Slow dynamics of turbulent flows in a von Kármán experiment

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Outline

[1/5] Why?

[2/5] Where? The experiment

[3/5] The very slow regime

[4/5] The slow regime

[5/5] FIONA and SHREK

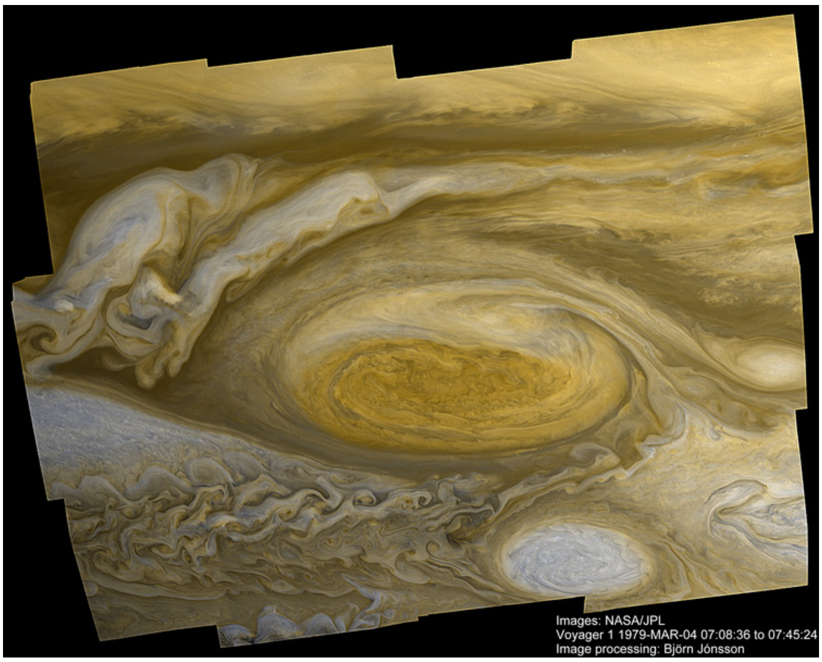
Why? → Turbulence

Large structures are ubiquous in turbulent flows:

[1/5] Motivation: Why?

Why? → Turbulence

Large structures are ubiquous in turbulent flows:



Why? $\rightarrow$ Turbulence

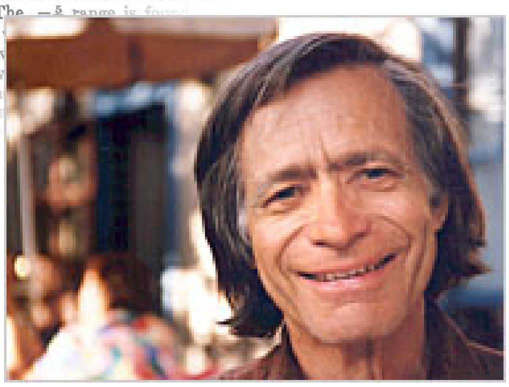
Large structures are ubiquous in turbulent flows:

THE PHYSICS OF FLUIDS VOLUME 10, NUMBER 7 JULY 1967

Inertial Ranges in Two-Dimensional Turbulence

Robert H. Kraichnan
Peterborough, New Hampshire
(Received 1 February 1967)

Two-dimensional turbulence has both kinetic energy and mean-square vorticity as inviscid constants of motion. Consequently it admits two formal inertial ranges, $E(k) \sim \epsilon^{2/3}k^{-5/3}$ and $E(k) \sim$ ... where ϵ is the rate of cascade of kinetic energy per unit mass, η is the rate of cascade of mean-... vorticity, and the kinetic energy per unit mass is $\int_0^\infty E(k)\, dk$. The ... backward energy cascade, from higher to lower wavenumbers k, ... The -3 range gives an upward vorticity flow and zero-energy flow ... resolved by the irreducibly triangular nature of the elementary wav... -3 range gives a nonlocal cascade and consequently must be m... energy is fed in at a constant rate to a band of wavenumbers $\sim k_i$ ... it is conjectured that a quasi-steady-state results with a $-\frac{5}{3}$ range ... $k \gg k_i$, up to the viscous cutoff. The total kinetic energy ... range pushes to ever-lower k, until scales the size of the ... energy dissipation by viscosity decreases to zero if kine... parameters unchanged.



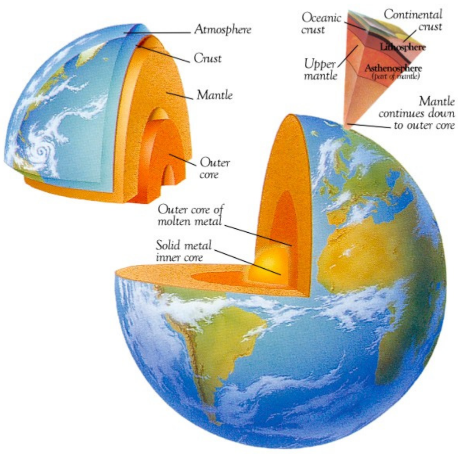
Why? $\rightarrow$ MHD

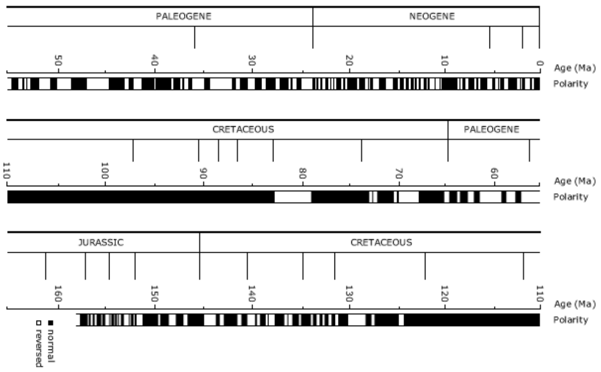
Dynamo action:

Self-generation of a Magnetic field
in a moving conducting fluid

Driving force: Convection, coriolis, propellers...

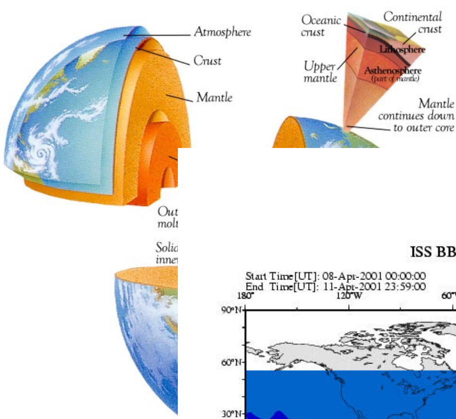
Why? $\rightarrow$ Earth's magnetic field

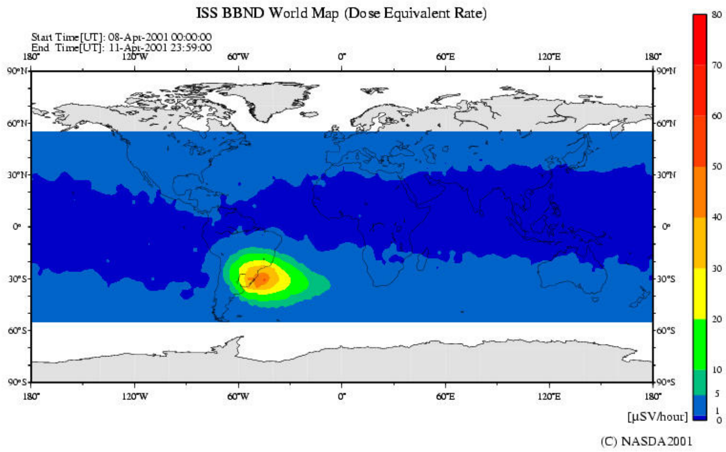


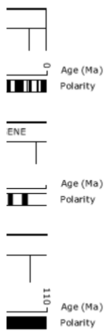


[1/5] Motivation: Why?

Why? $\rightarrow$ Earth's magnetic field







Why? → The experiment

We decided to analyze these slow dynamics experimentally...

⇒von Kármán flow in a closed cylinder

[1/5] Motivation: Why?

Why? → The experiment

Fundamental and applied research:

- MHD interest (Dynamo action)
- Applied research: Mixing problems.

Turbulence "test bench"

- Homogeneous / Isotropic vs. Inhomogeneous / anisotropic:
- Lagrangian vs. Eulerian statistics
- Structure functions,...

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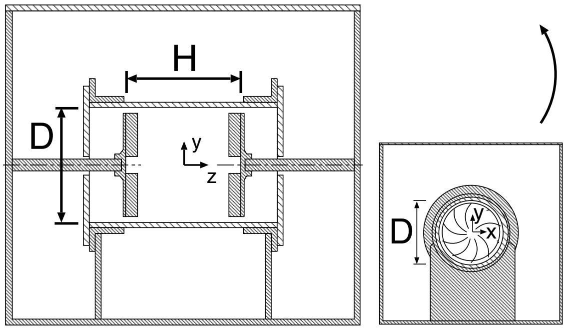
Water Experiment

Experimental setup:

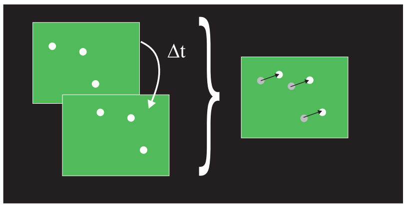


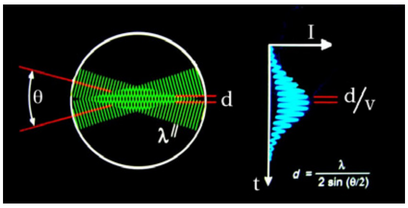
- Cylindrical volume
  $D = 0.1 - 0.4$m, $H = 0.1 - 0.5$m
- Two counter rotating impellers
- Frequency: $f = 1 - 20$Hz $\leftarrow$ fluctuations below 1‰

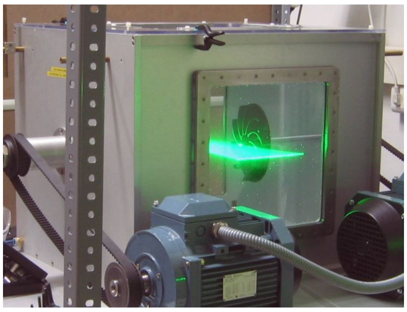
[2/5] Experimental setup: Where?

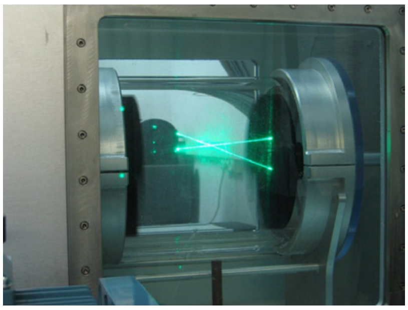
Water Experiment

PIV (spatial evolution) $\Leftrightarrow$ LDA (temporal evolution)









spatial resolution $\uparrow$
temporal resolution $\downarrow$

temporal resolution $\uparrow$
spatial resolution $\downarrow$

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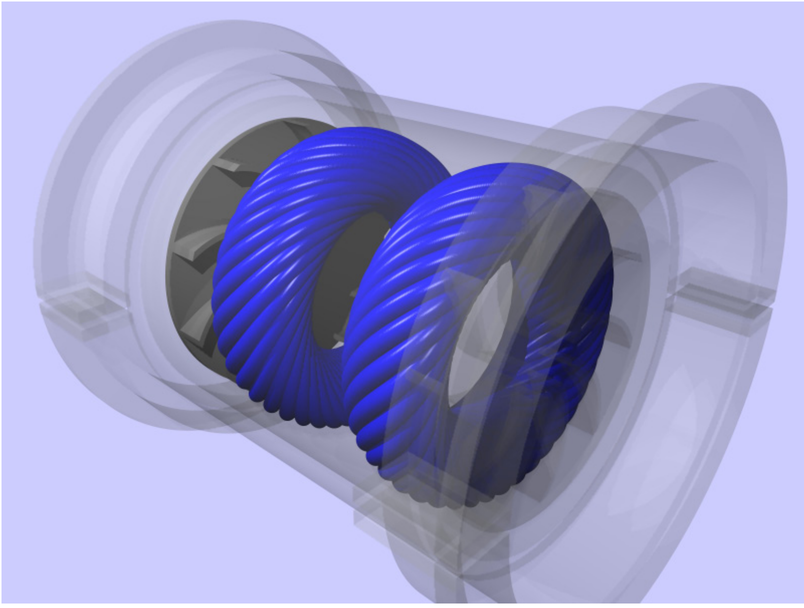
Expected flow characteristics

The mean flow recovers all the symmetries??

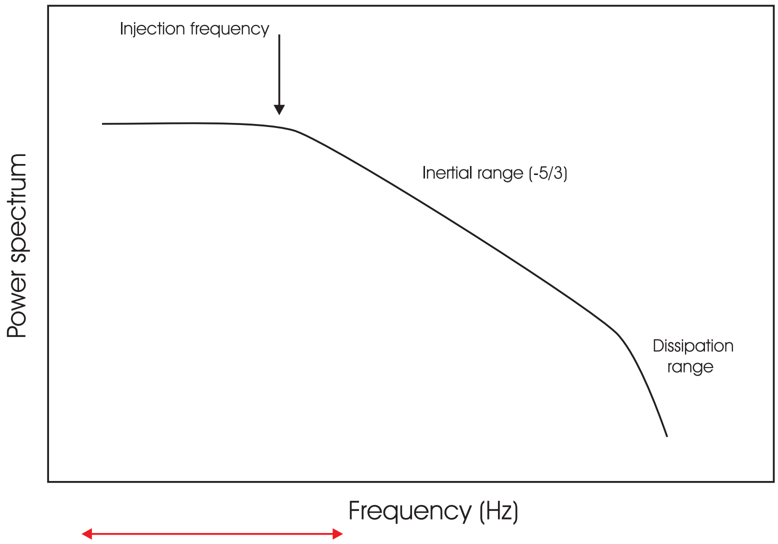


Anisotropic and very slow fluctuations?

[2/5] Experimental setup: Where?

Expected flow characteristics

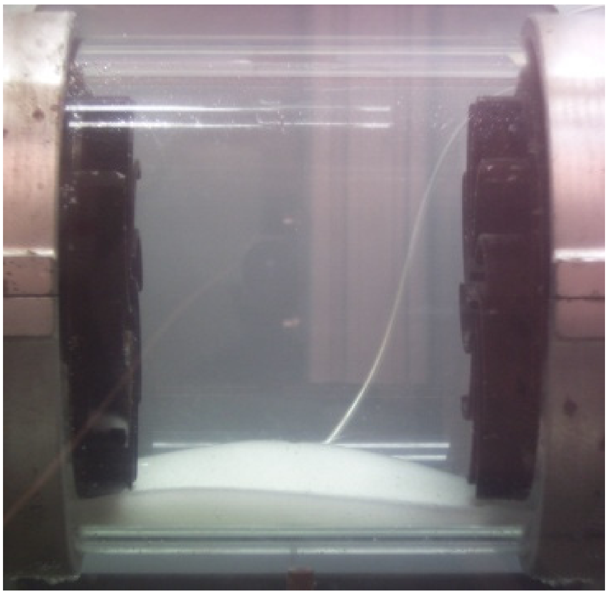
Power spectrum??



Here we will focus on the slow behaviour

Large Re: Flow Visualization

$Re = 2.5\ 10^5$



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Large Re: Flow Visualization

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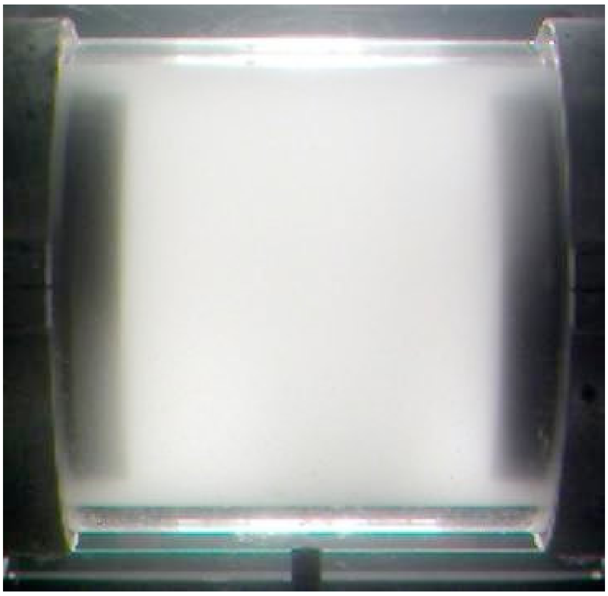

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Large Re: Flow Visualization

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[2/5] Experimental setup: Where?

Large Re: Flow Visualization

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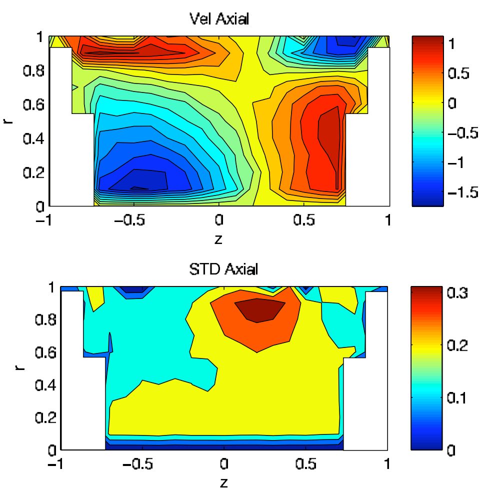

Large Re: Flow Visualization

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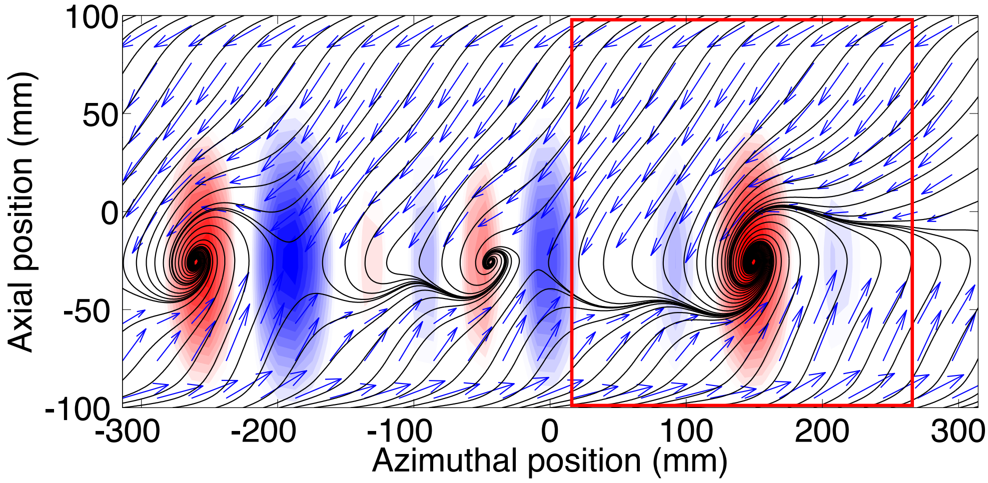
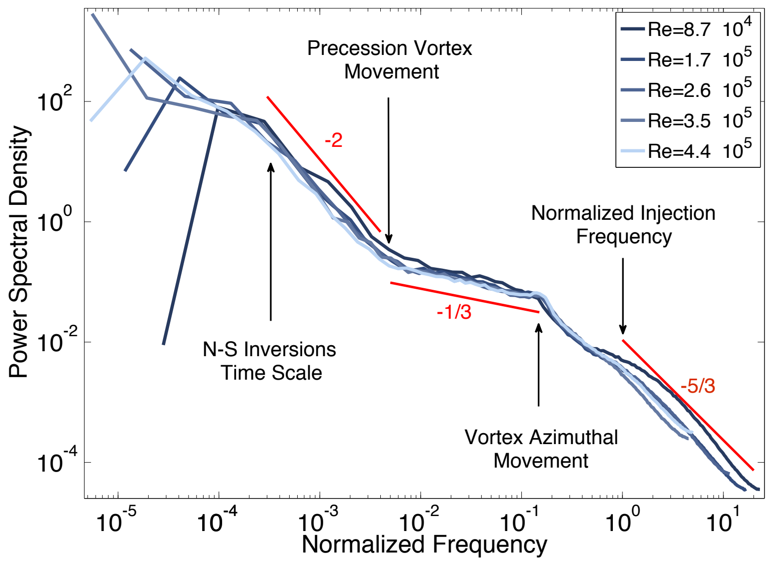


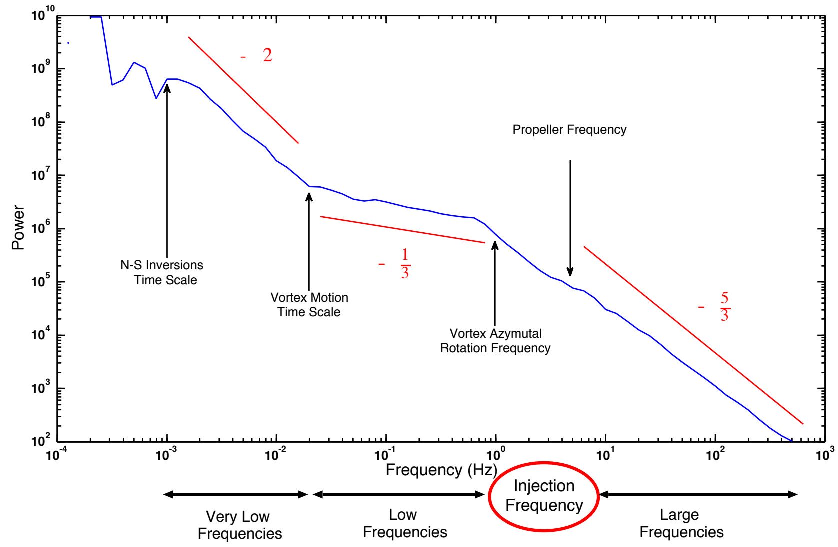
Power spectrum

Many different time scales below the injection scale.



Power spectrum

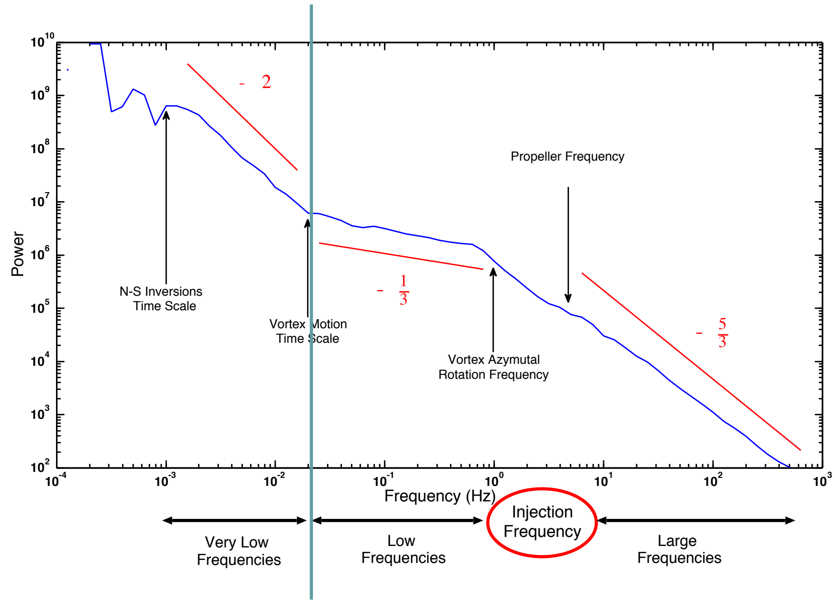
We can stablish three diferent ranges:



[2/5] Experimental setup: Where?

Power spectrum

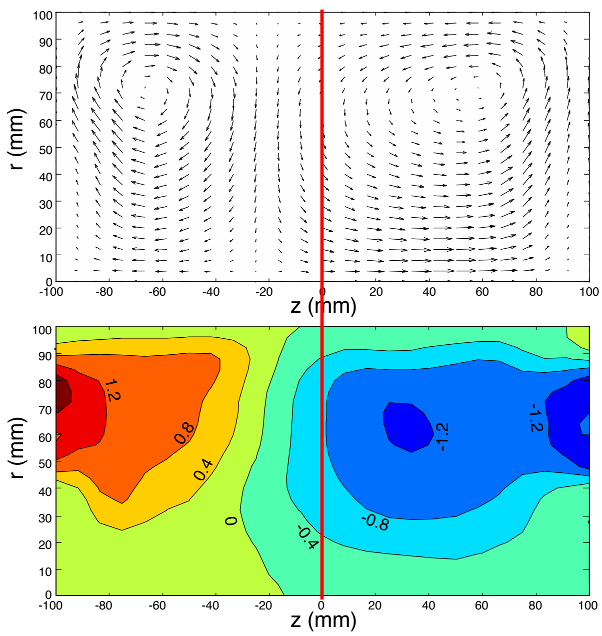
We will start with the range of very low frequencies:

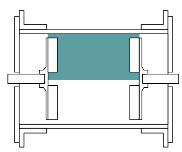


[3/5] Very Slow Regime
Reversals and Mean Flow Dynamics

Large Re: Two possible solutions

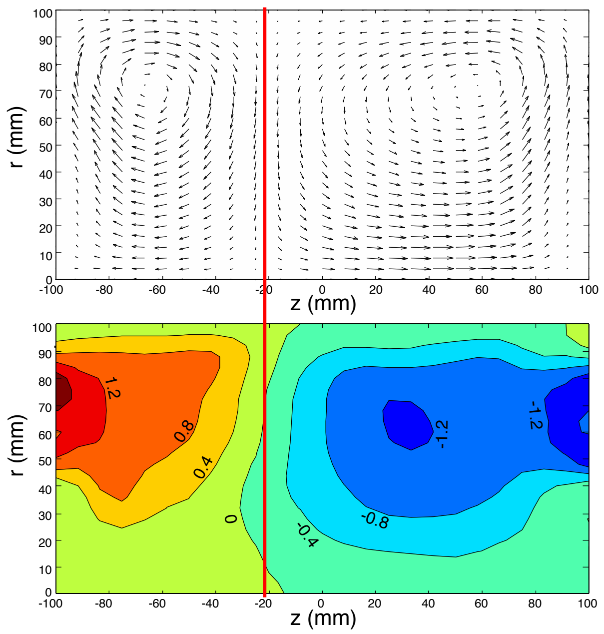
Measured velocity flow ($Re = 3\ 10^5$)
Time averaged $\rightarrow$Not symmetric around $z = 0$!

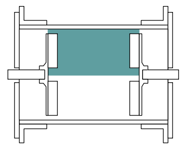




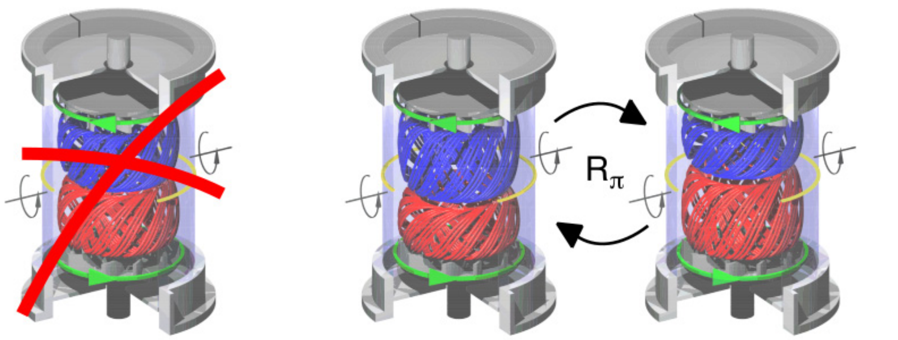
Large Re: Two possible solutions

Measured velocity flow ($Re = 3\ 10^5$)
Time averaged $\rightarrow$Here, the shear layer is around $z = -20$!





Large Re: Two possible solutions



This behavior appears **only** if the stability is better than 0.1 %.

With a random fluctuation of 1-2%, a fast dynamics appears between both solutions and a “symmetric” flow is recovered!!

Large Re: Two possible solutions

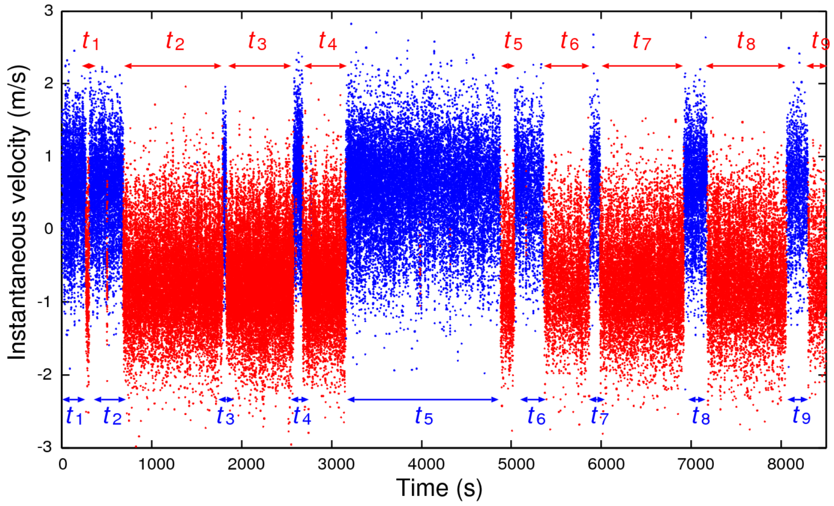
Reversals also present in Rayleigh Bénard convection:

Univ. Twente, D. Lohse group

[3/5] Very Slow Regime

Large Re: Two possible solutions

$$Re = 2.5\ 10^5;\ f_{N,S} = \pm 7.76 Hz;\ \Delta = \frac{f_N - f_S}{f_N + f_S} = 0$$



“Symmetric behaviour”, both states are visited

Turbulence rate $\simeq$100%

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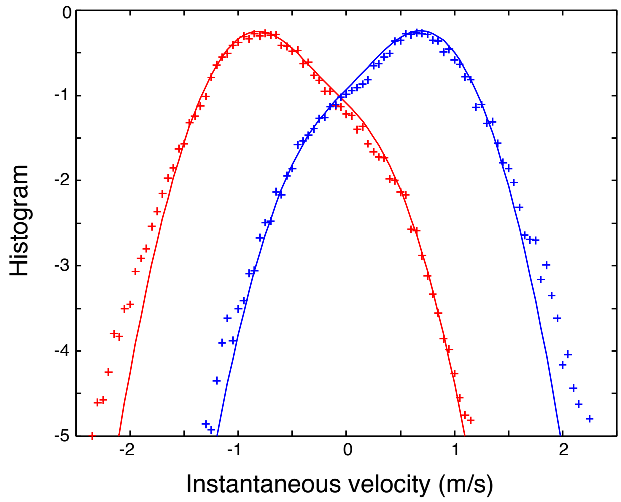
Large Re: Two possible solutions

Histograms:

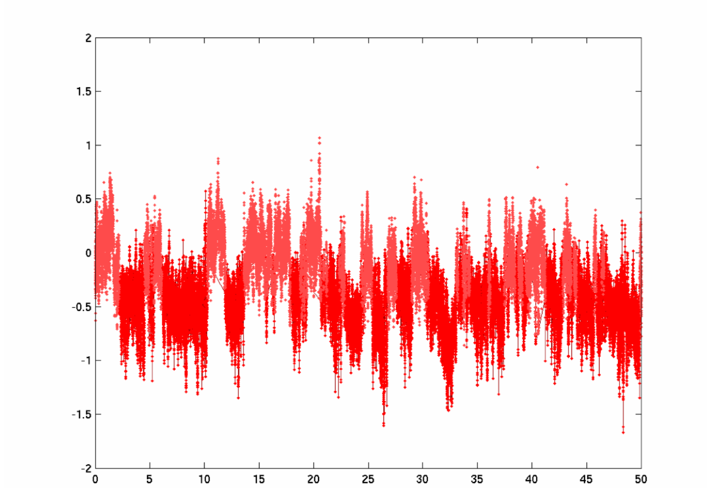
Same shape for both states



[3/5] Very Slow Regime

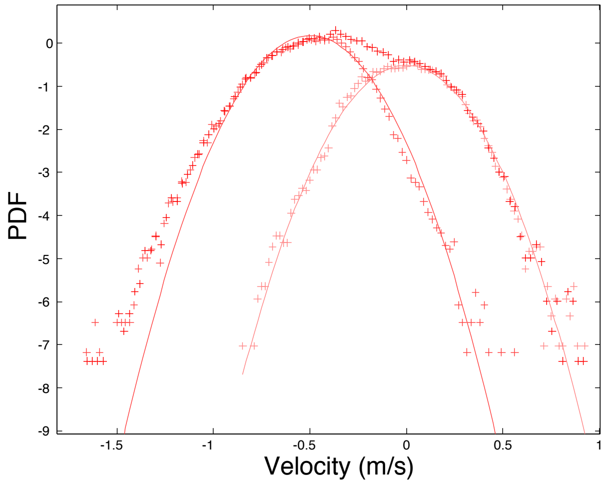
Large Re: Two possible solutions

Actually, this shape is due to another dynamics with another time-scaling,...



Large Re: Two possible solutions

...and two gaussians are distinguished inside each state.



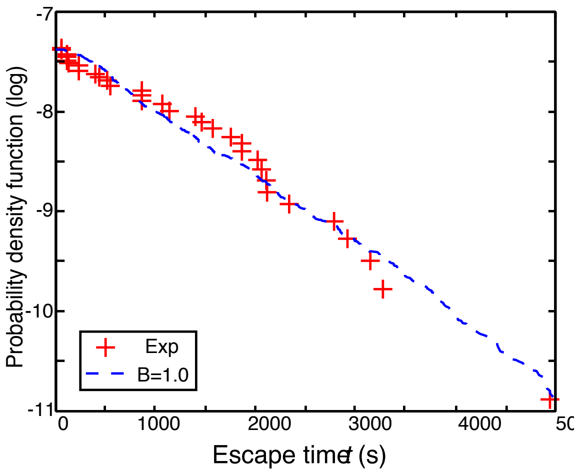
$$p_{N,S}(u_\theta) = \frac{A_0}{\sqrt{2\pi}\sigma_0} \exp\left(-\frac{u_\theta^2}{2\sigma_0^2}\right) + \frac{A_{N,S}}{\sqrt{2\pi}\sigma_{N,S}} \exp\left(-\frac{(u_\theta - u_{N,S})^2}{2\sigma_{N,S}^2}\right)$$

[3/5] Very Slow Regime

Large Re: Two possible solutions

Escape times (Kramer's escape rate):

$$\rho(t) = \frac{1}{T_0} \exp\left(-\frac{t}{T_0}\right)$$



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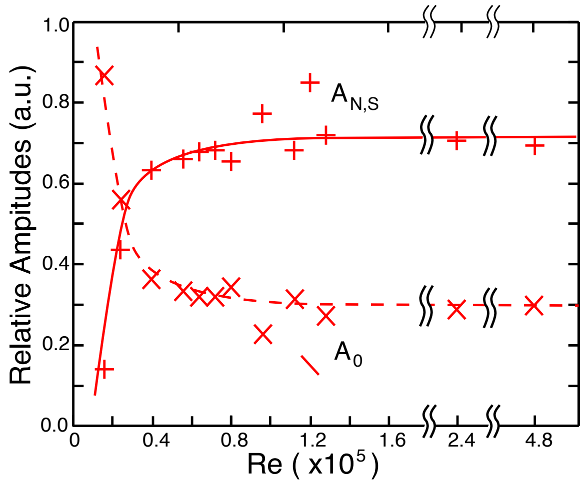
Large Re: Two possible solutions

Experimental Amplitudes:

A_N, A_S and A_0



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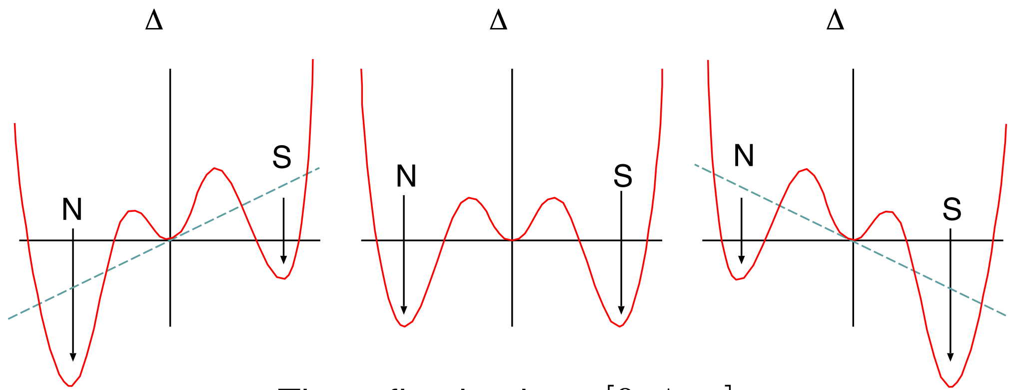
Large Re: Two possible solutions

Toy model: three well potential with additive noise

$$\dot{u}_\theta = \varepsilon u_\theta + g u_\theta^3 - u_\theta^5 + \kappa\Delta + \sqrt{2B}\xi(t)$$

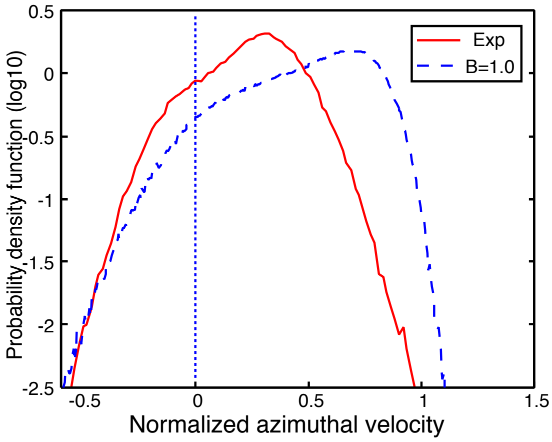
where B is the noise level ("turbulence rate")
and $\chi(t)$ is a noise distribution



Three fixed points: $[0, \pm u_0]$

Large Re: Two possible solutions

Model results:



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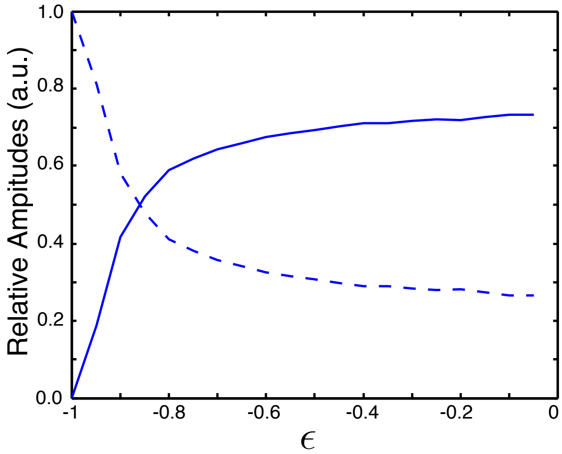
[3/5] Very Slow Regime

Large Re: Two possible solutions

Model results:



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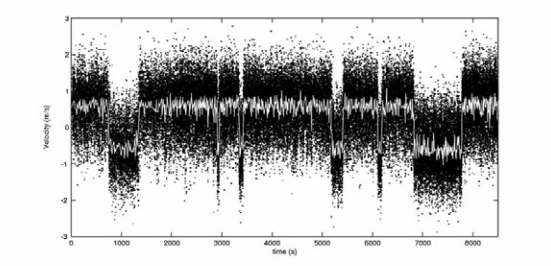
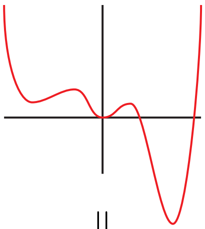
Hysteresis

Predicts hysteresis ⇒ recovered on the experiment!

$$\Delta = \frac{f_N - f_S}{f_N + f_S} = +0.0017$$

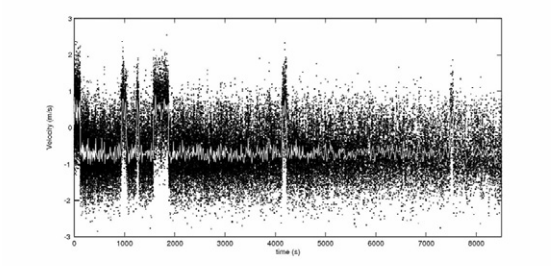


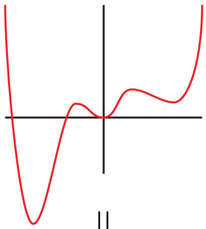


⇓

South state wins!!

$$\Delta = \frac{f_N - f_S}{f_N + f_S} = -0.0017$$





⇓

North State wins!!

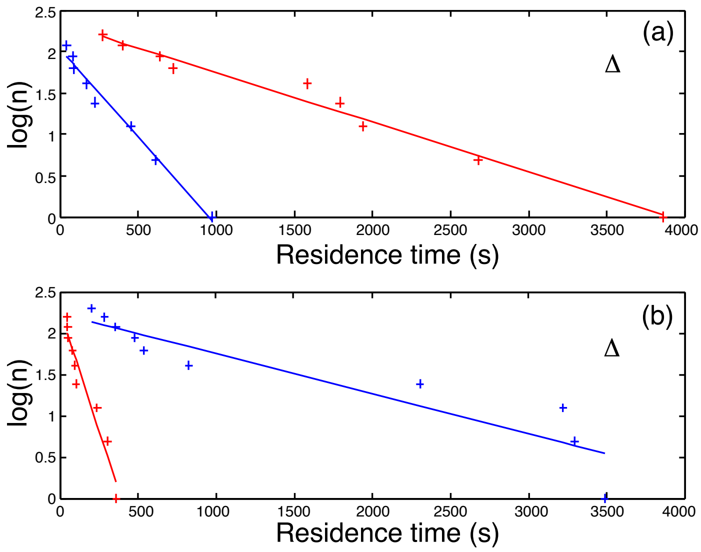
Burguete and de la Torre, IJBC 19 (2009) 2695

[3/5] Very Slow Regime

Hysteresis

Escape time *vs.* Δ

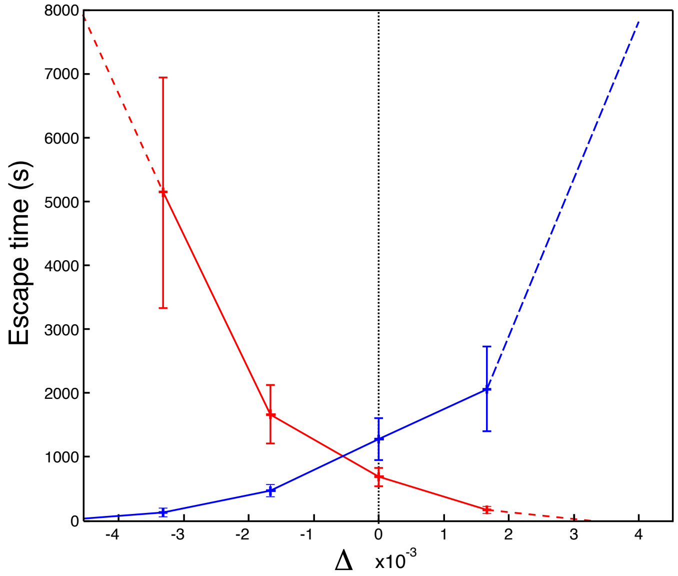
(North and South states have different residence times)



Burguete and de la Torre, IJBC 19 (2009) 2695

Hysteresis

Escape time *vs.* Δ
(North and South states have different residence times)



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Coloured Noise

Two wells + Colour noise $\rightarrow$ 3 states

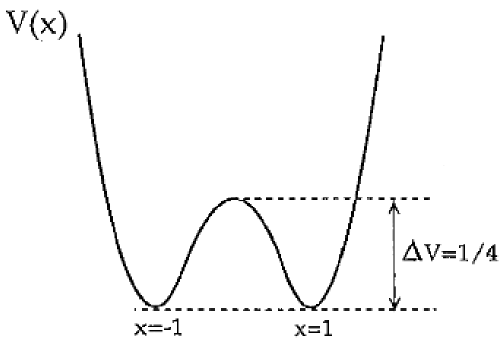

Figure 6.1. The double-well potential [Eq. (6.3)] is shown in normalized coordinates.

P. Hanggi, P. Jung, Adv. Chem. Phys. Volume LXXXIX, John Wiley & Sons (1995).

Coloured Noise

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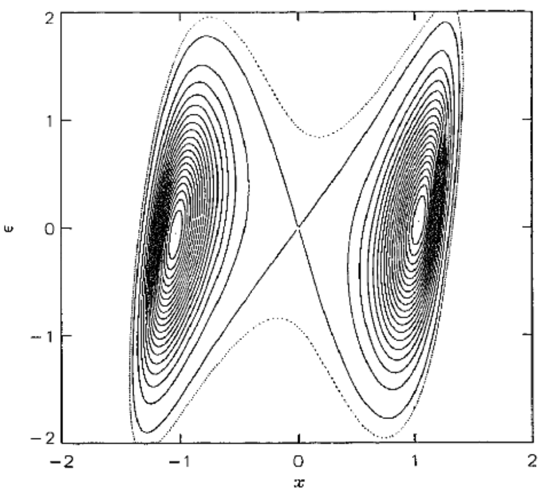
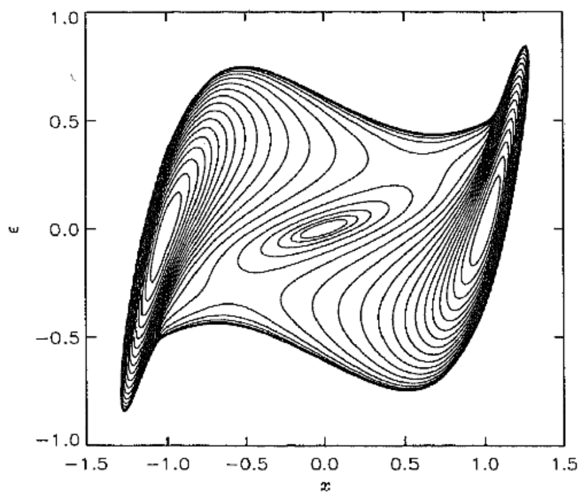
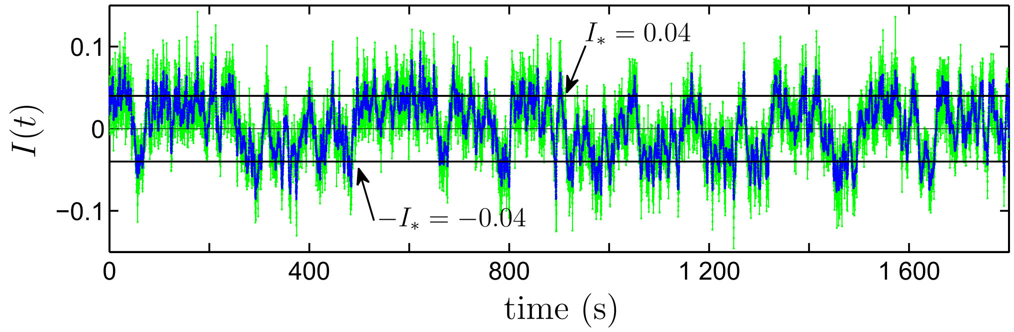




P. Hanggi, P. Jung, Adv. Chem. Phys. Volume LXXXIX, John Wiley & Sons (1995).

Susceptibility to Simmetry breaking

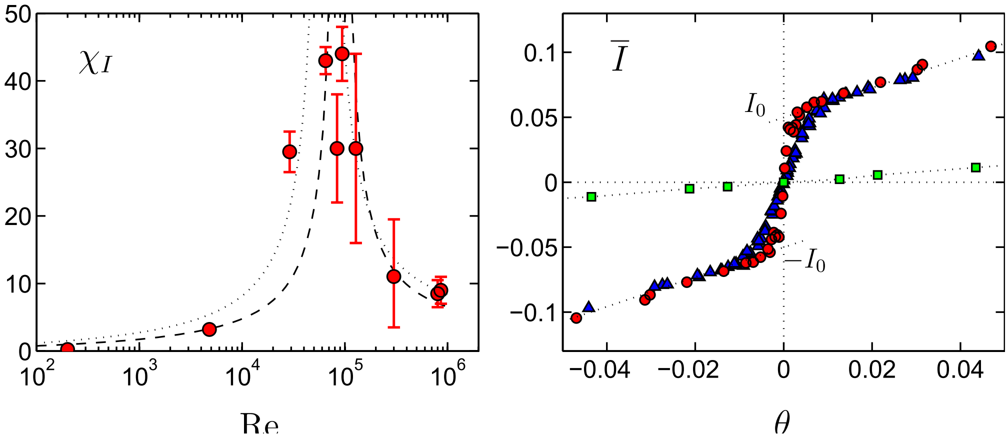


$$I(t) = \frac{1}{V}\int_V \frac{r u_\theta(t)}{\pi R^2 f} dv$$

$$\chi = \left.\frac{\partial \bar{I}}{\partial \Delta}\right|_{\Delta=0}$$

P.-P. Cortet, A. Chiffaudel, F. Daviaud, and B. Dubrulle, Phys Rev Lett 105 (2010) 214501

Susceptibility to Simmetry breaking

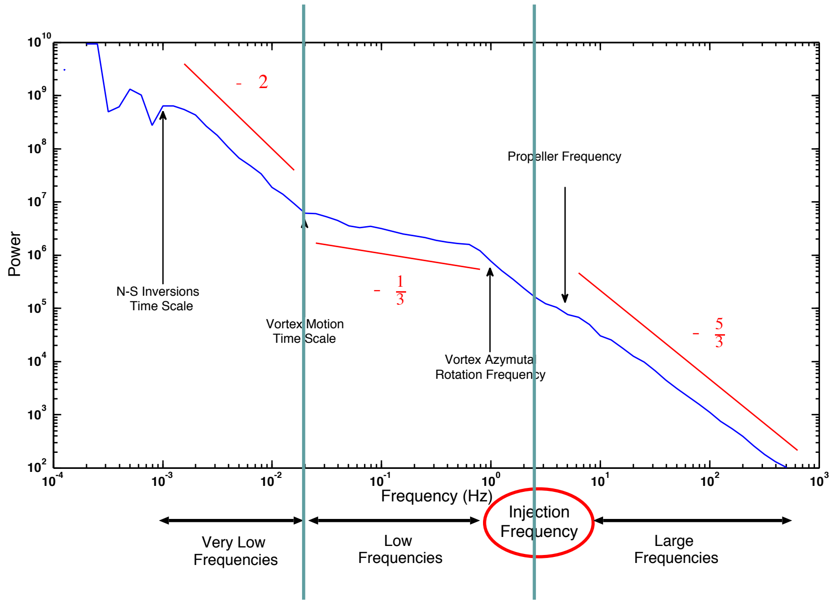


P.-P. Cortet, A. Chiffaudel, F. Daviaud, and B. Dubrulle, Phys Rev Lett 105 (2010) 214501

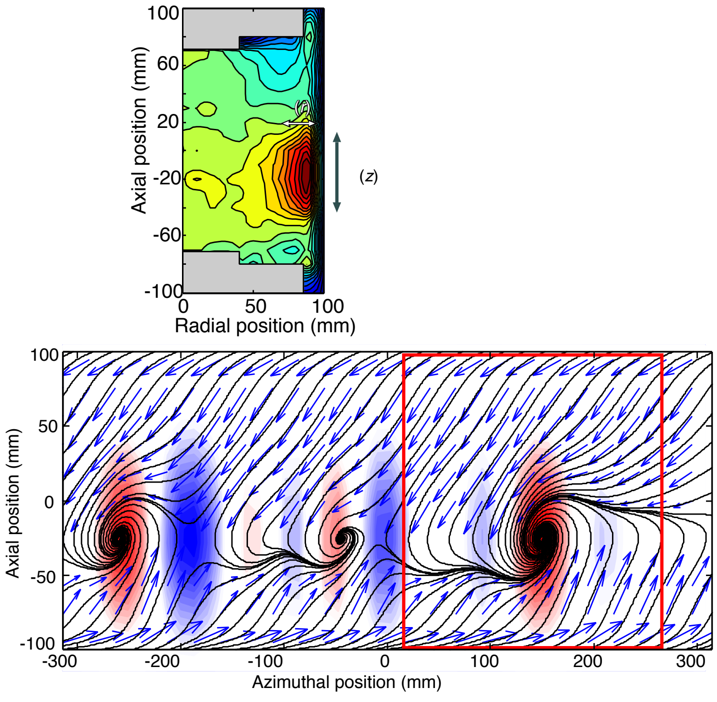
[4/5] Slow Regime
Torque transmision

Power spectrum

Now we move into the **intermediate range:**

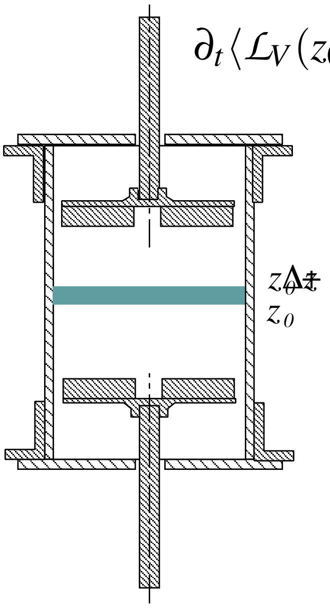


Large Re: Equatorial Vortices



Mean flow destabilization

Torque transmission:



$$\partial_t \langle \mathcal{L}_V(z_0) \rangle = \left[\int_V \partial_t \langle \mathcal{L} \rangle dV \right] = \left[\int_V \rho \langle r v_z \partial_z v_\theta \rangle \, dV \right]_{z_0} =$$

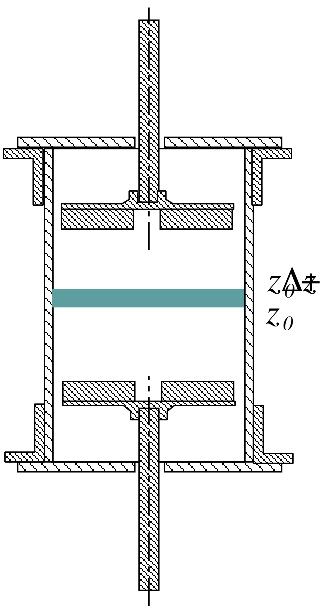
$$= \left[\int_A \rho \langle r u_\theta u_z \rangle dA + \int_A \rho r U_\theta U_z dA \right]_{z_0 + \Delta z/2} -$$

$$- \left[\int_A \rho \langle r u_\theta u_z \rangle dA + \int_A \rho r U_\theta U_z dA \right]_{z_0 - \Delta z/2}$$

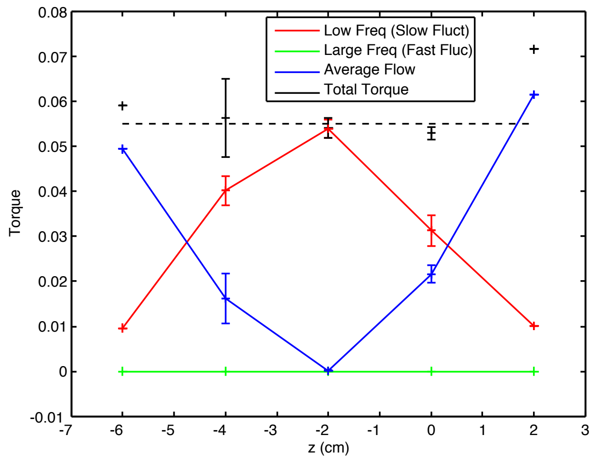
[4/5] Slow Regime

Mean flow destabilization

Torque transmission:



Contributions from mean flow and fast and low frequencies



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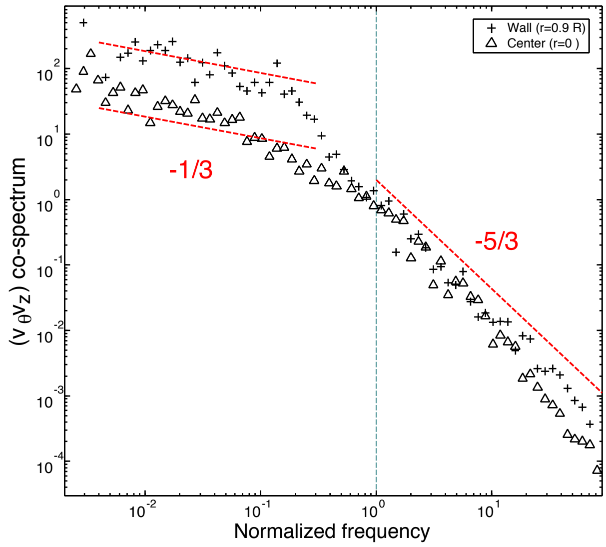
Mean flow destabilization

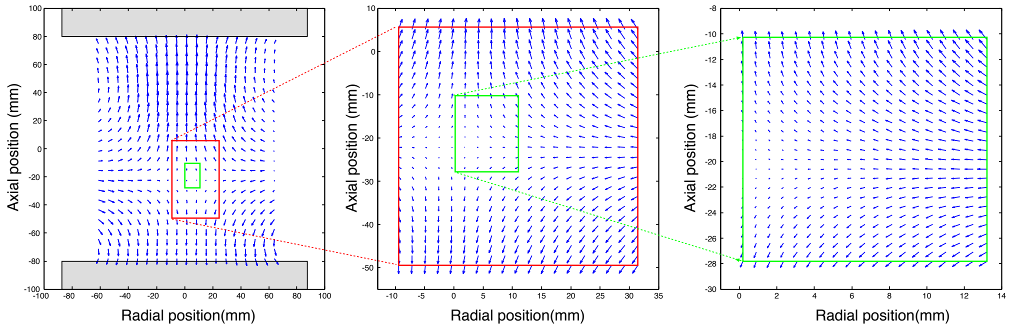
Co-spectrum $\rightarrow$ $u_\theta u_z$



[4/5] Slow Regime

Mean flow destabilization

And the spatial behaviour? $\rightarrow$ PIV



Mean flow destabilization

Dimensional analysis:

$$\partial_t \mathcal{L}_V^2 \sim \int_V 2r^2 v_\theta v_z \partial_z v_\theta dV$$

$$\Rightarrow \qquad \varepsilon_L \propto L^2 (L^3/T^3)/L = L^4/T^3$$

$$\mathcal{L}_V^2 = \int \mathcal{L}_F^2(k) dk$$

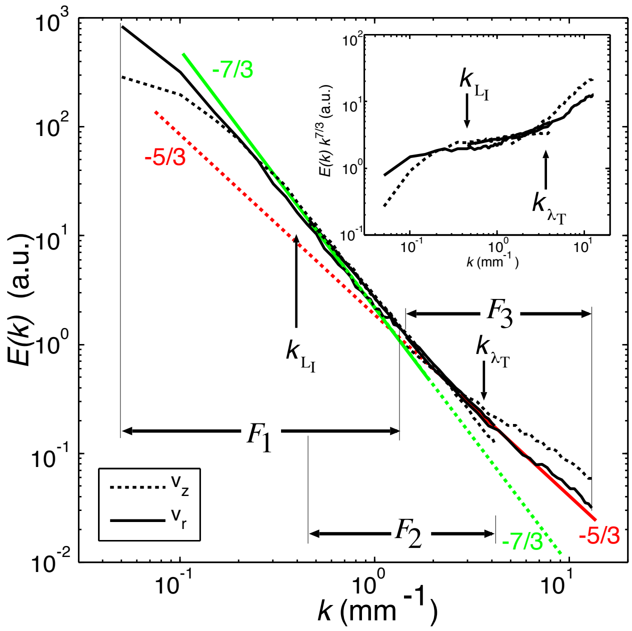
$$\Rightarrow \qquad \mathcal{L}_F^2(k) \propto \varepsilon_L^{2/3} k^{-7/3}$$

$$\mathcal{E}_V = \mathcal{L}_V^2 / I_V$$

$$\Rightarrow \qquad \mathcal{E}(k) \propto \mathcal{R}^{-2} \varepsilon_L^{2/3} k^{-7/3}$$

Mean flow destabilization

Dimensional analysis: $\mathcal{E}(k) \sim I^{-1} \varepsilon_L^{2/3} k^{-7/3}$

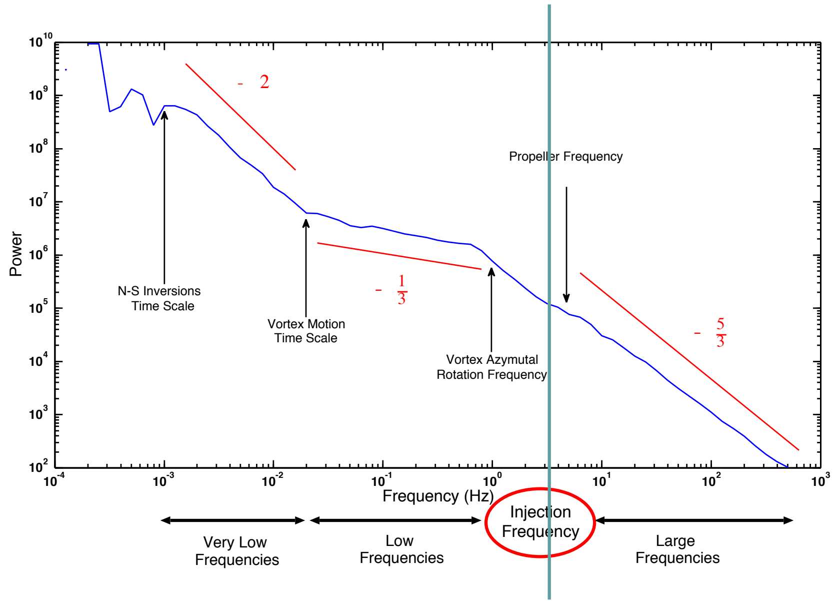


Spatial spectra
↓
two cascades

M. Lopez-Caballero, J. Burguete, PRL 110, 124501 (2013)

Power spectrum

...and the inertial range:



Inertial range

From PIV measurements of the velocity flow we can determine:

For a $Re = 1.75\,10^5$

- Integral scale $L_I = 15$ mm
  $\rightarrow$ on the order of the interblade spacing
- Dissipative scale $\eta = 30\ \mu$m
- Energy dissipation rate $\varepsilon = 1.1$ W/kg
- Taylor microscale= 1.8 mm
- $Re_\lambda = 900$

...and many other characteristics of the turbulent fluctuations

[5/5] A cute couple:
FIONA and SHREK

To infinity and beyond!! .... $Re = 10^8$!

Flow Instability Observation usiNg Anemometers

on the

Superfluid High REynolds
von Kármán facility

at SBT / CEA-Grenoble / France

SHREK facility was developped as a joint effort by:
SBT, CEA-Grenoble; Intitut Néel, CNRS, Grenoble;
SPEC, CEA-Saclay; ENS-Lyon, Lyon;
LEGI, U. Joseph Fourier, Grenoble;

France

To infinity and beyond!! .... $Re = 10^8$!

Flow Instability Observation usiNg Anemometers

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Superfluid High REynolds
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FIONA & SHREK

[5/5] Fiona and Shrek

To infinity and beyond!! .... $Re = 10^8$!

Some numbers...

Temperature [K]	2.3	1.9
Kinematic viscosity [m^2/s]	$2\,10^{-8}$	$9.43\,10^{-9}$
Frequency [Hz]	2	1
Velocity (prop rim) [m/s]	4.8	2.4
Reynolds number Re	$9.6\,10^{7}$	10^{8}

To infinity and beyond!! .... $Re = 10^8$!

2000 l of fluid He (normal or superfluid) experiment: 1.8 $\leftrightarrow$ 3K

[5/5] Fiona and Shrek

To infinity and beyond!! .... $Re = 10^8$!

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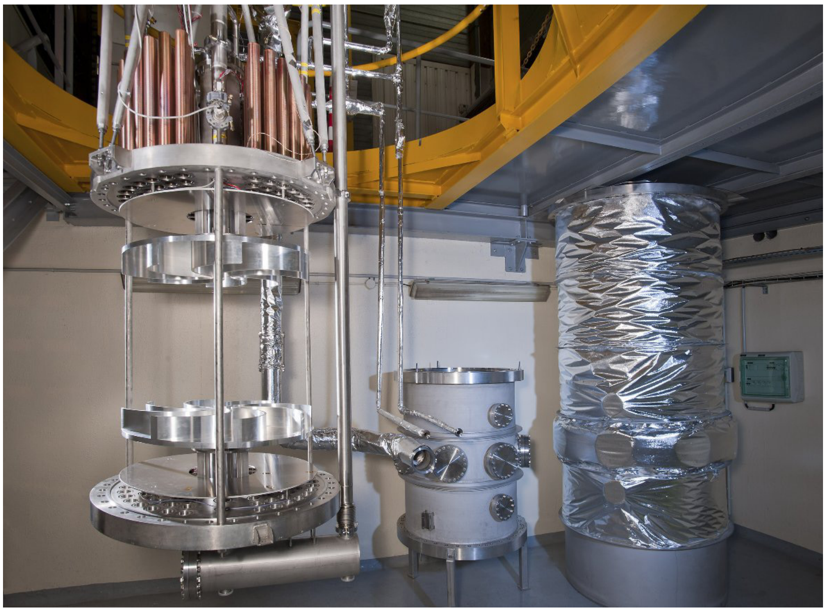

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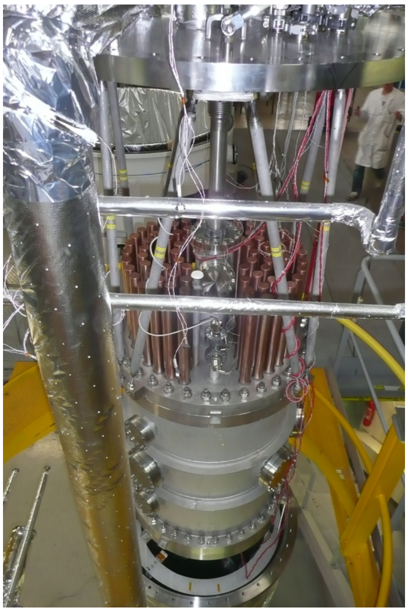

[5/5] Fiona and Shrek

To infinity and beyond!! .... $Re = 10^8$!

2000 l of fluid He (normal or superfluid) experiment: 1.8 $\leftrightarrow$ 3K

NCTR3, Les Houches, March 18th, 2014 26

To infinity and beyond!! .... $Re = 10^8$!

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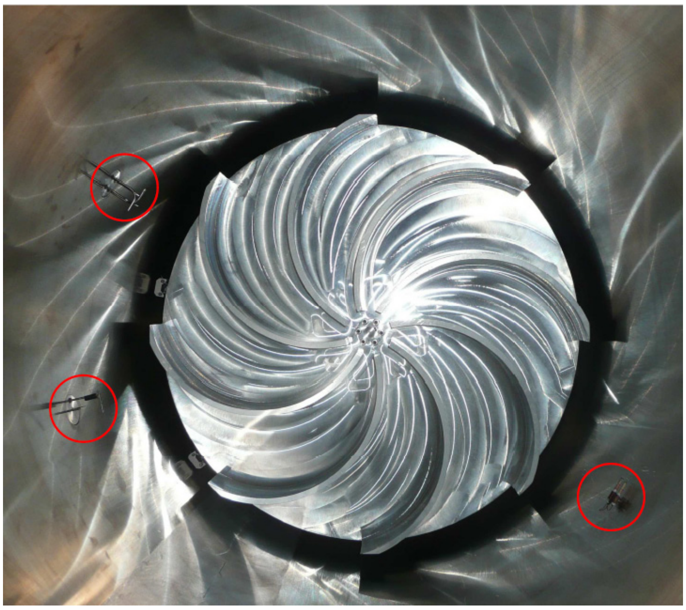

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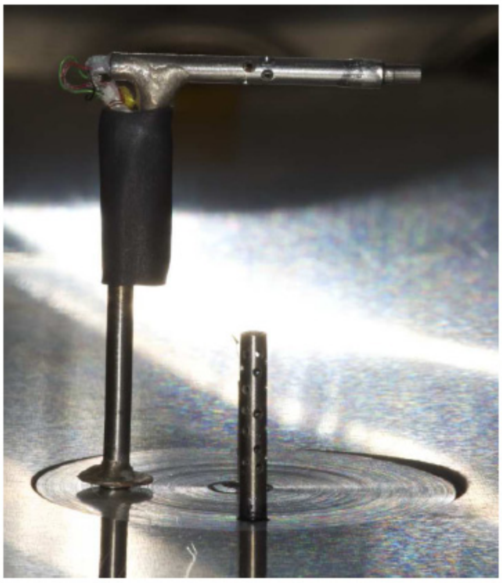

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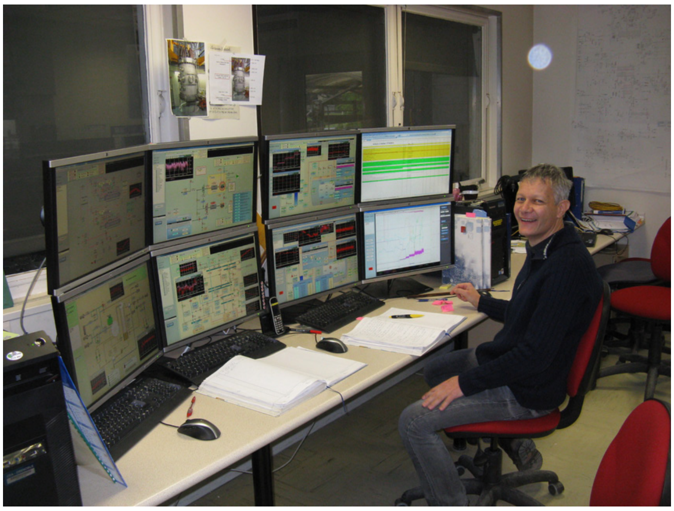

[BONUS] MHD
Effects on the Dynamo Action

Problem formulation

Governing Equations (MHD approx.):

$$\frac{\partial \vec{B}}{\partial t} + (\vec{u}\cdot\nabla)\vec{B} = \left(\vec{B}\cdot\nabla\right)\vec{u} + \eta\nabla^2\vec{B}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{1}{\rho\mu_0}\left(\nabla\times\vec{B}\right)\times\vec{B} + \vec{F}_{ext}$$

$$\nabla\cdot\vec{u} = 0 \qquad \nabla\cdot\vec{B} = 0$$

Adimensional numbers:

$$Rm = \frac{UL}{\eta} = UL\mu_0\sigma \qquad Re = \frac{UL}{\nu} \qquad Pm = \frac{Rm}{Re}$$

Problem formulation

Governing Equations (MHD approx.):

$$\frac{\partial \vec{B}}{\partial t} + (\vec{u}\cdot\nabla)\vec{B} = \left(\vec{B}\cdot\nabla\right)\vec{u} + \eta\nabla^2\vec{B}$$

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$$\nabla\cdot\vec{u} = 0 \qquad \nabla\cdot\vec{B} = 0$$

Adimensional numbers:

But, $\nu \ll \eta$ for most neutral conducting fluids $\Rightarrow$ $Re \gg Rm$
$\Rightarrow$Fully developped turbulence !

Typically, $\nu \sim 10^{-5}\eta$, so for a $Rm = 100$ we need a $Re = 10^7$!

Dynamo Experiments

Homogeneous dynamos:

- von Kármán Sodium (Cylindrical geometry)
  (CEA Saclay + CEA Cadarache + ENS Paris + ENS Lyon)
  → Successful!! PRL **98** (2007) 044502 (Iron propellers)
  → Unsuccessful with stainless steel propellers
- University of Wisconsin (Spherical geometry)
- University of Maryland (Spherical geometry)
- University of Perm (Toroidal geometry)
- New Mexico (TC dynamo)
- Others

Our approach

1st Step:

Below the dynamo threshold ($\vec{B}=0$), the conducting fluid is equivalent to any other fluid with similar hydrodynamic properties
⇒ We use a water experiment to determine $\vec{u}$:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\,\vec{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{1}{\rho\mu_0}\left(\nabla\times\vec{B}\right)\times\vec{B} + \vec{F}_{ext}$$

2nd Step:

We analyze the effect of this flow numerically in a kinematic code:

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{u}\times\vec{B}\right) + \eta\nabla^2\vec{B}$$

Kinematic dynamo

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{u} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

The usual “weak” aproximation:

- Axisymmetric, stationary flow
  (preserving the equatorial symmetry).

[BONUS] MHD

Kinematic dynamo

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{u} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

The usual “weak” aproximation:

- Axisymmetric, stationary flow
  (preserving the equatorial symmetry).

Here, we will consider:

(a) Axisymmetric, but with two symmetric solutions
and “periodic” reversals (very low frequencies)

(b) Non-axisymmetric flows, without reversals
(low frequencies)

Kinematic dynamo

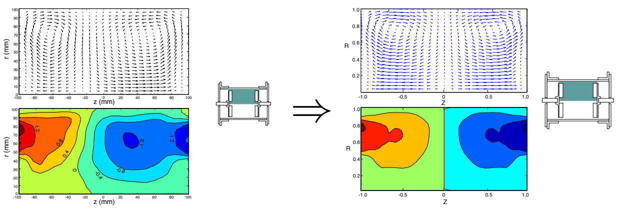
- Pseudo-spectral code:
  Finite differences in r and Fourier in θ, z

$$\vec{B}(\vec{s},t) = \sum_{n,m} \vec{b}_{n,m}(r)\exp\left[i(m\theta + n2\pi z/H)\right]$$

- Rm definition: $Rm = \max\left\{U(r,\theta,z)\right\} R/\eta$
- Magnetic energy growth rates:

$$E_{m,n} = e^{\sigma_{m,n} t}$$

- We only considere the symmetric part:



Effect on the dynamo threshold

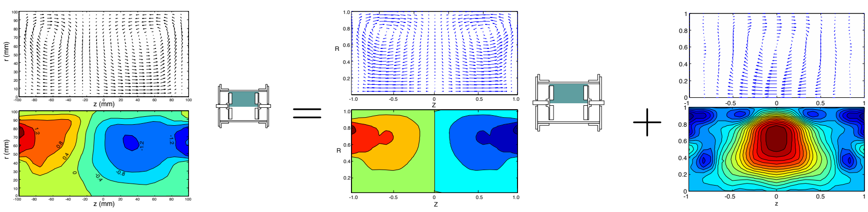
(a) Equatorial symmetry broken:

- Slowly evolving axisymmetric flows:

$$\vec{V}_\omega(t) = \vec{V}_S + \mathcal{A}_{mod}\vec{V}_D\cos(\omega t) = \frac{\vec{V}_N + \vec{V}_S}{2} + \mathcal{A}_{mod}\frac{\vec{V}_N - \vec{V}_S}{2}\cos(\omega t)$$

V_N and V_S are the velocity fields where the N or S side dominates. (In the following, $\mathcal{A}_{mod} = 1$)

For example, for $t = 0$:



Effect on the dynamo threshold

- Rm definition used:

$$Rm = \max_{0 \le t < T} \{V_\omega(t)\} R/\eta$$

- Magnetic energy growth rates:

$$E_{m,n} = e^{\sigma_{n,m}(t)t}$$

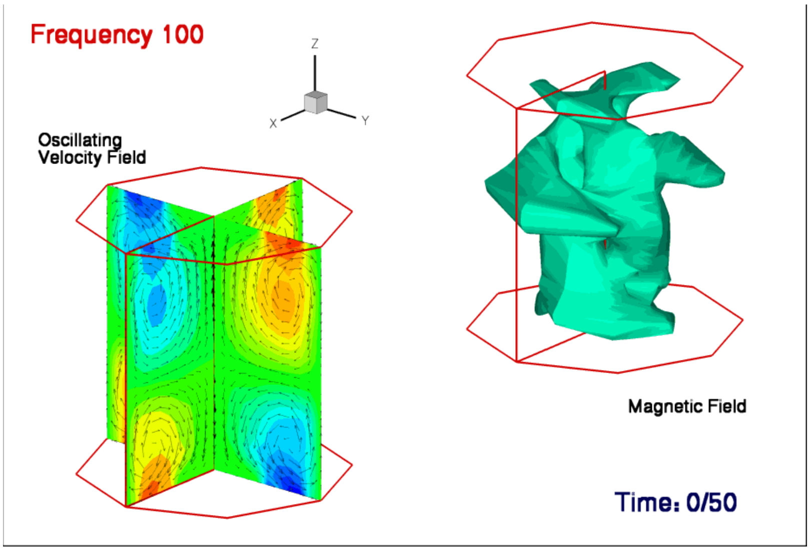
$$\Downarrow$$

$$\langle \sigma_{n,m} \rangle = \frac{1}{T} \int_{T=\frac{2\pi}{\omega}} \sigma_{n,m}(t) dt > 0$$

[BONUS] MHD

Effect on the dynamo threshold

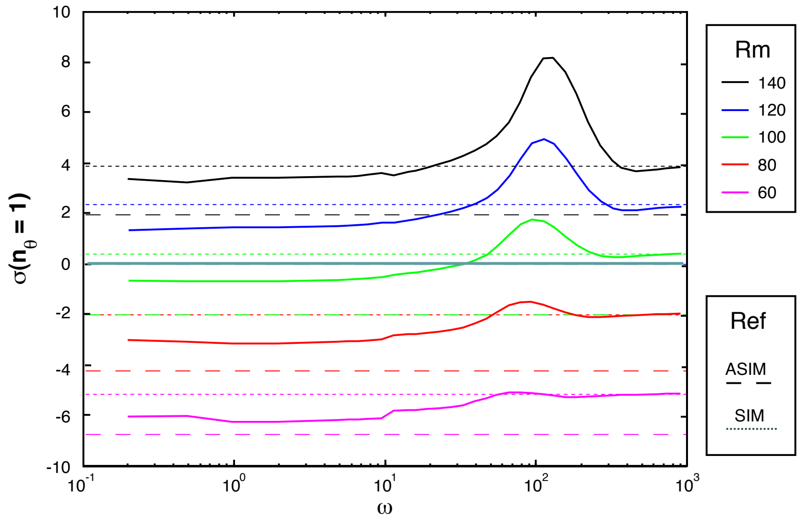
Time-dependent velocity field:



[BONUS] MHD

Effect on the dynamo threshold

Growth rates *vs.* the frequency

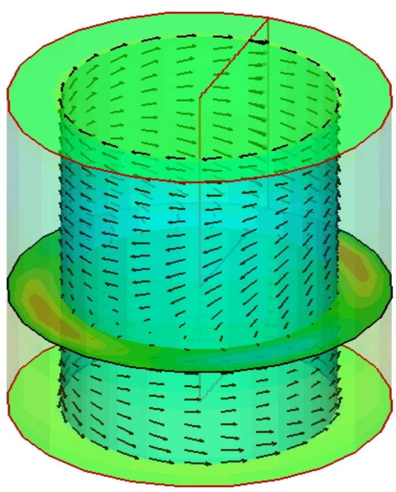


[BONUS] MHD

Effect on the dynamo threshold

(b) MHD analysis of real 3D flows (equatorial vortices):

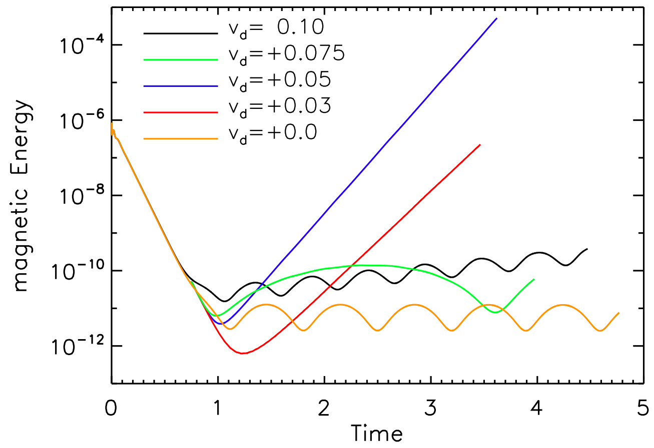
- Large scales can be very important$\rightarrow$ vortices



Effect on the dynamo threshold

(b) MHD analysis of real 3D flows (equatorial vortices):

(3D kinematic dynamo)



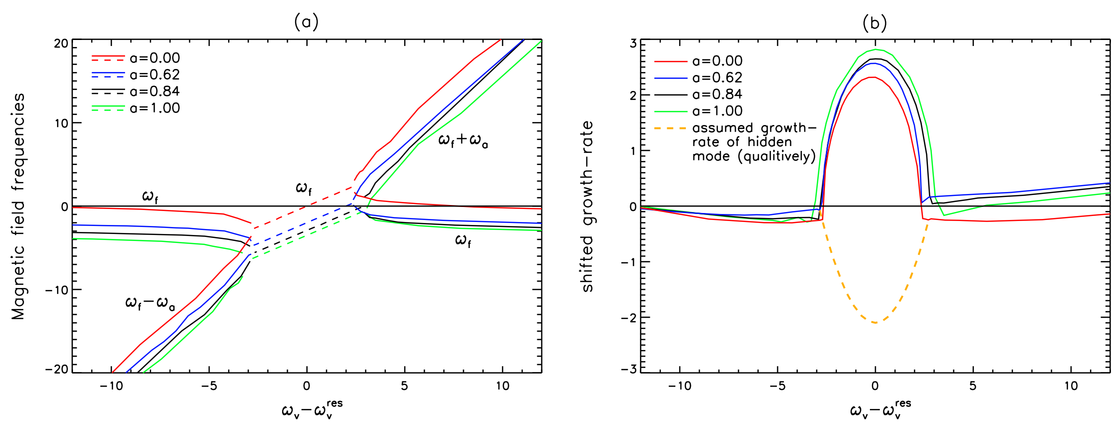
PRE **87** (2013) (accepted) A. Giesecke, F. Stefani, J. Burguete

[BONUS] MHD

Effect on the dynamo threshold

(b) MHD analysis of real 3D flows (equatorial vortices):

(3D kinematic dynamo)



$$\ddot{x}+\omega_0^2(1+2\varepsilon\cos(\tilde{\omega}t))x=0$$

Conclusions

Main characteristics of this flow:

- $Re \simeq 10^{5-6}$
- Turbulence rate: 50-100%
- Broken symmetries: Non-stationary,
  Non axisymmetric (actually, m≥ 2)
  Equatorial symmetry is broken (two mirrored states)
- Random reversals
- Inverse cascade due to angular momentum transport

Different Time scales:

- Dissipation $\sim 10^{-5}$s
- Injection ~ 0.1s
- Vortex motion ~ 10s
- North – South inversion dynamics $\sim 10^{3}$s

Conclusions

These turbulent flows have **very slow** dynamics

- Symmetry breakings (equatorial and axisymmetry),
- Vortices
- Random inversions$\rightarrow$ simple model
- many involved time-scales...

The dynamo threshold is **very sensitive** to the fluid flow
(kinematic approach)

- Strong resonances appear for small windows of U_{pol}/U_{tor}.
- Equatorial vortices and reversals can help or destroy the
  dynamo depending on their respective time-scales.