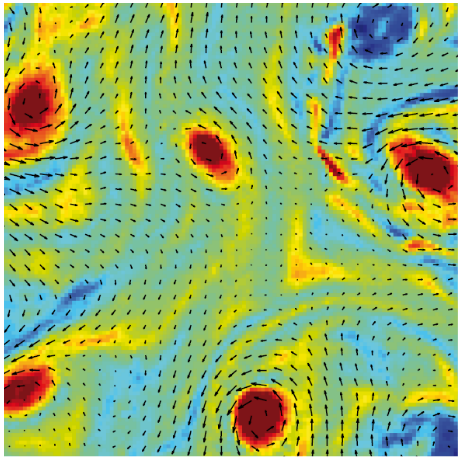
Energy transfers in turbulence under rotation
Some experiments



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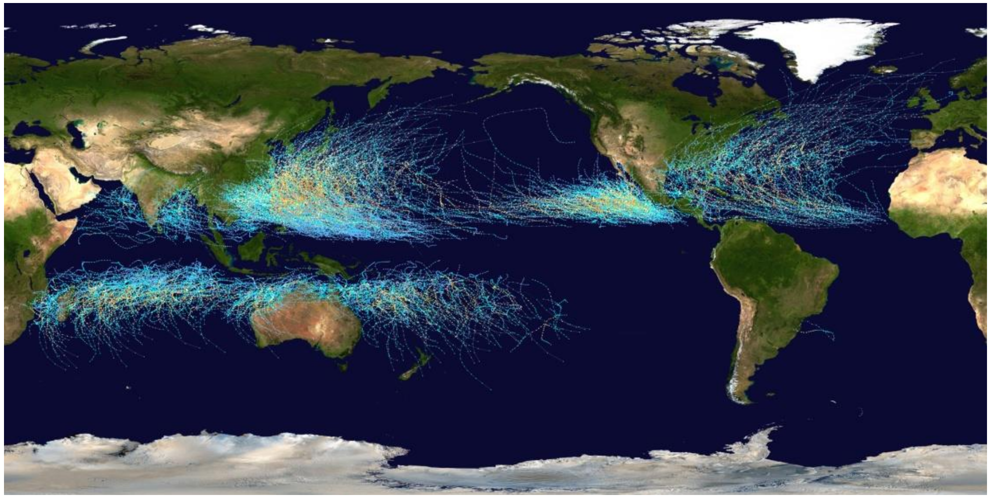
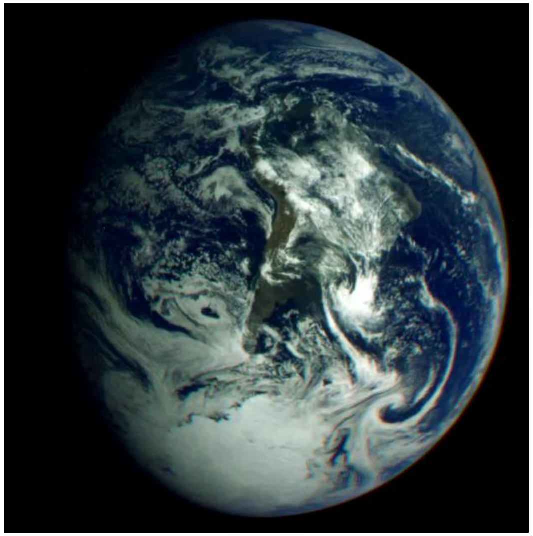
Hydrodynamics under rotation

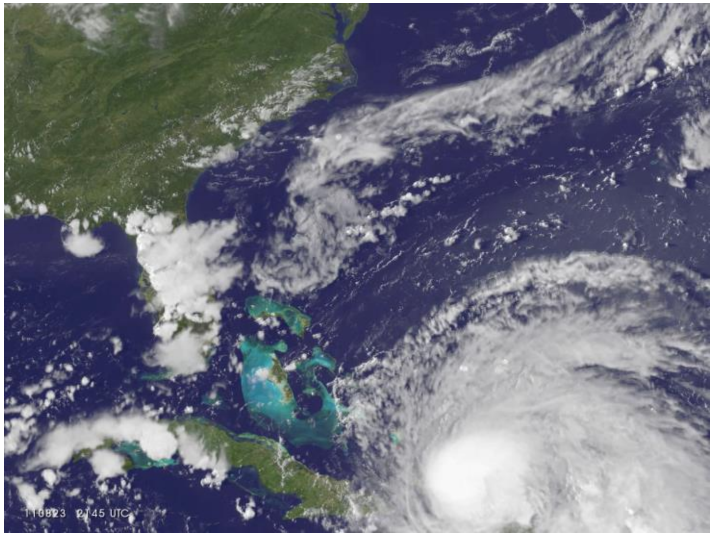
Geo and astrophysical flows



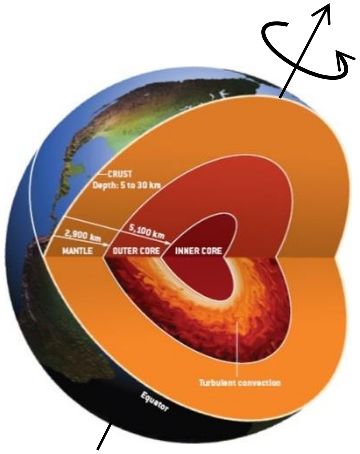
Earth rotation from Galileo (1990)



Hurricane Irene (2011)

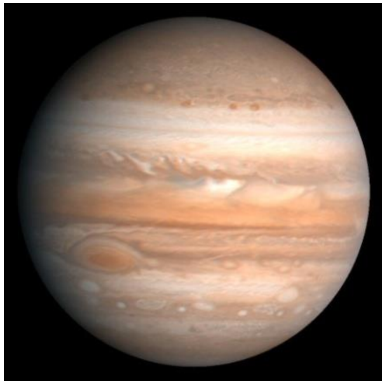


Hydrodynamics under rotation



Geo and astrophysical flows

- Geophysics
  (ocean, atmosphere, liquid core)
- Astrophysics
  (gazeous planets, galaxies)



Physical ingredients

- Stratification → Internal gravity waves
- Magnetism → Alvén waves, Dynamo action
- Thermal convection
- <u>**Rotation → Internal inertial waves**</u>
- <u>**Turbulence**</u>

Model experiments

Hydrodynamics under rotation

Navier-Stokes equation in a rotating frame

$\vec{\Omega}$ 

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p - 2\vec{\Omega} \times \vec{u} + \nu \Delta \vec{u}$$

with $p = p_0 + \frac{1}{2}\rho(\vec{\Omega} \times \vec{r})^2$

Coriolis force

Reynolds number $Re = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|\nu \Delta \vec{u}|} \sim \frac{U L_\perp}{\nu}$

Rossby number $Ro = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{U}{2\Omega L_\perp}$

Temporal Rossby number $Ro_t = \frac{|\partial\vec{u}/\partial t|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{\sigma}{2\Omega}$

In a non-dimensional version

$$Ro_t \frac{\partial \vec{u}}{\partial t} + Ro(\vec{u} \cdot \vec{\nabla})\vec{u} = -\vec{\nabla} p - \vec{e_z} \times \vec{u} + Ro\ Re^{-1} \Delta \vec{u}$$

Turbulence under rotation

Navier-Stokes equation in a rotating frame

$\vec{\Omega}$ 

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p - 2\vec{\Omega} \times \vec{u} + \nu \Delta \vec{u}$$

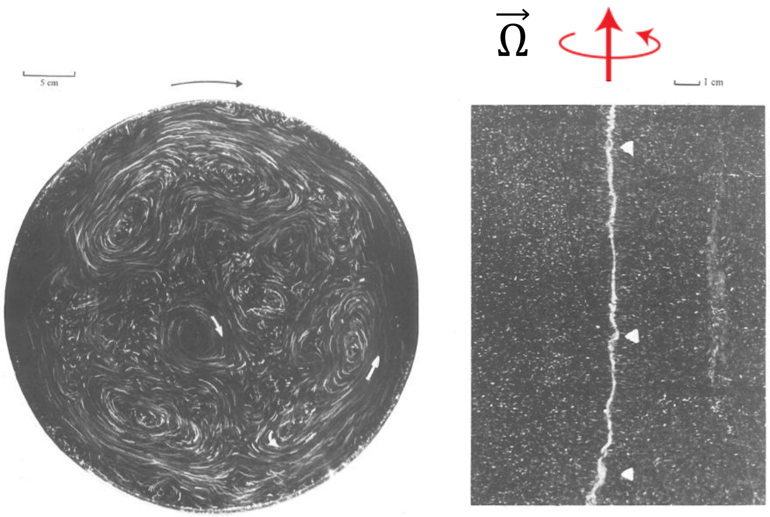
Reynolds number $Re = \frac{|(\vec{u} \cdot \vec{\nabla})\vec{u}|}{|\nu \Delta \vec{u}|} \sim \frac{U L_\perp}{\nu} \gg 1 \rightarrow$ The flow is turbulent.

Rossby number $Ro = \frac{|(\vec{u} \cdot \vec{\nabla})\vec{u}|}{|2\vec{\Omega} \times \vec{u}|} \sim \frac{U}{2\Omega L_\perp} \lesssim 1 \rightarrow$ Rotation interacts with non-linearities.

Temporal Rossby number $Ro_t = \frac{|\partial \vec{u}/\partial t|}{|2\vec{\Omega} \times \vec{u}|} \sim \frac{\sigma}{2\Omega} \simeq Ro$

This interaction drives turbulence toward an anisotropic 2D states invariant along $\vec{\Omega}$

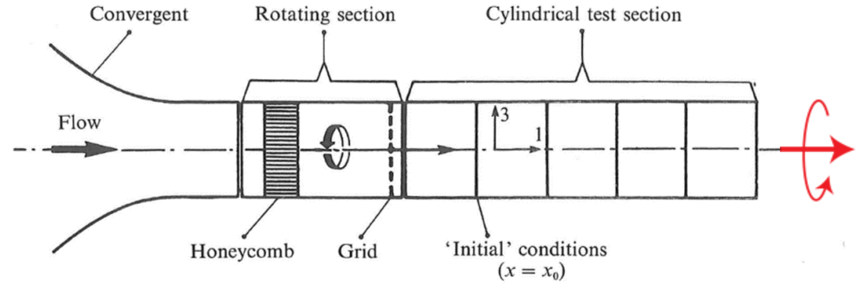
Two-dimensionalisation of turbulence by rotation



Hopfinger, Browand & Gagne, JFM (1982)

Oscillated grid in a rotating tank

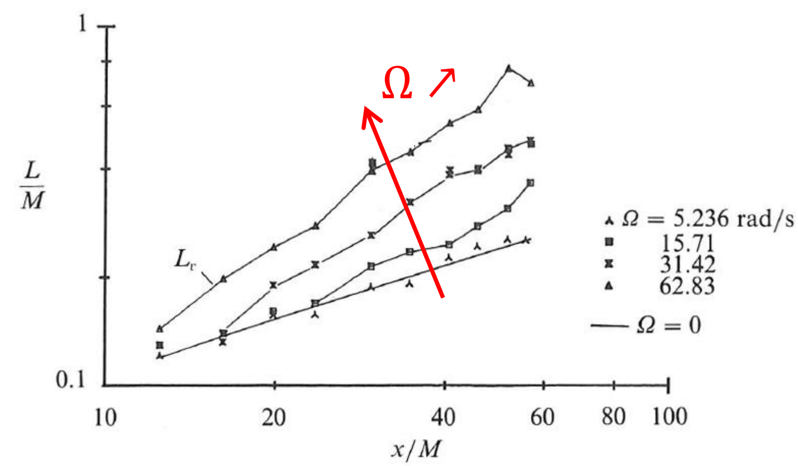
Emergence of columnar vortices aligned with the rotation axis



Jacquin, Leuchter, Cambon & Mathieu, JFM (1990)

Hot wire measurements in a rotating grid turbulence in a wind tunnel

Growth of axial integral scale with Ω



Rotation drives turbulence toward a 2D state

$$Ro_t \frac{\partial \vec{u}}{\partial t} + Ro(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}p - \vec{e_z}\times\vec{u} + Ro\, Re^{-1}\Delta\vec{u}$$

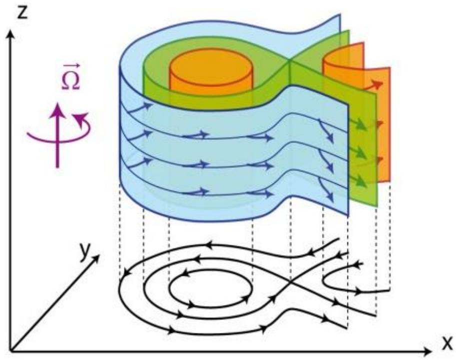
1. In the limit $Ro = Ro_t = 0$, NS becomes

$$\frac{1}{\rho}\vec{\nabla}p = -2\vec{\Omega}\times\vec{u}$$

Taking its curl gives $(\vec{\Omega}\cdot\vec{\nabla})\vec{u} = \vec{0}$

Taylor-Proudman Theorem = Geostrophic equilibrium

$Ro = 0$ → 2D 3C flow, but no turbulence



2D-3C flow

2. Whatever Ro and Re, for a pure 2D 3C flow, i.e. $\vec{u} = (\partial_y\psi, -\partial_x\psi, u_z)$

the Coriolis force can be absorbed in the pressure gradients

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla}\tilde{p} + \nu\Delta\vec{u}$$

with $\tilde{p} = p + 2\rho\Omega\psi$

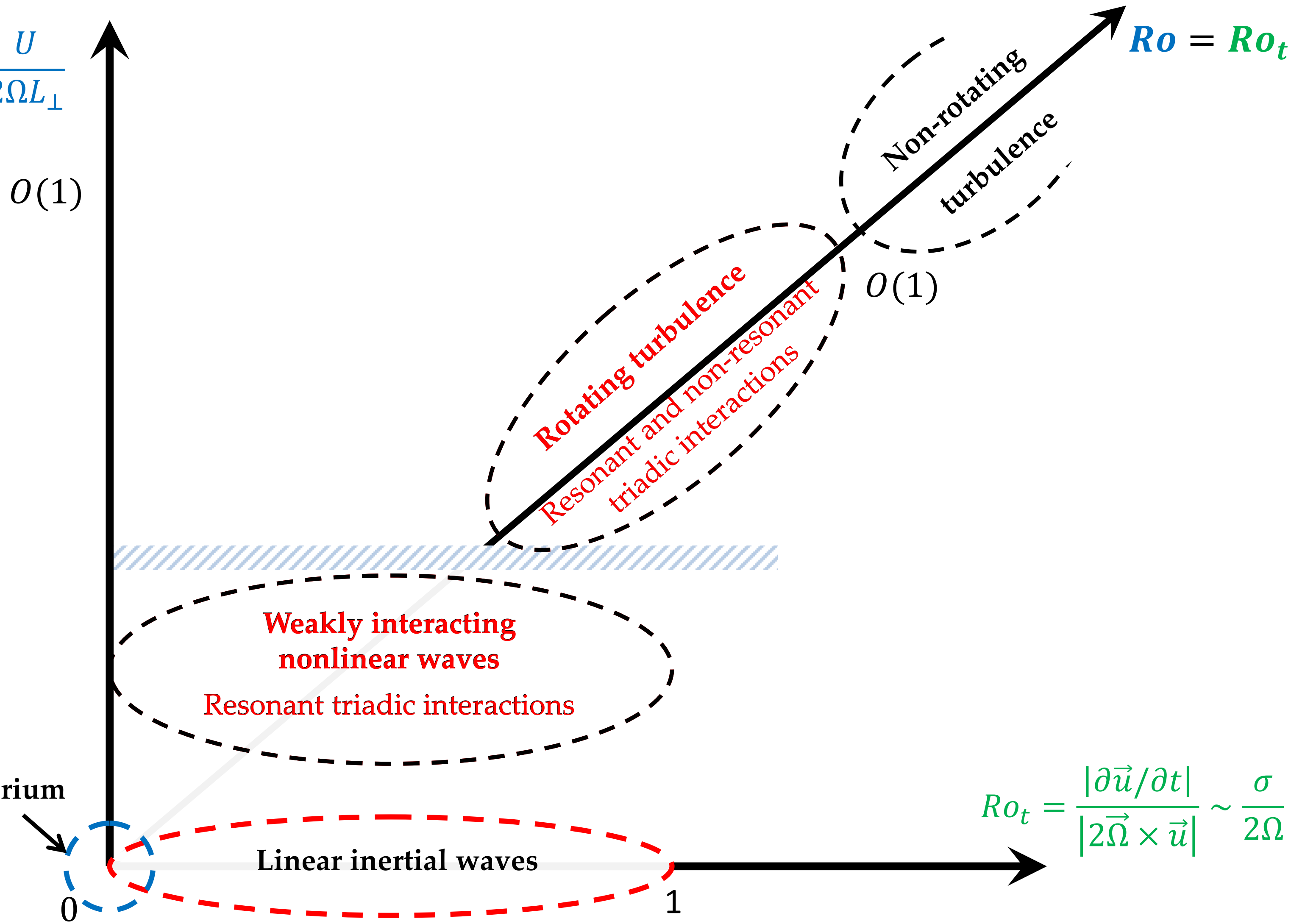
2D 3C flow → The dynamics is no more affected by rotation

→ " Turbulence affected by rotation " is neither exactly 2D, nor at $Ro = 0$

Hydrodynamics under rotation

$$Ro_t \frac{\partial \vec{u}}{\partial t} + Ro(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}p - \vec{e_z}\times\vec{u} + Ro\, Re^{-1}\Delta\vec{u}$$

$$Ro = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{U}{2\Omega L_\perp}$$



$$Ro_t = \frac{|\partial\vec{u}/\partial t|}{|2\vec{\Omega}\times\vec{u}|} \sim \frac{\sigma}{2\Omega}$$

Inertial waves in fluids under rotation

Navier-Stokes equation in a rotating frame

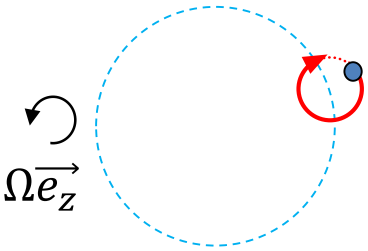
$$\frac{\partial \vec{u}}{\partial t} + \cancel{(\vec{u} \cdot \vec{\nabla})\vec{u}} = -\cancel{\frac{1}{\rho}\vec{\nabla} p} - 2\vec{\Omega} \times \vec{u} + \cancel{\nu \Delta \vec{u}}$$

Restoring action of the Coriolis force

$$\frac{\partial \vec{u}}{\partial t} = -2\vec{\Omega} \times \vec{u}$$

For velocities in the plane $\perp \vec{\Omega}$

$\vec{\Omega} = \Omega \vec{e_z}$

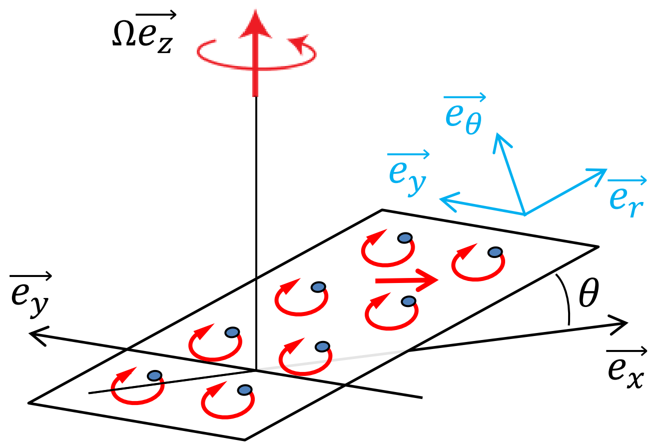


Anticyclonic circular translation
at frequency $\sigma = 2\Omega$

Inertial waves in fluids under rotation

Velocity in a plane tilted of θ

$$\vec{u} = u_r \vec{e_r} + u_y \vec{e_y}$$



Coriolis force

$$\vec{F_c} = -2\vec{\Omega} \times \vec{u} = \boxed{2\Omega u_y \cos(\theta)\vec{e_r} - 2\Omega u_r \cos(\theta)\vec{e_y}} - 2\Omega u_y \sin(\theta)\vec{e_\theta}$$

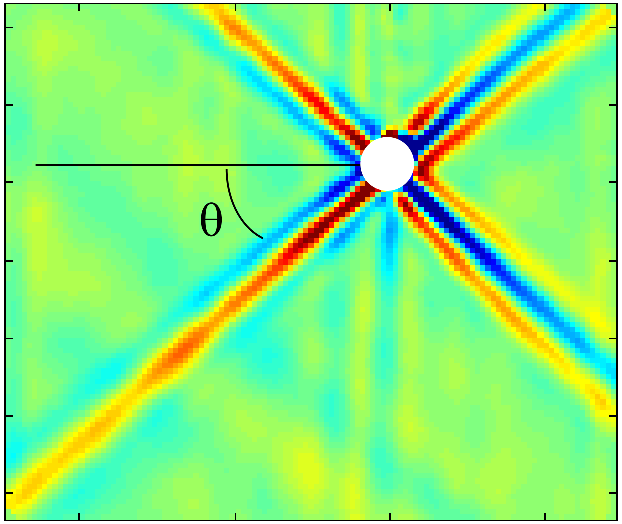
$$\begin{cases} \dfrac{\partial u_r}{\partial t} = 2\Omega\cos(\theta)\, u_y \\ \dfrac{\partial u_y}{\partial t} = -2\Omega\cos(\theta) u_r \end{cases}$$

Anticyclonic circular translation of the plane tilted of θ at the frequency $\sigma = 2\Omega\cos(\theta)$

Inertial waves in fluids under rotation

$\vec{\Omega}$

2D point source



Cortet, Lamriben & Moisy, POF (2010)

Dispersion relation

$$\frac{\sigma}{2\Omega} = \cos(\theta) = Ro_t$$

→ Propagation along $\theta = \mathrm{acos}\,\frac{\sigma}{2\Omega}$

$$Ro_t = \frac{|\partial \vec{u}/\partial t|}{|2\vec{\Omega} \times \vec{u}|} \sim \frac{\sigma}{2\Omega}$$

Inertial waves in fluids under rotation

Coriolis force

$$\vec{F_c} = -2\vec{\Omega}\times\vec{u} = 2\Omega u_y\cos(\theta)\vec{e_r} - 2\Omega u_r\cos(\theta)\vec{e_y}\boxed{-\,2\Omega u_y\sin(\theta)\vec{e_\theta}}$$

$$\frac{1}{\rho}\vec{\nabla}p = 2\Omega u_y\sin(\theta)\vec{e_\theta}$$

Component along $\vec{e_\theta}$ taken by the pressure gradients

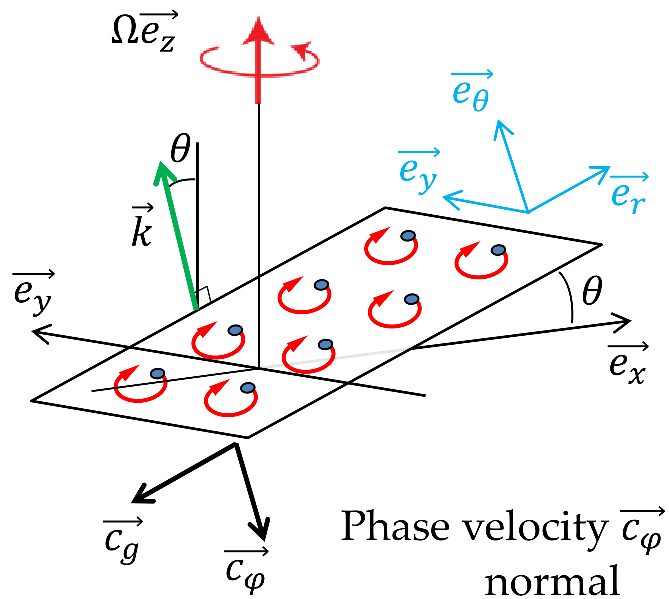
« $u_0\cos(\varphi(\vec{x},t))$ »

$$\vec{\nabla}\varphi = \rho\frac{u_0}{p_0}2\Omega\sin(\theta)\,\vec{e_\theta} \rightarrow \vec{\nabla}\varphi\perp\vec{u}$$

For a monochromatic wave

$$\vec{k} = \begin{pmatrix} k \\ m \end{pmatrix} = \vec{\nabla}\varphi$$

$\rightarrow \boxed{\dfrac{m}{\kappa} = \dfrac{\sigma}{2\Omega}}$ and $\kappa = \sqrt{k^2+m^2}$ arbitrary



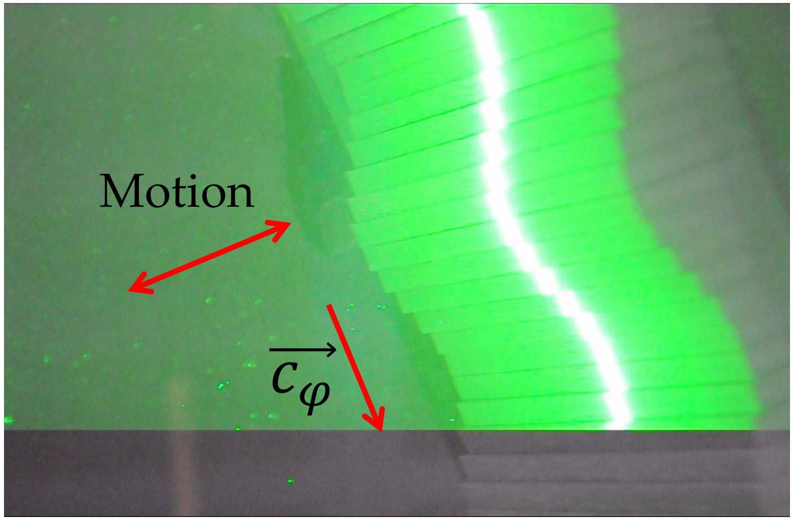
Phase velocity $\vec{c_\varphi}\parallel\vec{\nabla}\varphi$
normal
to group velocity $\vec{c_g}\parallel\theta$

Excitation of a plane inertial wave

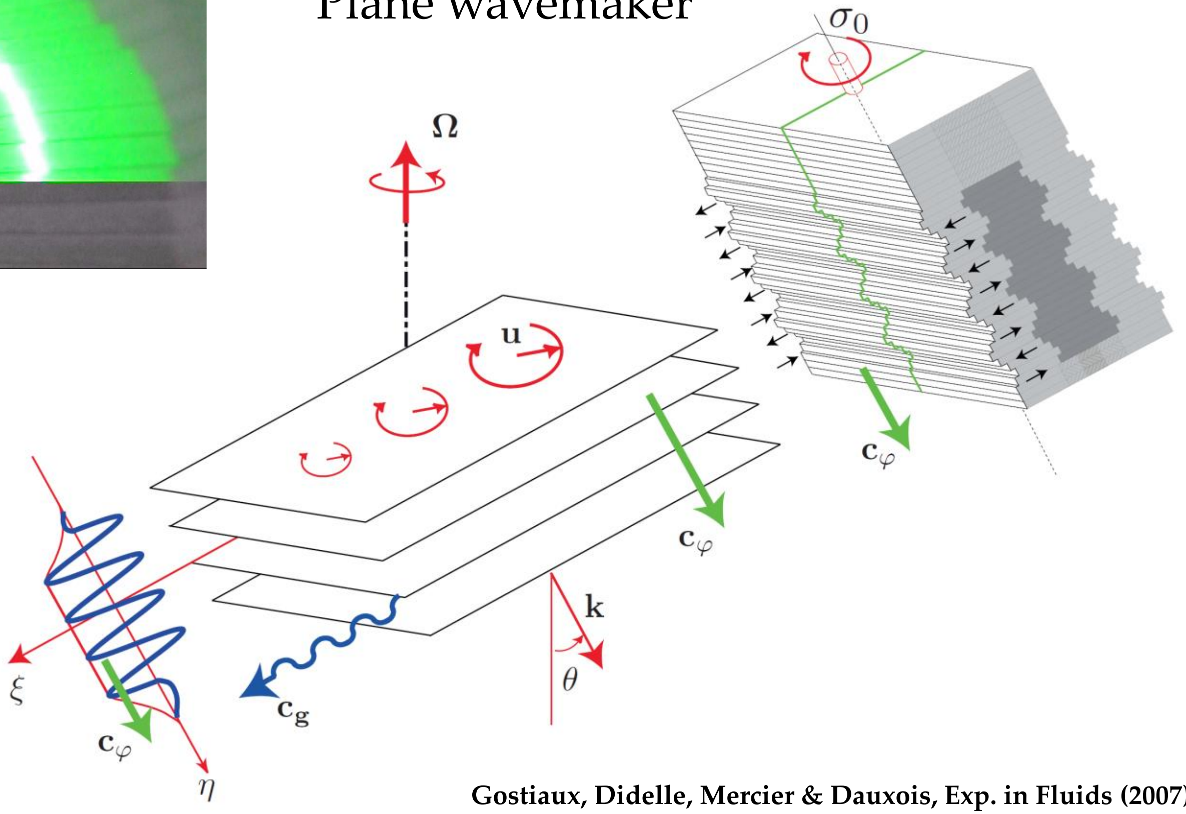
with F. Moisy

and

G. Bordes, T. Dauxois

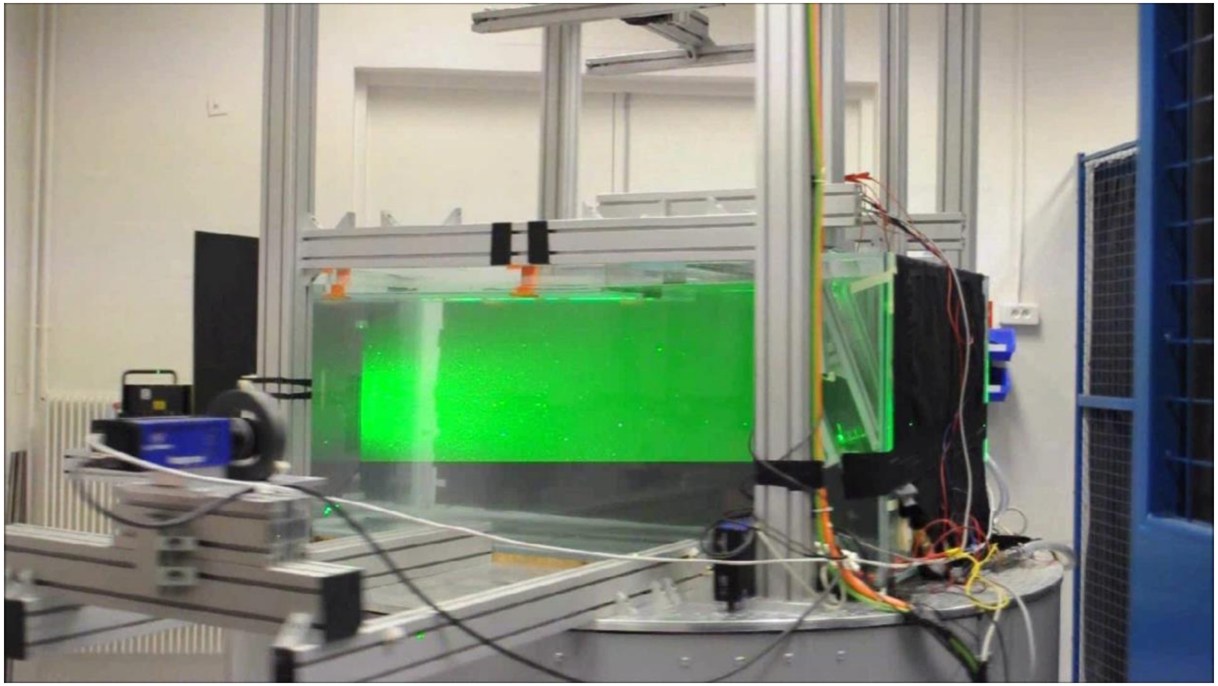


Plane wavemaker



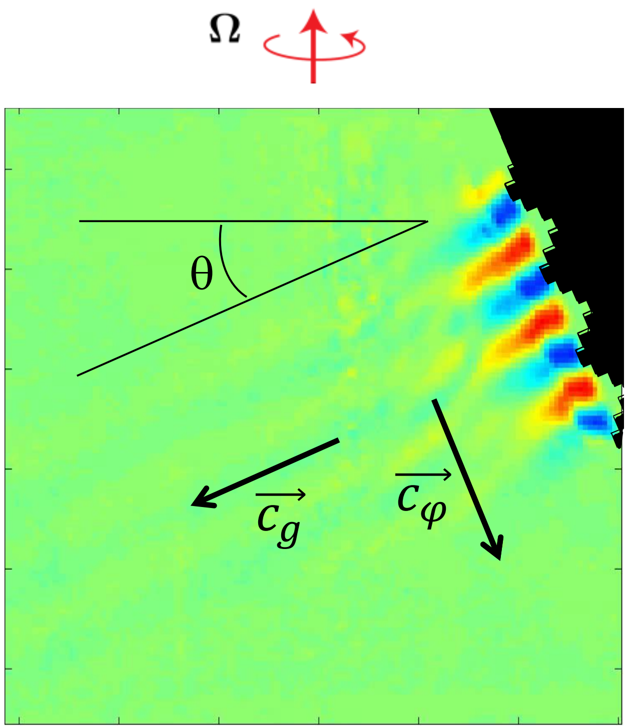
Gostiaux, Didelle, Mercier & Dauxois, Exp. in Fluids (2007)
Mercier, Martinand, Mathur, Gostiaux, Peacock & Dauxois, JFM (2010)

Rotating platform with PIV on board



Excitation of a plane inertial wave

Bordes, Moisy, Dauxois & Cortet, POF (2012)



Dispersion relation

$$\frac{\sigma}{2\Omega} = \frac{m}{\kappa} = \cos(\theta)$$

→ Propagation along $\theta = \text{acos}\,\frac{\sigma}{2\Omega}$

→ No link between frequency σ and wavelength

→ Phase velocity normal to group velocity

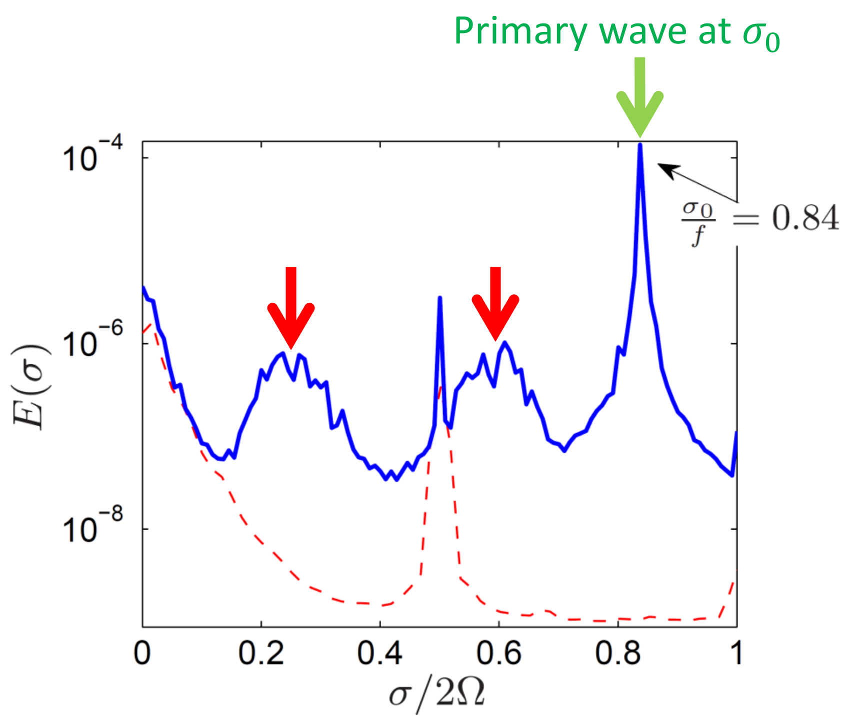
Subharmonic instability

Temporal energy spectrum

$$E(\sigma) = \langle |\hat{\boldsymbol{u}}_\sigma|^2 \rangle_{x,z}$$

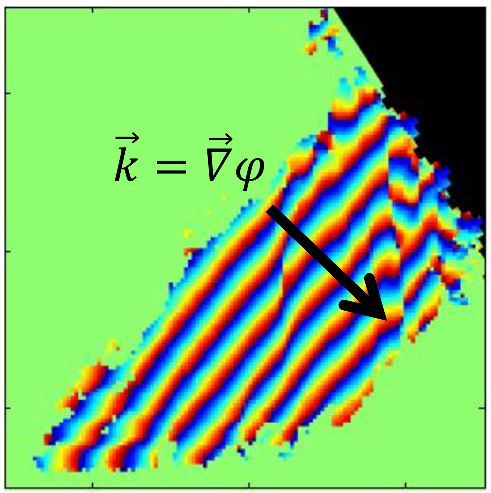
Temporal Fourier transform

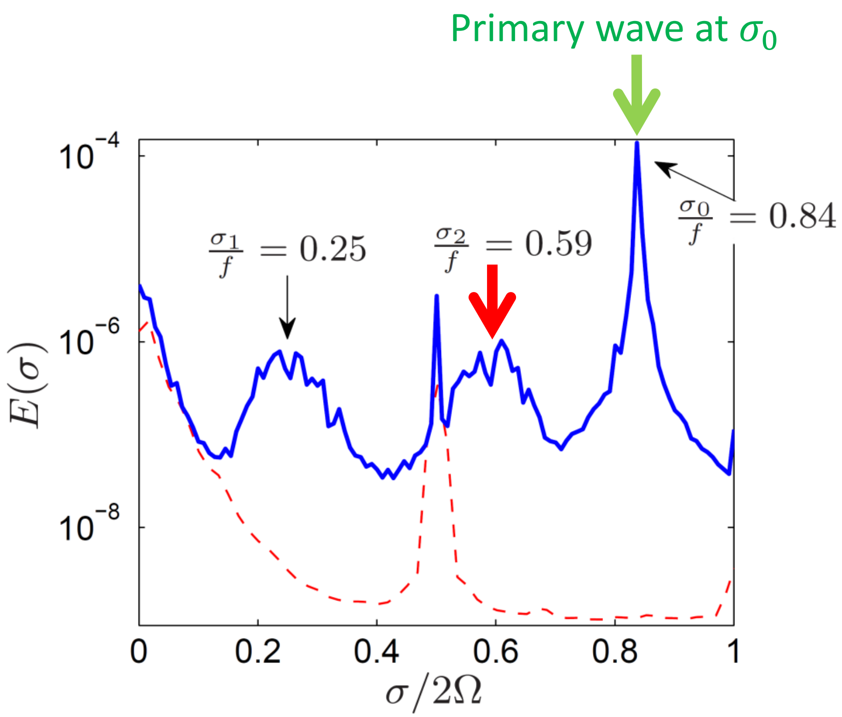
$$\hat{\boldsymbol{u}}_\sigma(x,z) = \frac{1}{2\pi} \int \boldsymbol{u}(x,z,t) e^{i\sigma t} dt$$



Growth of two subharmonic waves

Temporal Fourier filtering





The subharmonic waves
are plane waves

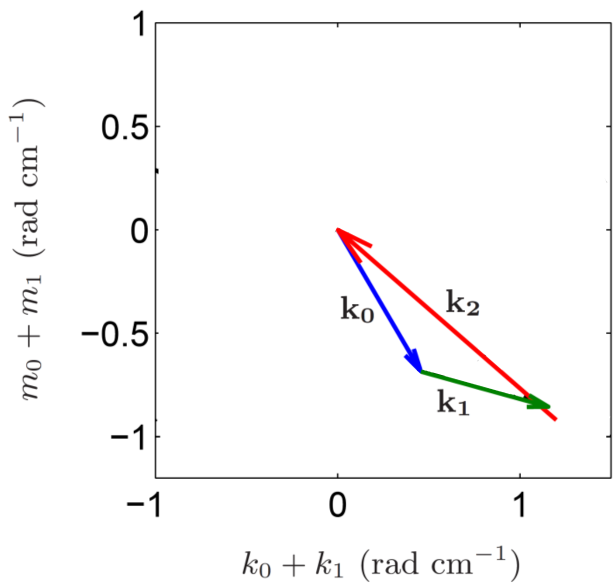
Triadic resonance of plane waves

Resonance condition for a triad of plane waves

$$\sigma_1 + \sigma_2 + \sigma_3 = 0$$

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{0}$$

σ_0/f	$(\sigma_1+\sigma_2)/f$	σ_1/f	σ_2/f
0.64	0.64	0.19	0.45
0.71	0.71	0.21	0.50
0.84	0.84	0.25	0.59
0.91	0.94	0.27	0.67
0.95	0.97	0.29	0.68
0.98	0.98	0.32	0.66
0.99	1.00	0.34	0.66

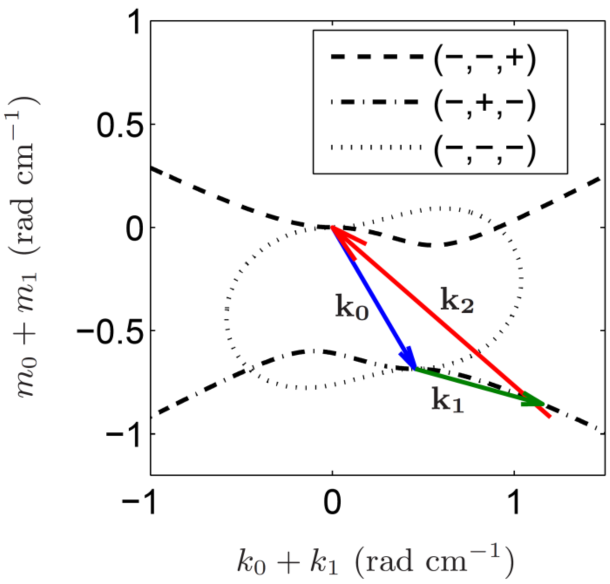


Triadic resonance of plane waves of inertia

Resonance condition + dispersion relation

$$\begin{cases} \sigma_1 + \sigma_2 + \sigma_3 = 0 \\ \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{0} \end{cases} + \quad \frac{\sigma}{2\Omega} = s\frac{m}{\kappa} \qquad \text{avec } \boldsymbol{k} = \binom{k}{m} \qquad \text{et} \qquad s = \pm 1$$

$$\Rightarrow \quad s_0 \frac{m_0}{\sqrt{k_0^2 + m_0^2}} + s_1 \frac{m_1}{\sqrt{k_1^2 + m_1^2}} + \ldots - s_2 \frac{m_0 + m_1}{\sqrt{(k_0 + k_1)^2 + (m_0 + m_1)^2}} = 0.$$



Experimental triad

Decomposition of Navier-Stokes equation in helical modes

Cambon & Jacquin, JFM (1989)

Waleffe, POF (1992)

Helical mode $$\mathbf{u}(\mathbf{x},t) = A(t)\,\mathbf{h_s}\,e^{i(\mathbf{k}\cdot\mathbf{x}-\sigma_s t)} + \text{c.c.}$$

with $$\mathbf{h}_{s_\mathbf{k}}(\mathbf{k}) = \frac{\mathbf{k}}{|\mathbf{k}|} \times \frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|} + i s_\mathbf{k} \frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|}$$

→ Consider a velocity field made of an assembly of helical modes

$$\mathbf{u} = \sum_\mathbf{k} A_\mathbf{k}(\mathbf{k},t)\,\mathbf{h}_s(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x}-\sigma t)}$$

Navier-Stokes equation becomes

$$\left(\frac{\partial}{\partial t} + \nu\kappa^2\right) A_\mathbf{k} = \frac{1}{2}\sum_{\mathbf{p},\mathbf{q}} C_{\mathbf{kpq}} A^*_\mathbf{p} A^*_\mathbf{q} e^{i(\sigma_k+\sigma_p+\sigma_q)t}$$

with $$C_k = C_{\mathbf{kpq}} = [s_q\kappa_q - s_p\kappa_p]\,(\mathbf{h}^*_{s_p}\times\mathbf{h}^*_{s_q})\cdot\mathbf{h}^*_{s_k}/2$$

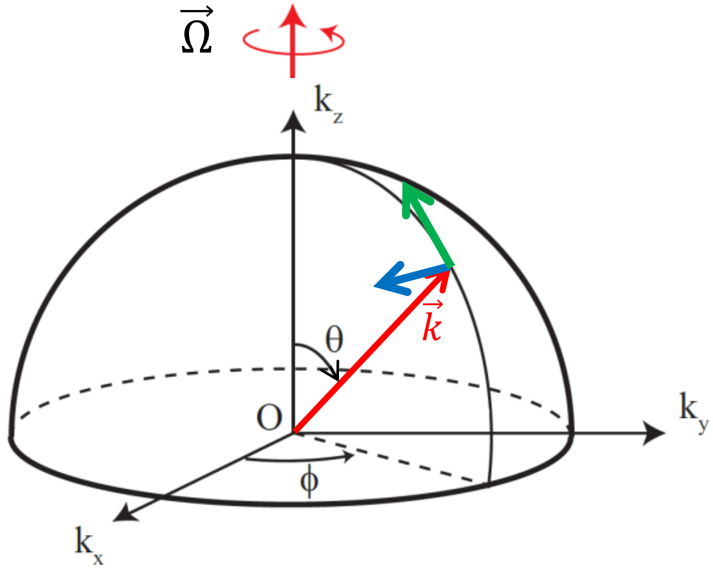
A helical mode = a plane inertial wave

Under rotation, if we take $\mathbf{e_z}$ along $\mathbf{\Omega}$,

$$\mathbf{u}(\mathbf{x},t) = A(t)\,\mathbf{h_s}\,e^{i(\mathbf{k}\cdot\mathbf{x}-\sigma_s t)} + \text{c.c.}$$

with

$$\mathbf{h}_{s_\mathbf{k}}(\mathbf{k}) = \boxed{\frac{\mathbf{k}}{|\mathbf{k}|}\times\frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|}} + \boxed{i s_\mathbf{k}\frac{\mathbf{k}\times\mathbf{e_z}}{|\mathbf{k}\times\mathbf{e_z}|}}$$



describes a plane inertial wave

Resonant triad of plane inertial waves

Restricting to 3 inertial waves

with initially $A_0 \neq 0$ et $A_{1,2} = 0$

$$\left(\frac{\partial}{\partial t} + \nu\kappa^2\right) A_{\mathbf{k}} = C_{\mathbf{k}} A^*_{\mathbf{p}} A^*_{\mathbf{q}} e^{i(\sigma_{\mathbf{k}}+\sigma_{\mathbf{p}}+\sigma_{\mathbf{q}})t}$$

(i) The interaction $(\mathbf{k}, \mathbf{p}, \mathbf{q})$ possible for $\mathbf{k} + \mathbf{p} + \mathbf{q} = \mathbf{0}$

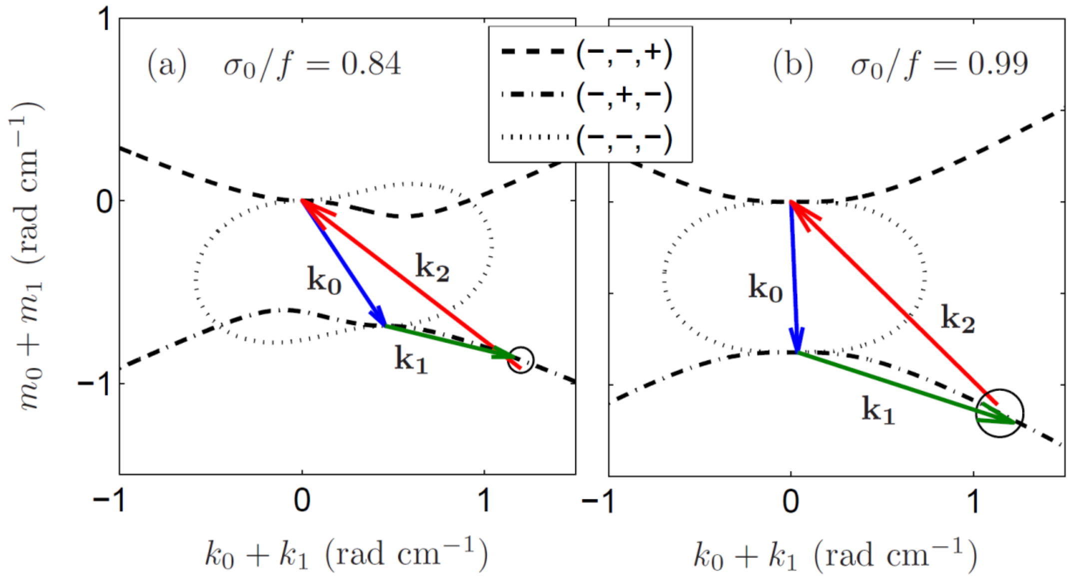
(ii) For $A_{1,2}(t)$ to grow, $\sigma_1 + \sigma_2 + \sigma_3 = 0$

$$\Rightarrow \begin{cases} \dfrac{dA_1}{dt} = C_1 A_0^* A_2^* - \nu\kappa_1^2 A_1 \\ \dfrac{dA_2}{dt} = C_2 A_0^* A_1^* - \nu\kappa_2^2 A_2 \end{cases} \Rightarrow \quad A_{1,2}(t) = B_{1,2}\left(e^{\gamma_+ t} - e^{\gamma_- t}\right)$$

with the growth rate

$$\gamma_\pm = -\frac{\nu}{2}(\kappa_1^2 + \kappa_2^2) \pm \sqrt{\frac{\nu^2}{4}(\kappa_1^2 - \kappa_2^2)^2 + C_1 C_2 |A_0|^2}$$

Most instable triad of inertial waves



Energy transfers toward modes with more horizontal wavevectors $\frac{k_\parallel}{k_\perp} \to 0$

→ **Two-dimensionalisation mecanism for weakly non-linear interaction**

Resonant triad of plane inertial waves

$$\left(\frac{\partial}{\partial t} + \nu\kappa^2\right) A_{\mathbf{k}} = C_{\mathbf{k}} A^*_{\mathbf{p}} A^*_{\mathbf{q}} e^{i(\sigma_{\mathbf{k}}+\sigma_{\mathbf{p}}+\sigma_{\mathbf{q}})t}$$

(i) The interaction $(\mathbf{k},\mathbf{p},\mathbf{q})$ possible for $\mathbf{k}+\mathbf{p}+\mathbf{q}=\mathbf{0}$

(ii) For $A_{1,2}(t)$ to grow, $\sigma_1+\sigma_2+\sigma_3=0$

$$\Rightarrow \begin{cases} \dfrac{dA_1}{dt} = C_1 A_0^* A_2^* - \nu\kappa_1^2 A_1 \\ \dfrac{dA_2}{dt} = C_2 A_0^* A_1^* - \nu\kappa_2^2 A_2 \end{cases}$$

This framework makes sense if one have a separation of timescales

$$\gamma \ll \sigma_{1,2,3}$$

Since $\gamma \sim \kappa A \;\; \rightarrow \;\; Ro \ll Ro_t$

Moreover, in order to have a good phase mixing, the resonance needs to be precise to

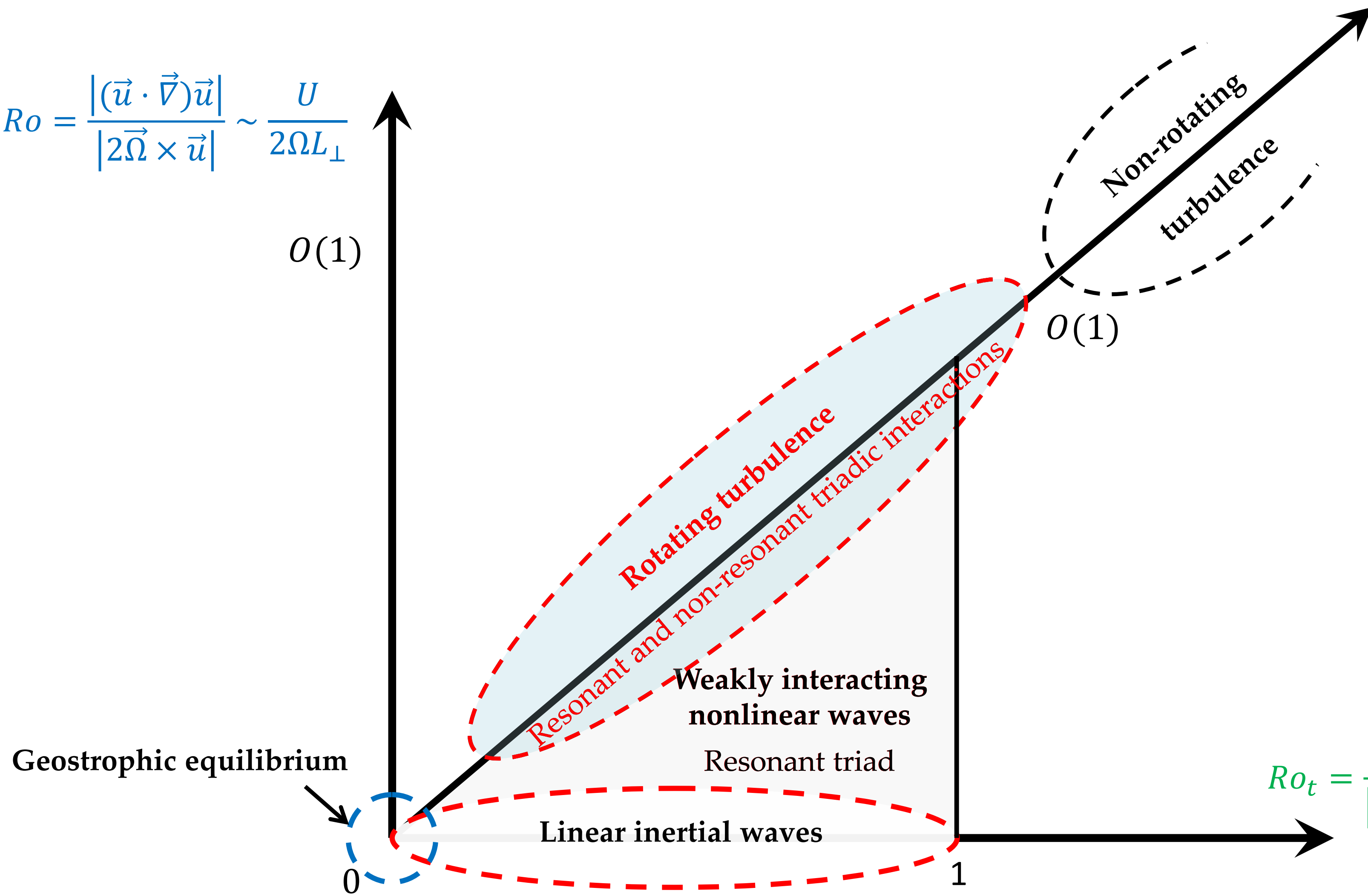
$$\sigma_1+\sigma_2+\sigma_3 \ll \gamma = \Omega\, Ro$$

→ When $Ro \nearrow$ more non-resonant triads are allowed to exchange energy

Hydrodynamics under rotation

$$Ro_t \frac{\partial \vec{u}}{\partial t} + Ro(\vec{u} \cdot \vec{\nabla})\vec{u} = -\vec{\nabla} p - \vec{e_z} \times \vec{u} + Ro\, Re^{-1} \Delta \vec{u}$$

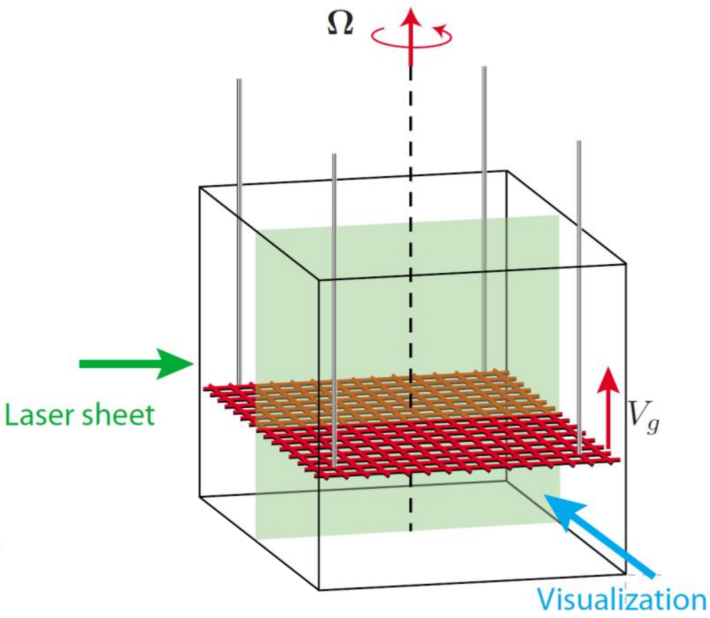
$$Ro = \frac{|(\vec{u} \cdot \vec{\nabla})\vec{u}|}{|2\vec{\Omega} \times \vec{u}|} \sim \frac{U}{2\Omega L_\perp}$$

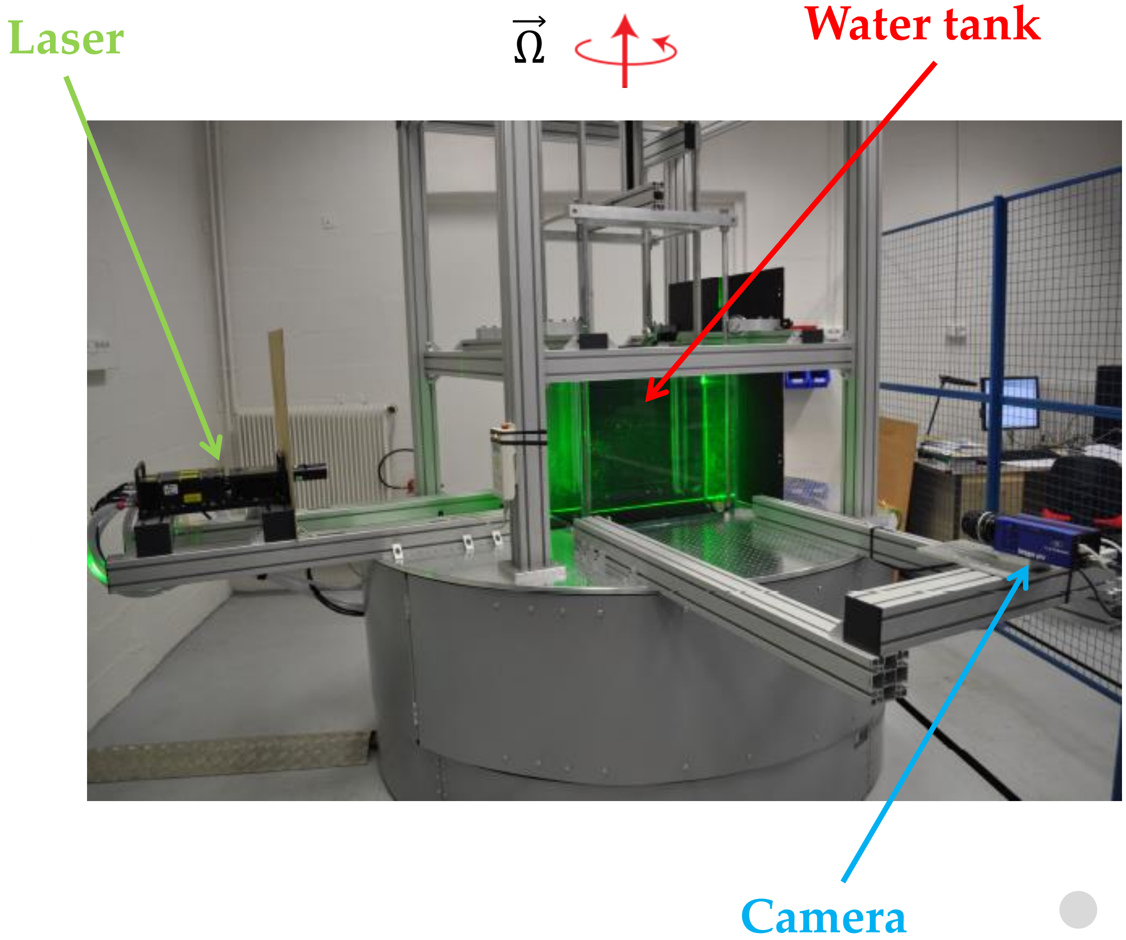


$$Ro_t = \frac{|\partial \vec{u}/\partial t|}{|2\vec{\Omega} \times \vec{u}|} \sim \frac{\sigma}{2\Omega}$$

with F. Moisy
C. Lamriben

Lamriben, Cortet & Moisy, PRL (2011)





Turbulence decay, $Ro(t)$

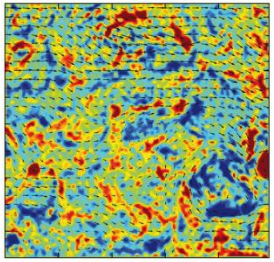
600 decays of the turbulence for each of the studied rotation rates

$$Re(t) = 1000 \rightarrow 200$$

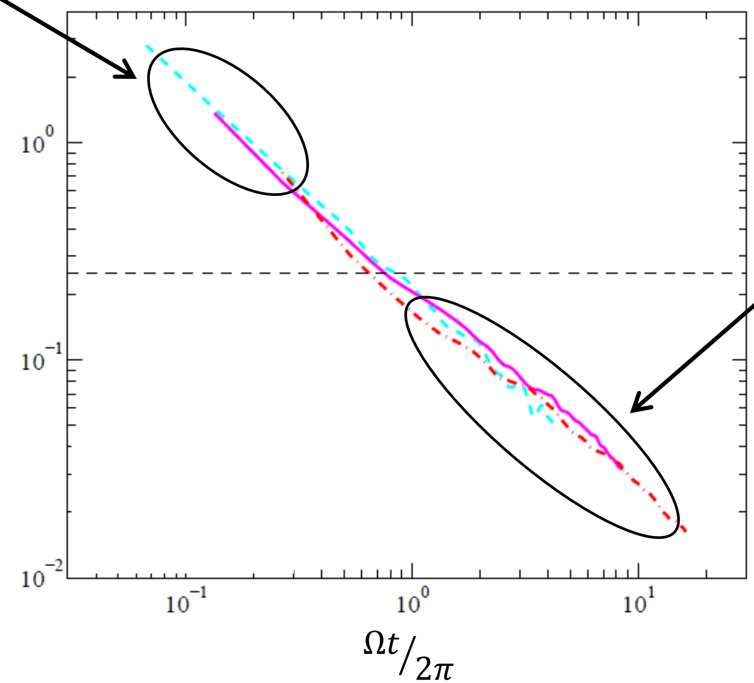
$Ro \gtrsim 1$

Rotation does not matter

$$Ro(t) = \frac{u'_x}{2\Omega L_\perp} \quad \text{with } u'_x = \sqrt{\langle u_x(t)^2 \rangle}$$

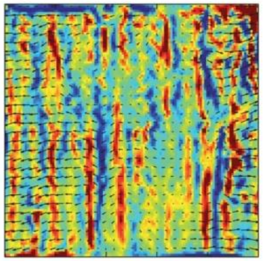


Isotropic



$Ro \lesssim 1$

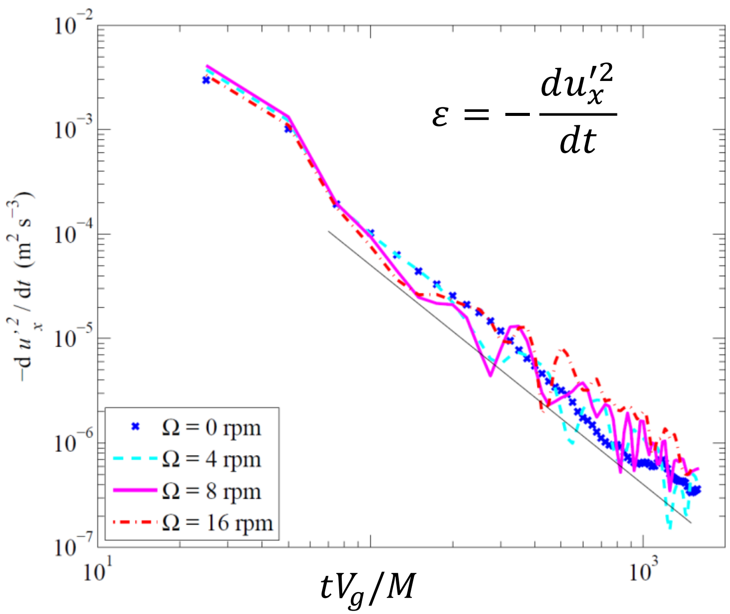
Rotation is in action



Anisotropic

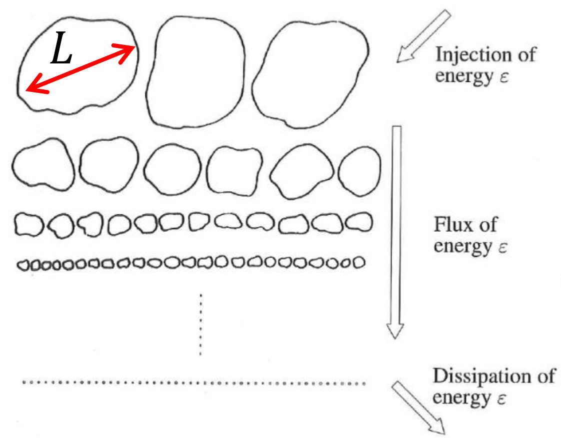
Invariance along z

Turbulence decay, Energy dissipation rate

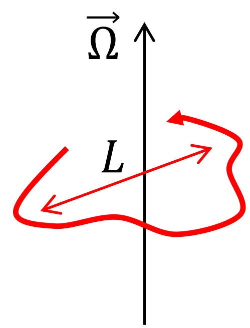


$$\varepsilon = \frac{u_x'^2}{\tau_{tr}} = \nu_T \frac{u_x'^2}{L^2}$$

- without rotation $\varepsilon_{3D} = \frac{u_x'^3}{L}$ $\quad \begin{cases} \boldsymbol{\nu_T = u_x' L} \\ \tau_{tr} = \tau_{nl} = \frac{L}{u_x'} \end{cases}$



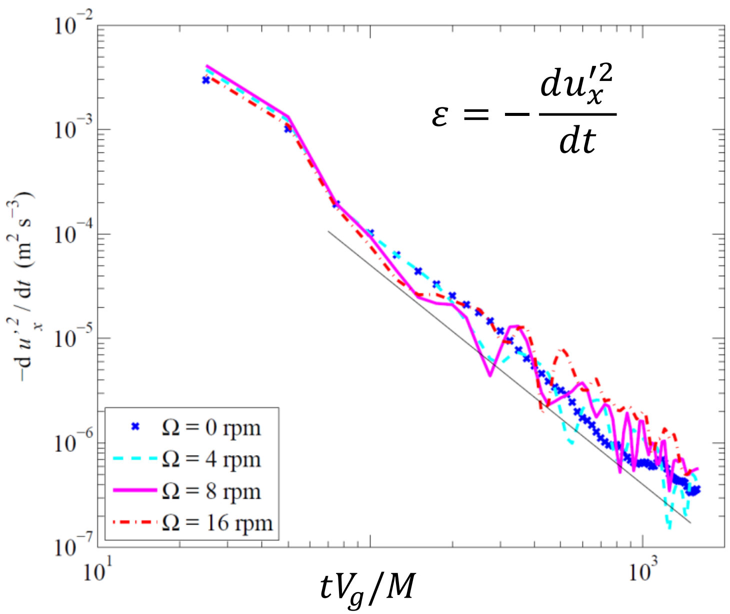
Radius of rotation of inertial wave $L \sim \frac{u}{\Omega}$ $\quad \begin{cases} \boldsymbol{\nu_T = u_x' \frac{u_x'}{\Omega}} \\ \tau_{tr} = \Omega\, \tau_{nl}^2 \end{cases}$



- with rotation $\boldsymbol{\varepsilon_{rot} = \frac{u_x'^3}{L_\perp} Ro = \varepsilon_{3D}\, Ro}$

Jacquin, Leuchter, Cambon & Mathieu, JFM (1990)
Squires, Chasnov, Mansour & Cambon (1994)
Zhou, POF (1995)

Turbulence decay, Energy dissipation rate



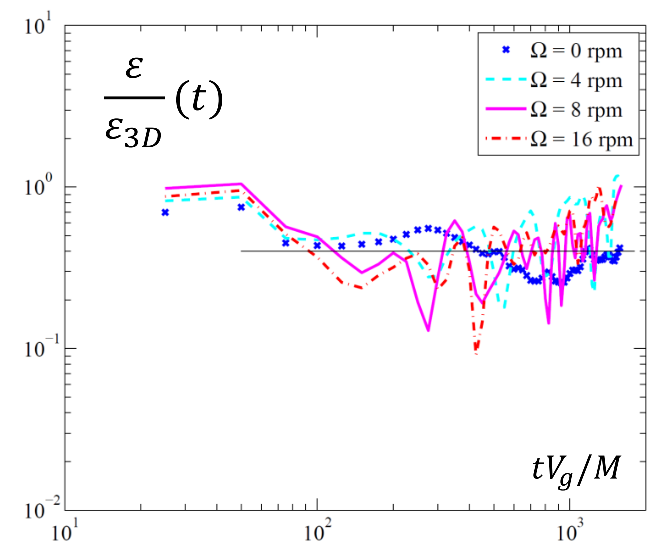
$$\varepsilon = \frac{u'^2_x}{\tau_{tr}} = \nu_T \frac{u'^2_x}{L^2}$$

- without rotation $\varepsilon_{3D} = \dfrac{u'^3_x}{L}$
- with rotation $\boldsymbol{\varepsilon_{rot} = \dfrac{u'^3_x}{L_\perp} Ro = \varepsilon_{3D}\, Ro}$

ε is not affected by rotation

→ Large scales are apparently not affected by rotation

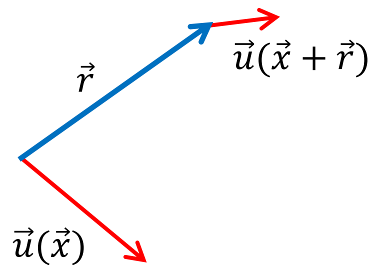
Which scales affected?

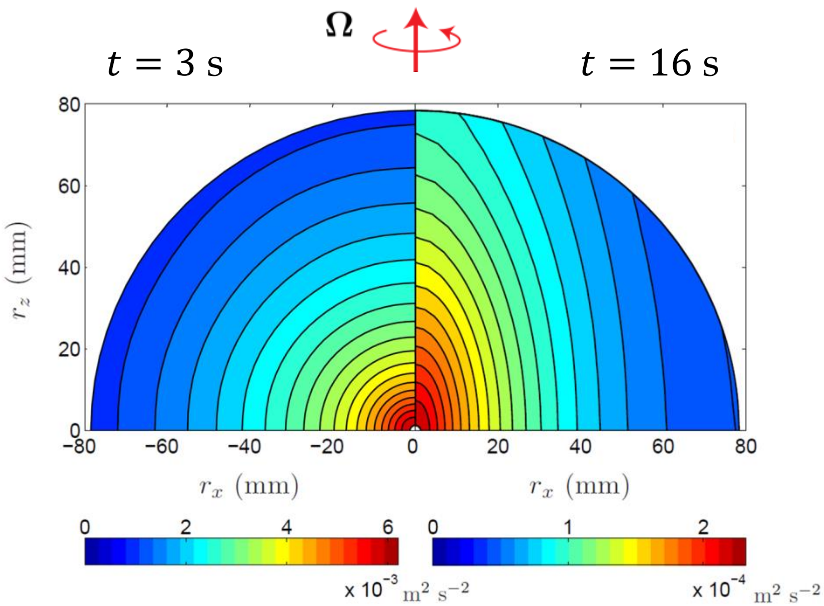


Two points velocity correlation = « Energy distribution in the space of scales »

$$R(\vec{r},t) = \langle \vec{u}(\vec{x}+\vec{r}) \cdot \vec{u}(\vec{x}) \rangle$$

where $\langle\ \rangle$ stands for spatial (over $\vec{x}$) and ensemble average





→ Isotropy

→ Toward an anisotropic 2D state

How this anisotropy of $R(\vec{r},t)$ is built by energy transfers?

How it is distributed among scales?

Kármán-Howarth-Monin equation

Conservation of $R(\vec{r},t)$ in the space of scales

$$< \mathrm{NS}\big(u_i(\vec{x}+\vec{r})\big)\mathrm{NS}\big(u_i(\vec{x})\big) > \;\rightarrow\; \frac{1}{2}\frac{\partial R}{\partial t} = \underbrace{\frac{1}{4}\vec{\nabla}\cdot\vec{F}}_{\Pi(\vec{r},t)} + \underbrace{\nu\Delta R}_{-D(\vec{r},t)}$$

Rotation has disappeared

$R(\vec{r},t) = \langle \vec{u}(\vec{x}+\vec{r})\cdot\vec{u}(\vec{x})\rangle$

« Energy distribution »

$\vec{F}(\vec{r},t) = \langle(\delta\vec{u})^2\delta\vec{u}\rangle$

« Energy flux density »

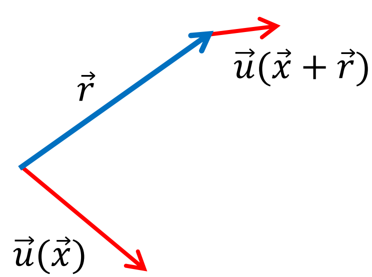
with $\delta\vec{u}(\vec{x},\vec{r}) = \vec{u}(\vec{x}+\vec{r}) - \vec{u}(\vec{x})$ the velocity increment

3D isotropic turbulence

$$\Pi(r) = -\varepsilon$$

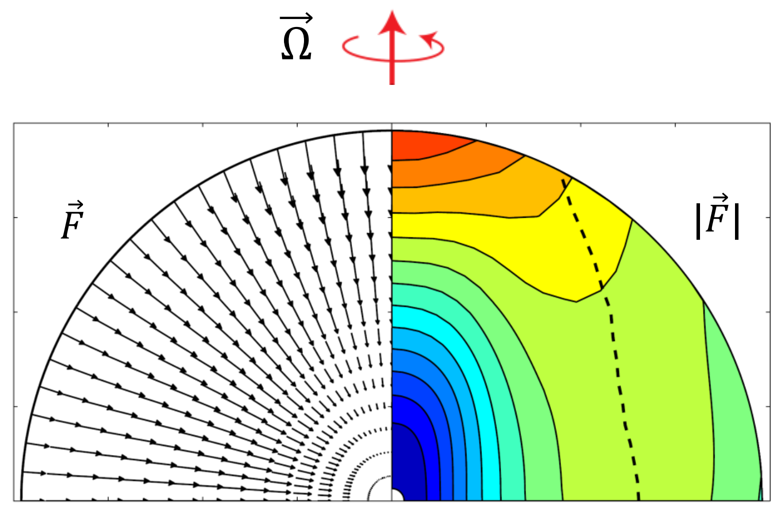
equivalent to $\langle(\delta u_L)^3\rangle = -\frac{4}{5}\varepsilon r$

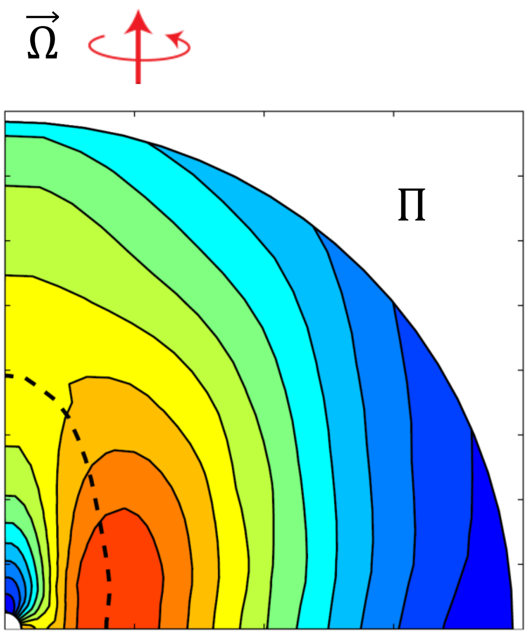
(Kolmogorov 1941, Monin & Yaglom 1975)



KHM equation with rotation

$$\frac{1}{2}\frac{\partial R}{\partial t} = \underbrace{\frac{1}{4}\vec{\nabla}\cdot\vec{F}}_{\Pi(\vec{r},t)} + \underbrace{\nu\Delta R}_{-D(\vec{r},t)}$$

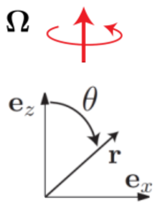




→ Strong anisotropy of the energy transfers $\Pi(\vec{r},t)$ between spatial scales

→ $\vec{F}$ remains nearly radial but is dependant on the orientation of $\vec{r}$

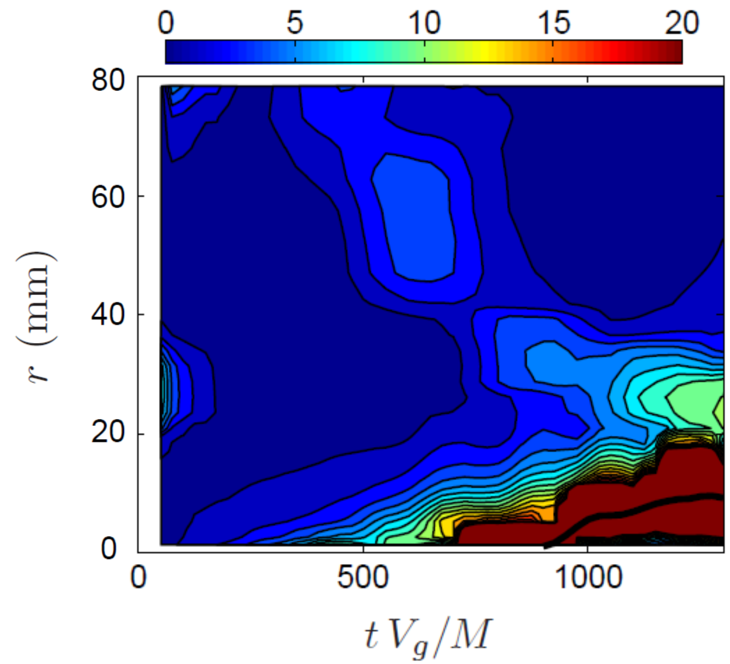
Scale dependent anisotropy of energy transfers



Horizontal to vertical ratio

of energy flux

$$a_{\Pi}(r,t) = \frac{\Pi(r,\theta=\pi/2,t)}{\Pi(r,\theta=0,t)}$$



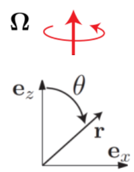
The anisotropy of energy transfers $\Pi(\vec{r},t)$ is stronger at small scales, i.e. decreases with r

→ Consistent with the independence of ε with Ω.

Scale dependent anisotropy of energy distribution

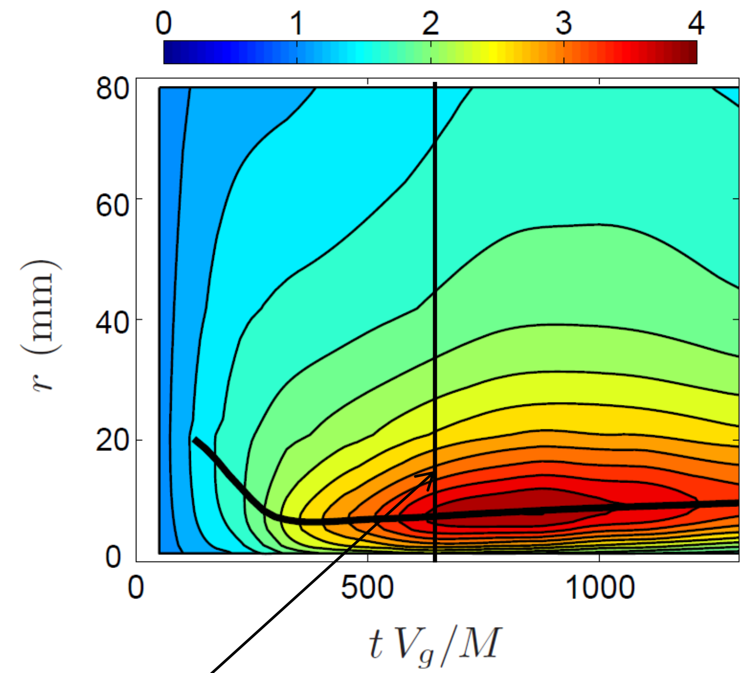
Horizontal to vertical ratio

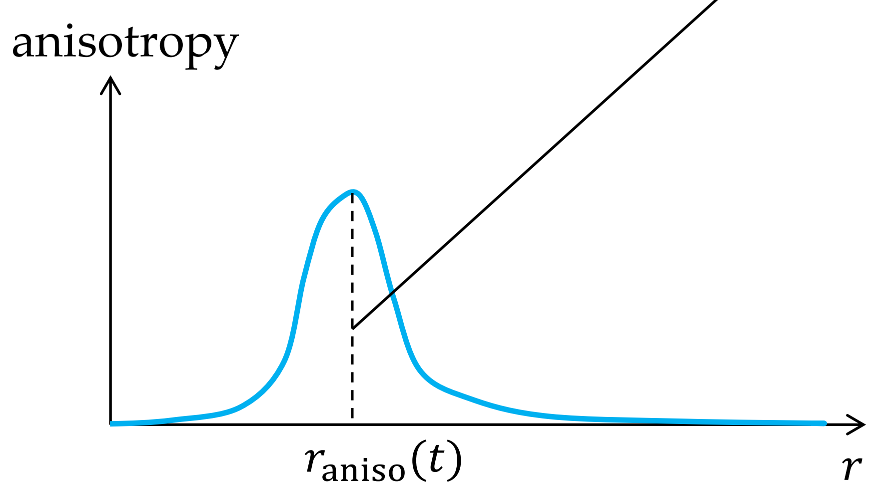
of energy density



$$a_E(r,t) = \frac{E(r,\theta=\pi/2,t)}{E(r,\theta=0,t)}$$

with $E(\vec{r},t) = 2\langle\vec{u}^2\rangle - 2R(\vec{r},t)$



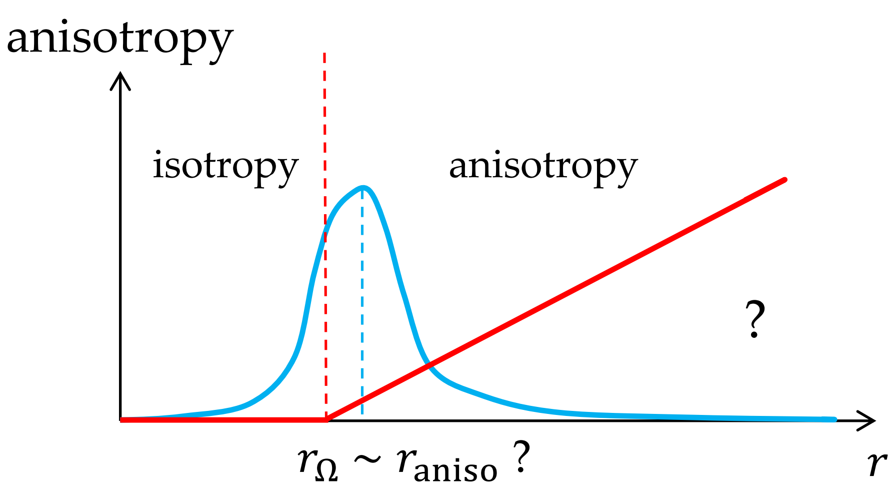


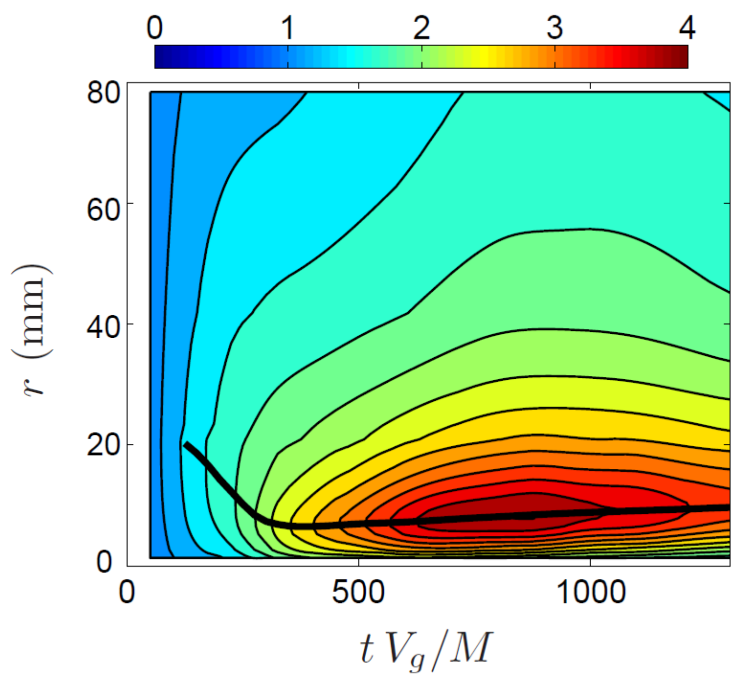
Scale dependent anisotropy of energy distribution

Zeman, POF (1994)

$$R_O(r) \sim \frac{u(r)}{\Omega\, r} \sim \left(\frac{r_\Omega}{r}\right)^{\frac{2}{3}} \quad \text{with } r_\Omega = \left(\frac{\varepsilon}{\Omega^3}\right)^{1/2} \quad \text{the Zeman scale}$$

$r < r_\Omega$ are not affected by rotation → isotropic

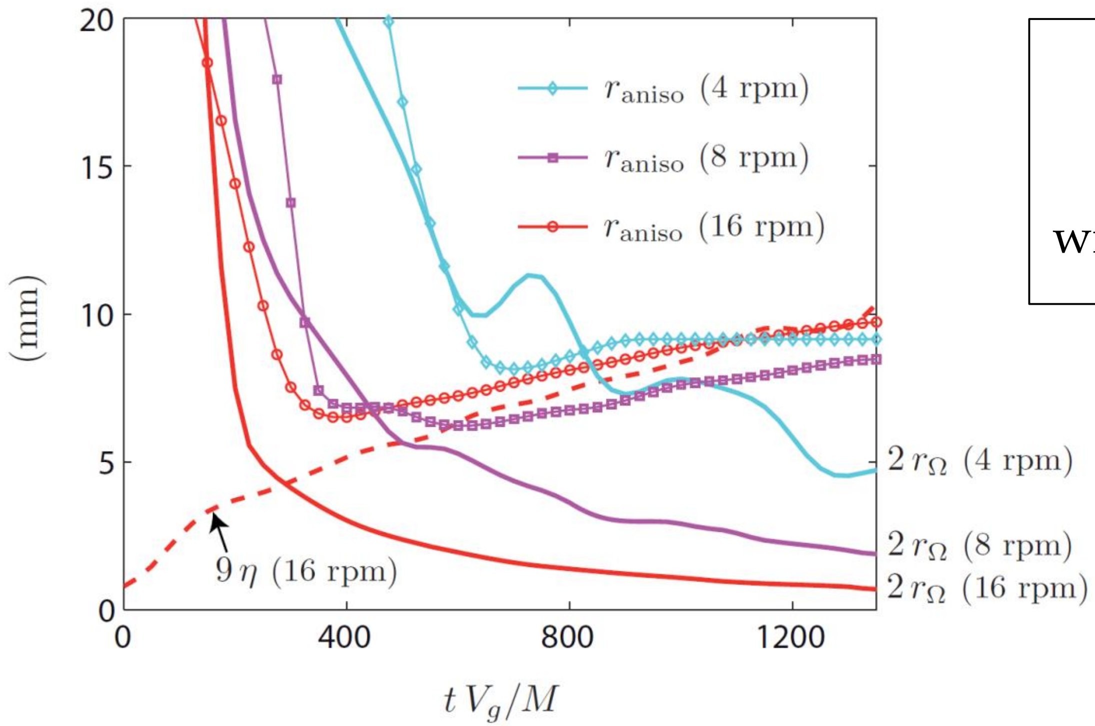




→ The Zeman scale r_Ω is maybe a good description of the scale r_{aniso}.

→ But, the Zeman Rossby number seems not relevant for $r > r_\Omega$.

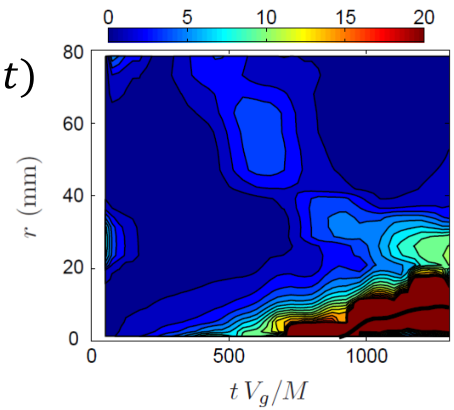
The scale of maximum anisotropy



→ $r_{\mathrm{aniso}} = \max\{2r_\Omega, 9\eta\}$

with $r_\Omega = \left(\frac{\varepsilon}{\Omega^3}\right)^{1/2}$ and $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$

$a_\Pi(\vec{r}, t)$



$$\frac{1}{2}\frac{\partial R}{\partial t} = \underbrace{\frac{1}{4}\vec{\nabla}\cdot\vec{F}}_{\Pi(\vec{r},t)} + \underbrace{\nu\Delta R}_{-D(\vec{r},t)}$$

Energy transfers $\Pi(\vec{r}, t)$ are more anisotropic at small scales but for $\mathrm{r} < \eta$ they are no more at play → $r_{\mathrm{aniso}} \sim \eta$

A new Rossby number above the Zeman scale ?

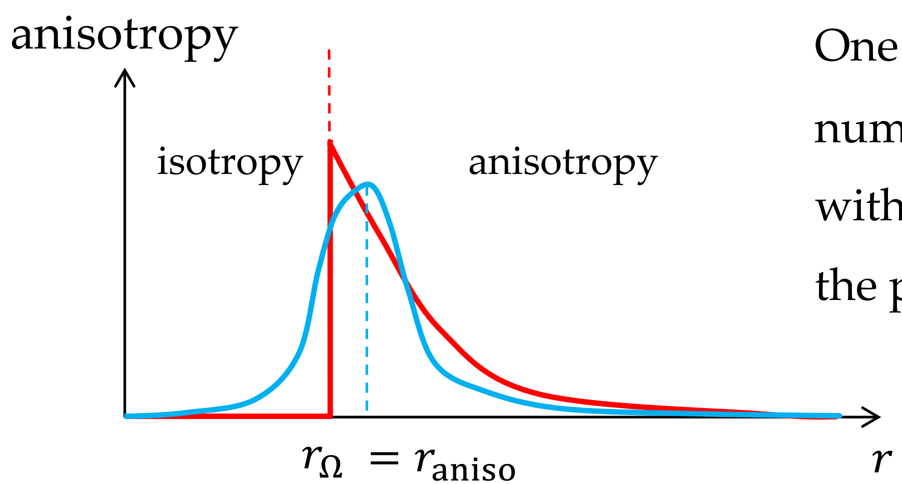
→ For $r < r_\Omega$, the flow is not affected by rotation.

→ For $r > r_\Omega$, the Zeman Rossby number $R_O(r) \sim \left(\frac{r_\Omega}{r}\right)^{\frac{2}{3}}$ fails to interpret data.

However, it is based on 3D isotropic scalings!!

→ One needs to build a Rossby number increasing with r, in which case:

The larger the scale, the larger the Rossby number, the less affected by rotation



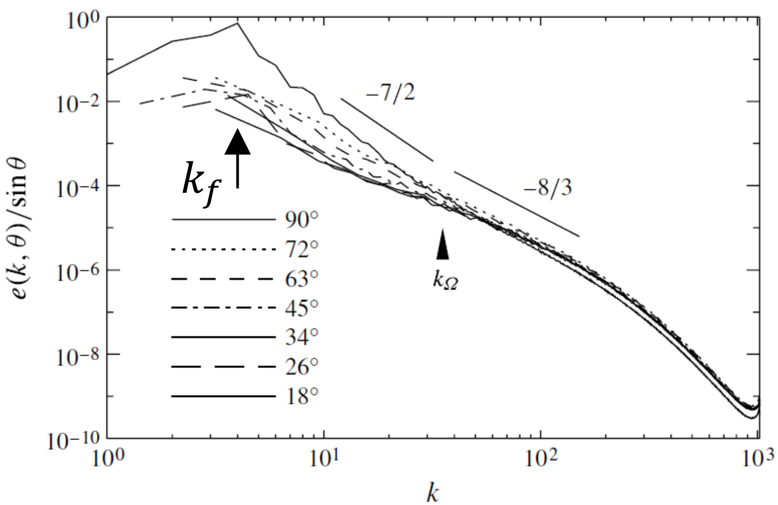
One can note that scale dependant Rossby numbers $R_O(r) \sim \frac{\sigma(r)}{2\Omega}$ associate a frequency $\sigma(r)$ with a scale r, which is highly incompatible with the physics of inertial waves!

Two-dimensionalisation of turbulence by rotation

Mininni, Rosenberg & Pouquet, JFM (2012)

DNS of forced turbulence under rotation with helical forcing

$Re = 3 \times 10^4$ and $Ro = 0{,}07$

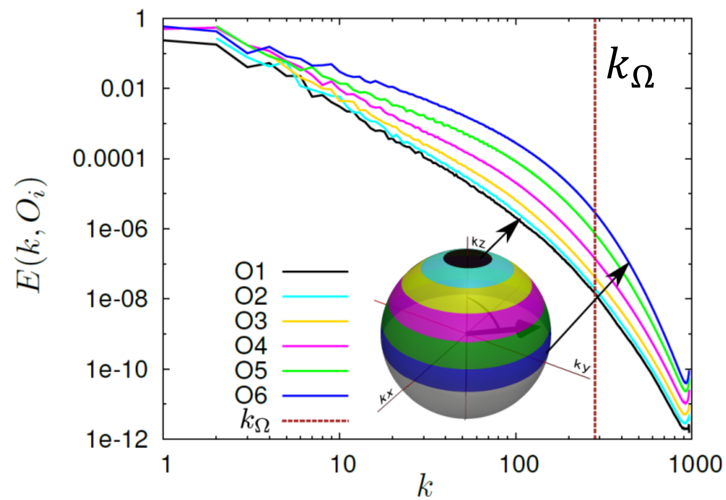


Delache, Cambon & Godeferd, POF (2014)

DNS of the decay of turbulence under rotation

Initially isotropic and non-helical

$Re = 5 \times 10^3$ and $Ro = 0{,}02$



Results are contrasted

Anisotropy of rotating turbulence

The scale dependance of the anisotropy of rotating turbulence is still not fully understood. It probably depends on

→ the helicity of the turbulence,

→ the isotropy of the forcing or of the initial turbulence,

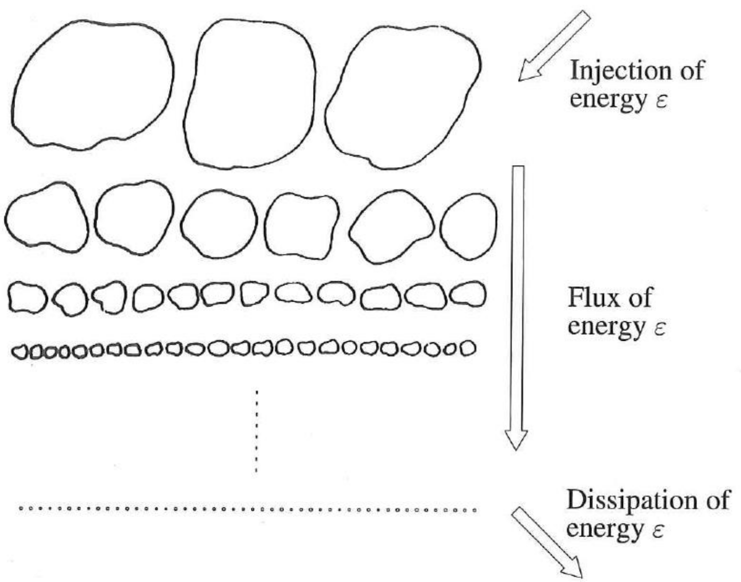
→ the nature of the forcing (monochromatic or broadband).

→ Talk given by Pablo Mininni tomorrow 8h45 !

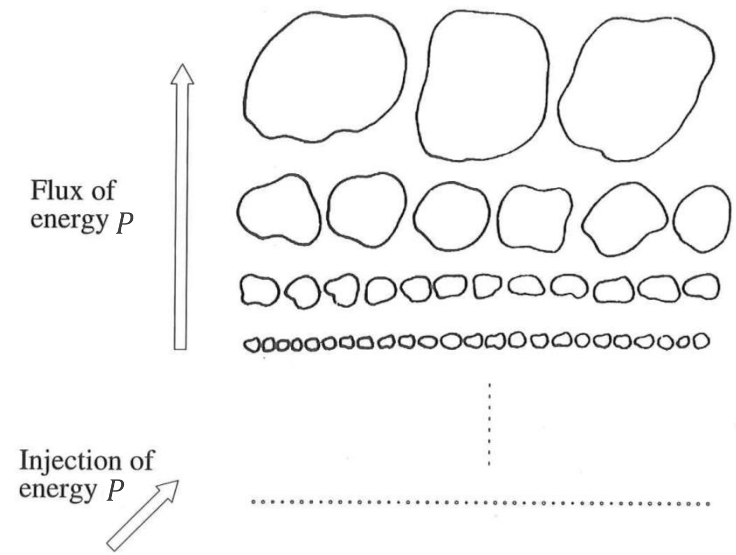
3D *vs.* 2D turbulence

Rotating turbulence tends to be 2D but is not exactly.

3D turbulence
Direct cascade of energy



2D turbulence
Inverse cascade of energy



What about the energy transfers in the horizontal plane $\perp \vec{\Omega}$?

An inverse cascade of energy?

Yarom, Vardi, Sharon, POF (2013)

Transient of a rotating turbulence experiment forced with an array of jets

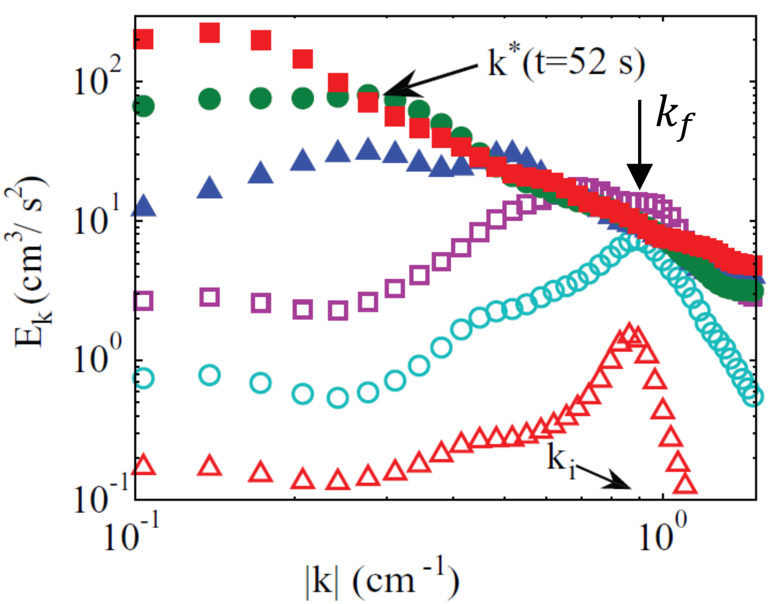
Turbulence is both unstationnary and inhomogeneous

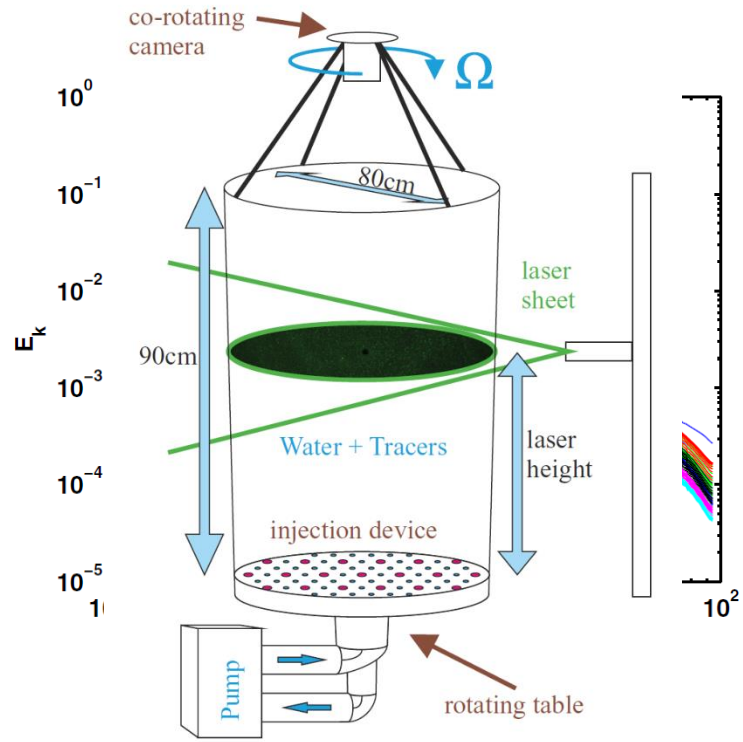
We are not sure where the energy comes from

Fluxes in scales or space?

Pouquet, Sen, Rosenberg, Mininni & Baerenzung, Phys. Scripta (2013)

Transient of a rotating LES forced at k_f





Kármán-Howarth-Monin equation

$$E(\vec{r},t) = \langle(\delta\vec{u})^2\rangle = \langle(\vec{u})^2\rangle - 2R$$

$$\Pi(\vec{r},t) = \frac{1}{4}\vec{\nabla}\cdot\langle(\delta\vec{u})^2\delta\vec{u}\rangle$$

$$\varepsilon = \frac{1}{2}\nu\langle\left(\partial_i u_j + \partial_j u_i\right)^2\rangle$$

$$\boxed{P_r = \langle\delta\vec{u}\cdot\delta\vec{f}\rangle/2 \sim P \text{ for } r > L_{\mathrm{inj}}}$$

$$\delta\vec{u}(\vec{x},\vec{r}) = \vec{u}(\vec{x}+\vec{r}) - \vec{u}(\vec{x})$$

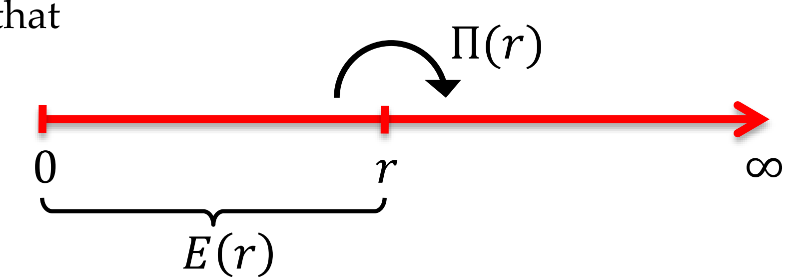
$$\boxed{\frac{1}{4}\frac{\partial E(\vec{r})}{\partial t} = -\Pi(\vec{r}) + \frac{1}{2}\nu\Delta E - \varepsilon + \boxed{P_r}}$$

For theory and numerics only

In the isotropic case (2D or 3D), an angular average gives that

→ $E(r)$ is the cumulated energy from 0 to r

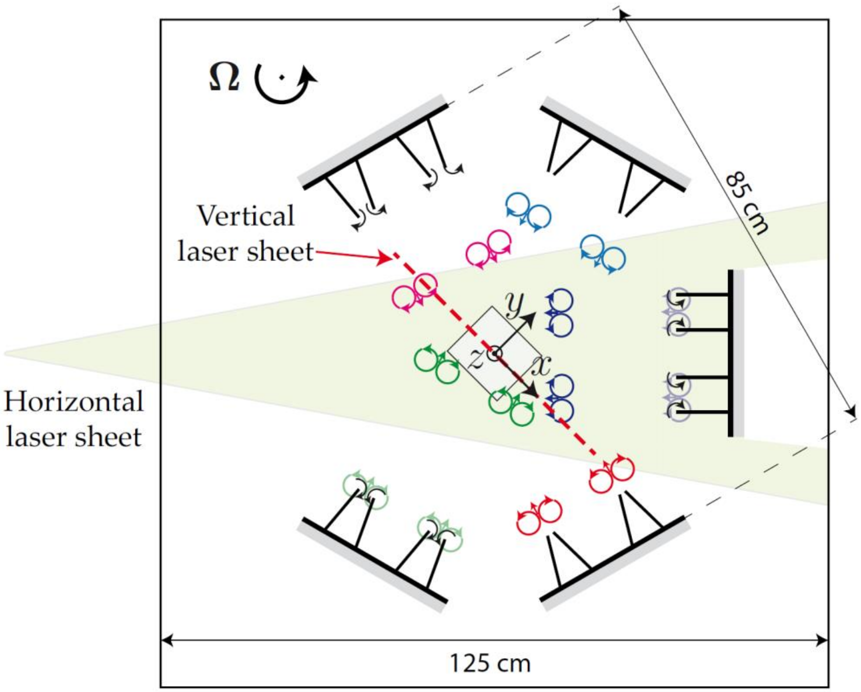
→ $\Pi(r_0)$ is the energy transferred from $r < r_0$ to $r > r_0$

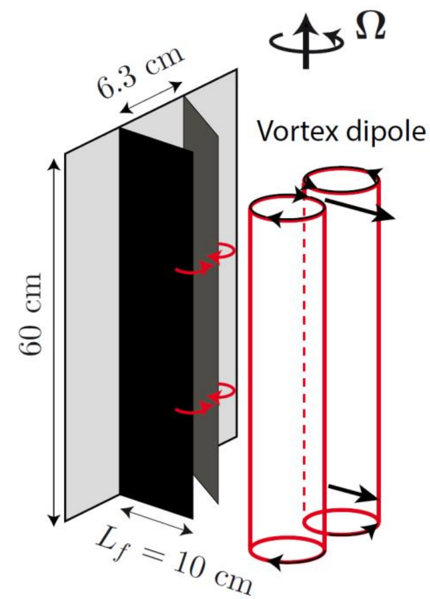


3D turbulence	2D turbulence
$\Pi(r) = -\varepsilon < 0$	$\Pi(r) = +P > 0$
equivalent to $\langle(\delta u_L)^3\rangle = -\frac{4}{5}\varepsilon r$	equivalent to $\langle(\delta u_L)^3\rangle = +\frac{3}{2}Pr$
(Kolmogorov 1941, Monin & Yaglom 1975)	(Kraichnan 1971, Bernard 1999, Lindborg 1999)
Forward cascade	Inverse cascade

A stationary turbulence under rotation

An arena of vortex dipole generators



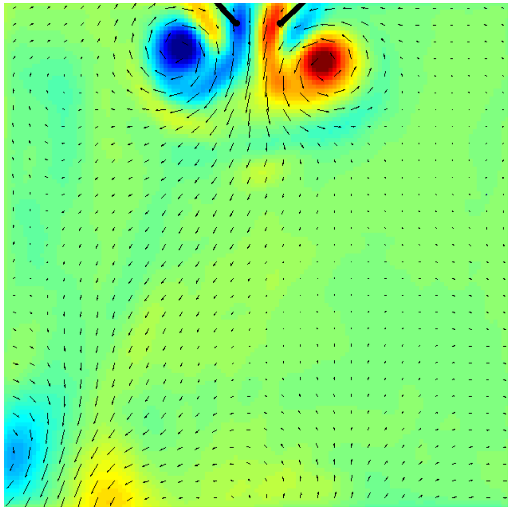


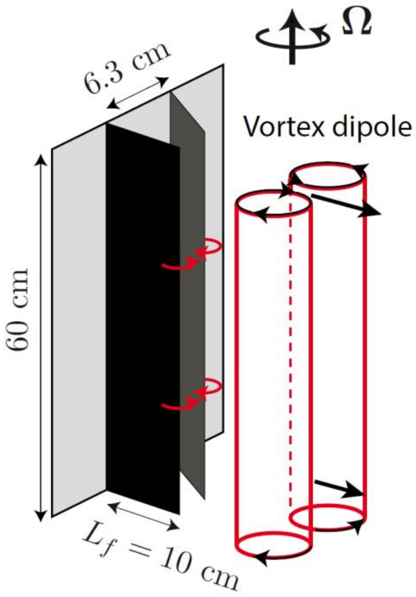
with A. Campagne, B. Gallet, F. Moisy

Generators designed at LadHyX, Ecole Polytechnique
by P. Augier, P. Billant, J.-M. Chomaz

A stationary turbulence under rotation

A vortex dipole flap generator

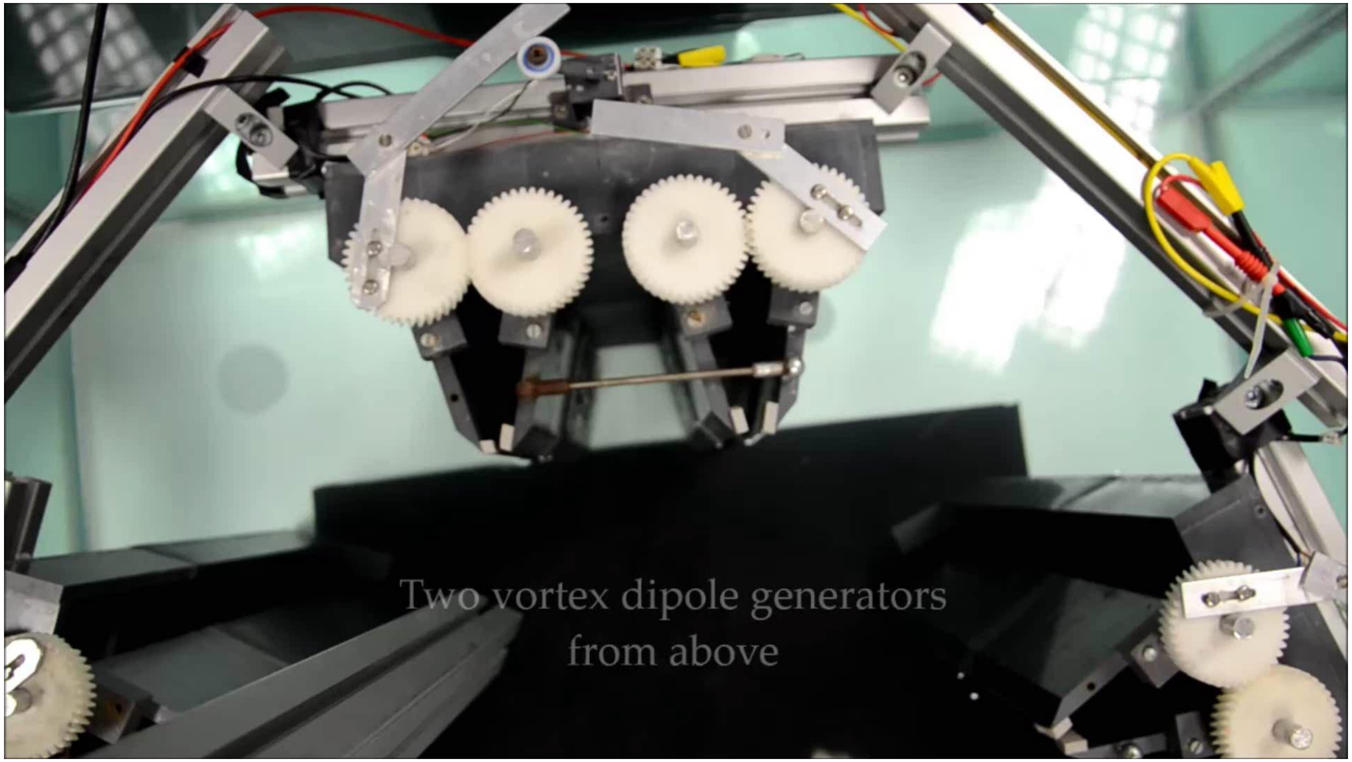




$$Re_f = \frac{\omega_f L_f^2}{\nu} \simeq 1000 \text{ and } Ro_f = \frac{\omega_f}{2\Omega} \simeq [0.03; 0.22]$$

The large scale vortex dipole produces and carries
3D turbulent fluctuations towards the center of the arena

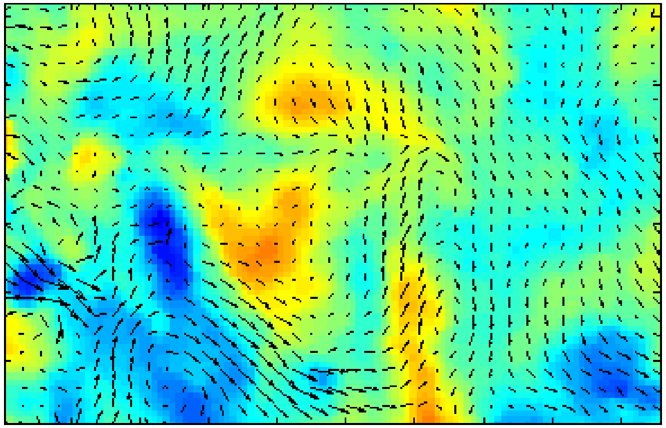
A stationary turbulence under rotation



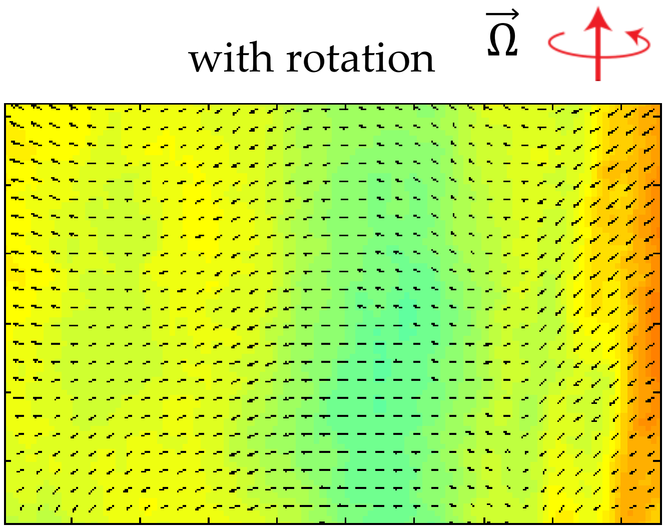
Stereoscopic PIV

Flow in the vertical plane

without rotation

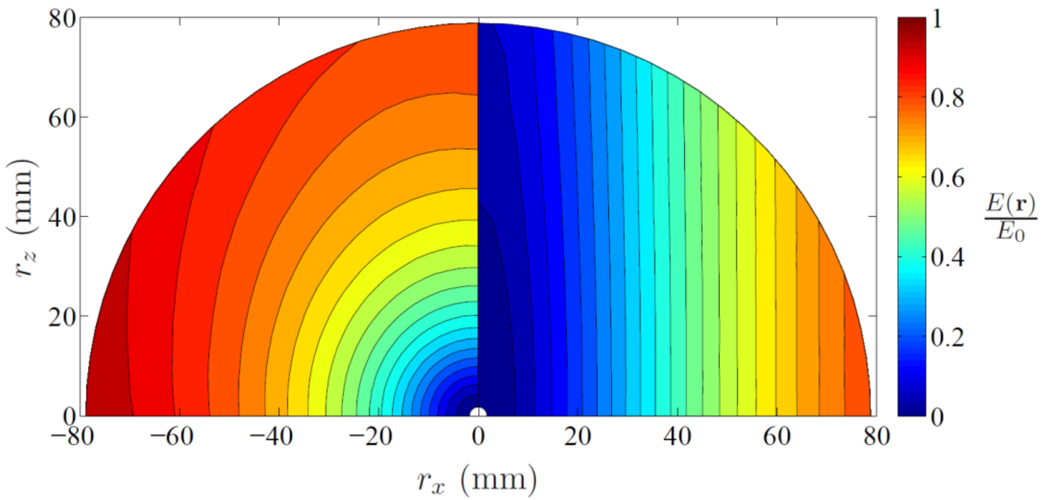


with rotation



$E(\vec{r},t) = \langle(\delta\vec{u})^2\rangle$

without rotation with rotation



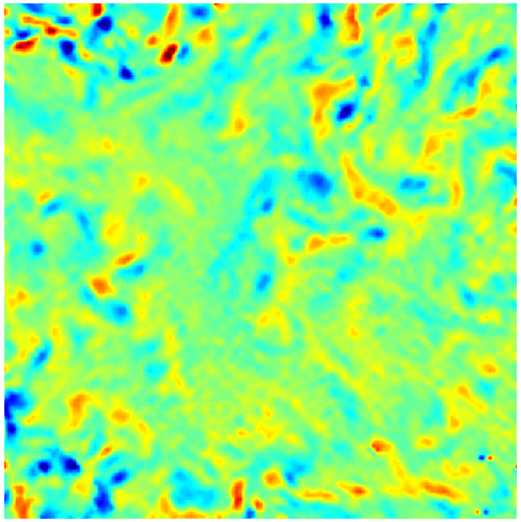
$Re \simeq 500$

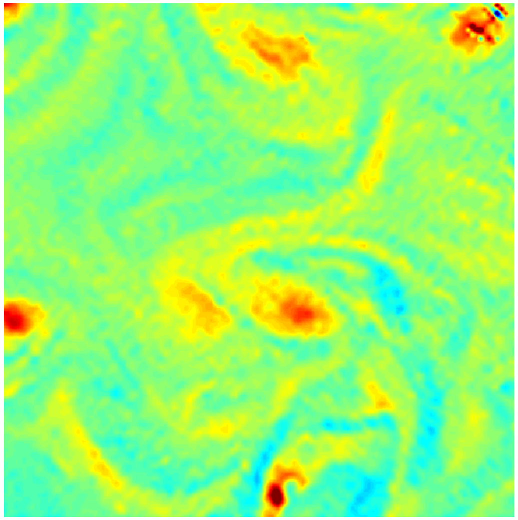
$Ro \in [0.04\ 0.15]$

Strong enhancement of the two-dimensionality by rotation

Flow in the horizontal plane

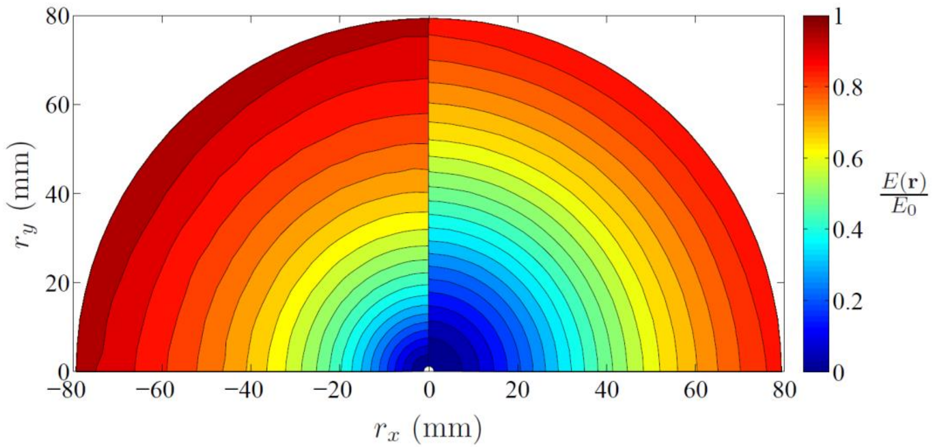
without rotation — Vertical vorticity ω_z — with rotation





Ω

without rotation — with rotation



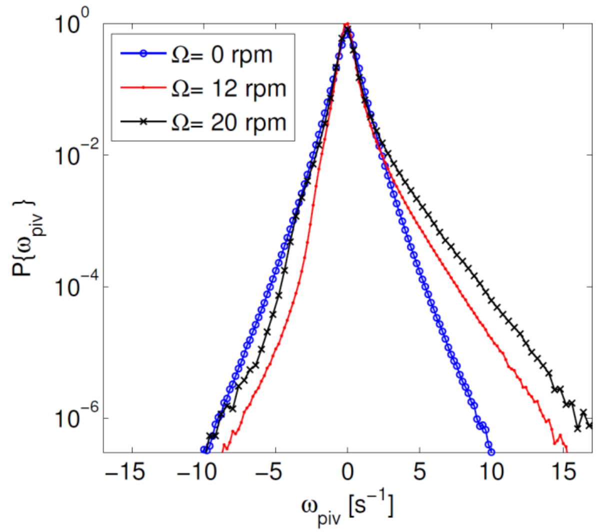
→ Larger structures

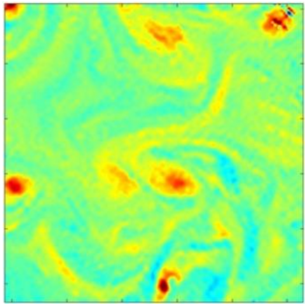
→ Merging of vortices

Signature of an inverse cascade

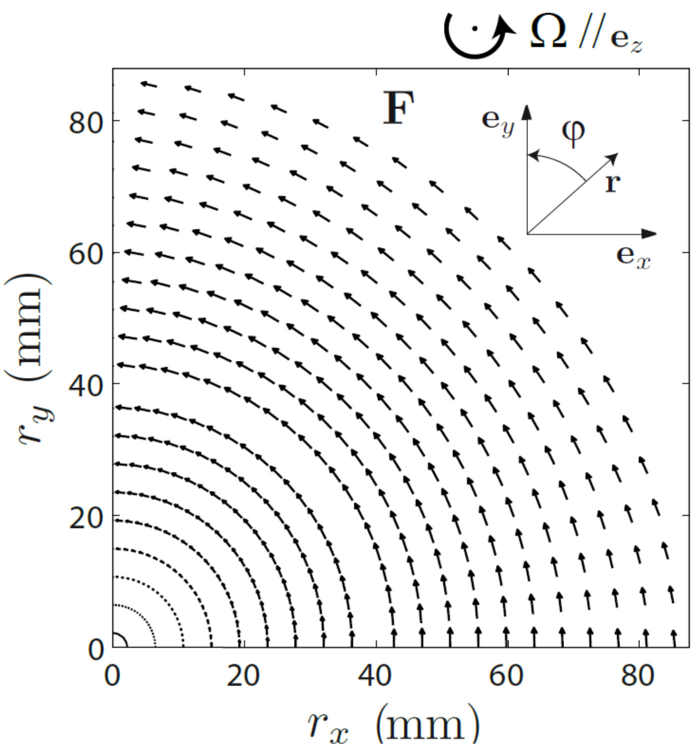
Good axisymmetry of the turbulence for all Ω

Energy flux density $\vec{F} = \langle\overline{(\delta\vec{u})^2\delta\vec{u}}\rangle$





Gallet, Campagne, Cortet & Moisy, POF (2014)



The energy flux density is azimuthal because of the cyclone/anticyclone asymmetry

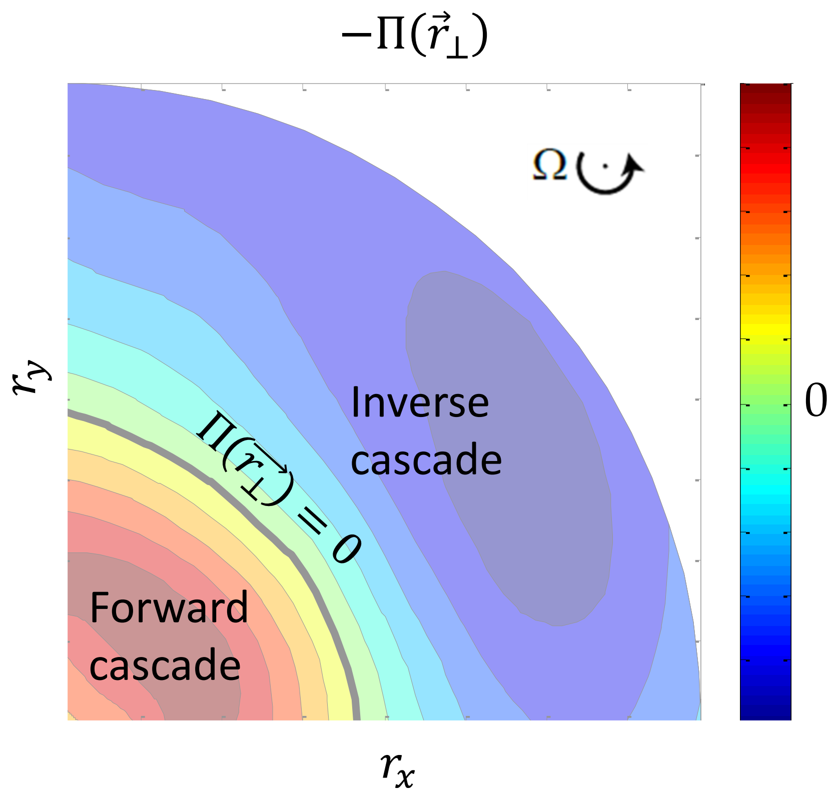
However, it does not matter for energy transfers because of axisymmetry

$$4\Pi(\vec{r}) = \vec{\nabla}\cdot\vec{F} \simeq \frac{1}{r_\perp}\partial_{r_\perp} r_\perp F_{r_\perp} + \partial_{r_\parallel} F_{r_\parallel}$$

$$\text{since } \frac{1}{r}\partial_\varphi F_\varphi \simeq 0$$

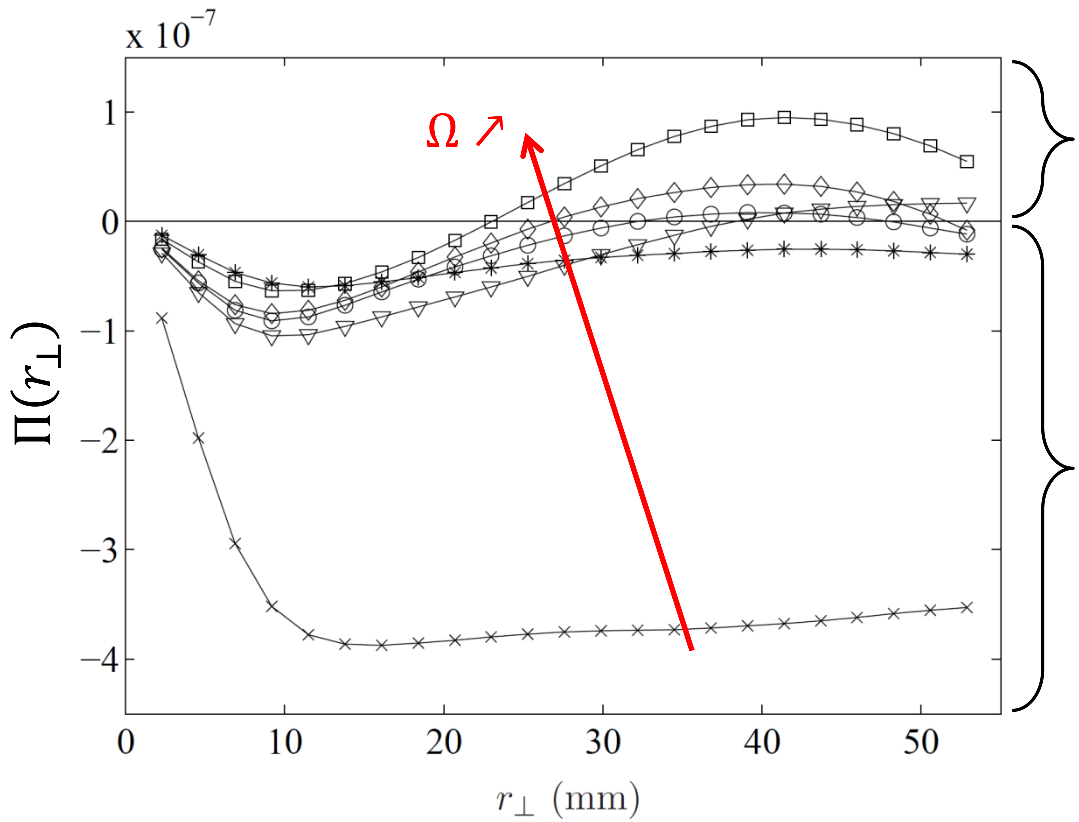
Energy flux $\Pi(\overrightarrow{r_\perp})$ in the horizontal plane

$$\Pi(\overrightarrow{r_\perp}) = \frac{1}{4}\vec{\nabla}\cdot\langle\overline{(\delta\vec{u})^2\delta\vec{u}}\rangle$$



Energy flux $\Pi(r_\perp)$ with an angular average

$$\Pi(r_\perp) = \langle \frac{1}{4} \vec{\nabla} \cdot \langle \overline{(\delta\vec{u})^2 \delta\vec{u}} \rangle \rangle_\varphi$$



$\Pi(r_\perp) > 0$
Inverse cascade

$\Pi(r_\perp) < 0$
Forward cascade

Decomposition of the energy flux in vertical and horizontal energy

$$\partial_t \vec{u} + (\vec{u}\cdot\vec{\nabla})\vec{u} = \frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega}\times\vec{u} + \nu\vec{\Delta}\vec{u}$$

$$\left\{\begin{array}{l} \overrightarrow{u_\perp} = \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix} \\ u_\parallel = u_z \end{array}\right.$$

in the 2D3C limit

$$\partial_t \overrightarrow{u_\perp} + (\overrightarrow{u_\perp}\cdot\overrightarrow{\nabla_\perp})\overrightarrow{u_\perp} = \frac{1}{\rho}\overrightarrow{\nabla_\perp}\tilde{p} + \nu\Delta_\perp\overrightarrow{u_\perp}$$

Inverse cascade of horizontal energy

$$E_\perp(r_\perp) = \langle(\delta u_\perp)^2\rangle$$

$$\Pi_\perp{}^{(\perp)}(r_\perp) = \frac{1}{4}\overrightarrow{\nabla_\perp}\cdot\langle\overline{(\delta\vec{u}_\perp)^2\delta\overrightarrow{u_\perp}}\rangle > 0$$

$$\partial_t u_\parallel + (\overrightarrow{u_\perp}\cdot\overrightarrow{\nabla_\perp})u_\parallel = \nu\Delta_\perp u_\parallel$$

$u_\parallel$ = passive scalar advected by the horizontal velocity

Direct cascade of vertical energy

$$E_\parallel = \langle(\delta u_\parallel)^2\rangle$$

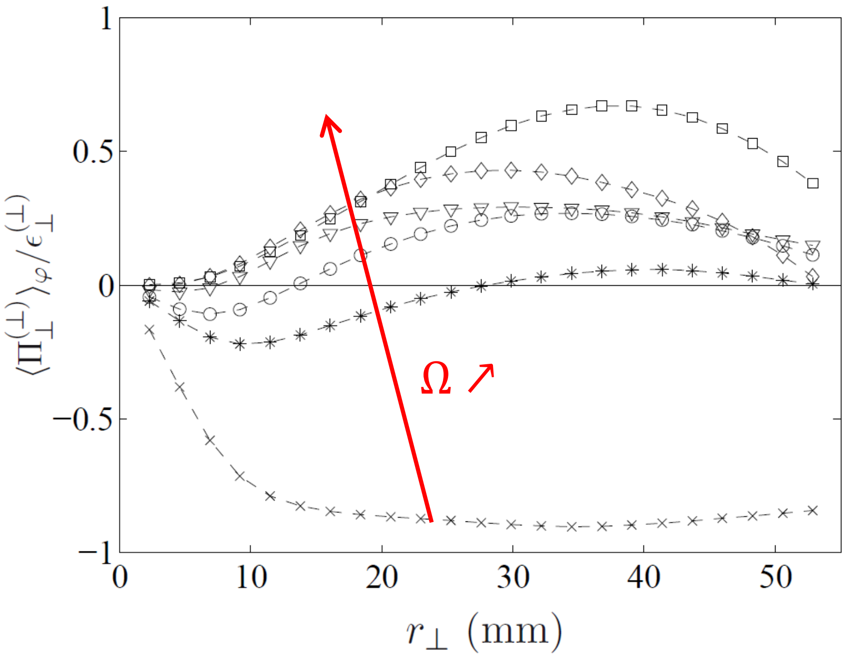
$$\Pi_\perp{}^{(\parallel)}(r_\perp) = \frac{1}{4}\overrightarrow{\nabla_\perp}\cdot\langle\overline{(\delta\vec{u}_\parallel)^2\delta\overrightarrow{u_\perp}}\rangle < 0$$

$$\Pi(r_\perp) = \Pi_\perp{}^{(\perp)}(r_\perp) + \Pi_\perp{}^{(\parallel)}(r_\perp)$$

Decomposition of the energy flux in vertical and horizontal energy

$$\Pi_\perp^{(\perp)}(r_\perp) = \frac{1}{4}\overrightarrow{\nabla_\perp} \cdot \langle \overline{(\delta\vec{u}_\perp)^2 \delta\overrightarrow{u_\perp}} \rangle > 0$$

$$\Pi_\perp^{(\parallel)}(r_\perp) = \frac{1}{4}\overrightarrow{\nabla_\perp} \cdot \langle \overline{(\delta\vec{u}_\parallel)^2 \delta\overrightarrow{u_\perp}} \rangle < 0$$



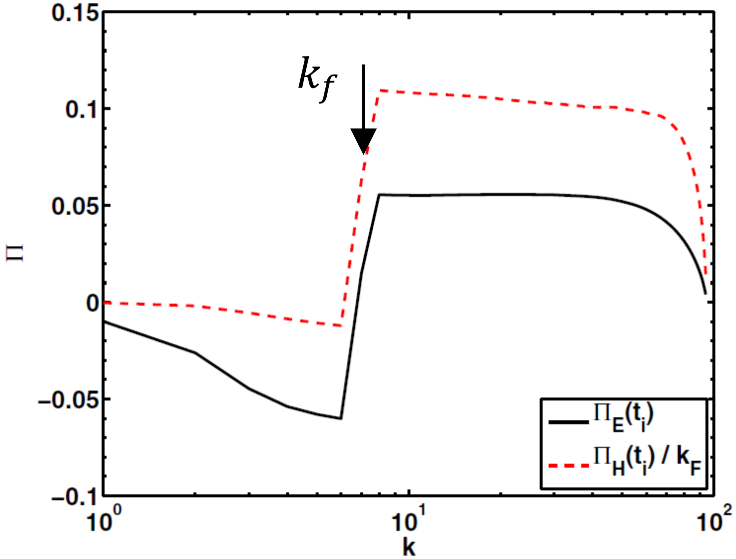
Double cascade of energy

3D isotropic forward energy transfer
and/or
stretching and folding of a passive scalar advected by $\overrightarrow{u_\perp}$

An inverse cascade of energy

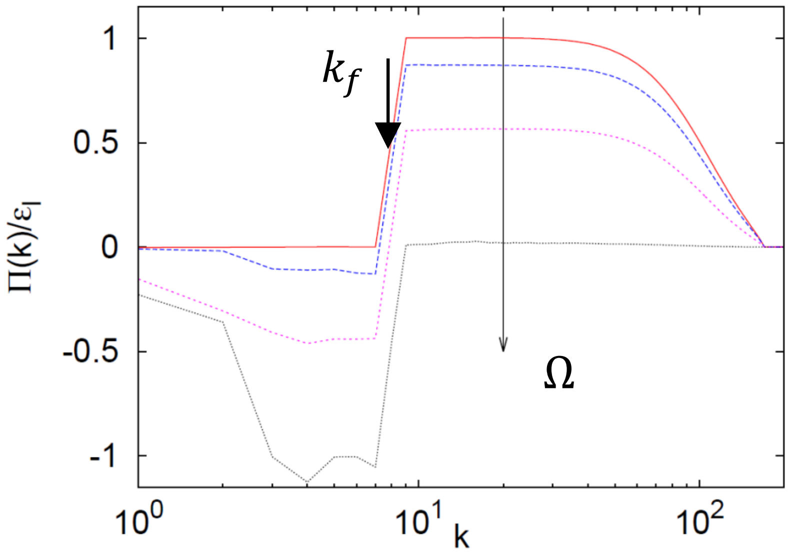
Pouquet, Sen, Rosenberg, Mininni & Baerenzung, Phys. Scripta (2013)

Rotating LES
Energy injection at k_f



Deusebio, Boffetta, Lindborg, Musacchio, ArXiv (2014)

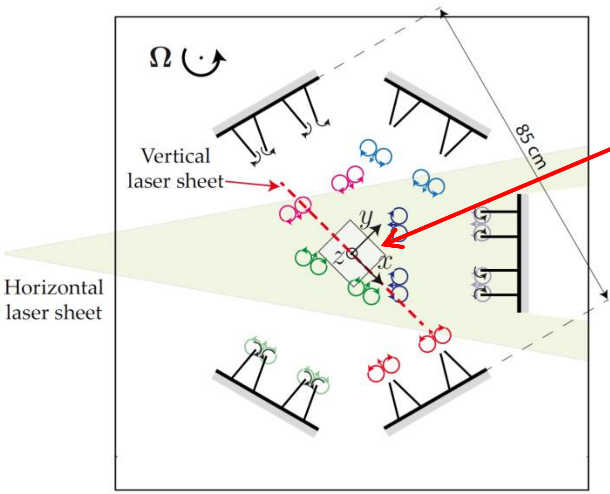
DNS of turbulence under rotation
Energy injection at k_f



Kármán-Howarth-Monin equation

$$\frac{1}{4}\cancel{\frac{\partial E(\vec{r})}{\partial t}} = -\Pi + D - \varepsilon + \cancel{P_r}$$

Where is the energy injection?



It is due to an energy flux through the frontier of the control volume

This flux is intrinsically related to inhomogeneities in the turbulence statistics even infinitely small (Our energy is homogeneous to 3%)

$$\boxed{\varepsilon = -\Pi + D + A + B}$$

with $A(\vec{r}) = -\frac{1}{4V_X}\oint_{S_X} \overline{(\delta\vec{u})^2\tilde{\vec{u}}} \cdot d\overrightarrow{S_X}$ with $\tilde{\vec{u}} = \frac{\vec{u}(\vec{x})+\vec{u}(\vec{x}+\vec{r})}{2}$

coarse-grained velocity

$$B(\vec{r}) = -\frac{1}{2V_X}\oint_{S_X} \overline{\delta p \delta\vec{u}} \cdot d\overrightarrow{S_X}$$

$$D(\vec{r}) = \frac{\nu}{2}\vec{\nabla}^2\langle\overline{(\delta\vec{u})^2}\rangle$$

Kármán-Howarth-Monin equation in the horizontal plane

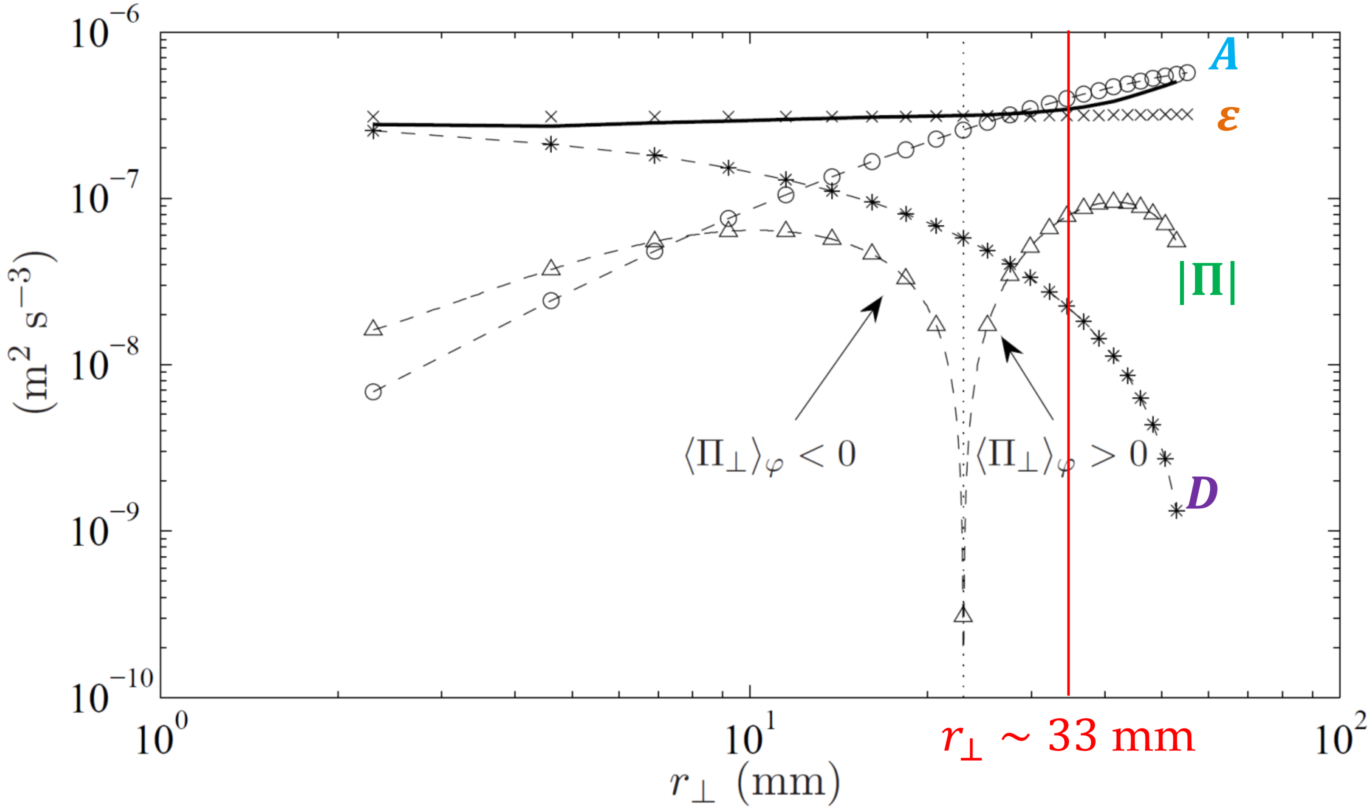
$$\boldsymbol{\varepsilon} = -\boldsymbol{\Pi} + \boldsymbol{D} + \boldsymbol{A} + \boldsymbol{B}$$

with

$$A = -\frac{1}{4V_X}\oint_{S_X} \overline{(\delta\vec{u})^2\vec{\tilde{u}}} \cdot d\vec{S_X}$$

$$B = -\frac{1}{2V_X}\oint_{S_X} \overline{\delta p\delta\vec{u}} \cdot d\vec{S_X} \quad ??$$

$$D = \frac{\nu}{2}\vec{\nabla}^2\langle\overline{(\delta\vec{u})^2}\rangle$$



Kármán-Howarth-Monin equation in the horizontal plane

$$\boldsymbol{\varepsilon} = -\boldsymbol{\Pi} + \boldsymbol{D} + \cancel{\boldsymbol{A}}\boldsymbol{\phi} + \cancel{\boldsymbol{B}}$$

$$\boldsymbol{\phi(r_\perp) = A + B}$$

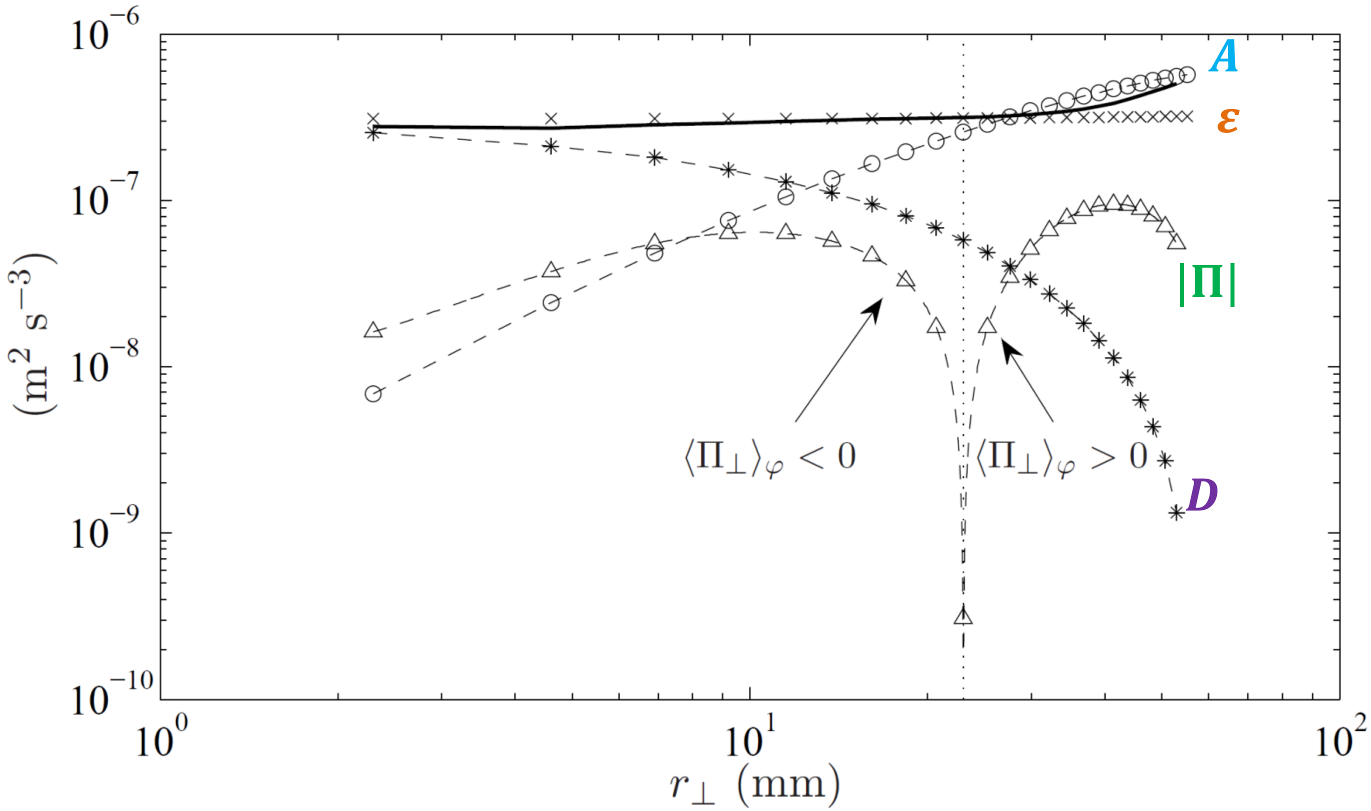
with $A = -\frac{1}{4V_X}\oint_{S_X} \overline{(\delta\vec{u})^2\tilde{\vec{u}}} \cdot d\overrightarrow{S_X}$

$\frac{d\phi(r_\perp)}{dr_\perp} > 0 \rightarrow$ energy injection

$B = -\frac{1}{2V_X}\oint_{S_X} \overline{\delta p \delta\vec{u}} \cdot d\overrightarrow{S_X}$??

$\frac{d\phi(r_\perp)}{dr_\perp} < 0 \rightarrow$ energy extraction

$D = \frac{\nu}{2}\vec{\nabla}^2\langle\overline{(\delta\vec{u})^2}\rangle$



***A* injects energy, ok!**

But, where does the energy go at the end of the inverse cascade?

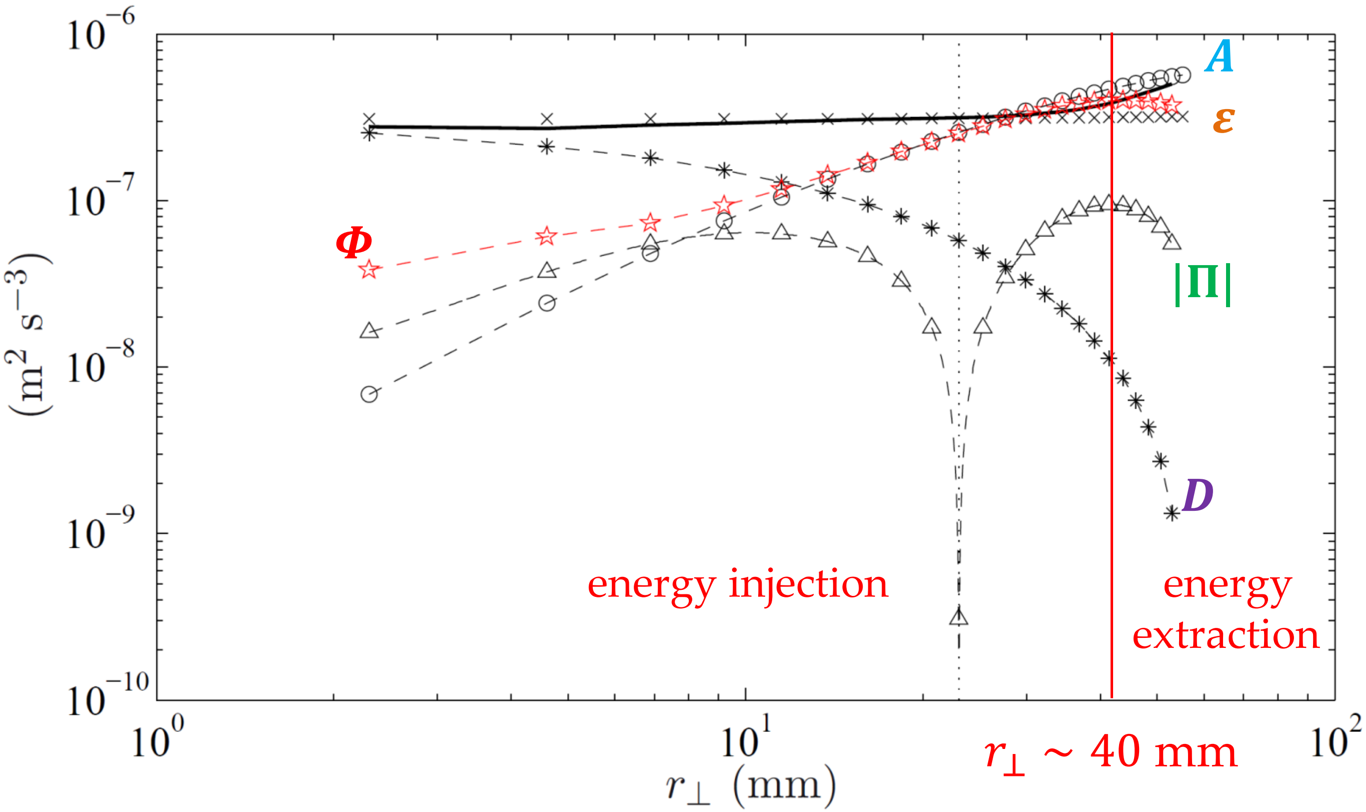
Kármán-Howarth-Monin equation in the horizontal plane

$$\boldsymbol{\varepsilon} = -\boldsymbol{\Pi} + \boldsymbol{D} + \boldsymbol{\phi}$$

$$\boldsymbol{\phi}(\boldsymbol{r}_\perp) = \boldsymbol{A} + \boldsymbol{B}$$

$$\frac{d\phi(r_\perp)}{dr_\perp} > 0 \rightarrow \text{energy injection}$$

$$\frac{d\phi(r_\perp)}{dr_\perp} < 0 \rightarrow \text{energy extraction}$$



Double cascade + injection of energy due to inhomogeneities

→ Direct measurement of energy transfers show that rotating turbulence can sustain a double cascade in the horizontal plane.

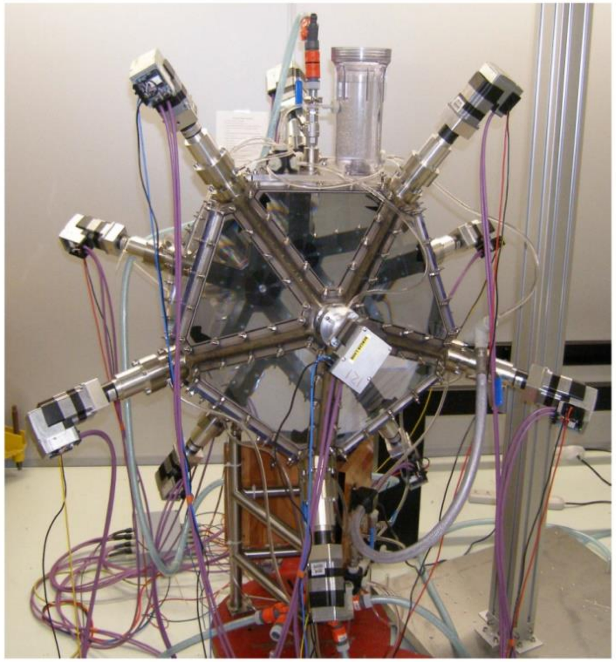
→ Inhomogeneities are necessary to explain the energy injection/extraction, even infinitely small.

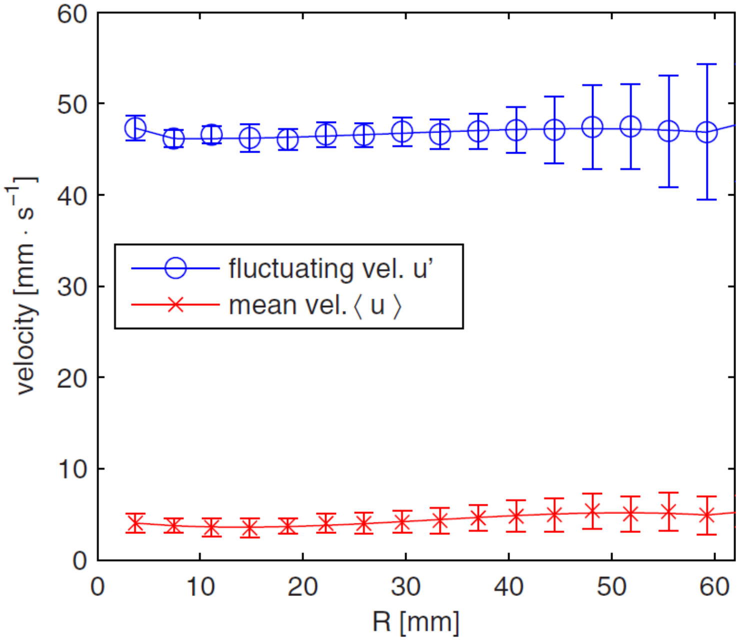
There is need to consider the inhomogeneities in the statistics to account for the energy injection in an experimental stationary turbulence…

… whatever the degree of homogeneity of the rms velocity.

In our case, we are homogeneous in energy to within 3%.

An experimental homogeneous and stationary turbulence ?





Lagrangian Exploration Module
(12 propellers organised around a icosahedron)

Zimmermann *et al.* Rev Scient Inst (2010)

Experimental turbulence

In the two presented experiments, we are either:

- homogeneous and decaying (no energy injection)
- inhomogeneous and stationary (energy injection)

These situations are not to avoid.

They correspond to the reality of turbulence

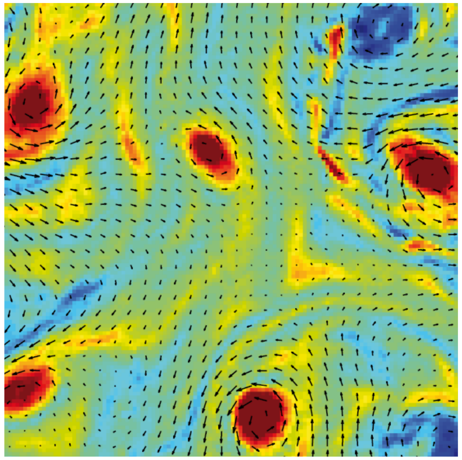
both in experiments and in nature.

Trivial but often forgotten!

→ It is a necessary challenge to consider energy transfers

in space and scales at the same time

Energy transfers in turbulence under rotation
Some experiments



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