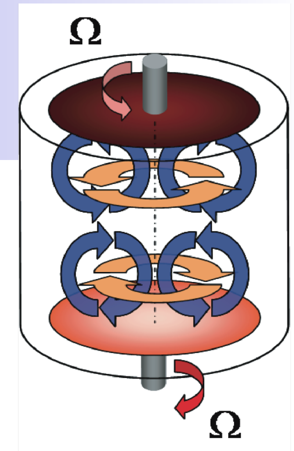
Dissipative scaling law

$$\langle a^2 \rangle = a_0 \epsilon^{3/2} \nu^{-1/2}$$

Heisenberg-Yaglom relation

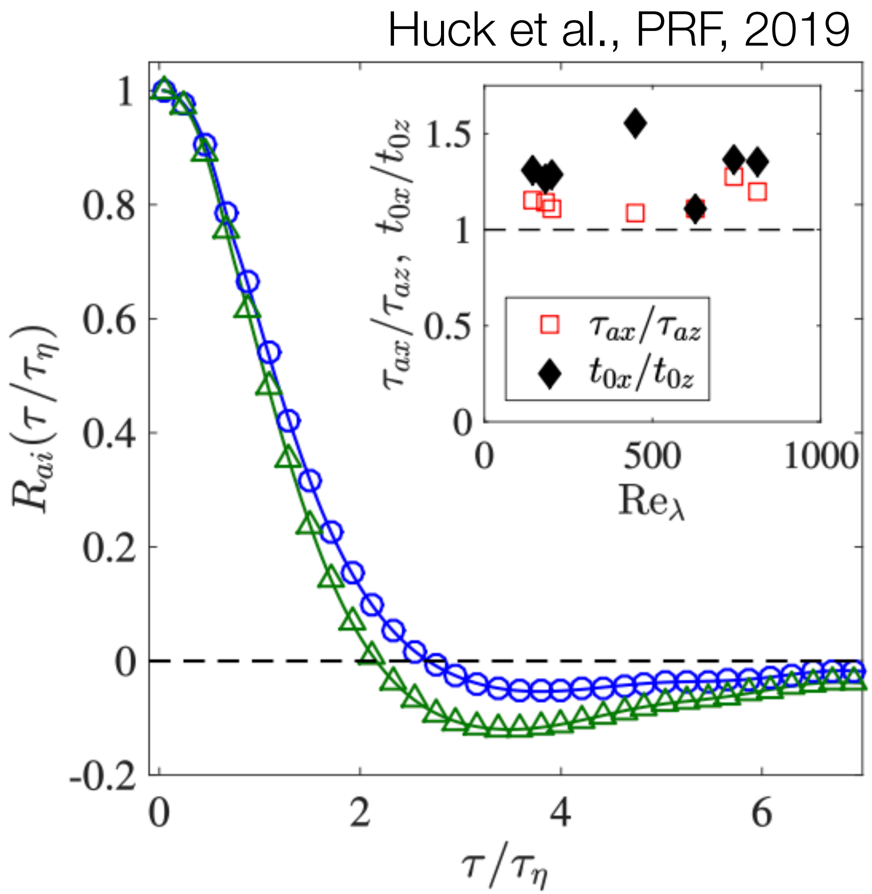
a_0 tends to a constant at high Reynolds number
Acceleration is more isotropic than velocity

Dynamics of fluid tracers



$$R_{a_i}(\tau) = \langle a_i(t) a_i(t+\tau) \rangle \,/\, \langle a^2 \rangle$$

Huck et al., PRF, 2019



Characteristic time-scale

$$\tau_a = \int_0^{t_0} R_{a_i}(\tau) d\tau \propto \tau_\eta$$

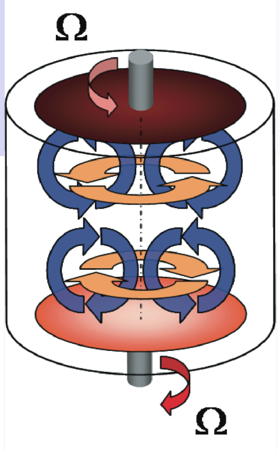
An other result

$$\frac{d}{d\tau}\langle (V(t+\tau) - V(t))^2 \rangle = 2\langle a^2 \rangle \int_0^{\tau} R_a(\tau') d\tau'$$

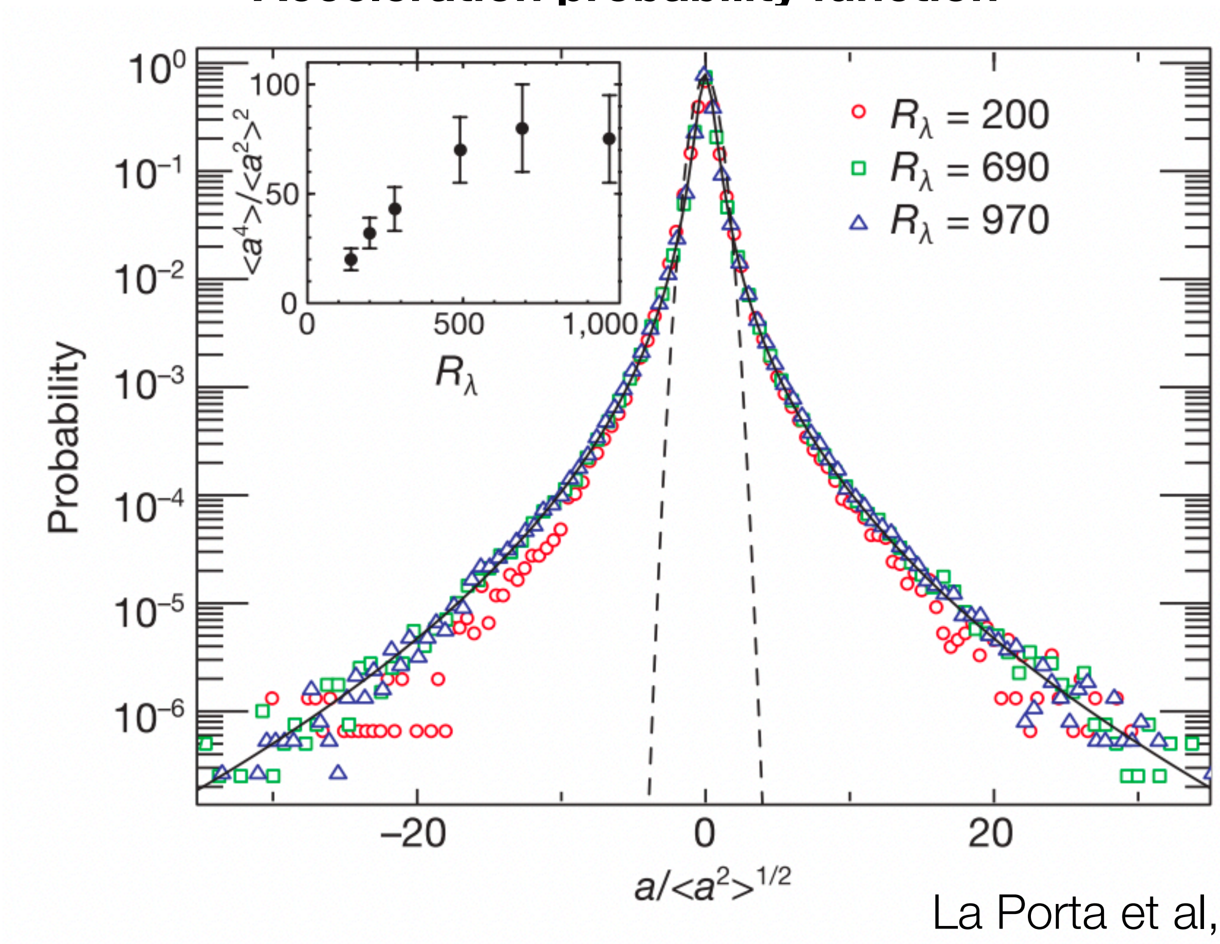
Thus $\displaystyle\int_0^{\infty} R_a(\tau') d\tau' = 0$

Acceleration time-scale are similar for different directions
Acceleration is more isotropic than velocity

Dynamics of fluid tracers



Acceleration probability function

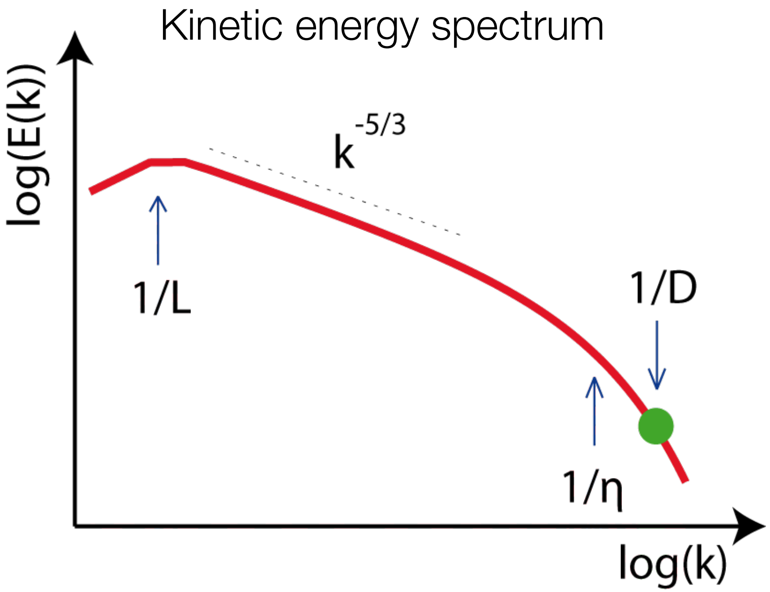


La Porta et al, Nature 2001

Acceleration statistics are strongly non Gaussian with extreme events

Intermittency increases with the Reynolds number

Dynamics of inertial particles



Tracers of flow motions

$$D \leq \eta \quad \& \; \rho_p \sim \rho_f$$

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

Inertial particles

$$\rho_p \neq \rho_f$$

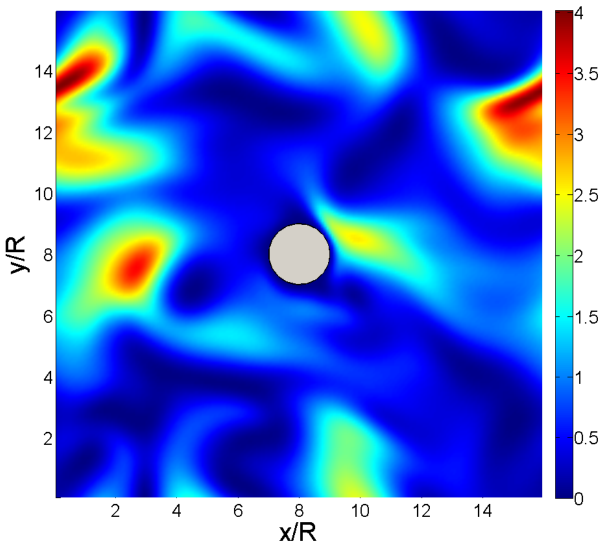
Bubbles in water
Water droplets in air

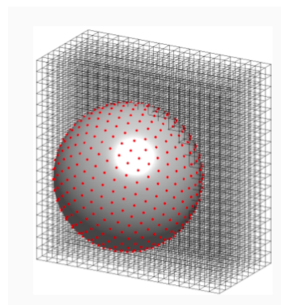
Material particles $D \geq 5\eta$

Dynamics of inertial particles

How to simulate particle motion numerically ?

What people should do





A. Naso

i) Set dimeter and density

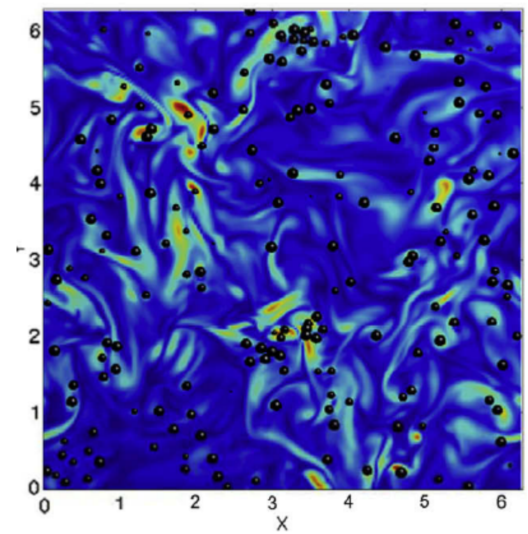
ii) Solve the flow around particles

ex : M. Uhlmann, KIT, Germany

$N_{\text{grid}} = 3000^3 \quad N_p = \mathcal{O}(10^4)$

$Re_\lambda \simeq 200$

What people want to do



$$\begin{cases} \dfrac{\partial \vec{u}}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\dfrac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{u} + \vec{f} \\ \vec{\nabla}\cdot\vec{u} = 0 \\ m\dfrac{d\vec{v}}{dt} = \vec{F}(\vec{v},\vec{u},\ldots) \end{cases}$$

$N_{\text{grid}} = 512^3$

$N_p = \mathcal{O}(10^7)$

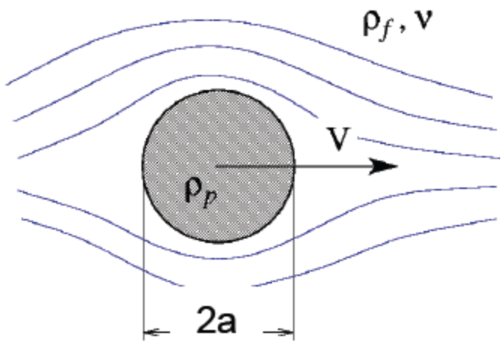
$Re_\lambda \simeq 200$

Dynamics of inertial particles

On the force exerted by the flow on a particle …

Compute the flow around the sphere :
(and integrate to get the force and torque …)

$$v = u + W$$



$$Re_p = \frac{a(u-v)}{\nu} \ll 1$$

$$\partial_t W = -\frac{1}{\rho_f}\nabla P + \nu \Delta W$$ + B.C.

Solution for the stationary problem : Stokes drag

$$-6\pi\nu\rho_f a\,(v-u)$$

For the linear case, no feed-back from rotation (no lift force)

Dynamics of inertial particles

On the force exerted by the flow on a particle ...

Non stationary case : $$\partial_t W = -\frac{1}{\rho_f}\nabla P + \nu \Delta W \quad + \text{B.C.}$$

Maxey & Riley 1983, Gatignol 1983

$$m_p \frac{dv}{dt} = m_f \frac{Du}{Dt} + (m_p - m_f) g$$ pressure gradient & gravity

$$-6\pi\nu\rho_f a\,(v-u)$$ viscous (Stokes) drag

$$-\frac{1}{2} m_f \left(\frac{dv}{dt} - \frac{du}{dt}\right)$$ added mass

$$-\int_0^t K(t-\xi)\left(\frac{dv}{d\xi} - \frac{du}{d\xi}\right) d\xi$$ history

Dynamics of inertial particles

A minimal model for inertial particles in a flow (no gravity)

$$a \ll \eta \qquad\qquad Re_p = \frac{a(u-v)}{\nu} \quad \text{not too large}$$

Minimal model accounting for inertia (beta-Stokes model)

$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\frac{Du}{Dt} + \frac{3\nu}{a^2}(u-v)\right)$$

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$$\tau_p = \frac{1}{3\beta}\frac{a^2}{\nu}$$

$$St \equiv \frac{\tau_p}{\tau_\eta} = \frac{1}{3\beta}\frac{a^2}{\eta^2}$$

Stokes number

$\beta = 3$ Bubble

$\beta = 1$ Neutrally buoyant particle

$\beta = 0$ Very dense particle

History term proved to be negligible for turbulent flows
(Burton et al. JFM 2005 : DNS around a fixed particle)

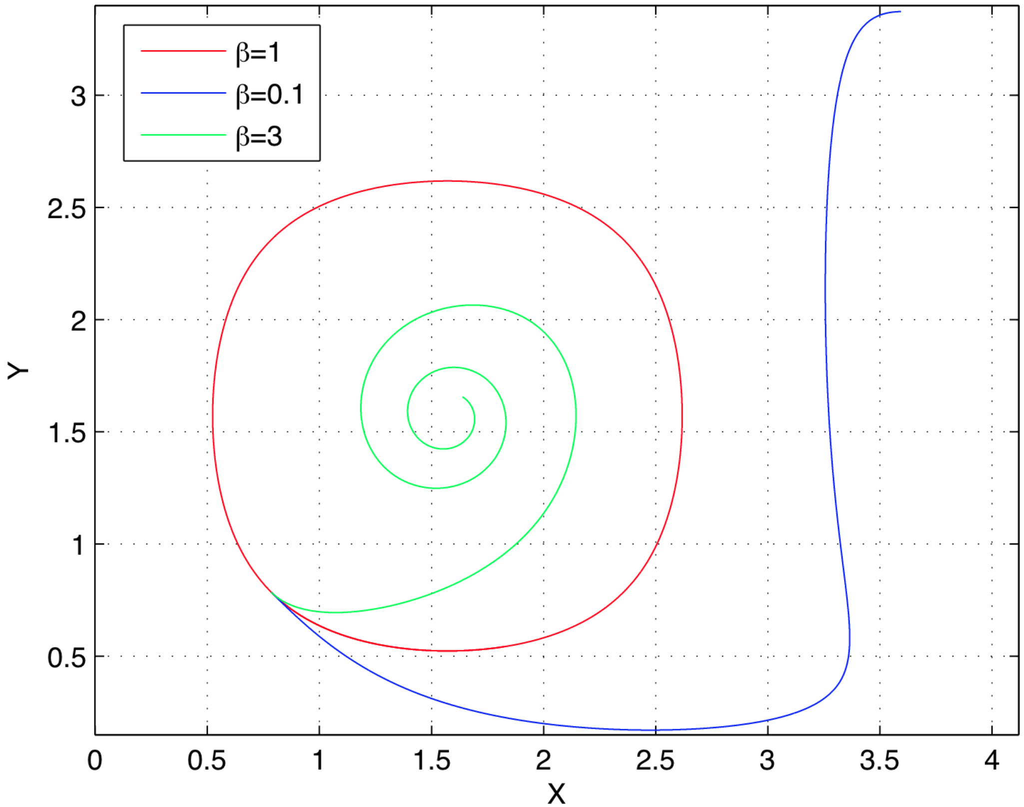
Dynamics of inertial particles

Some phenomenology

Heavy particles (smaller acceleration)

Bubbles (larger acceleration)

$$\frac{dv}{dt} = \beta \frac{Du}{Dt} + \frac{1}{\tau_p}(u - v)$$



Heavy particles ejection from vortices

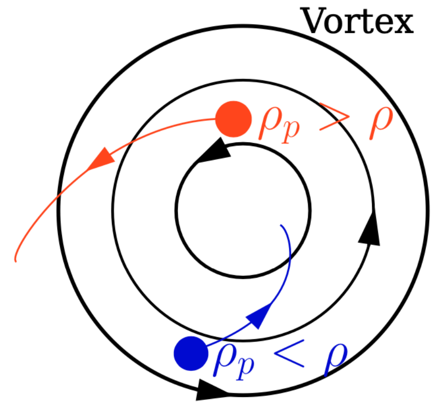
Light particles may be trapped

Particle segregation

Dynamics of inertial particles

More on phenomenology

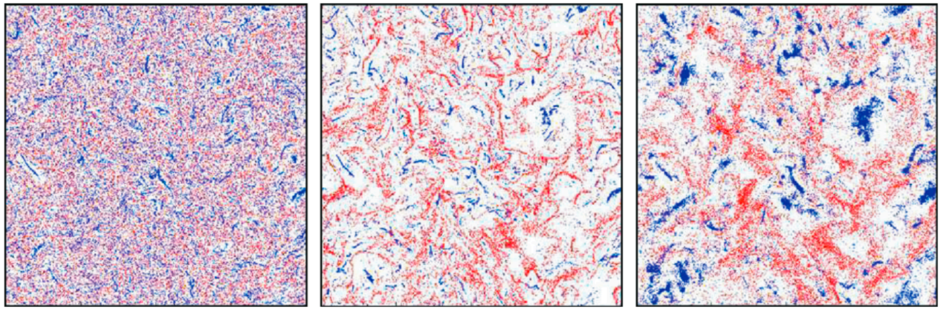
- Inertial particles $\begin{cases} \rho_p \neq \rho \\ \text{and / or} \\ d_p > \eta \end{cases}$ $\xrightarrow[\text{response time}]{\text{Non zero}}$ $\tau_p \neq 0$



Centrifugation Mechanism

- Compare τ_p and τ_η, Stokes number (inertia) : $St = \dfrac{\tau_p}{\tau_\eta}$

DNS from Calzavarini et al. (2008)



$St = 0.1$ $St = 0.6$ $St = 4$

Red : Heavy particle $\rho_p > \rho$

Blue : Light particle $\rho_p < \rho$

Flying Chairs

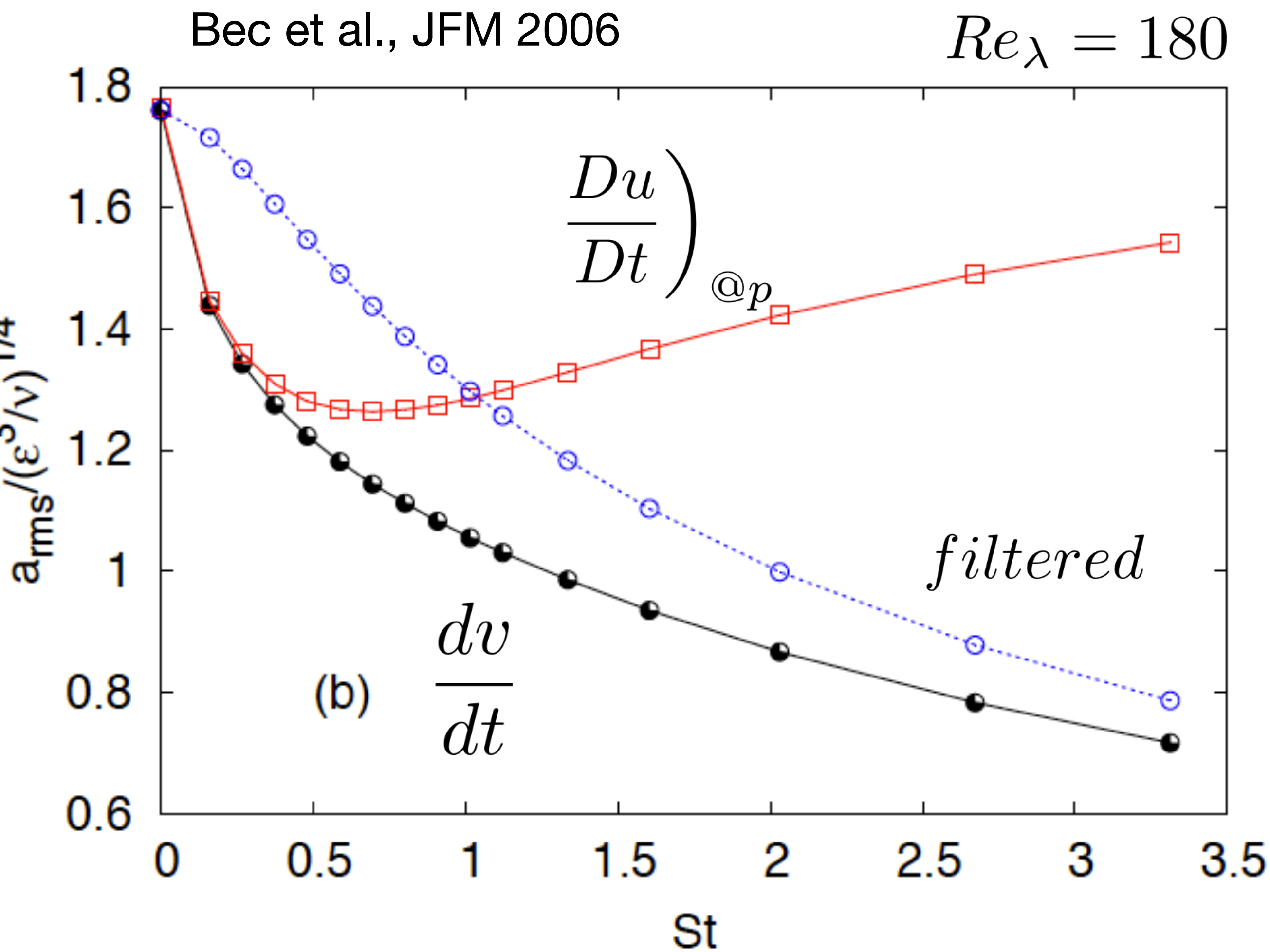
- Turbulence unmixes inertial particles $\longrightarrow$ $\vec{\nabla} \cdot \vec{v} \neq 0$

Particle segregation (low Stokes number)

$$\vec{v} \simeq \vec{u} + \tau_p(\beta - 1)\frac{D\vec{u}}{Dt}$$

Dynamics of inertial particles

Very small, very heavy, particles : influence of segregation



$$\frac{dv}{dt} = \frac{1}{\tau_p}(u - v)$$

One parameter

$$St = \frac{\tau_p}{\tau_\eta}$$

Filtering

$$a_p(\omega) \sim \frac{a_T(\omega)}{\sqrt{1 + \omega^2 \tau_p^2}}$$

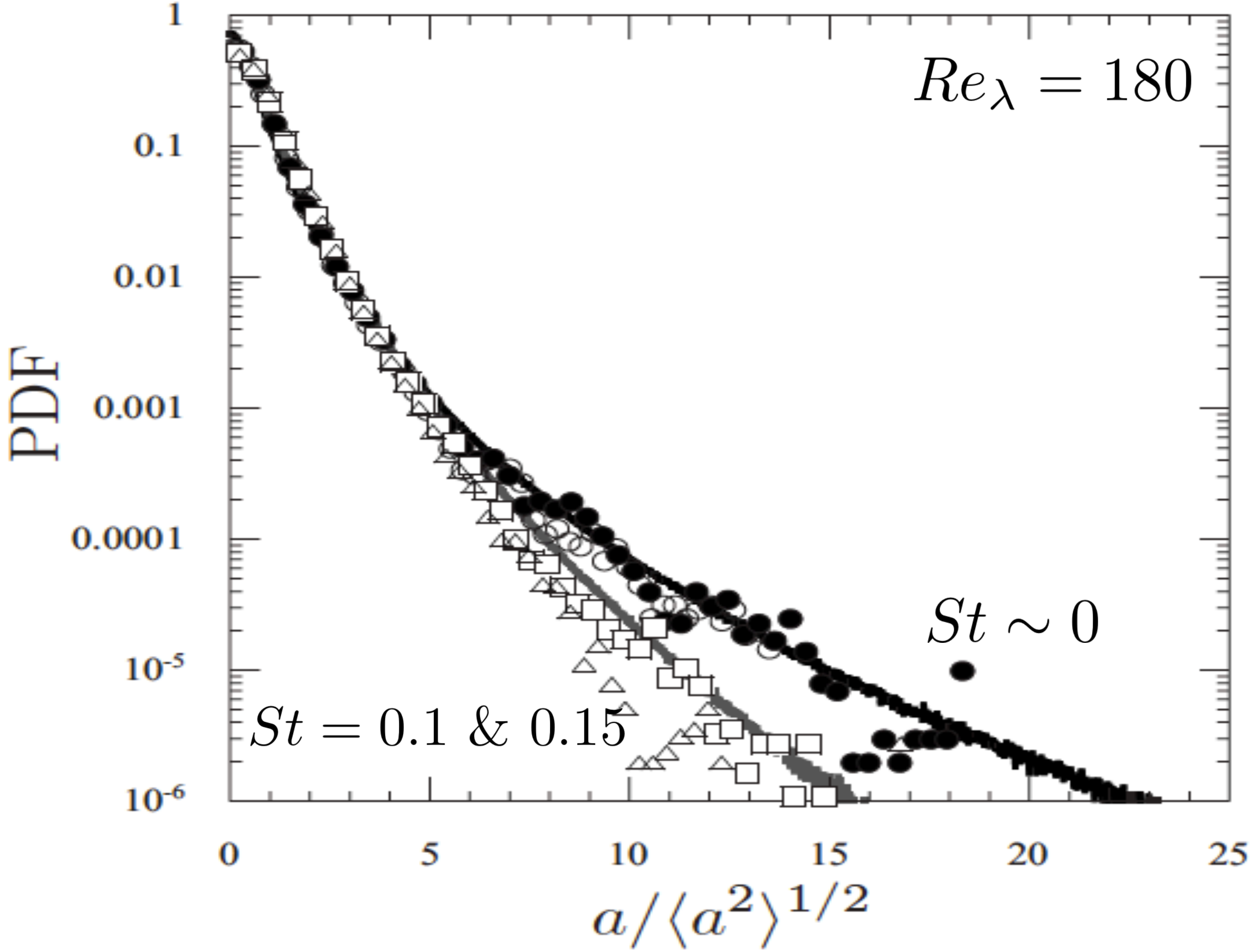
Selective sampling at small stokes numbers (St < 1)

Then more homogeneous distribution and temporal filtering

Dynamics of inertial particles

DNS vs Wind Tunnel Data

Warhaft group (Cornell) POF (2008)



Rough agreement on

$$a_p/a_T$$

at increasing St

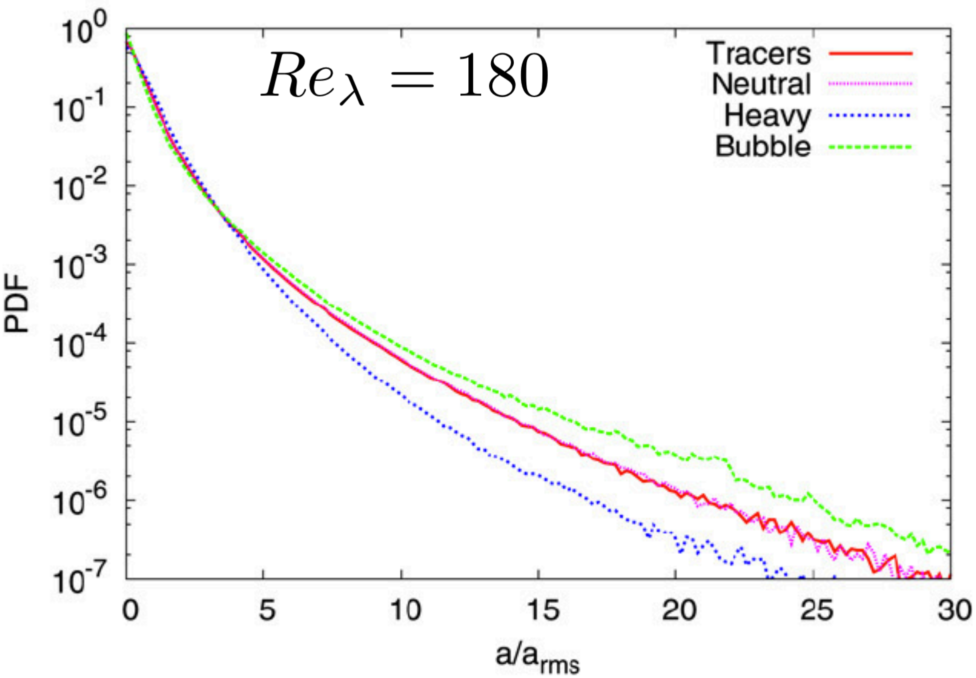
Pdf's very similar at same St (acceleration variance not given)

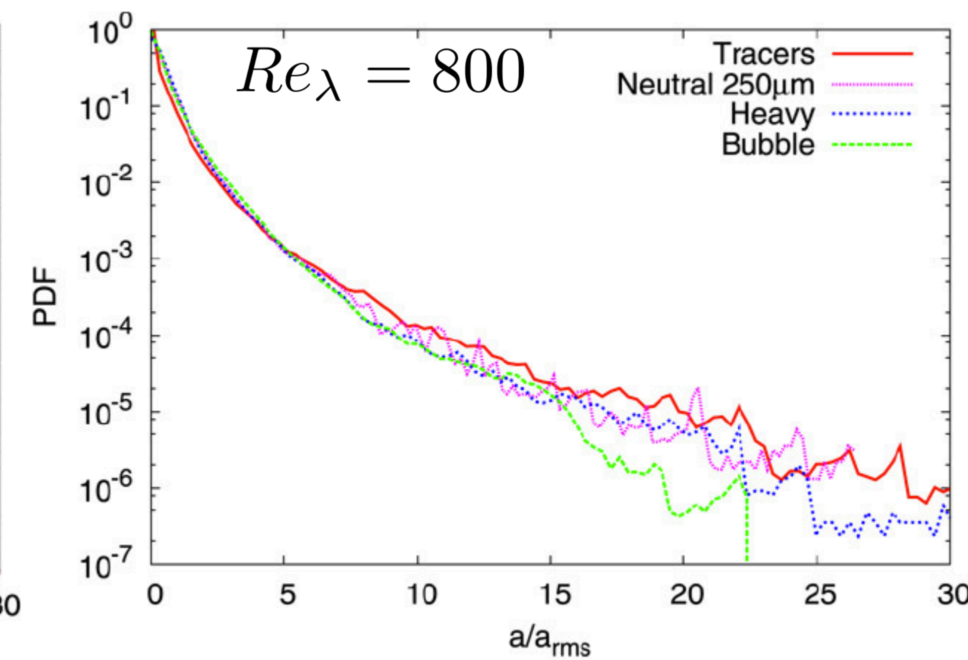
Model seems correct absen

Dynamics of inertial particles

Another comparison : DNS vs von Karman in water

Numerics (DNS) | Experiment (eLDV)





Bubbles, Glass beads, Tracers, Large neutrally buoyant $D/\eta \sim 15$

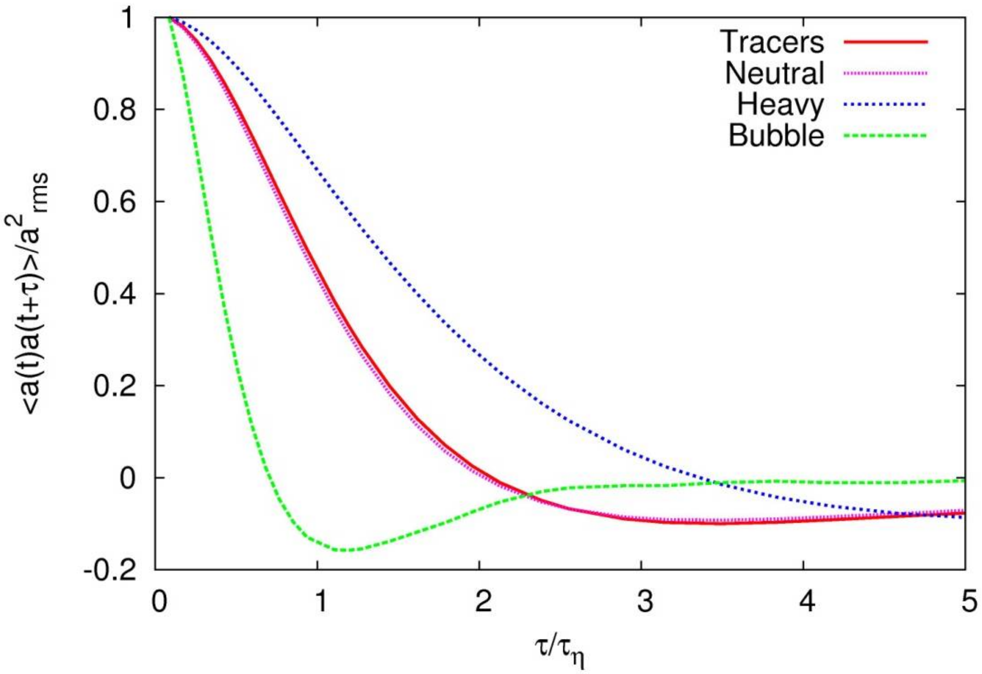
Volk, Calzavarini et al., Physica D (2008)

Dynamics of inertial particles

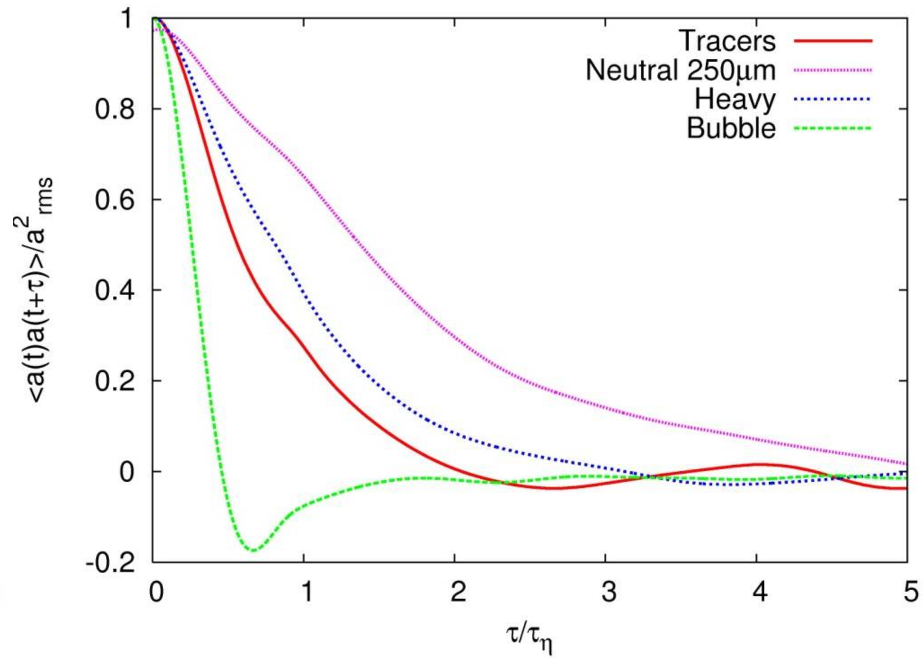
An other comparison : DNS vs von Karman in water

$$C_{aa}(\tau) = \langle a(t)a(t+\tau)\rangle / a^2_{rms}$$

Numerics (DNS)



Experiment (eLDV)



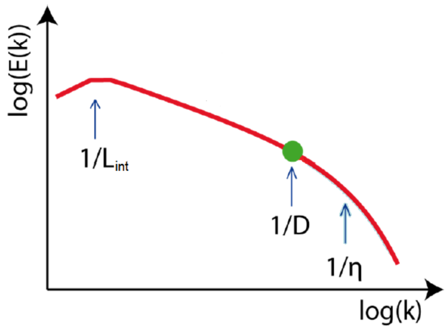
Model sounds physical for small particles
But large neutrally buoyant particles are not tracers
Increasing the size is not increasing St in the model …

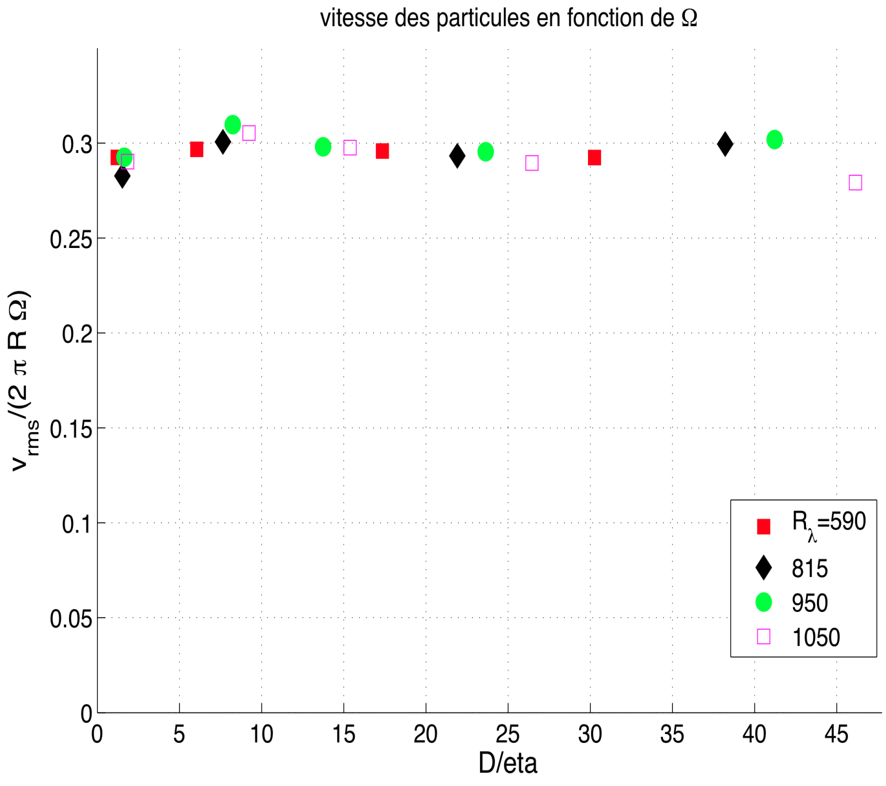
Large neutrally buoyant particles

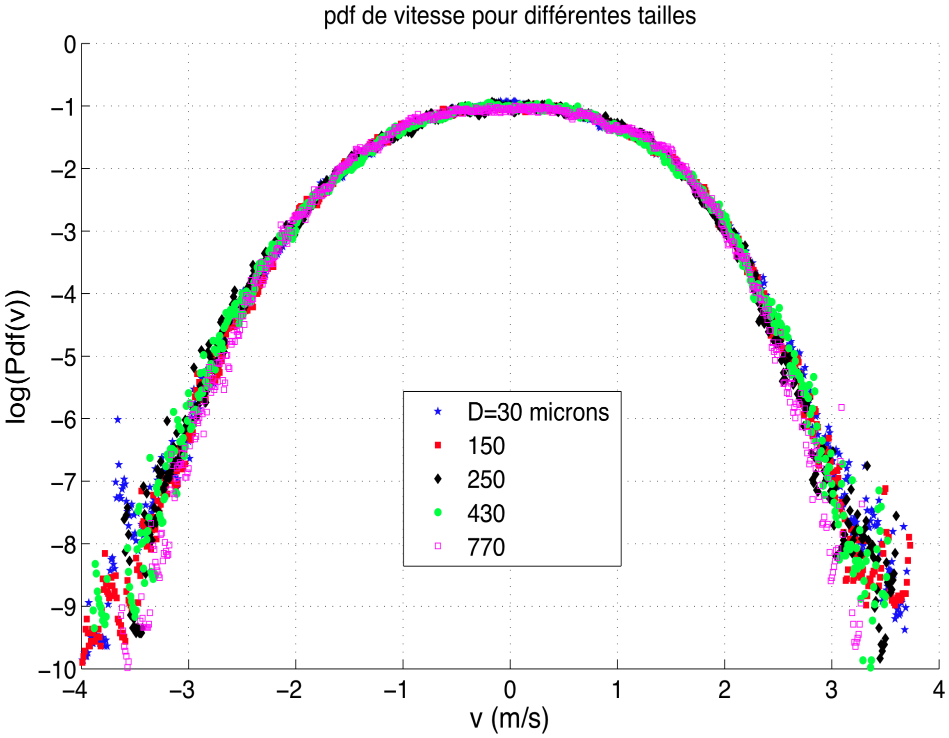
Main experiments

Qureshi et al. PRL 2007,
Brown et al. PRL 2009, Volk et al. JFM 2011

$$D/\eta \in [1, 60] \qquad Re_\lambda \in [180, 1000]$$





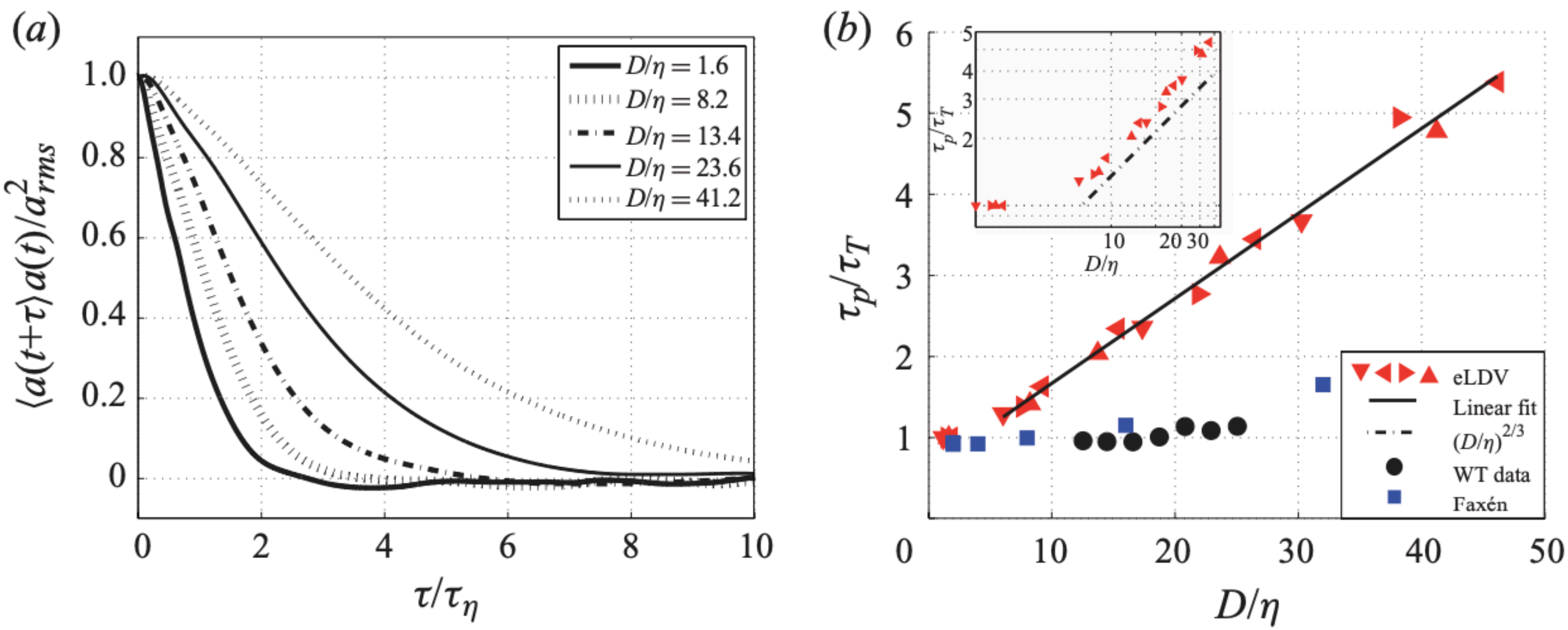


Weak impact on large scale statistics

Large neutrally buoyant particles

Acceleration variance

$$C_{aa}(\tau) = \langle a(t)a(t+\tau)\rangle / a_{rms}^2 \qquad \tau_p = \int_0^{T_0} C_{aa}(\tau)d\tau$$

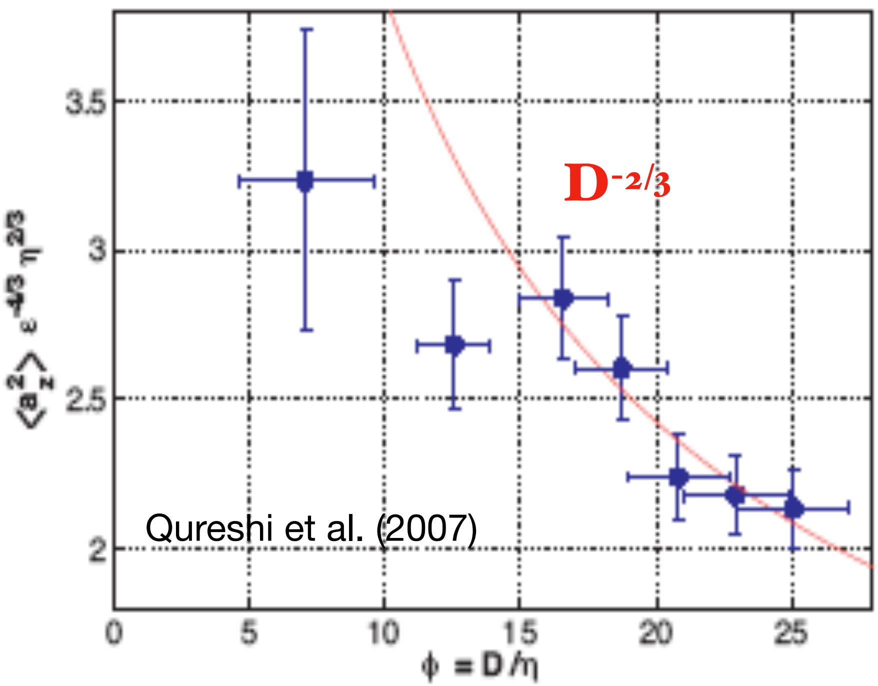


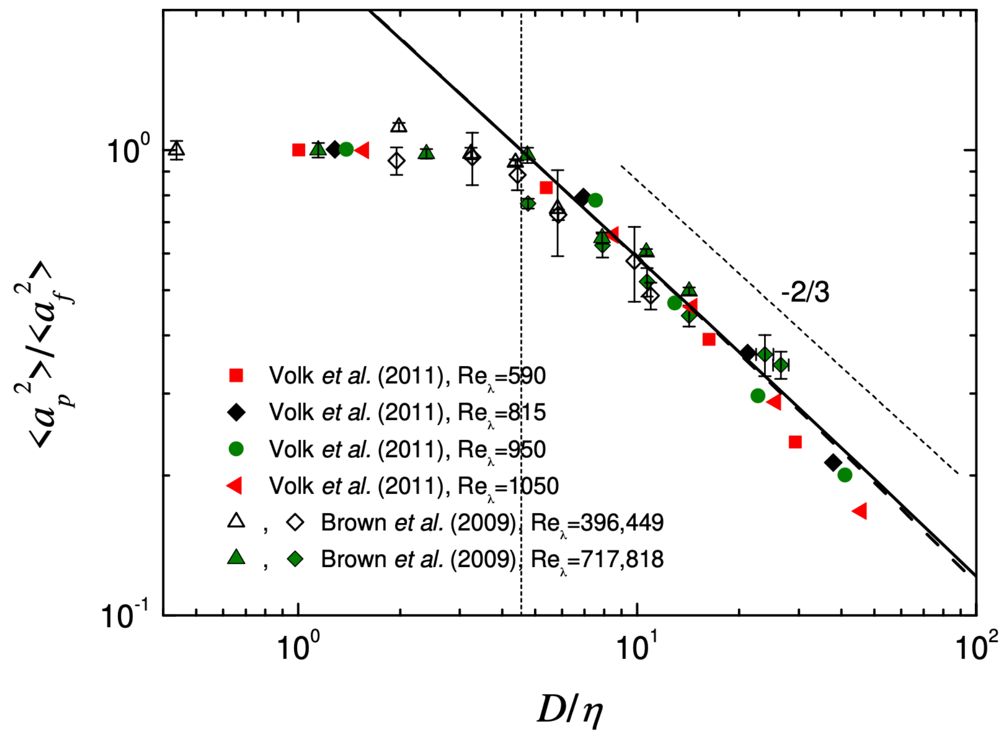
Major impact on small scale statistics $\tau_p \propto \tau_\eta \left(\frac{D}{\eta}\right)^\alpha$

$$\alpha = 2/3 \Leftrightarrow \tau_p \propto D^{2/3}\epsilon^{-1/3}$$

Large neutrally buoyant particles

Acceleration variance

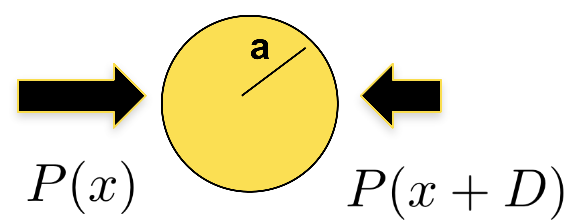




Dimensional prediction

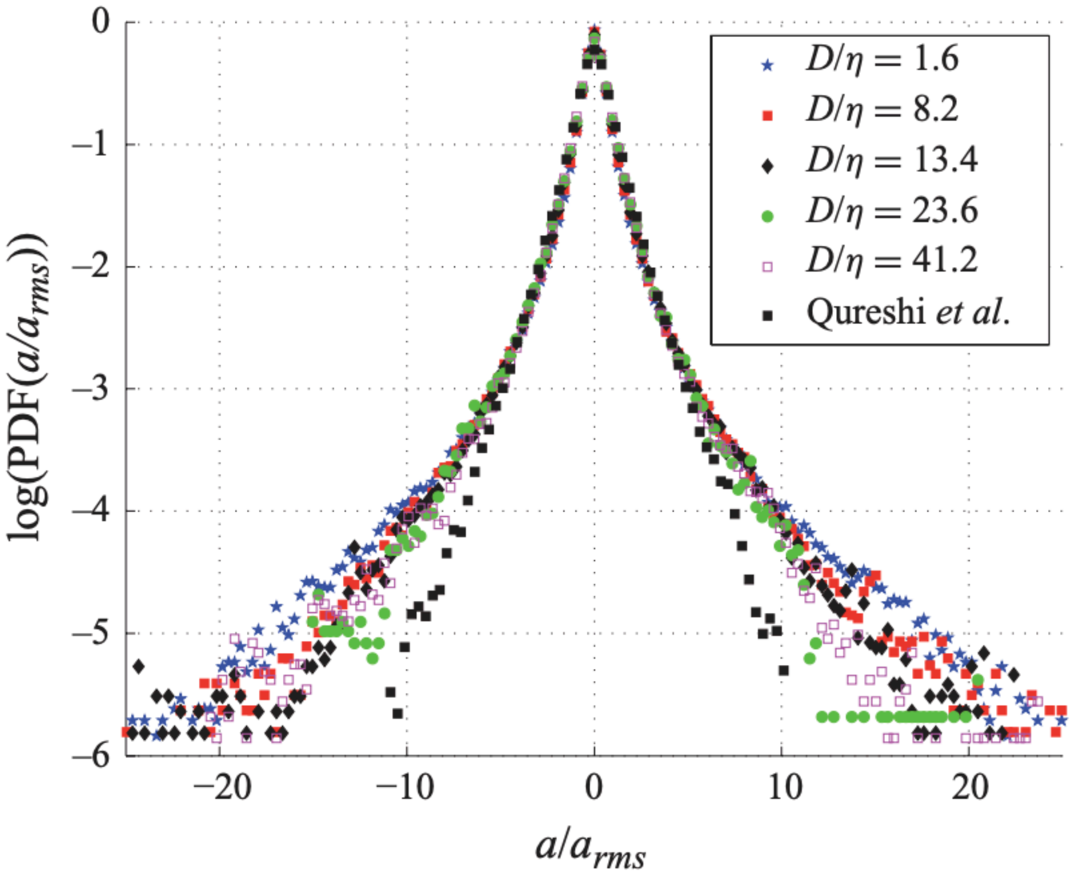
$$\vec{a} \propto \vec{\nabla}_D \, p$$

$$\langle a^2 \rangle \sim \left(\frac{\delta_D P}{D} \right)^2 \qquad \langle a^2 \rangle = a_0' \epsilon^{4/3} D^{-2/3}$$



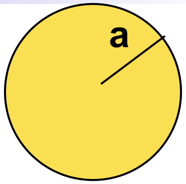
Large neutrally buoyant particles

Acceleration PDFs



The flatness decreases when the size increases, reduced intermittency

Large neutrally buoyant particles



Taking into account the size of the particles

Maxey & Riley, 1983

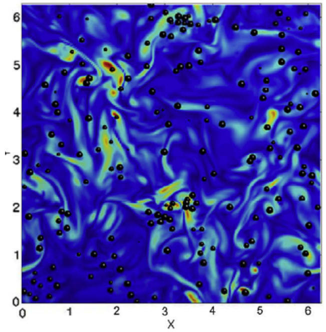
$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\frac{Du}{Dt} + \frac{3\nu}{a^2}(u - v)\right)$$

volume average

surface average

Gatignol, R. 1983

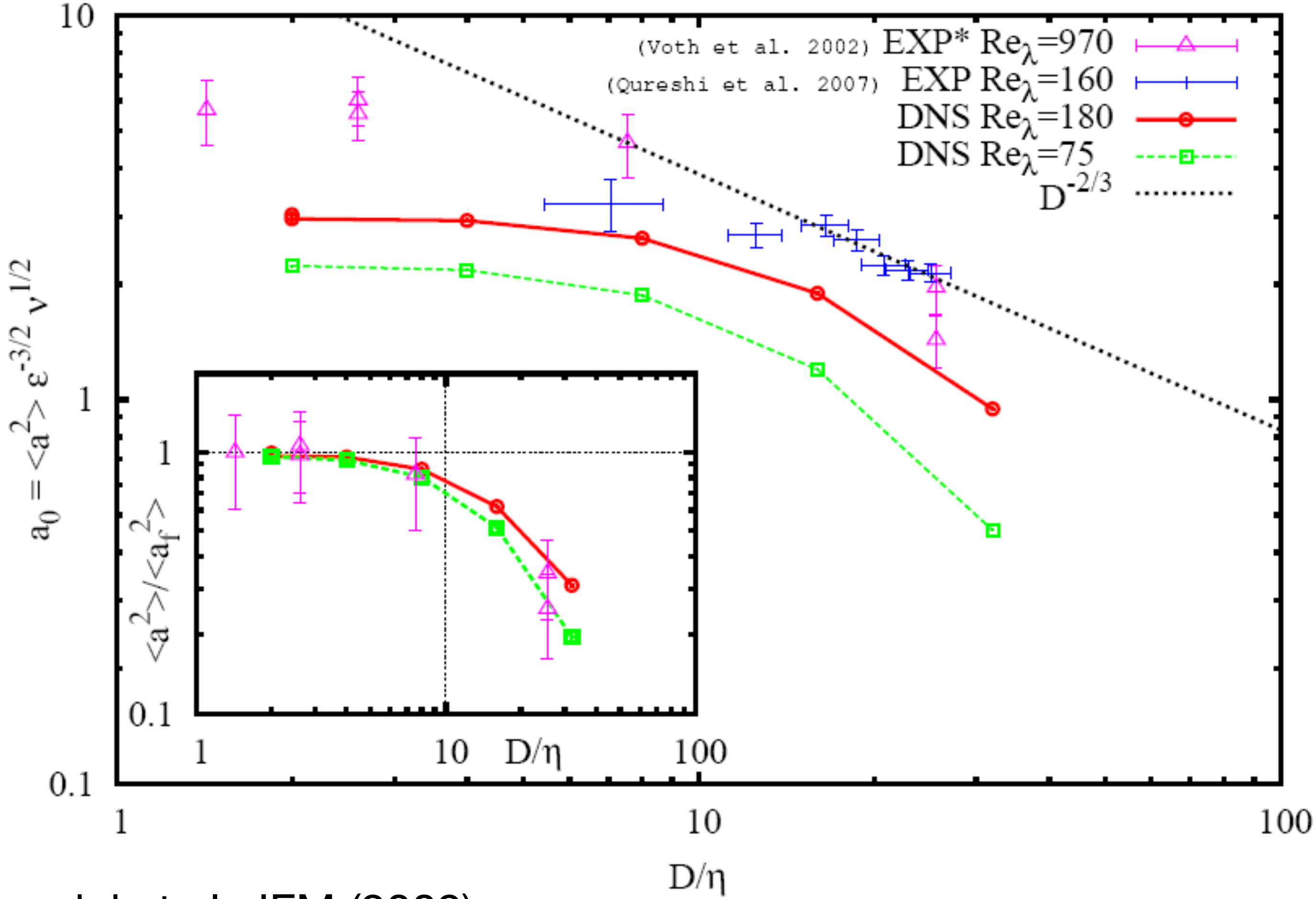
$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\left\langle \frac{Du}{Dt}\right\rangle_V + \frac{3\nu}{a^2}(\langle u\rangle_S - v)\right)$$



Computationally efficient
Spectral DNS + filtering in k space (gaussian kernel)
Particles move in spatially averaged fields

Large neutrally buoyant particles

Faxen model *vs* data

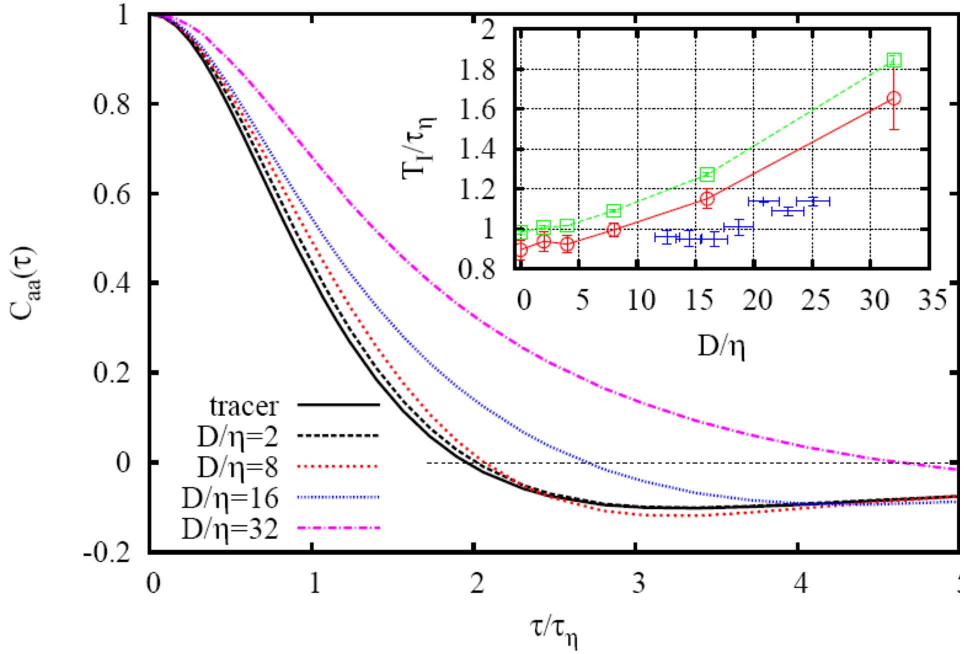


Calzavarini et al. JFM (2009)

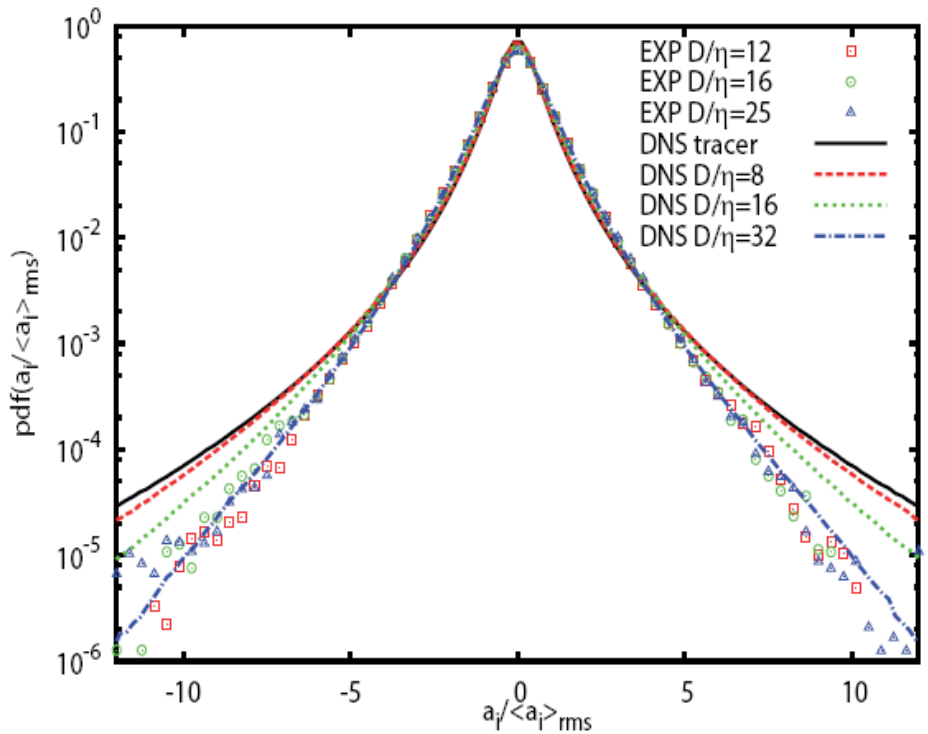
Large neutrally buoyant particles

Faxen model *vs* data

Autocorrelation



Acceleration PDFs

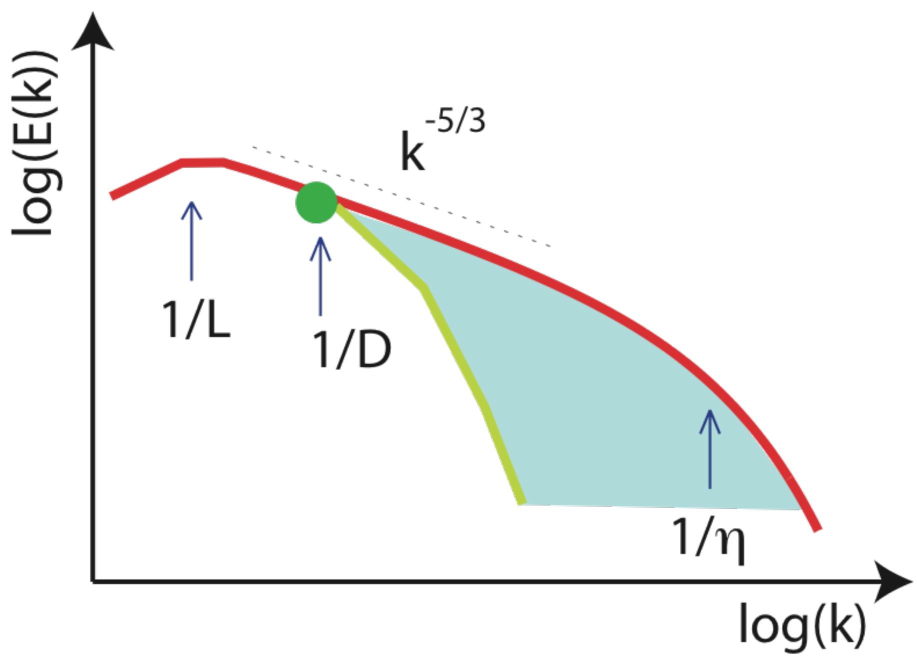


This model solves the main discrepancies

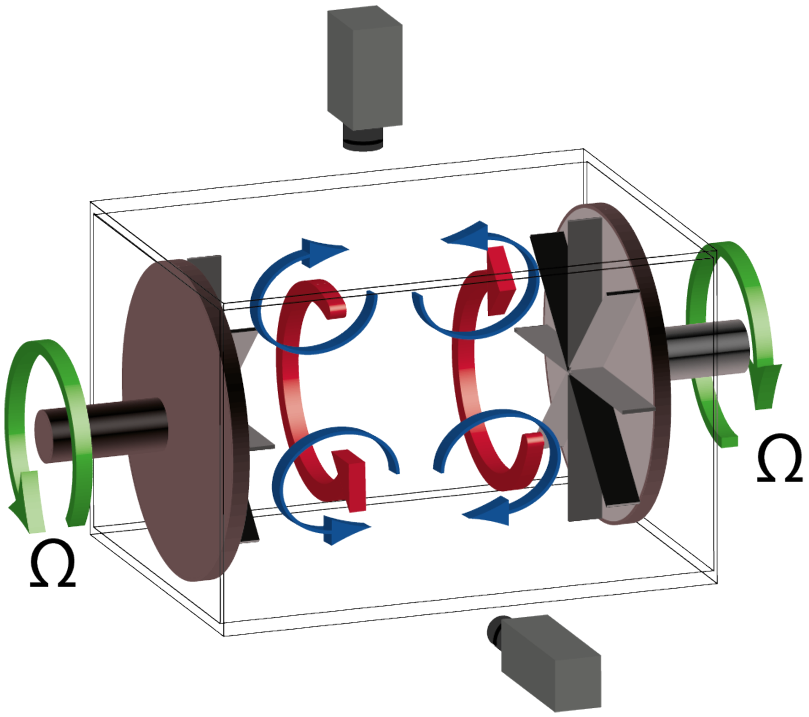
Increasing the size is taking into account the spatial extension of the particle

Improved with non linear drag : Calzavarini et al., Physica D (2012)

Dynamics of integral size particles



Integral size particles



$\Omega = 0.5 - 4.5 \text{ Hz}$

$R = 10 \text{ cm} \quad L \sim 3 \text{ cm}$

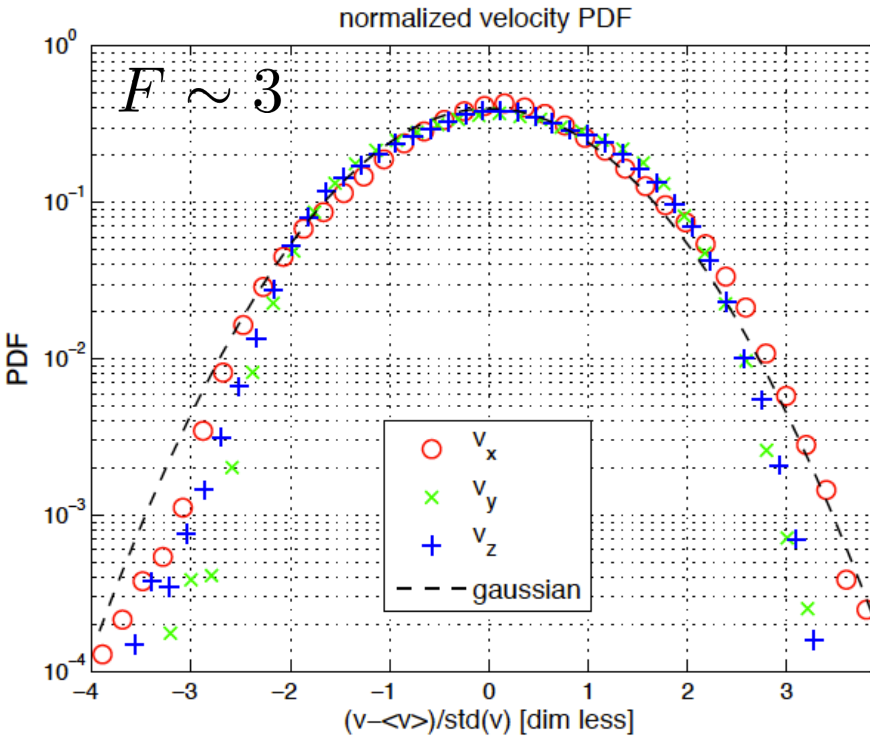
$Re = \frac{UR}{\nu} \in [10^4 - 10^6]$

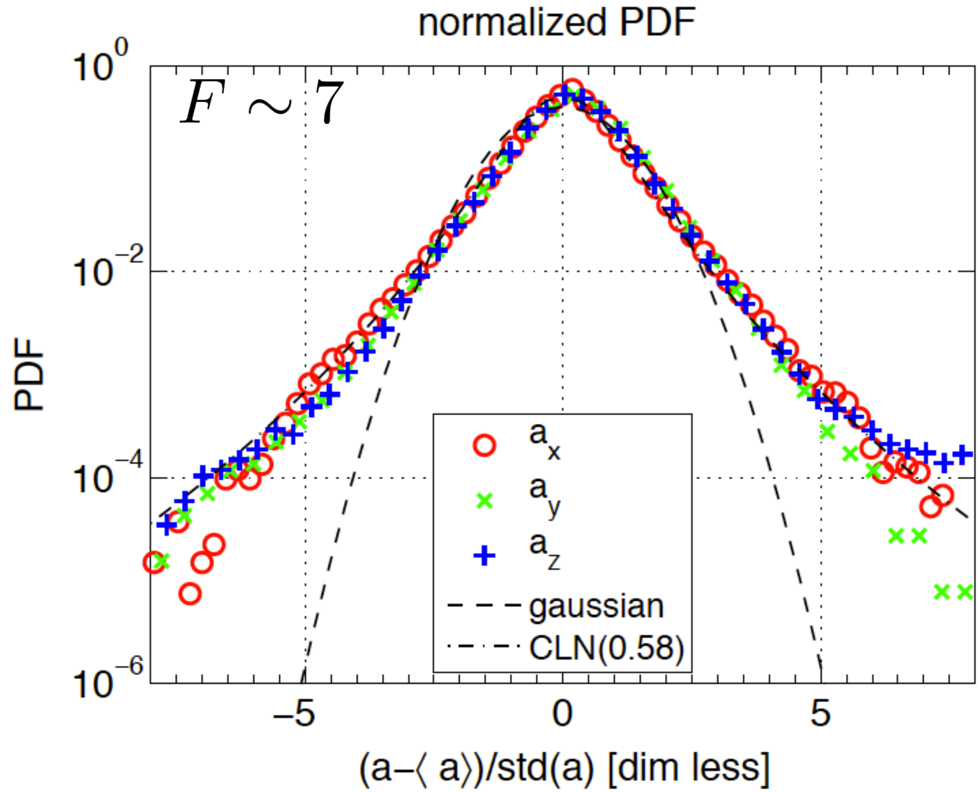
$\rho_p/\rho_f = [0.9,\ 1,\ 1.14]$

$D_{sphere} = 6 - 25 \text{ mm} \sim 0.2 - 0.8\ L$

Integral size particles

D/L = 0.6





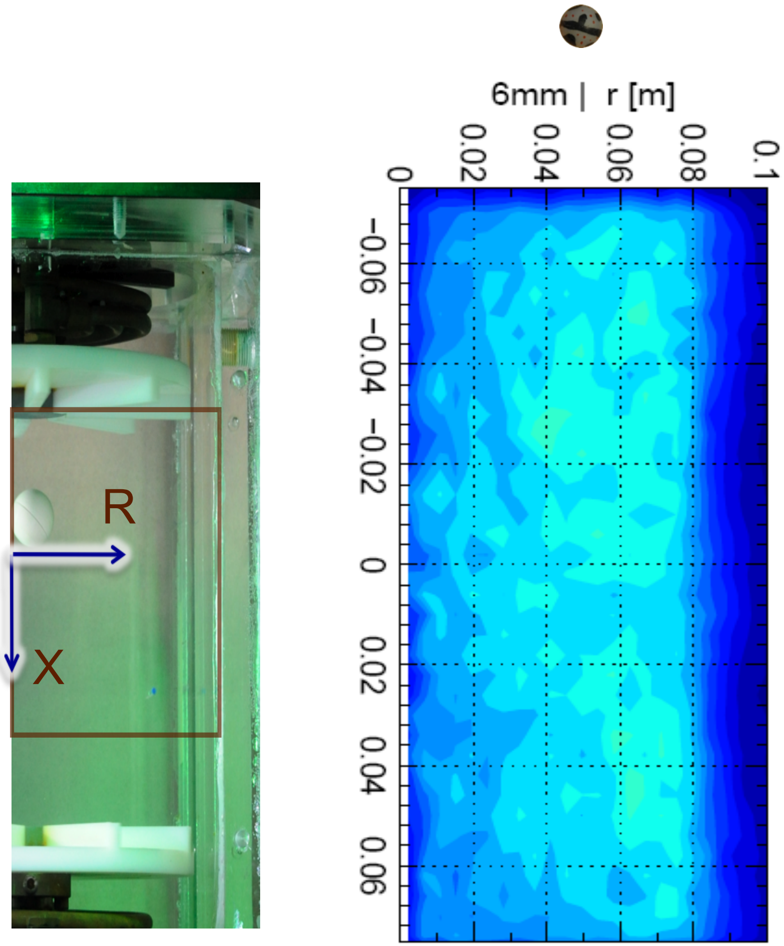
$$v_{rms} \propto 2\pi R f_{\text{imp}}$$

$$a_{rms} \propto 4\pi^2 R f_{\text{imp}}^2$$

Gaussian velocity PDF
Non gaussian acceleration PDF

Integral size particles

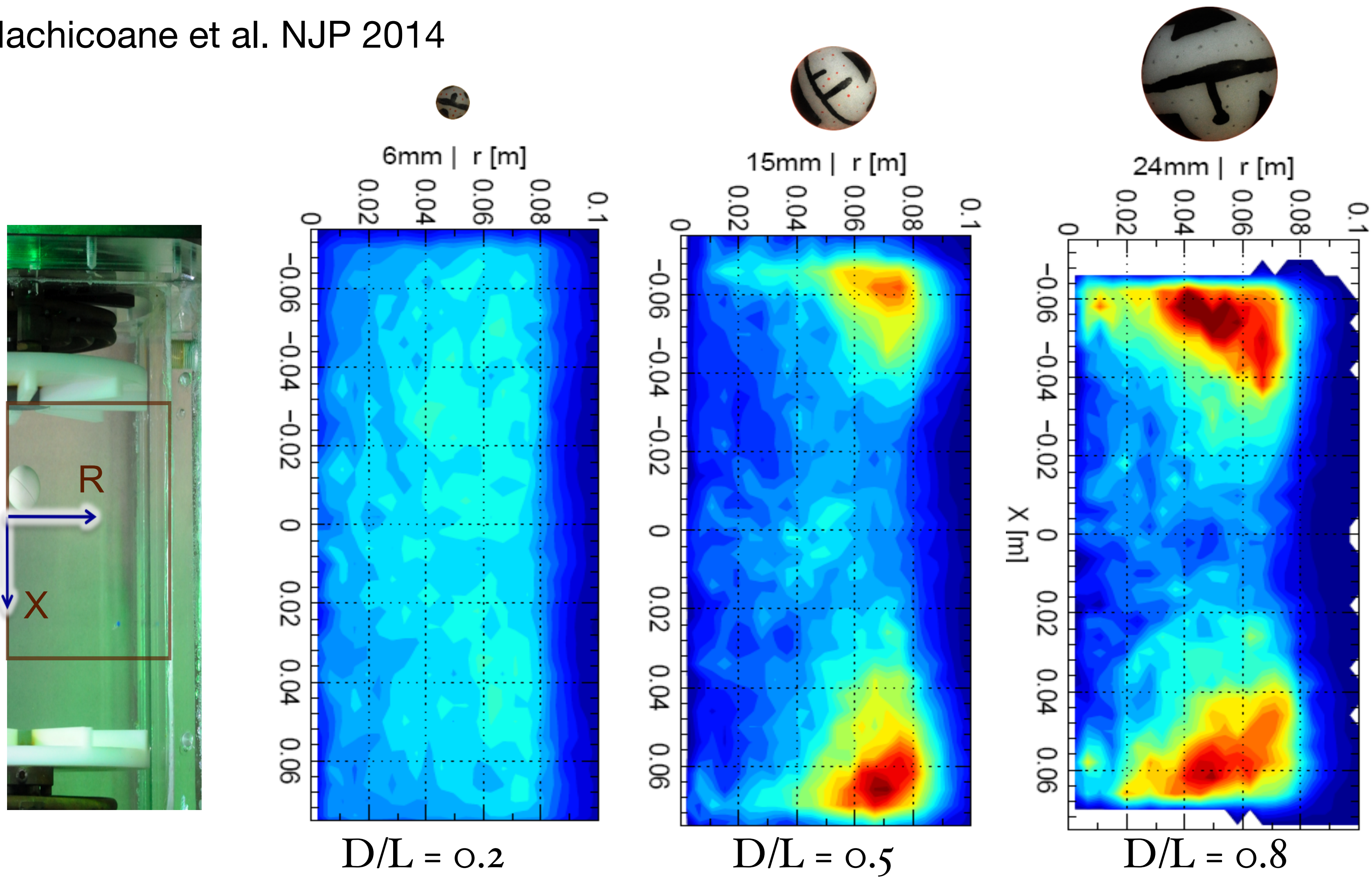
Machicoane et al. NJP 2014



Particles with D/L < 0.2 almost uniformly sample the flow

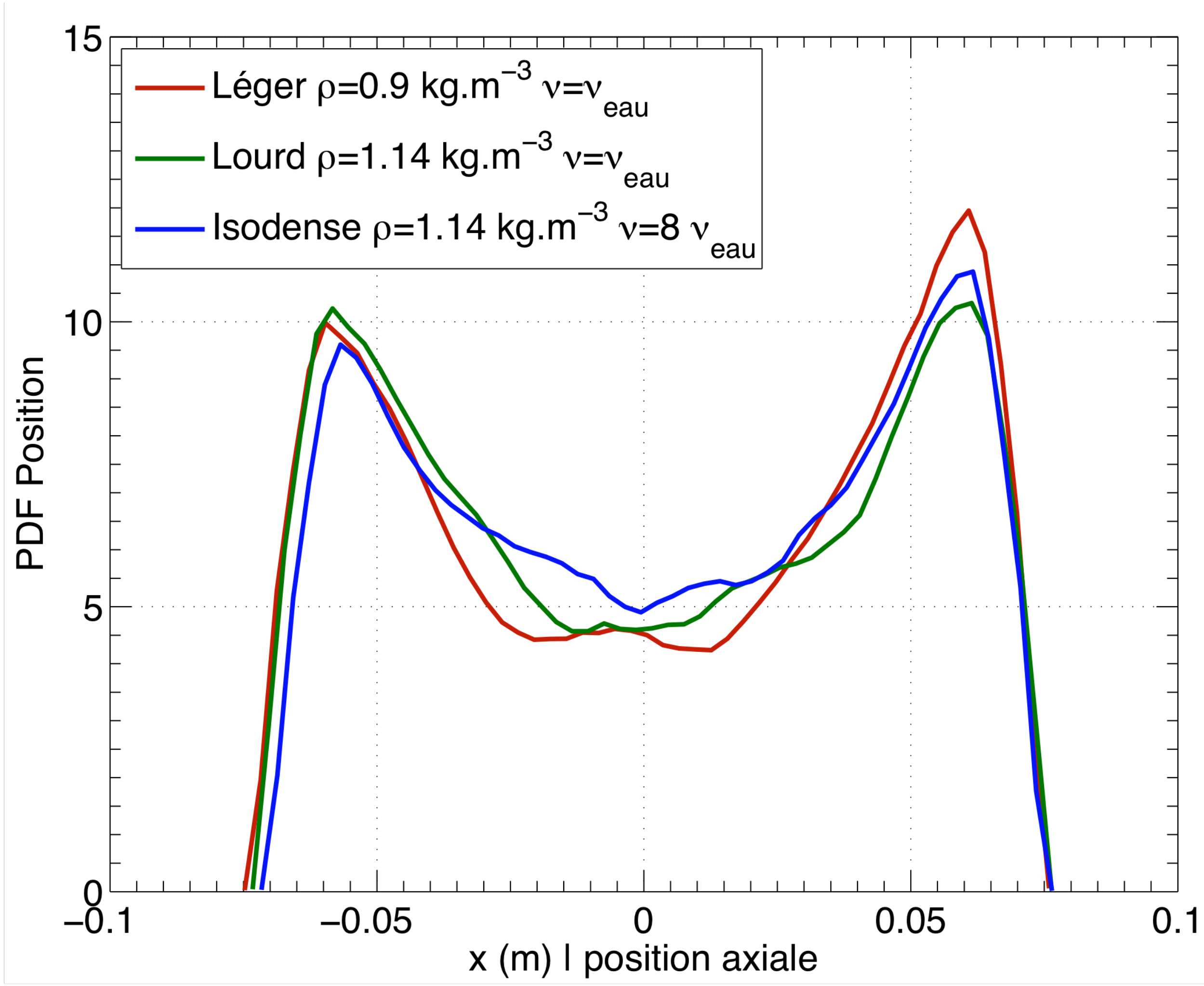
Integral size particles

Machicoane et al. NJP 2014



Large particles are trapped (D/L > 0.3)
Independent of Re, weak influence of density

Integral size particles



$$\beta = 3\rho_f/(2\rho_p+\rho_f)$$

PA / gly40%-eau β=1

$Re_\lambda \sim 200$

PA / water β=1.07

PP / water β=0.9

$Re_\lambda \sim 550$

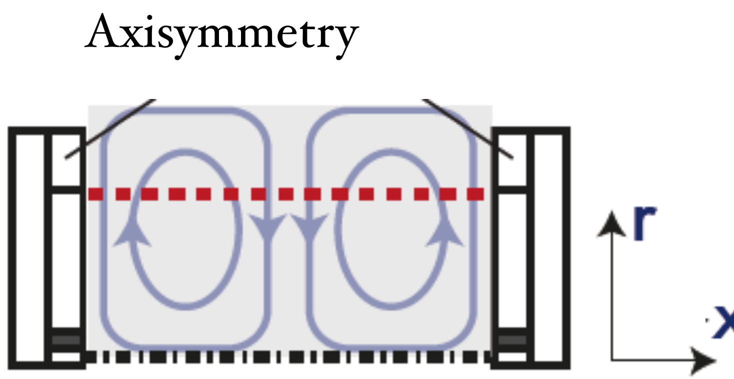
Modulated trapping : but the main effect comes from the size

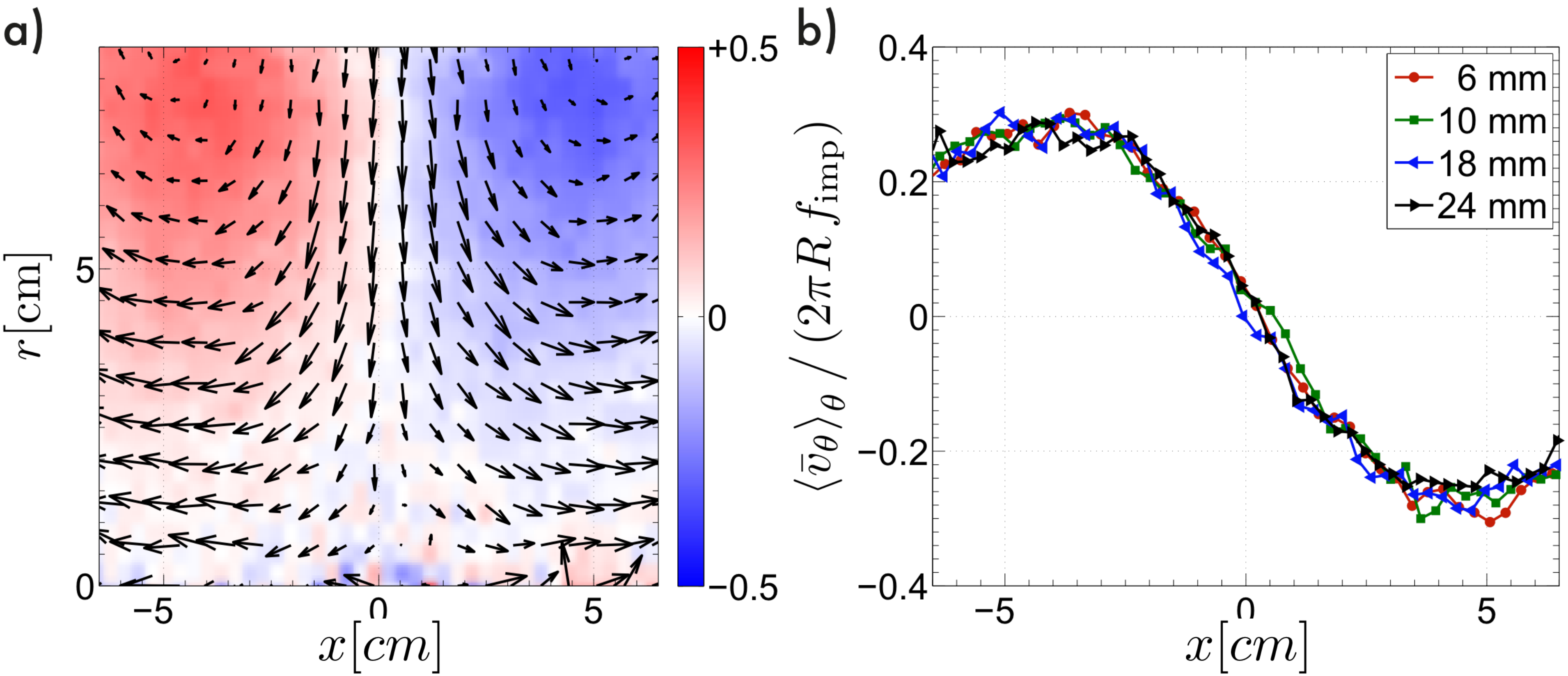
Integral size particles

Particle based mean flow :

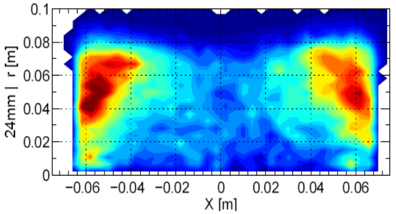
$$\overline{\mathbf{v}}_E(r,\theta,x) = (\overline{v}_r, \overline{v}_\theta, \overline{v}_x)$$

$$\langle \overline{\mathbf{v}}_E \rangle (r,\theta,x)$$

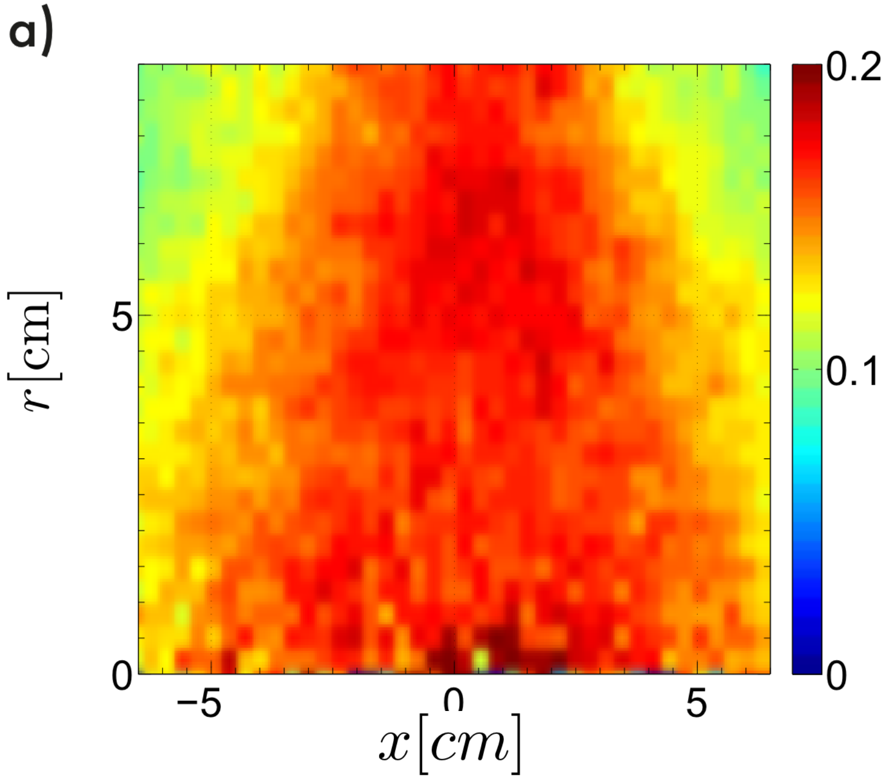


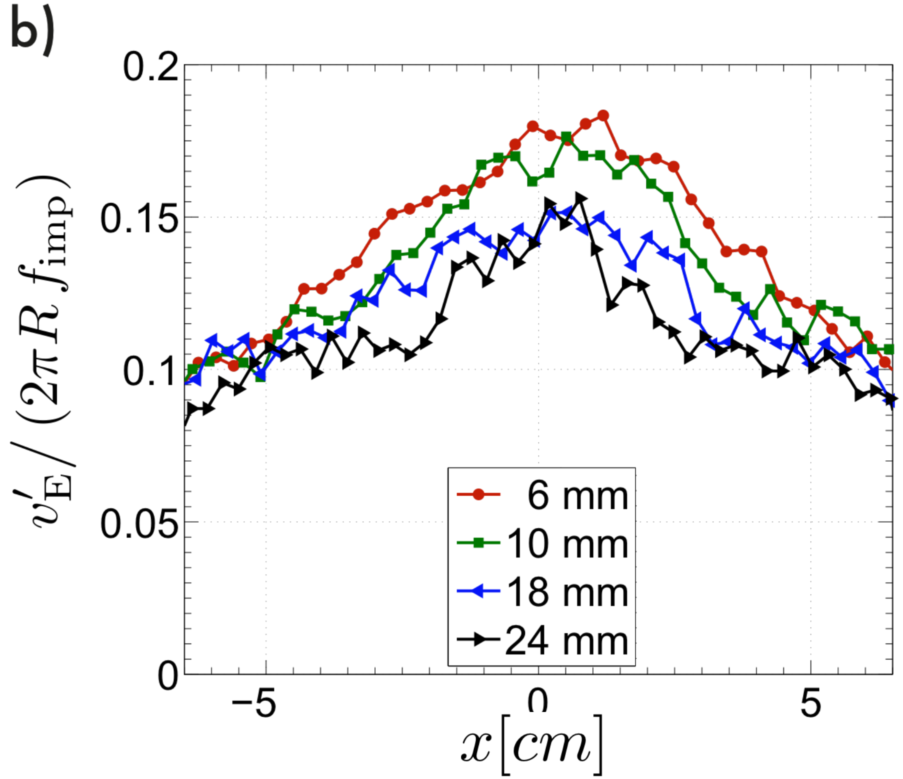


Integral size particles



Particle acceleration velocity



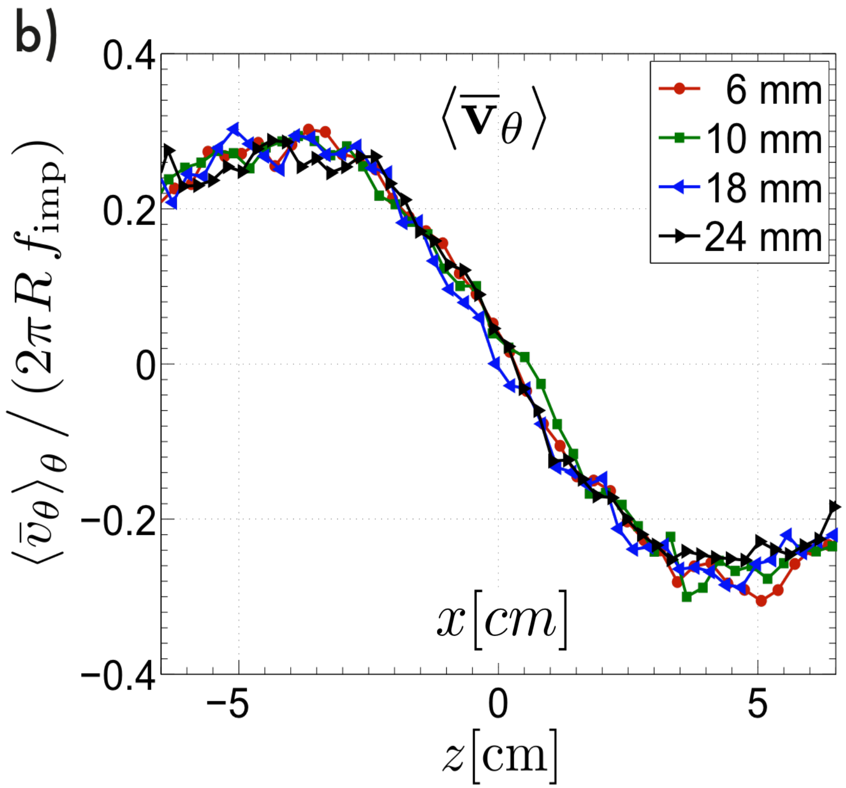


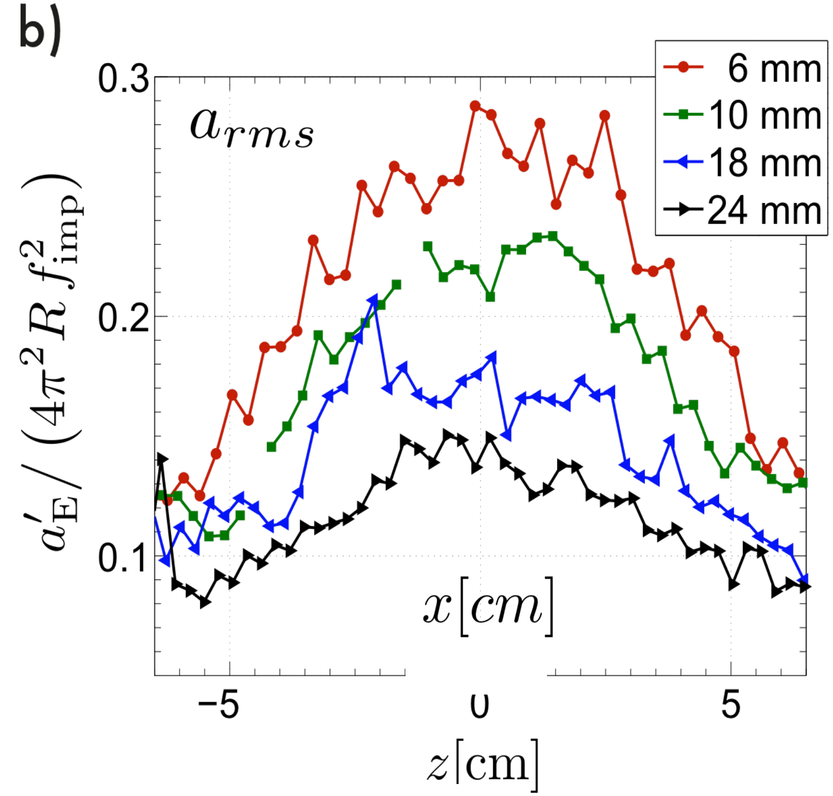
rms velocity decreases at increasing D

Very different from smaller particles (D in inertial range)

Integral size particles

An explanation for the trapping ?





Turbulence (acceleration) : $a_{rms} \sim 4\pi^2 R\Omega^2 (D/L)^{-1/2}$

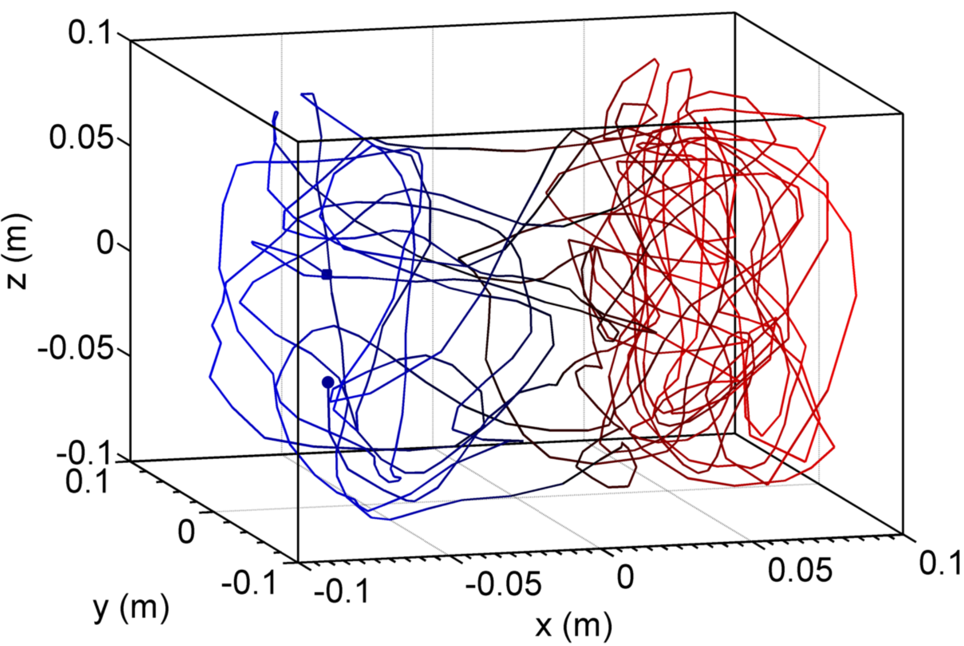
Trapping force : $F_{trap} \sim \beta \nabla \langle P \rangle \sim 0.3\beta 4\pi^2 R\Omega^2$

Ratio : $\beta a_{rms}/F_{trap} \sim [0.8,\ 0.3]$

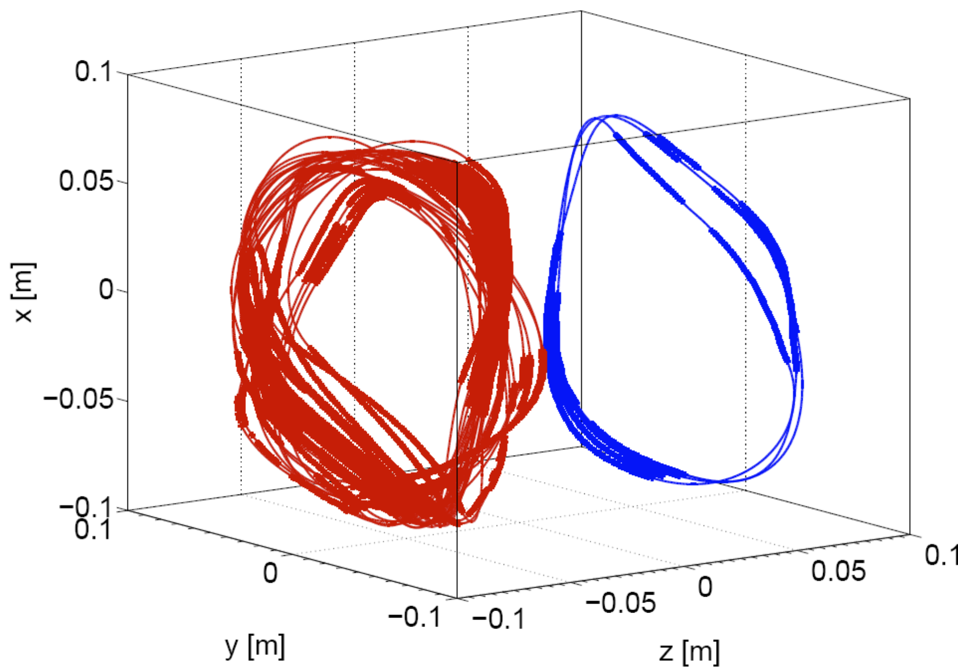
Integral size particles

Decreasing the Reynolds number

Fully developped, Re > 4000



Laminar, Re ~ 300

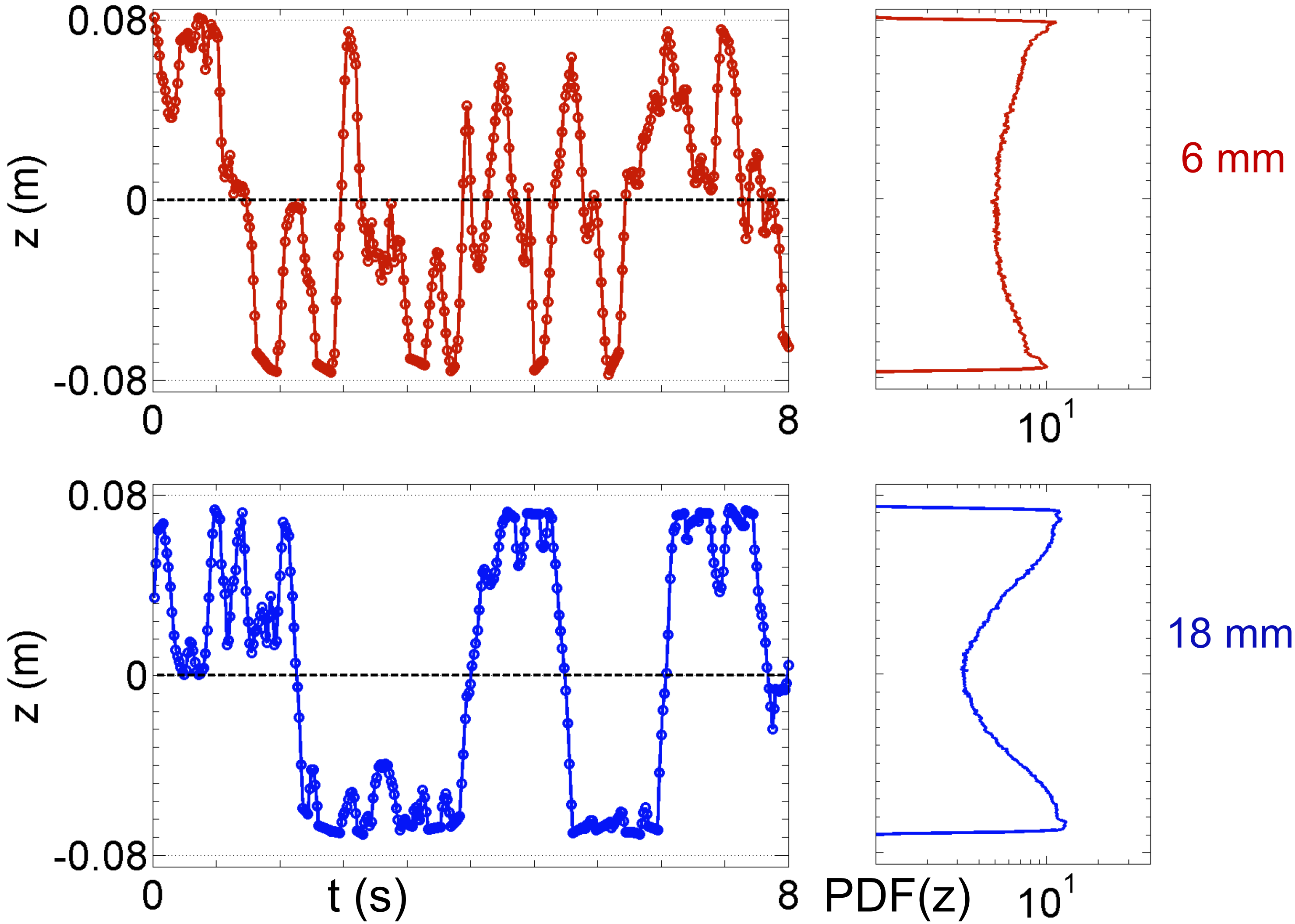


Rapid transition from turbulence to Lagrangian chaos

Fluctuations promote transitions between attractors

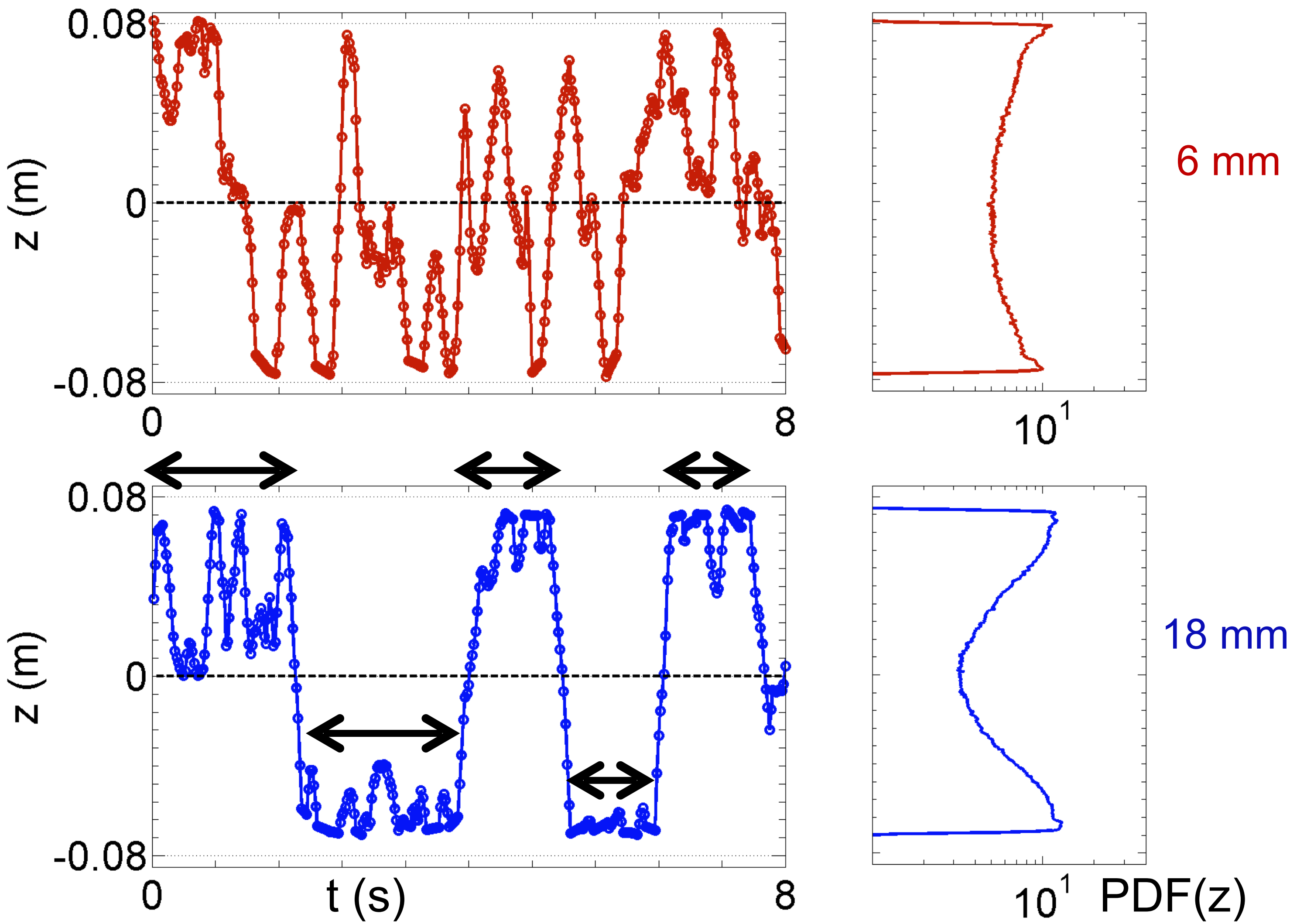
Integral size particles

Long time dynamics



Integral size particles

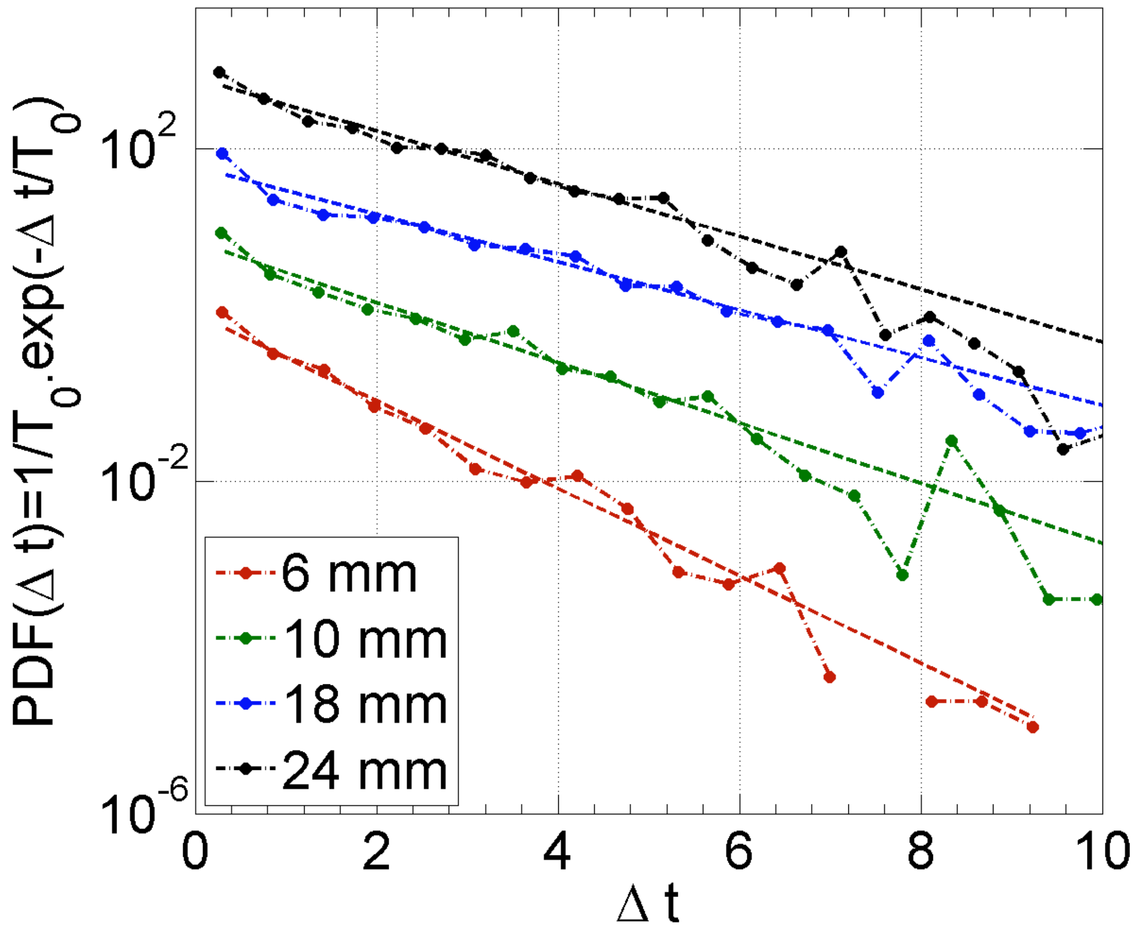
Long time dynamics



Residence time Δt

Integral size particles

Residence time distribution



Coming-and-going movements

Exponentially decreasing residence time PDF

$$PDF(\Delta t) \sim \exp(-\Delta t / T_0) \qquad T_0 = < \Delta t >$$

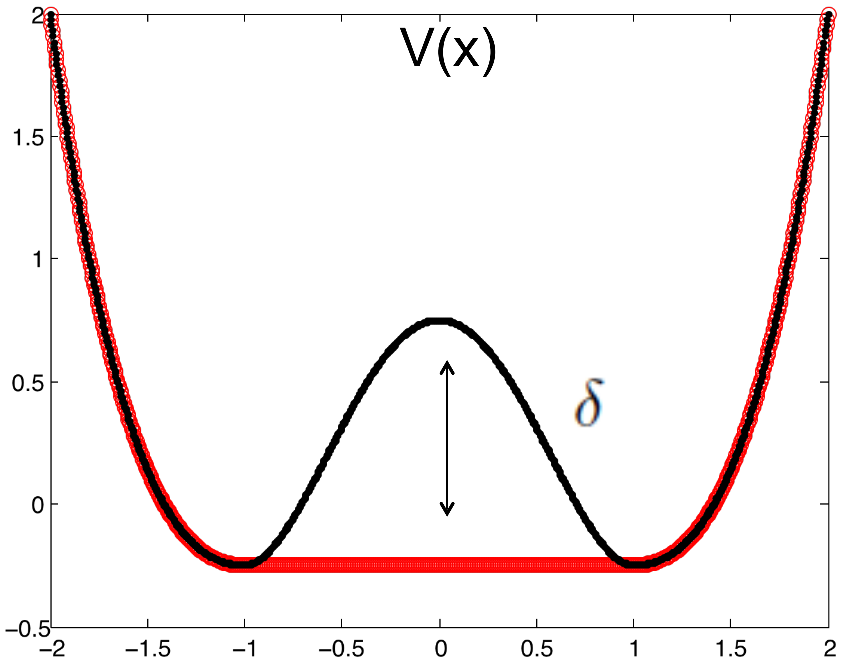
Integral size particles

A simple stochastic model

Over-damped particle in a double well V(x)

$$\frac{dx}{dt} = -\frac{dV}{dx} + v'$$

V(x) : confinement inside the vessel, trapping (barrier)
Bistable system driven by turbulent fluctuations v'(t).

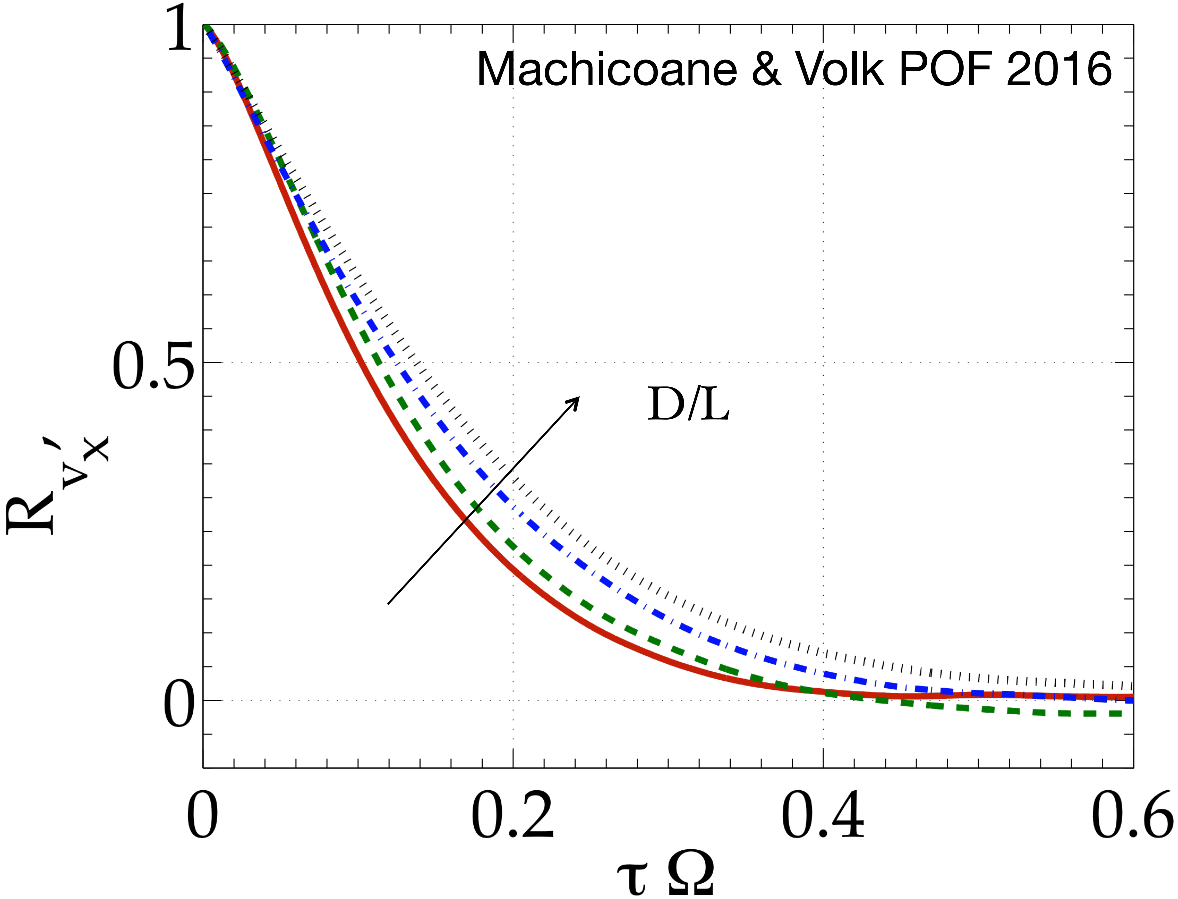


Integral size particles

A simple stochastic model

v’(t) is the Lagrangian fluctuating velocity (unbounded dynamics)

Experimental measurements : $v_i(t) = v_i'(t) + \overline{v}_i(\mathbf{x}(t))$



Nearly exponential, Langevin noise

$$du = -\frac{dt}{T_p}u + \sqrt{\frac{u_0^2}{T_p}}dW$$

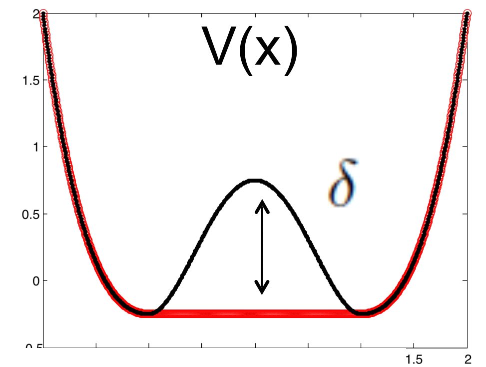
Forget dissipative dynamics

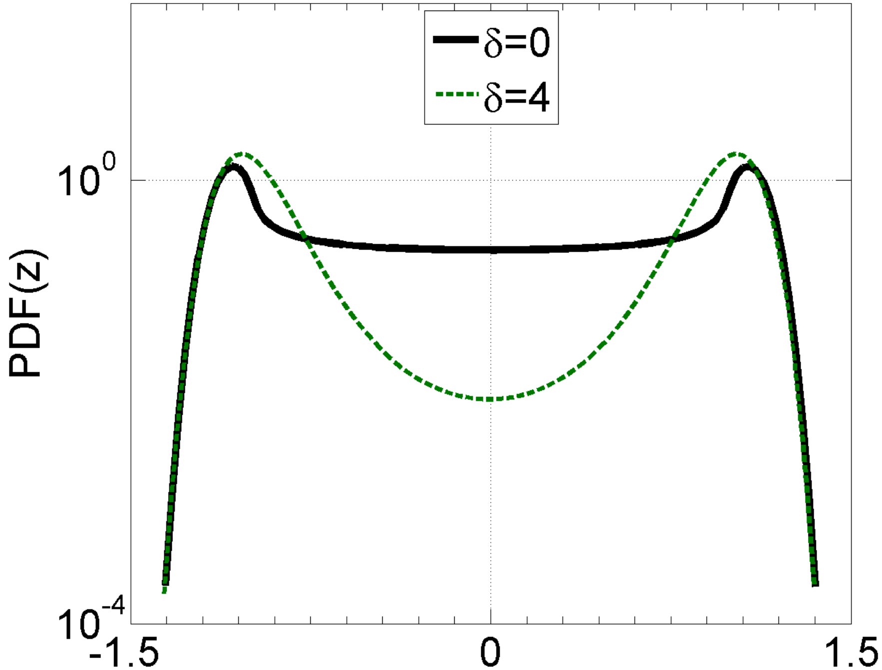
Integral size particles

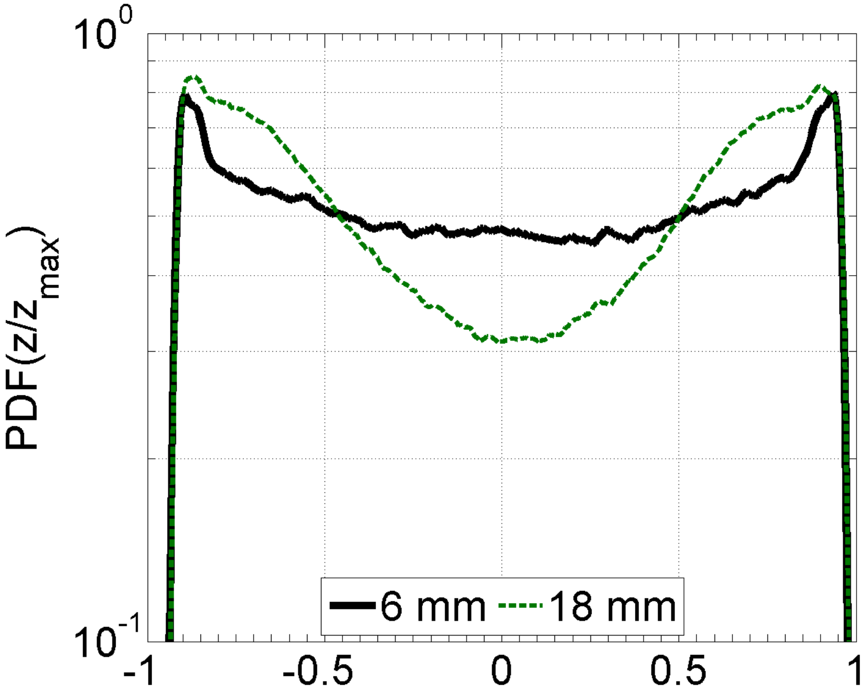
Position PDF

$T_p = 1 \quad u_0 = 1$

δ is the only parameter





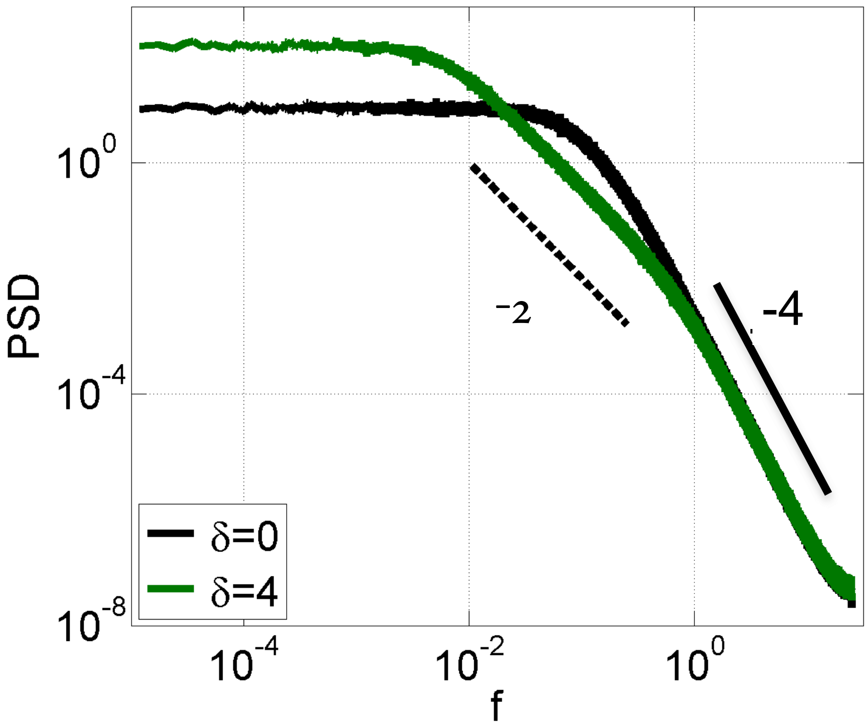


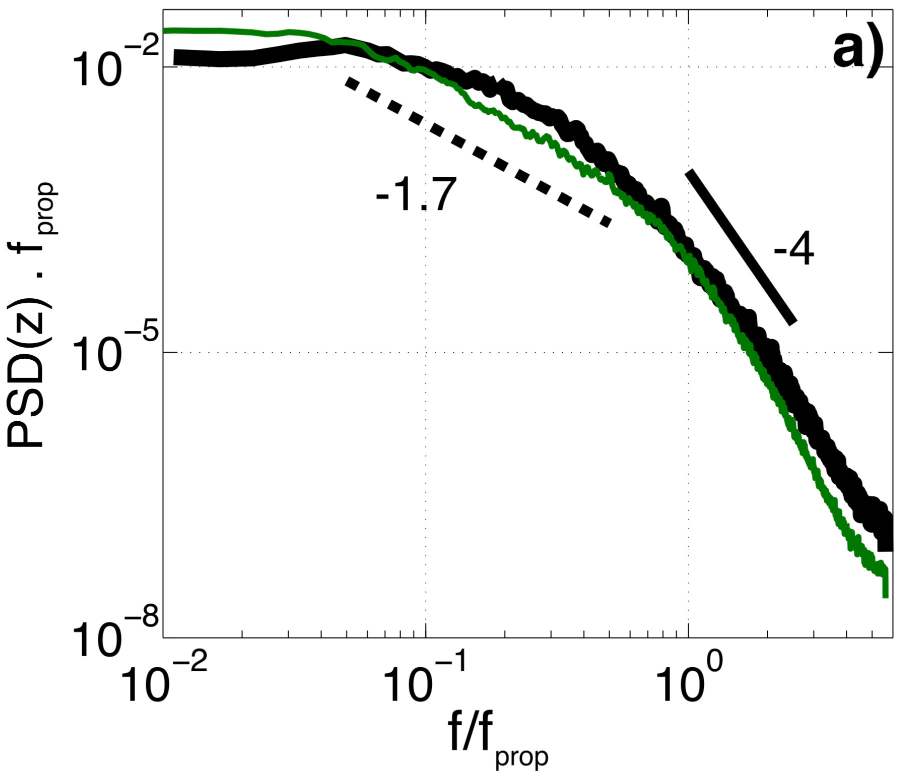
Good agreement on the PDFs

Exit times exponentially distributed

Integral size particles

Position spectra

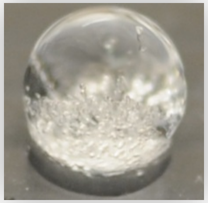


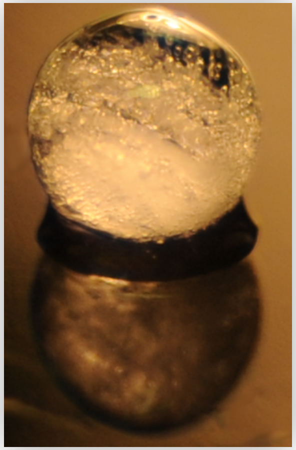


Emergence of a power law at intermediate frequencies

Reproduces also the evolution of position variance

Melting in (forced) turbulence





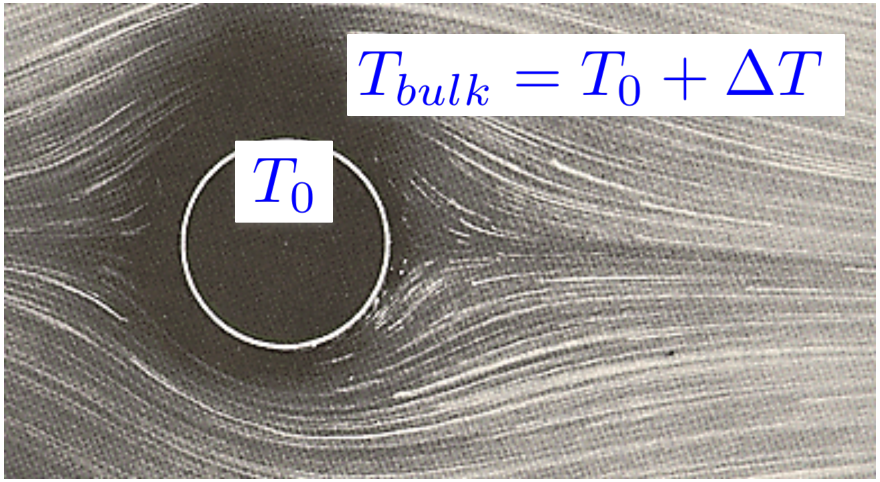
Heat transfer low Reynolds number

Fixed sphere in a uniform flow

Assume here a regime of forced convection

Fluid flow u(x,t) is governed by Navier-Stokes equation and $Re_D = \frac{UD}{\nu}$

Temperature transport equation :



$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \nabla^2 T$$

$$T(\vec{x} \in S, t) = T_0$$

Governed by $Pe_D = \frac{UD}{\kappa}$

Internal energy current

$$\vec{j}_{th} = -\lambda_{th} \vec{\nabla} T$$

Heat transfer low Reynolds number

Fixed sphere in a uniform flow

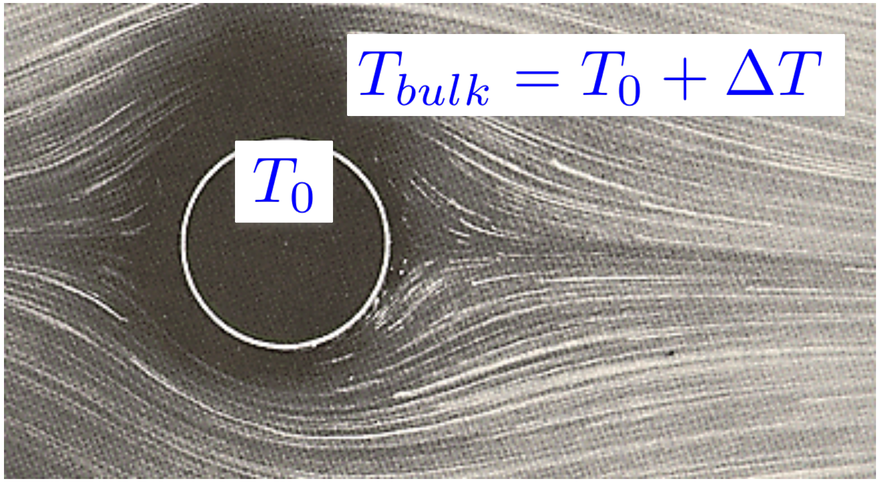
Assume here a regime of forced convection

Fluid flow u(x,t) is governed by Navier-Stokes equation and $Re_D = \frac{UD}{\nu}$

Temperature transport equation :

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \nabla^2 T$$

$$T(\vec{x} \in S, t) = T_0$$



Governed by $Pe_D = \frac{UD}{\kappa}$ or $Pr = \frac{\nu}{\kappa}$

Heat flux per unit surface

$$\phi = -\frac{\lambda_{th}}{S} \iint_S dS \; \vec{n} \cdot \vec{\nabla} T$$

$$\phi_{\text{diff}} \sim \lambda_{th} \frac{\Delta T}{D}$$

$$\phi = h(Re_D, Pr) \Delta T$$

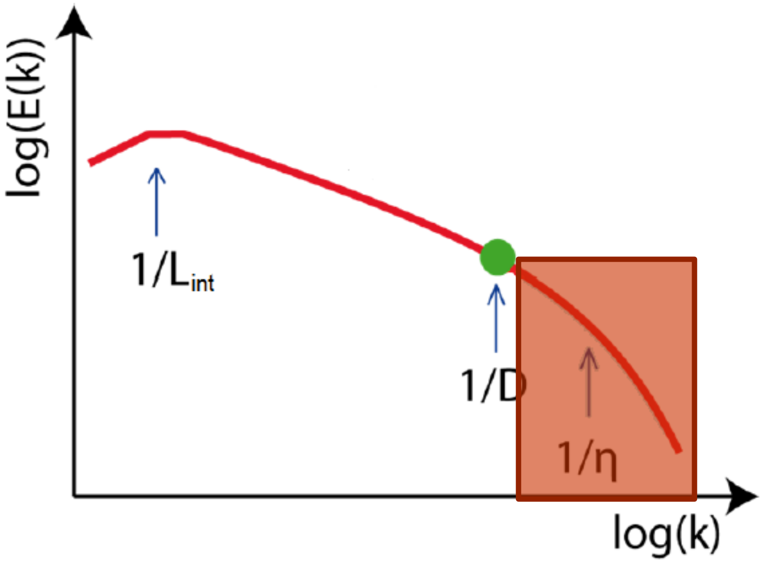
Nusselt number

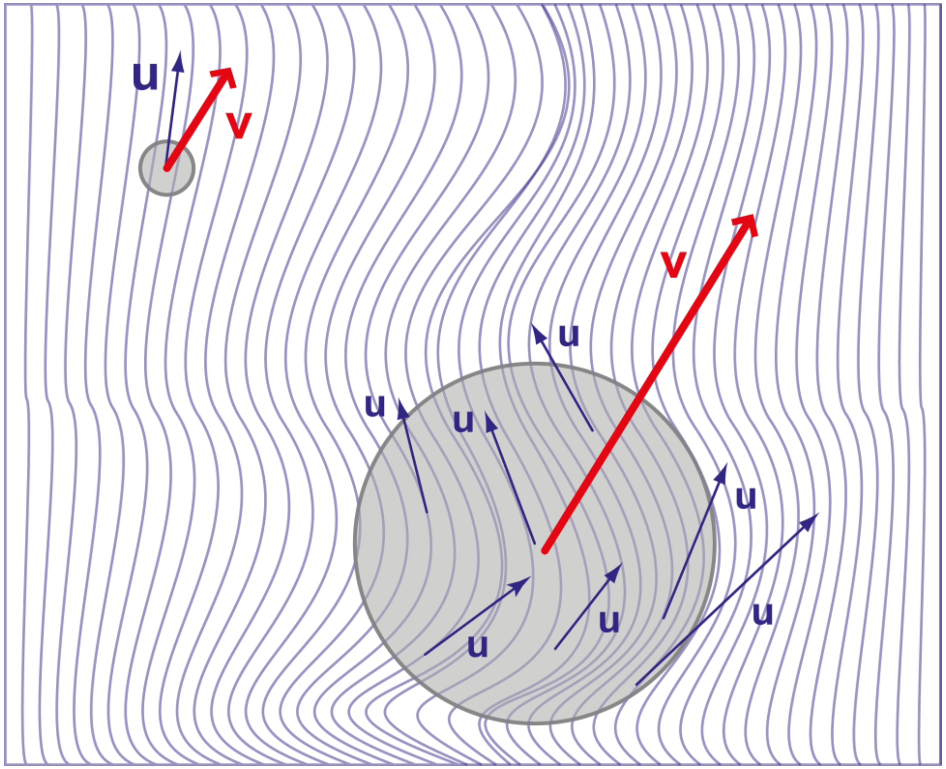
$$Nu = \frac{\phi}{\phi_{\text{diff}}} = \frac{hD}{\lambda_{th}}$$

Sphere (Re_D <<1) : Ranz & Marshall, 1952

$$Nu = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$$

Heat transfer at large Reynolds number





Big particles

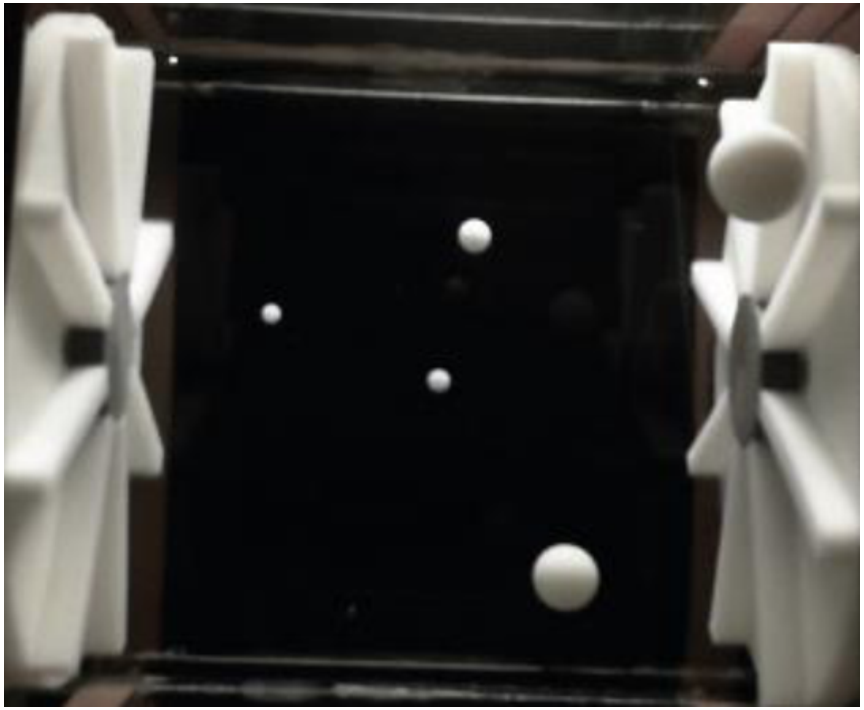
$$D \gg \eta \qquad Re_D \gg 1 \qquad Nu = f(Re_D, Pr, u'/U)$$

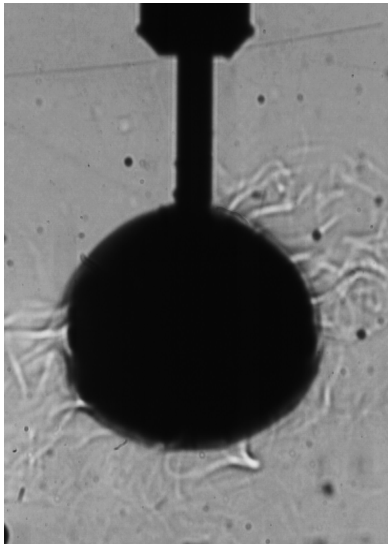
Experiments : $Nu \propto Re_D^{\alpha} Pr^{\beta}$ $\quad \alpha \in [0.5,\ 0.8]$

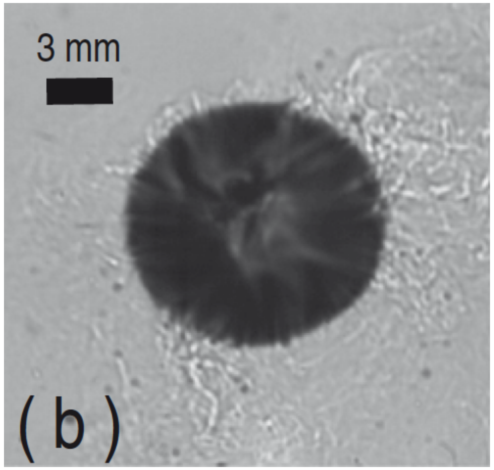
Increases with $Re_D \ \& \ u_{\mathrm{rms}}/U$

van der HeggeZijnen1958, Boon-Long1978, Birouk2006, …

Melting of ice balls





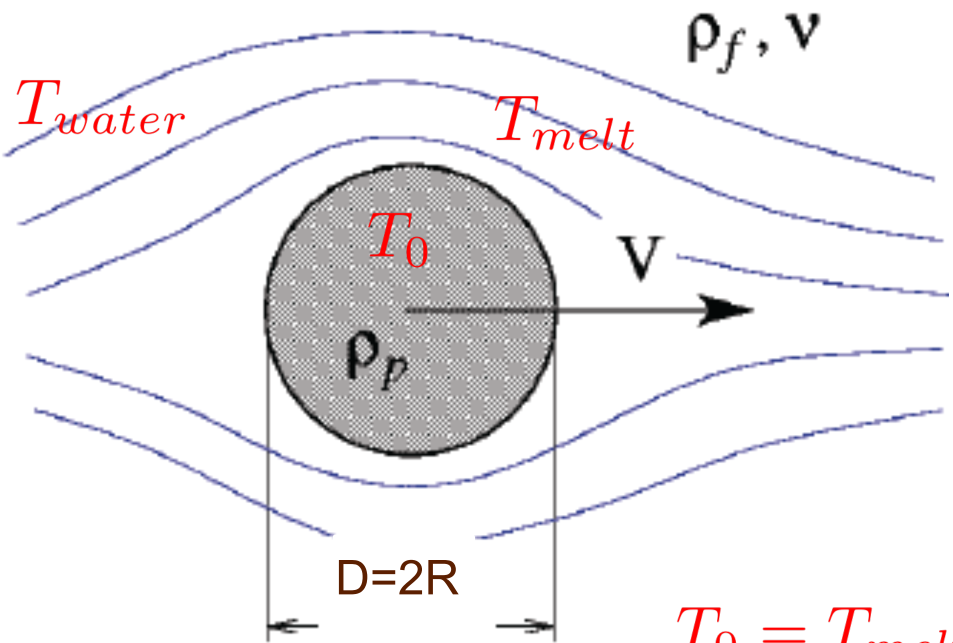


Will it melt faster :

Maintained fixed ?

Freely transported ?

Melting of ice balls



Energy balance

$$L_f \frac{dm}{dt} = \Phi_{\text{diff}} + \Phi_{\text{conv}}$$

<u>Stefan's equation:</u>

$T_0 = T_{melt}$

$$\rho_p L_f \frac{dR}{dt} = \lambda_{th} \left.\frac{\partial T}{\partial r}\right)_R - h(T_{water} - T_{melt})$$

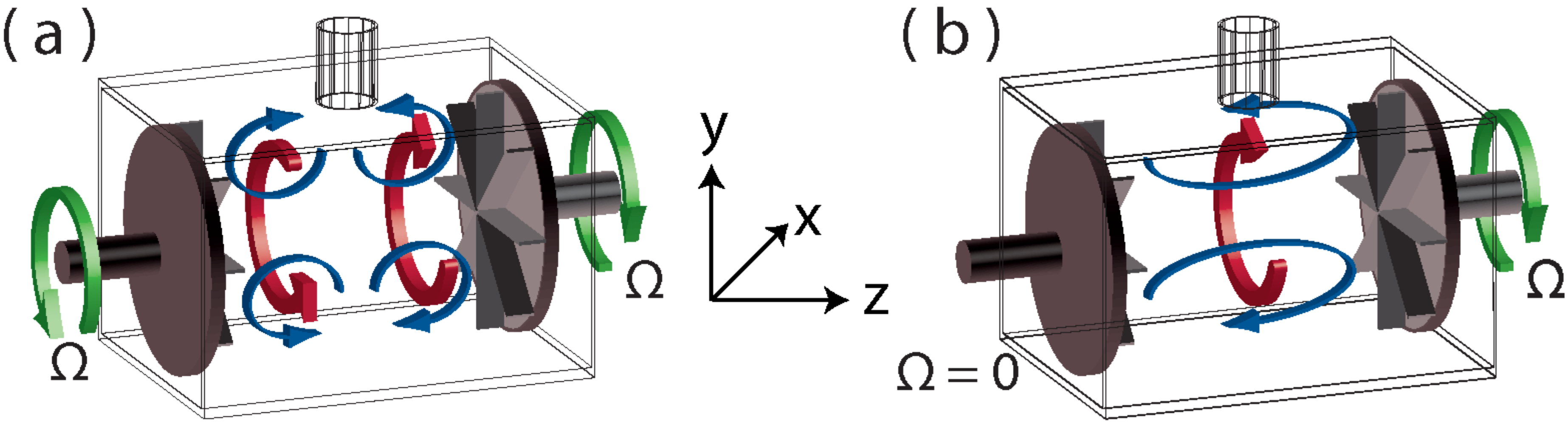
Latent heat (J/kg)

diffusion

convection

Record $R(t)$ → Measure $h(Re_D, Pr)$

Melting of ice balls



$\langle u \rangle = 0$

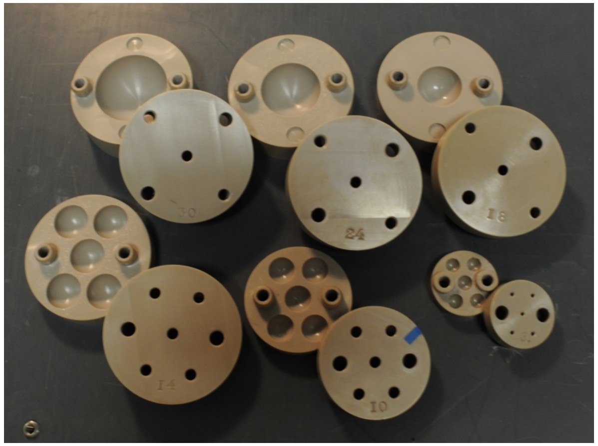
$u'/U \sim 0.2$

$U = 2\pi R_p \Omega \sim 3$ m/s

$R_p = 7.5$ cm

$u'_z / \langle u_z \rangle \sim 0.35$

$u'/U \sim 0.1$



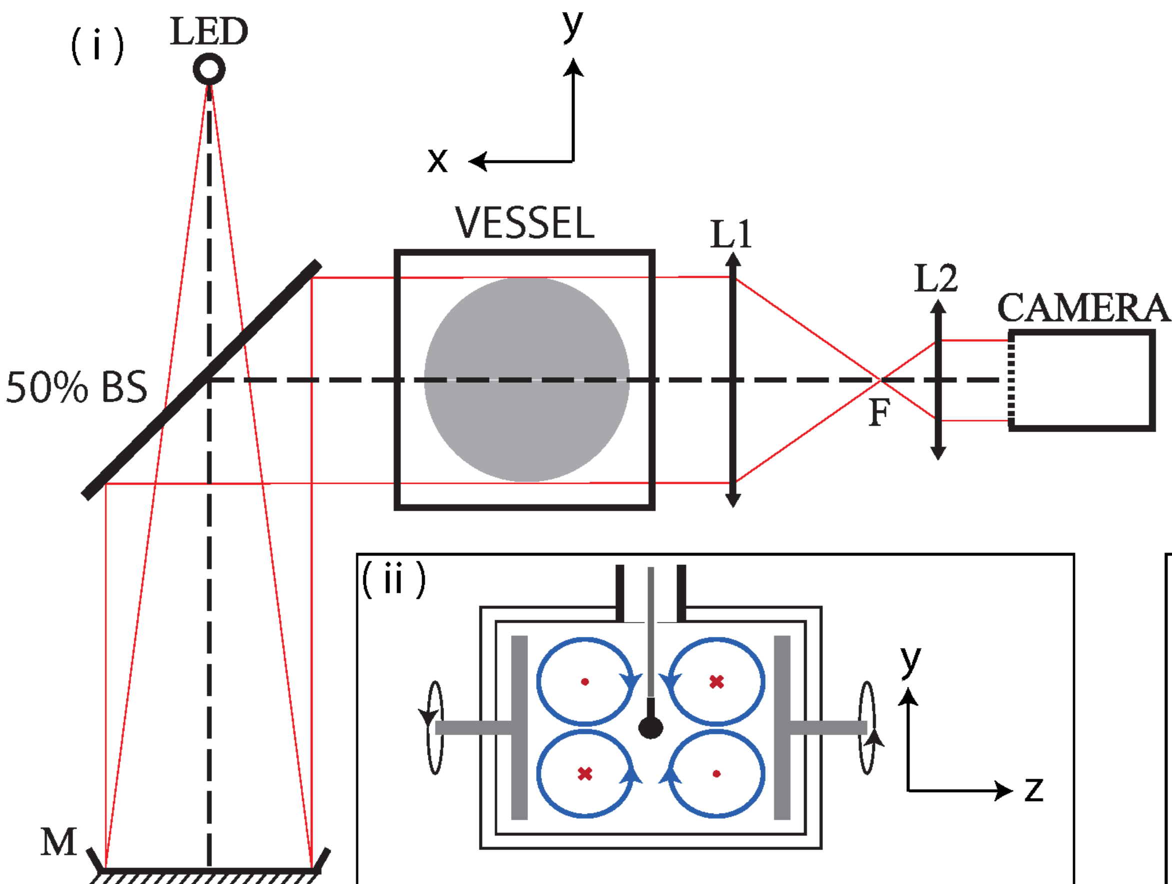
$Re \sim 2\ 10^5 \quad Re_\lambda \sim 700$

Controlled $T_{water} \in [1,\ 15]^\circ C$

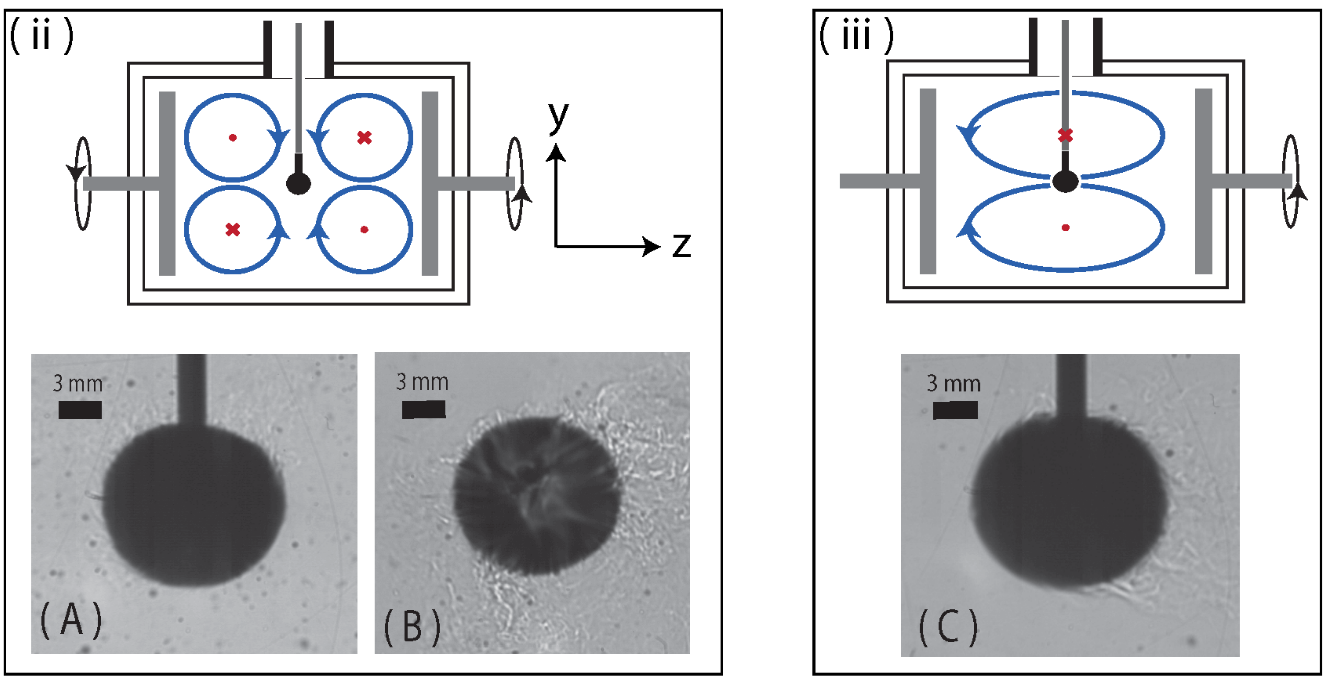
$10 \leq D \leq 30$ mm $\quad D \sim L$

$5\ 10^3 \leq Re_D \leq 75\ 10^3$

Melting of ice balls

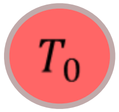


Afocal setup:
Object size independent of its position



Fixed ice balls, zero mean flow

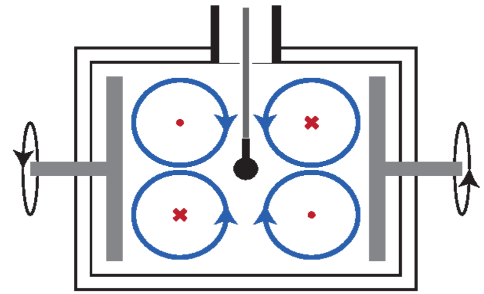
$$T_0 = T_{melt}$$

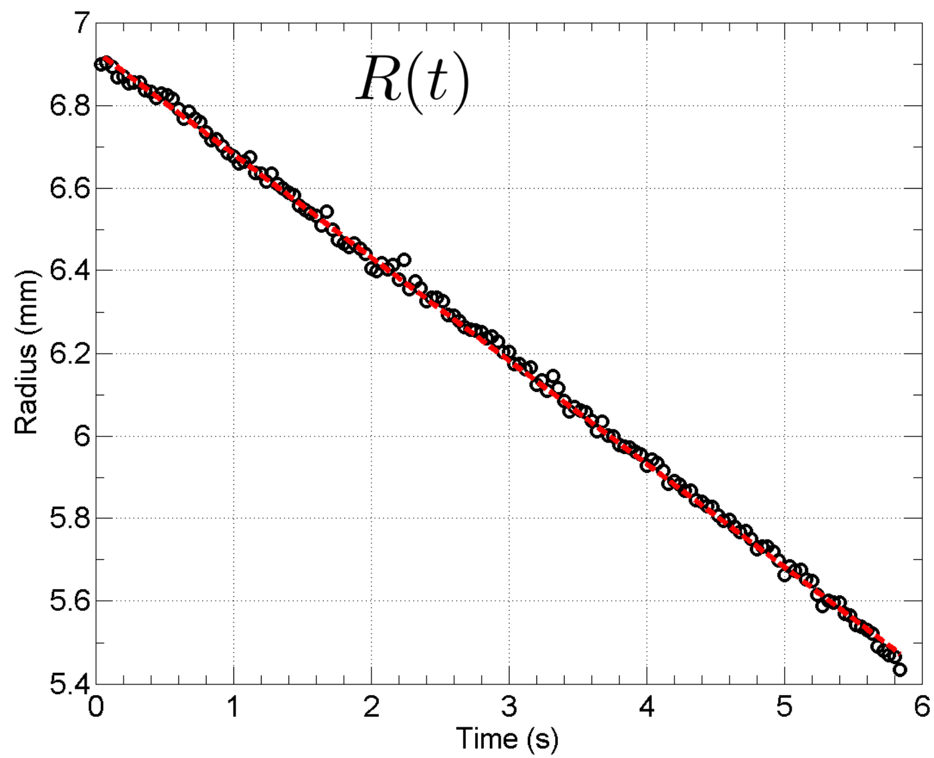


measured

$$\overline{\frac{dR}{dt}} = V_{melt}$$

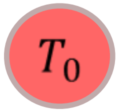
$$\rho_p L_f V_{melt} = \phi$$





Fixed ice balls, zero mean flow

$$T_0 = T_{melt}$$



measured

$$\overline{\frac{dR}{dt}} = V_{melt}$$

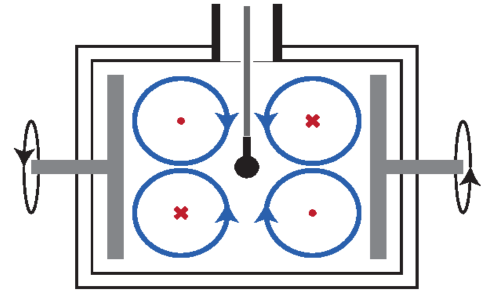
$$\rho_p L_f V_{melt} = \phi$$

$$\phi = h(T_{water} - T_{melt})$$

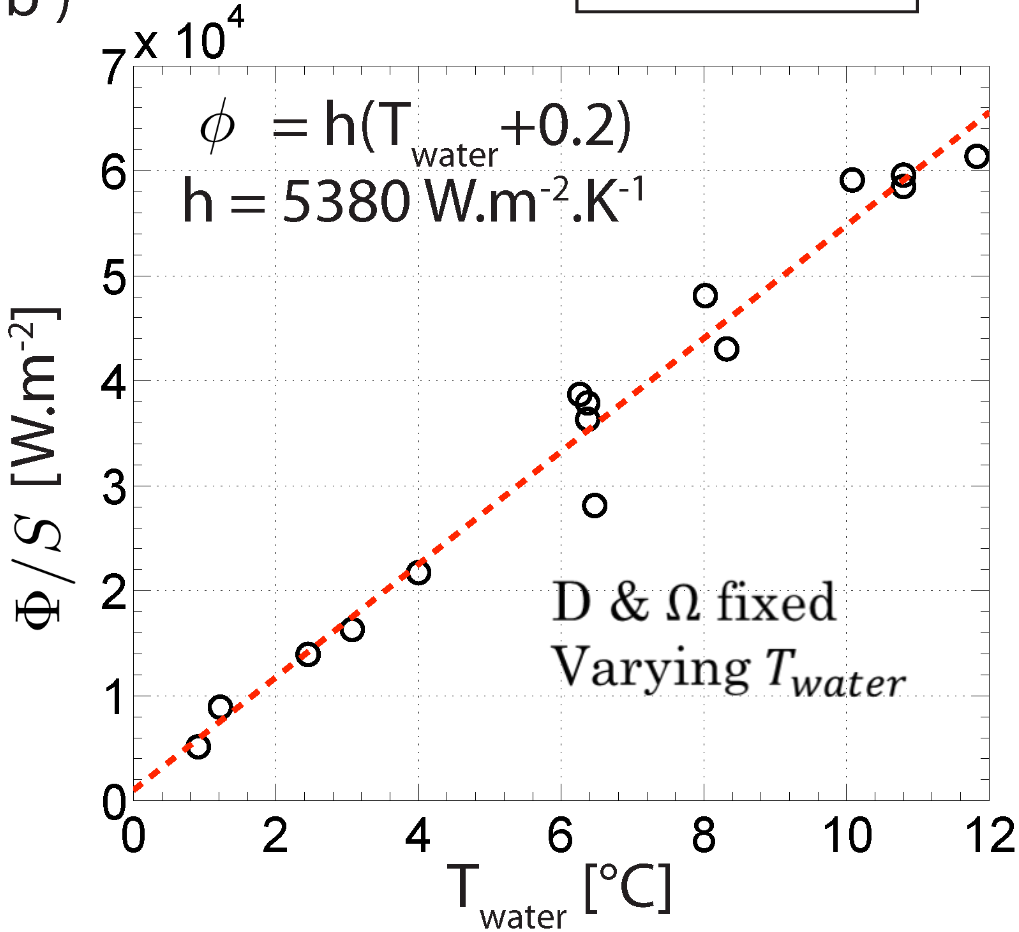
$$T_0 = T_{melt}$$

$$h = \frac{\rho_p L_f V_{melt}}{T_{water}}$$

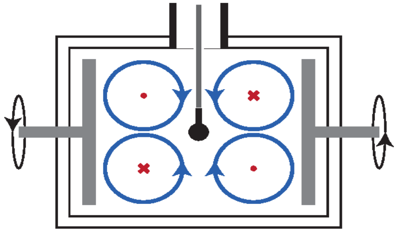
Natural convection neglected

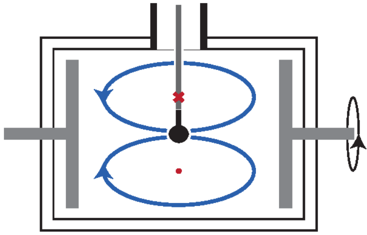


(b)



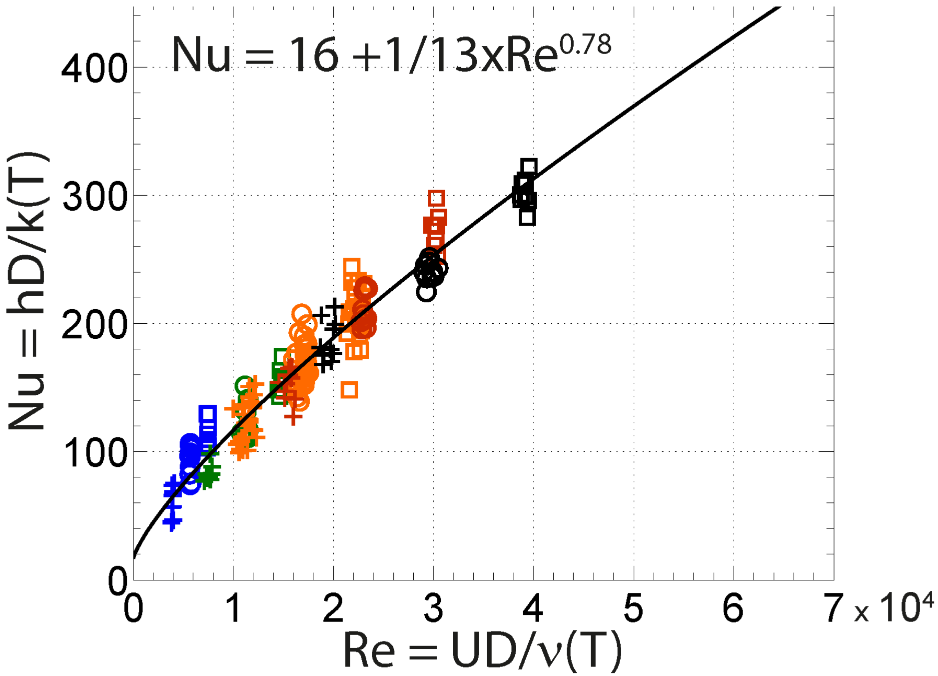
Fixed ice balls, the two cases

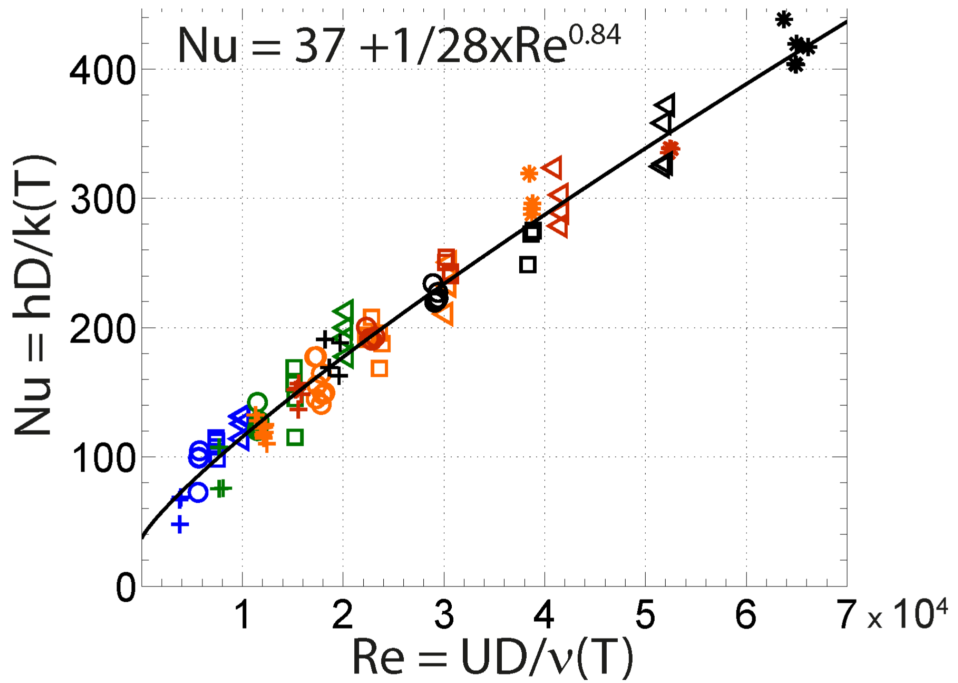




$\langle u \rangle = 0 \qquad u'/U \sim 0.2$

$u'_z/\langle u_z \rangle \sim 0.35 \qquad u'/U \sim 0.1$

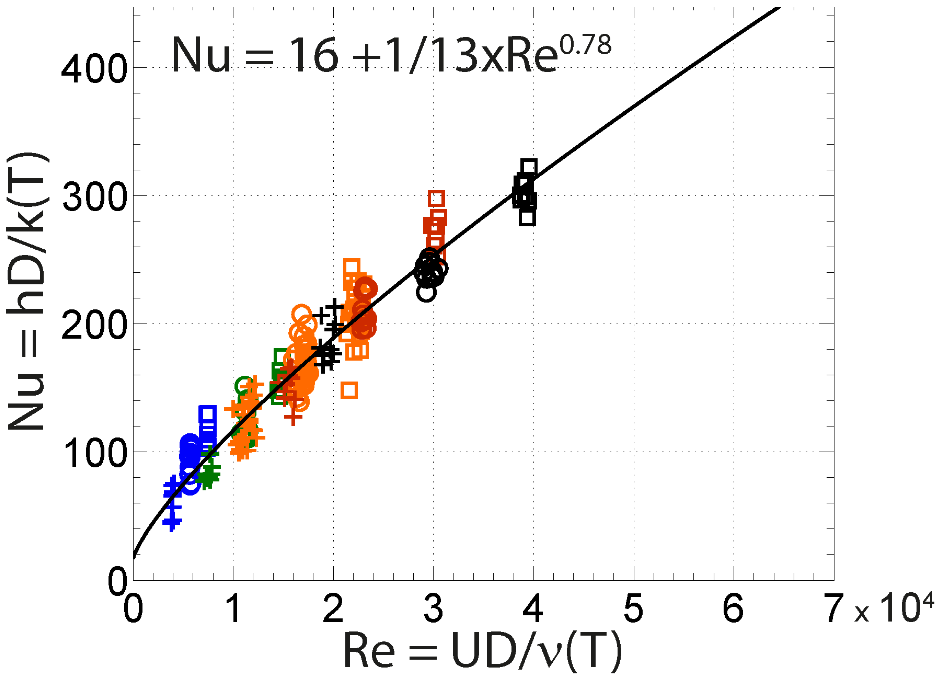




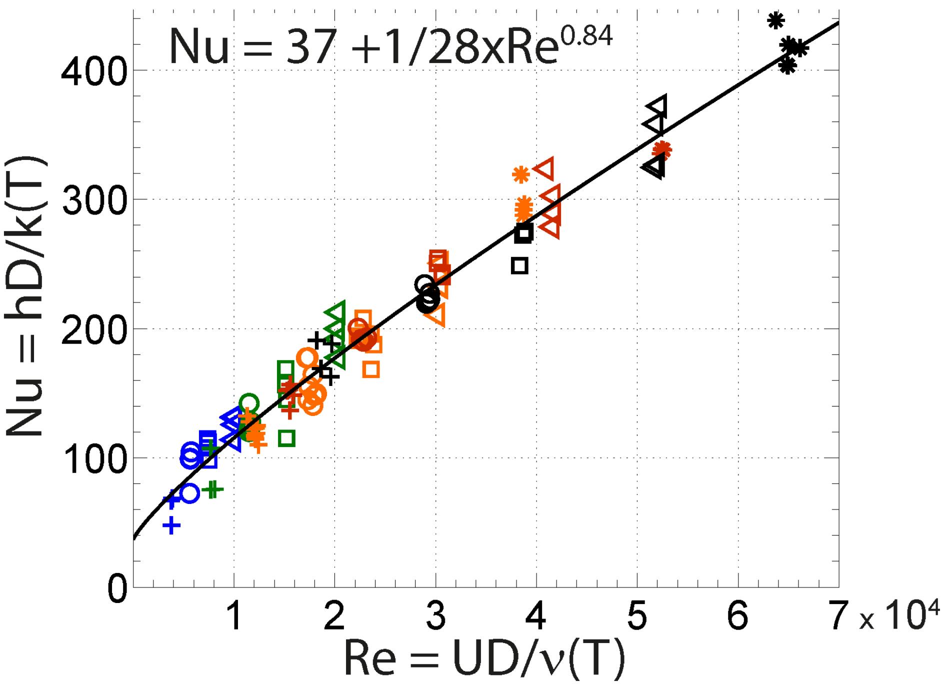
$Nu \sim Re_D^{0.8}$ for both cases

Fixed ice balls, the two cases

$\langle u \rangle = 0 \qquad u'/U \sim 0.2$



$u'_z/\langle u_z \rangle \sim 0.35 \qquad u'/U \sim 0.1$

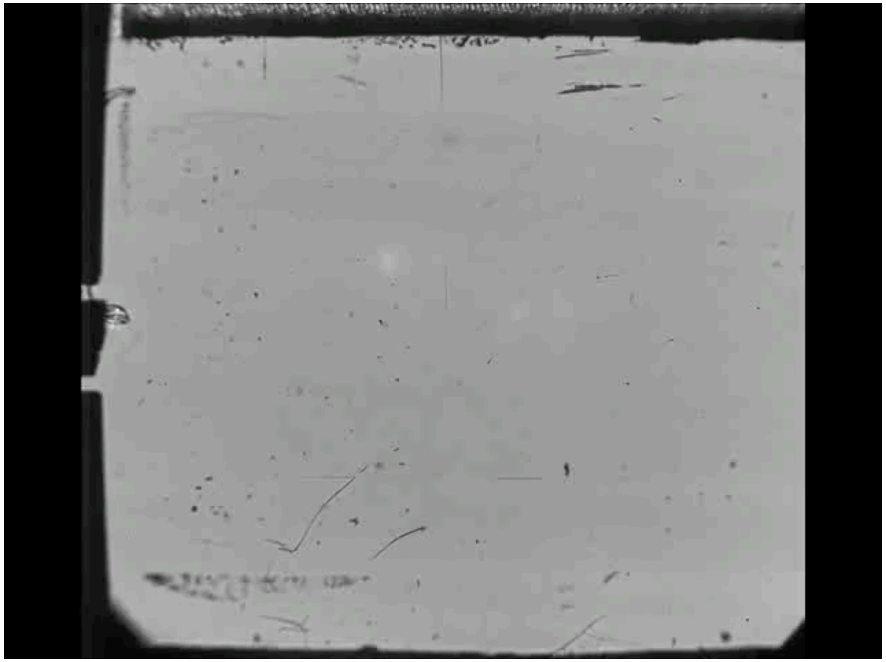


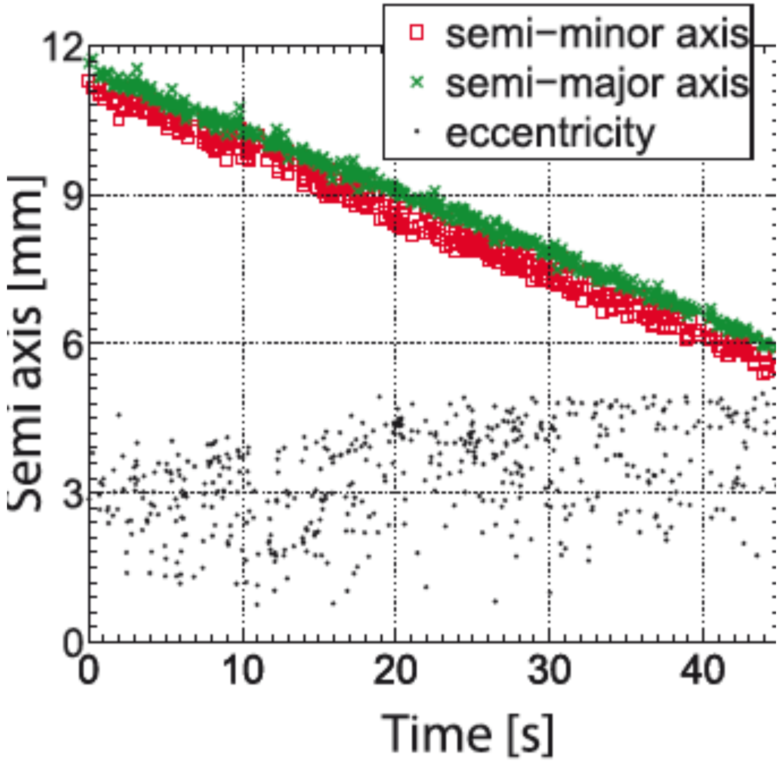
$$Nu \sim Re_D^{0.8}$$ for both cases

Same value of $$u_{trms} = \sqrt{u'^2 + \langle u \rangle^2} \propto U$$

$$Nu = f(u_{trms}, D, \cancel{u'/\langle u \rangle})$$

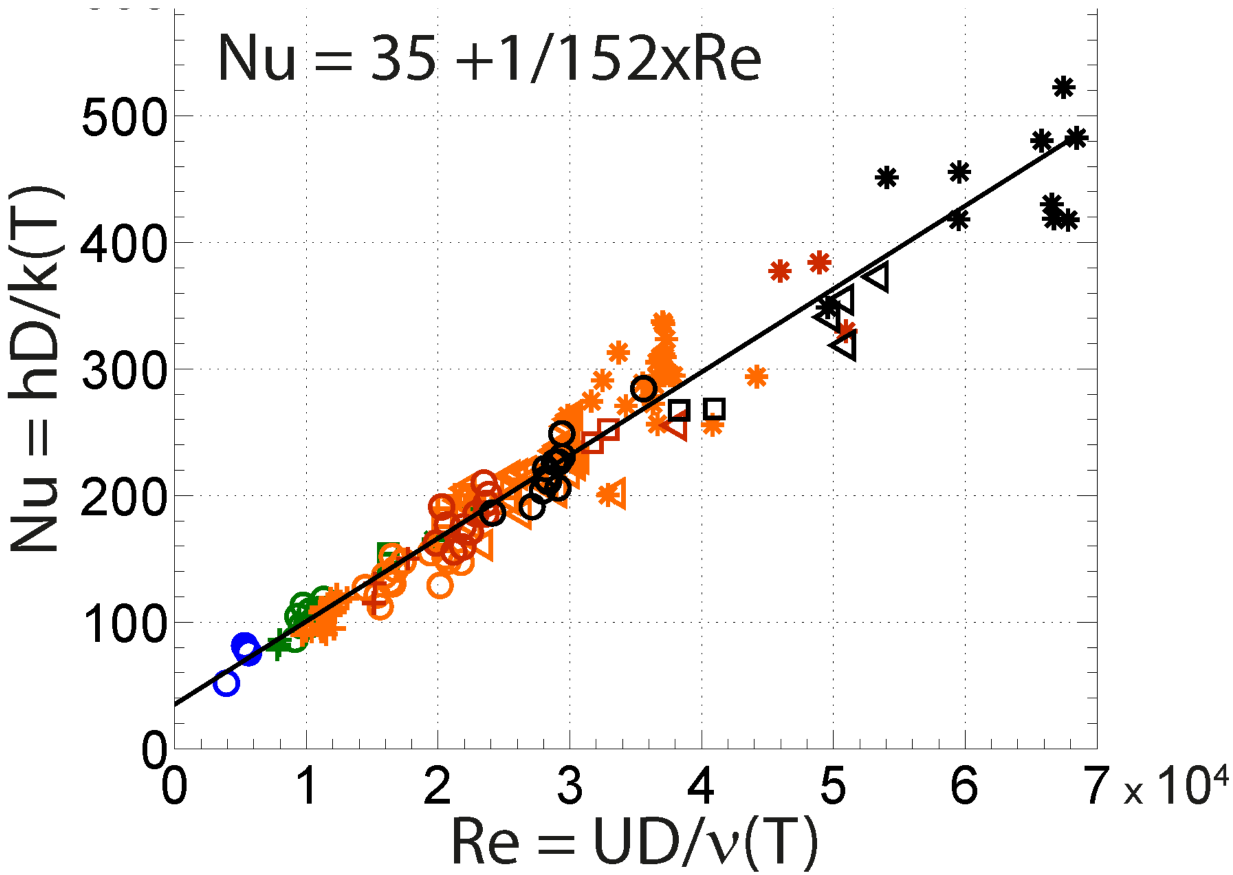
Fixed ice balls, the two cases

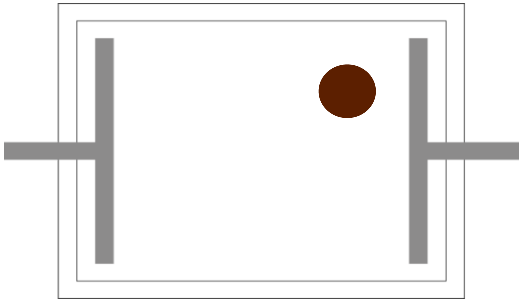




Linear evolution for $\triangle t \geq \frac{400}{\Omega}$, size no longer maters

Fixed ice balls, the two cases





Ultimate scaling

$$\phi \sim \rho C_p U \frac{\Delta T}{D}$$

$$\phi_{\mathrm{diff}} \sim \lambda_{th} \frac{\Delta T}{D}$$

$$Nu \propto Re_D Pr$$

N. Machicoane et al., POF 2013

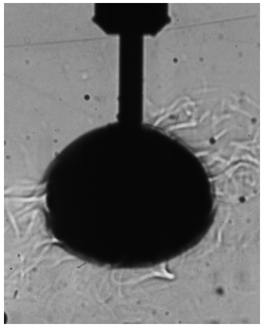
Take home message

- Fixed particles

$$Nu \sim Re_D^{0.8}$$

$$h = h(u_{trms}, D, \nu)$$

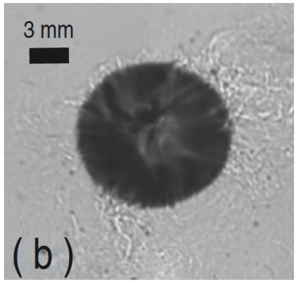
Anisotropic melting



- Free particles

$$Nu \sim Re_D$$

$$h = h(u_{rms}, \nu)$$



Some questions : what is the average sliding velocity

$$\tau_{melt} \gg \tau_{turb} \qquad \tau_{melt} \gg \frac{1}{\Omega}$$

Exploration of the flow on long times ?

Study particle trajectories …