Ferroturbulence

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CNRS URA 2464

Permanents
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Ph. D :
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First results of SHREK
SuperFluid Helium Turbulence

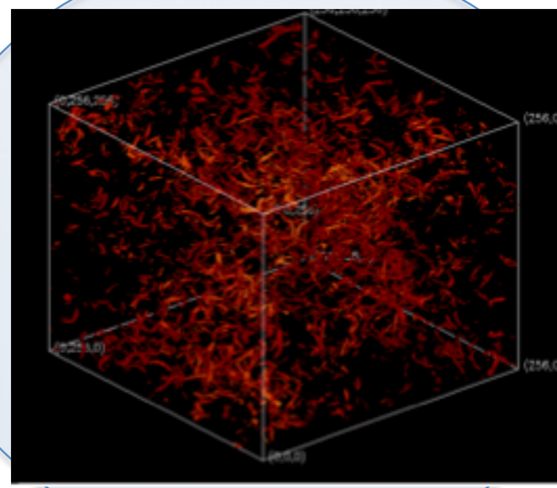
SHREK Collaboration

CEA/SBT: Rousset, Girard, Diribarne, …
NEEL: Roche, Gibert,Hebral, …
LEGI: Baudet, Bourgoin, Gagne,….
ENS Lyon: Castaing, Chevillard, Salort, …
CEA/SPEC**: Dubrulle**, Daviaud, Herbert, Saint-Michel,
Luth: Lehner

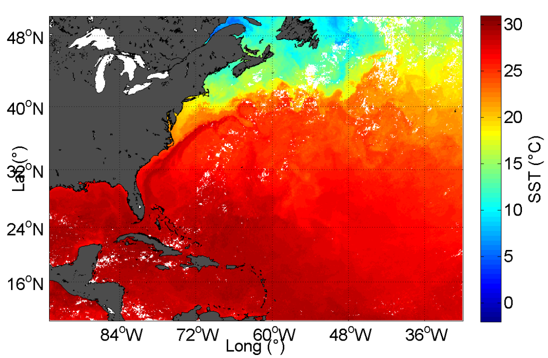
Turbulence

F and Diss

Energy Production

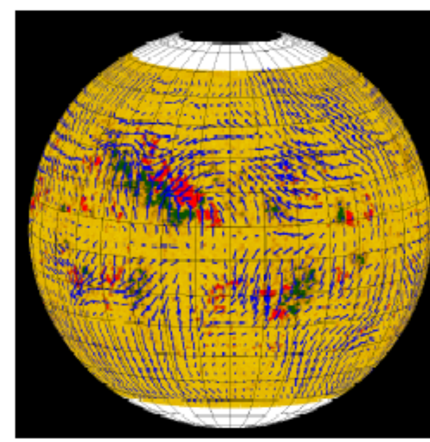


Transport



Climate

Turbulence is everywhere



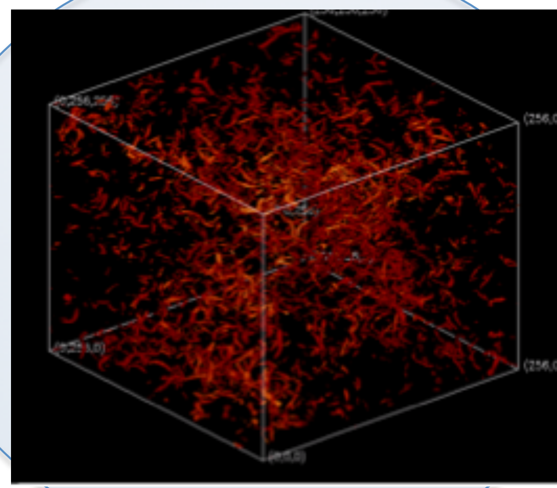
Astrophysical Objects

Turbulence

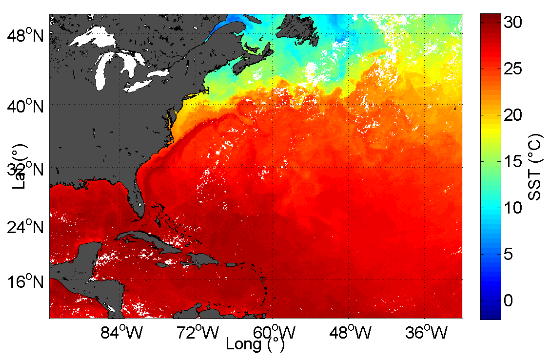
F
and
Diss

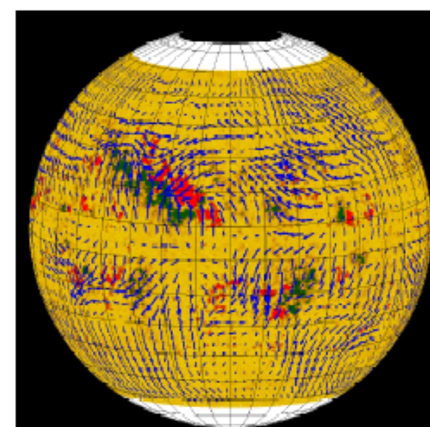
$$Re = \frac{LU}{\nu}$$

$Re = 10^9$



$Re = 10^9$



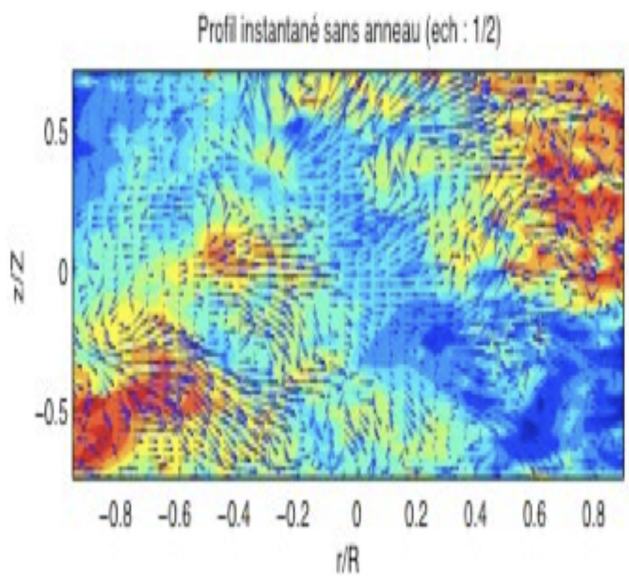


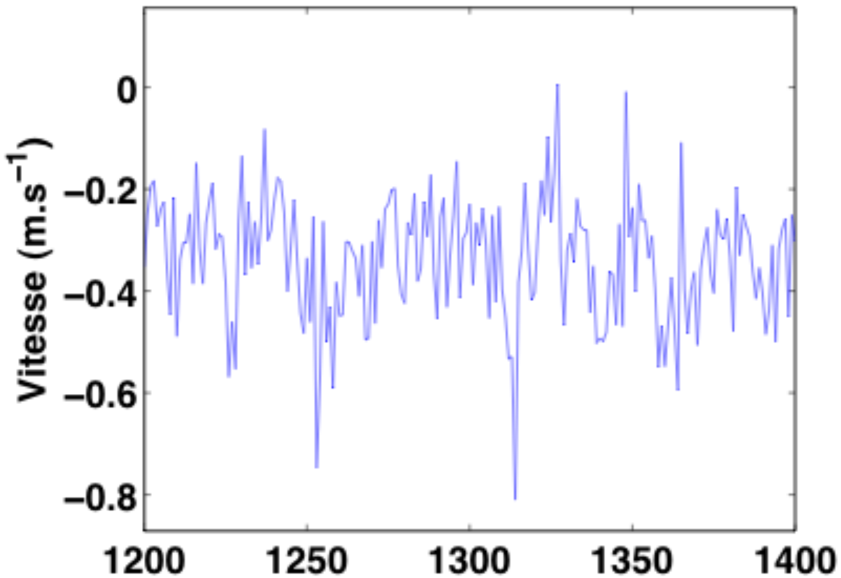
Main Equation known
But not understood

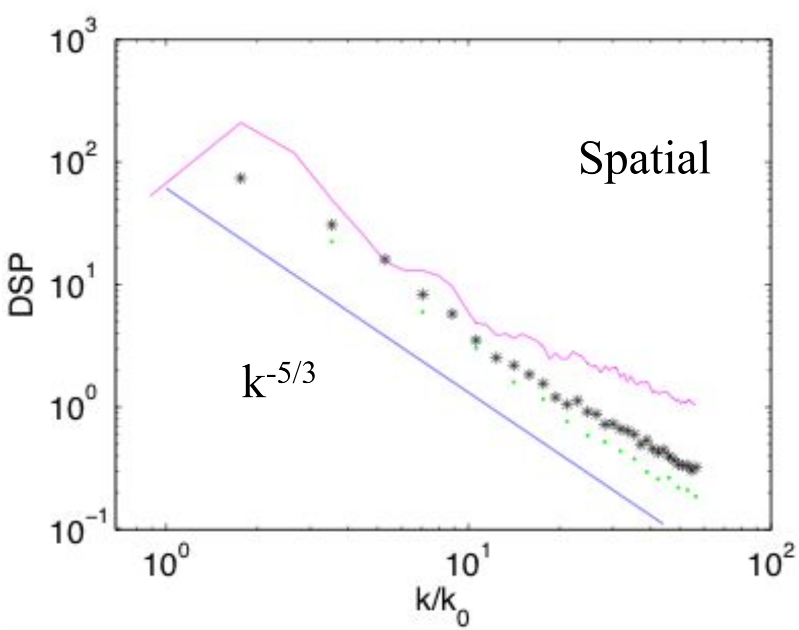
$$\partial_t u + u \bullet \nabla u = -\nabla p + \nu \Delta u + f$$

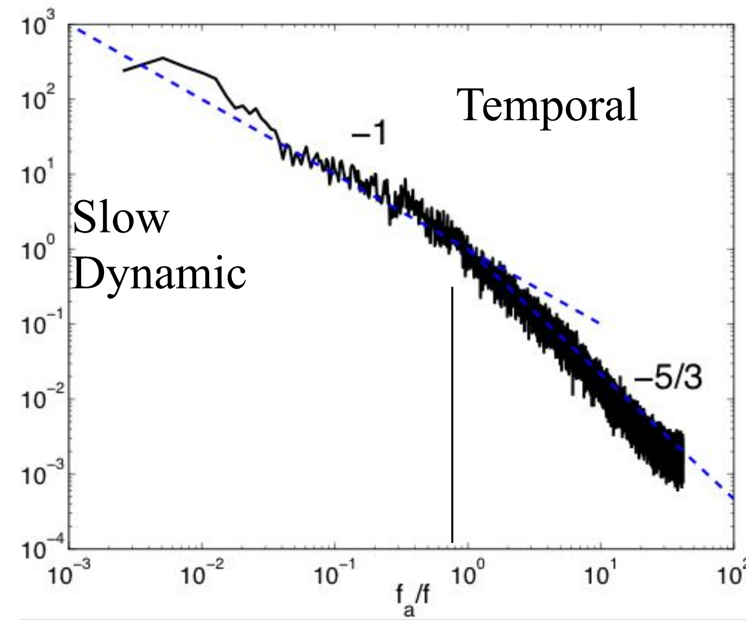
$Re = 10^{12}$

Spectra and scales







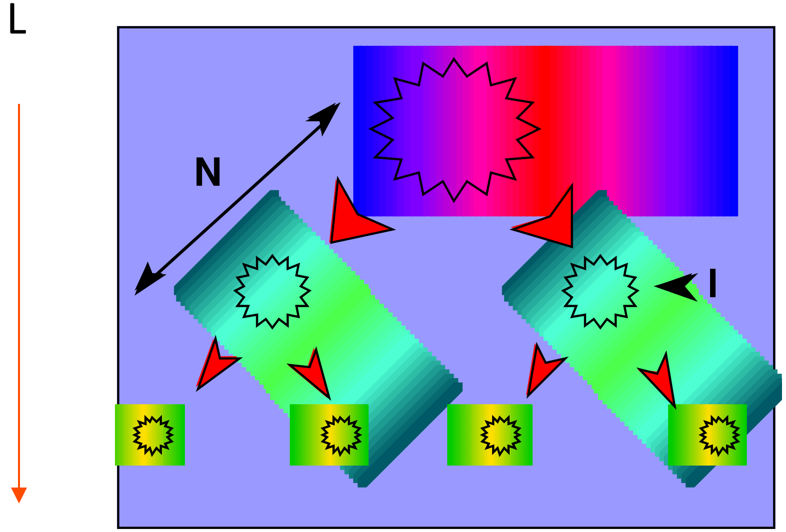


Turbulence phenomenology

Robust result
Kolmogorov spectrum

Interpretation (Kolmogorov 1941)
Energy cascade

L



$$\eta = \left(\nu^3/\varepsilon\right)^{1/4}$$

Constant energy transfer

$$\frac{du^2}{dt} = \varepsilon \cong \frac{u^3}{l} = cte$$

$$\Rightarrow u \propto (\varepsilon l)^{1/3}$$

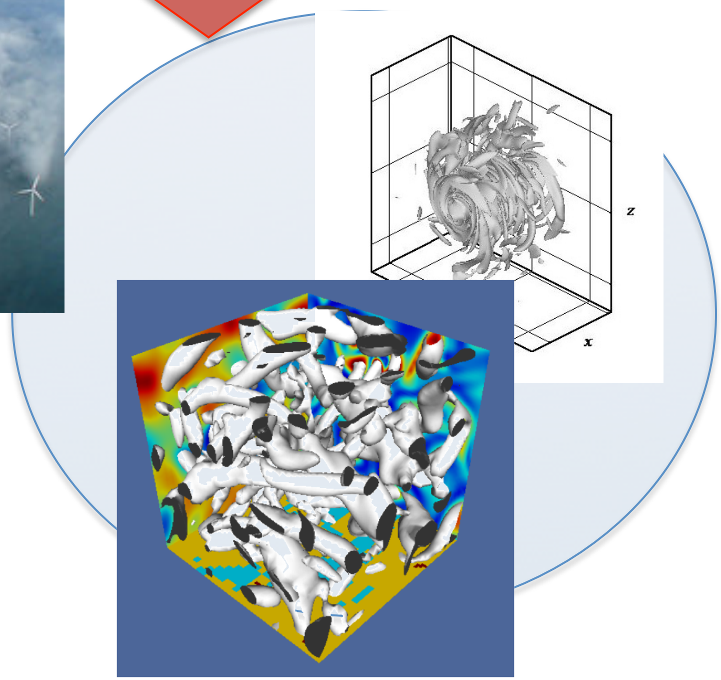
Degrees of freedom

$$N = \left(\frac{L}{\eta}\right)^3 \propto \mathrm{Re}^{9/4}$$

Turbulence
Organization vs degrees of freedom

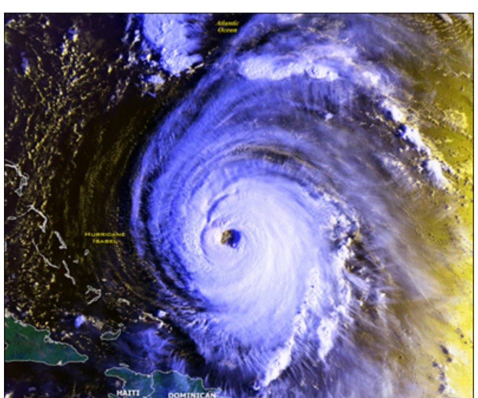
F
and
Diss

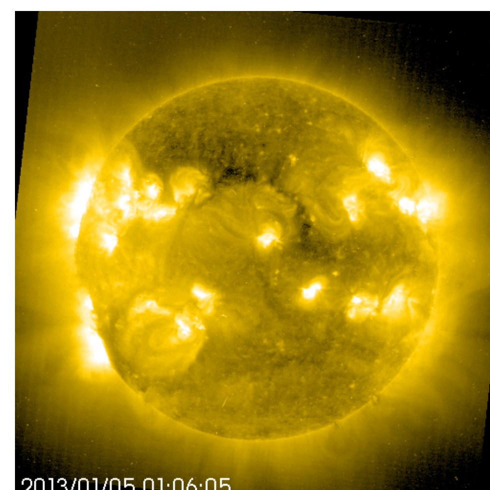
$$N = \mathrm{Re}^{9/4}$$



N=3 10^{20}

N=3 10^{20}





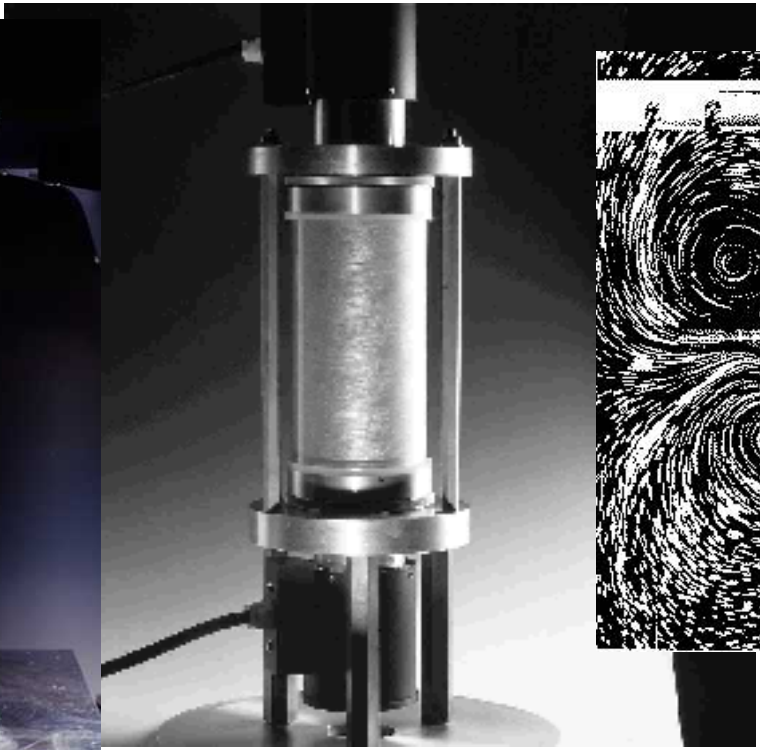
Out-of-equilibrium statistical physics!

N= 10^{27}

In Laboratory

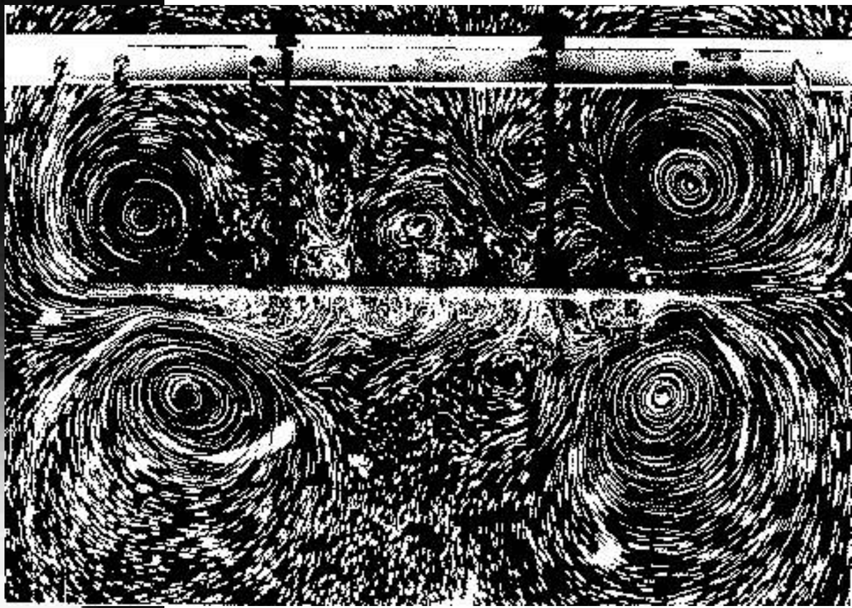
Von Karman

$$Re = 10^2 - 10^8$$



Taylor-Couette

$$Re = 10^5$$

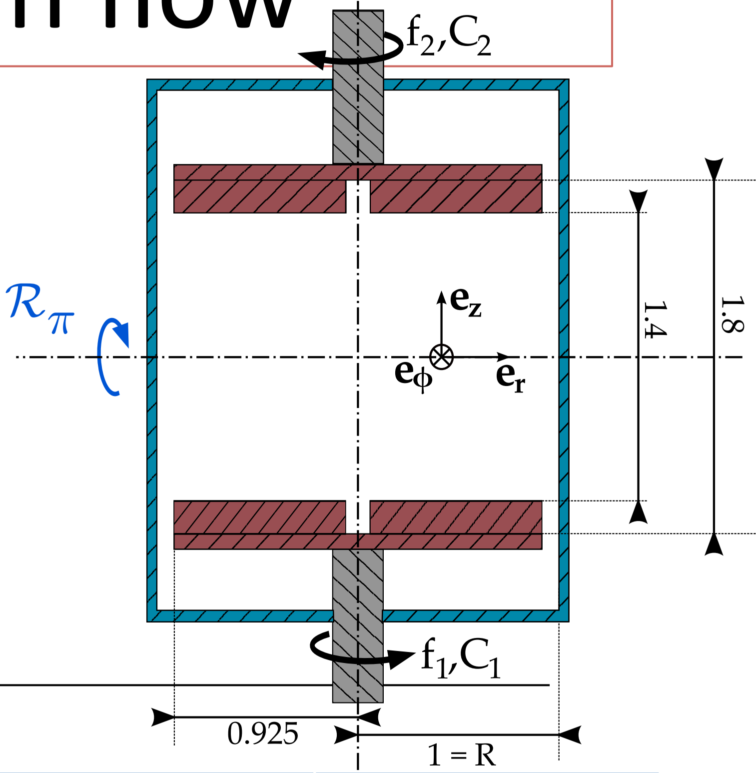


Turbulence behind oscillatory grid

$$Re = 10^3$$

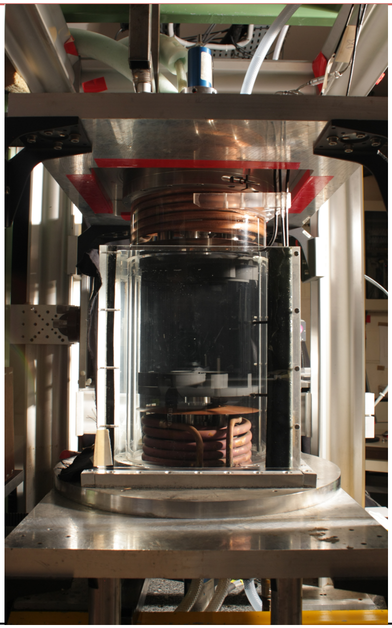
We can approach geophysical or astrophysical flows by experiments

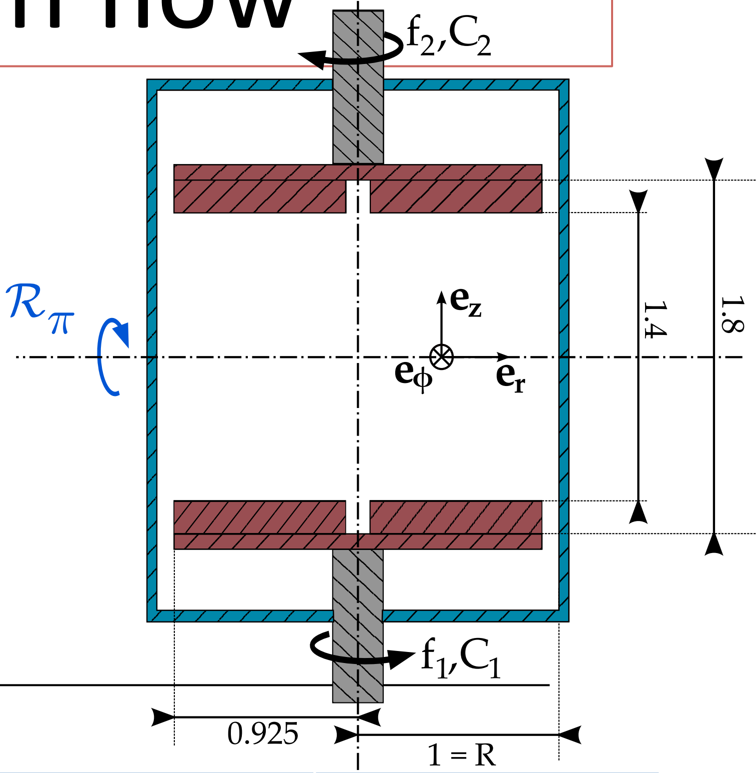
The von Karman flow



SetUp	Fluid	P(bars)	T(K)	Re
SHREK	HeI	1.1	2.62	10^8
SHREK	HeII	1.1	1.63	?
SHREK	N2	3.73	284	10^5
VKE VKE	H2O Glyc	1.8 1.8	300 300	10^5 10^2

The von Karman flow

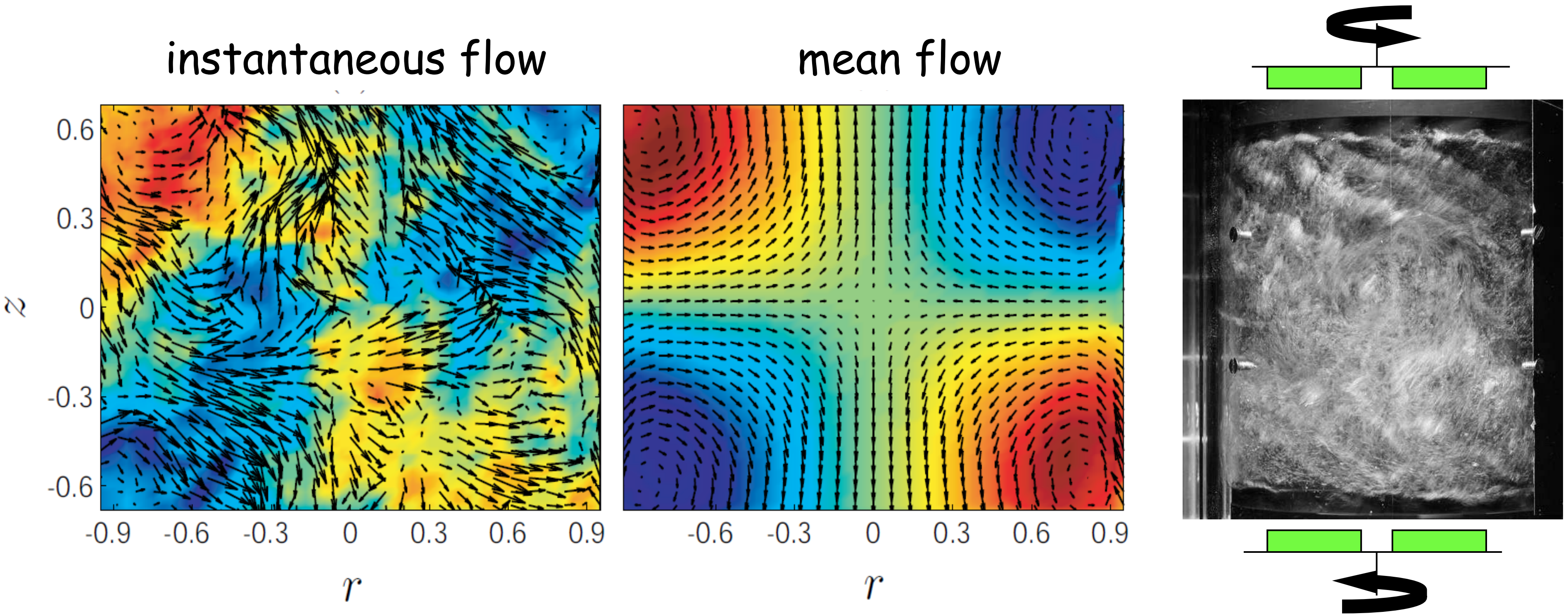




SetUp	Fluid	P(bars)	T(K)	Re
VKE	H2O	1.8	300	10^5
VKE	Glyc	1.8	300	10^2

Flow at large Re

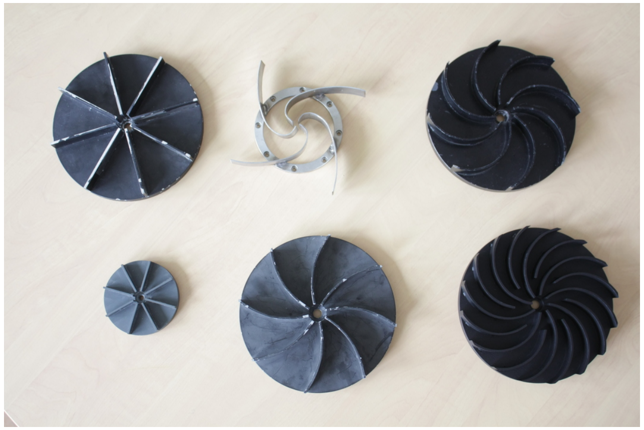
Fully developped turbulence at Re > *Re=10 000*

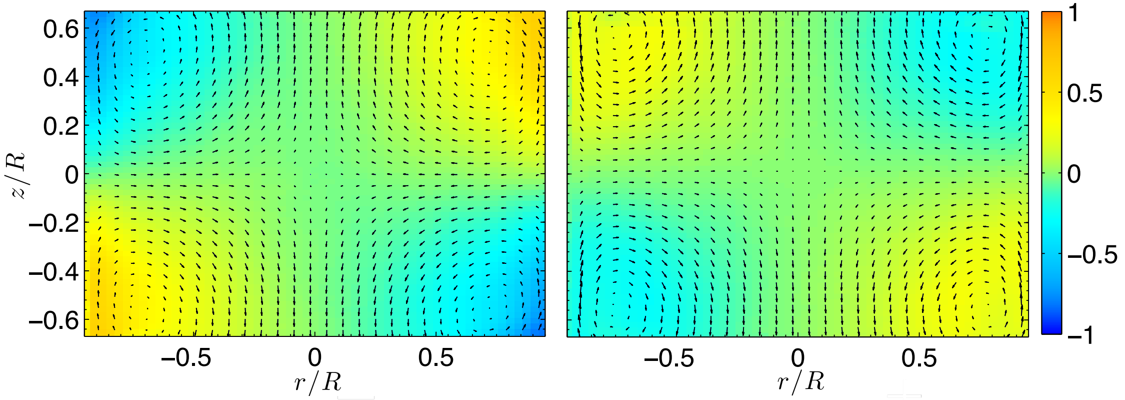


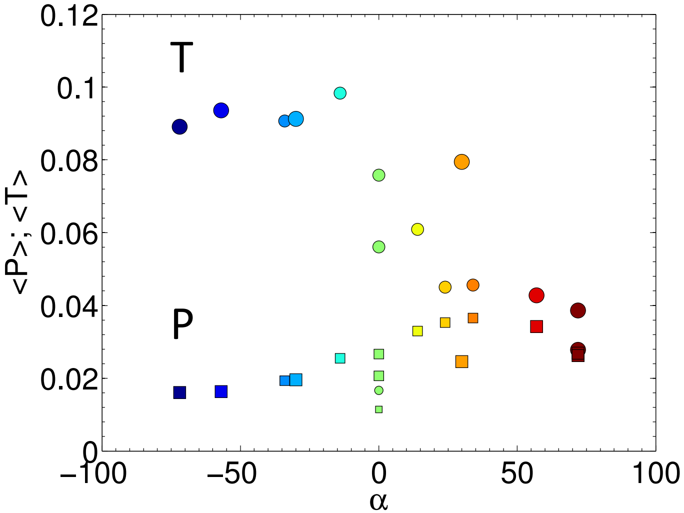
Universality of mean flow

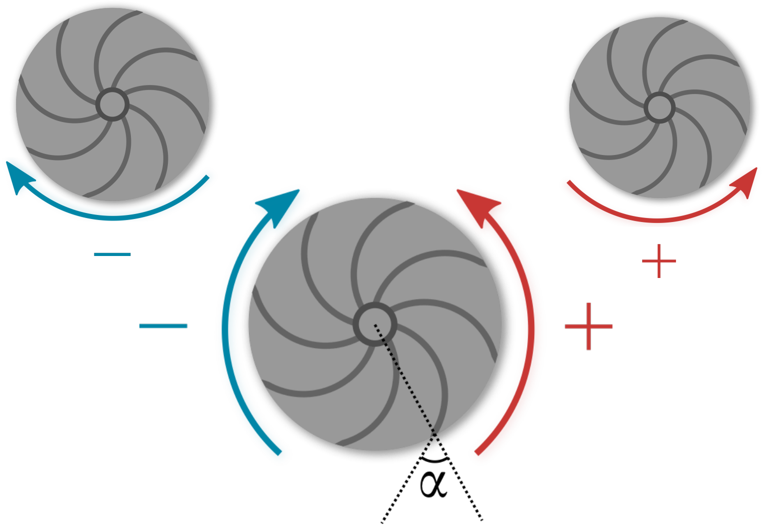
Different propellers at Re > *Re=10 000*

P=Poloidal=Arrows
T=Toroidal=Color

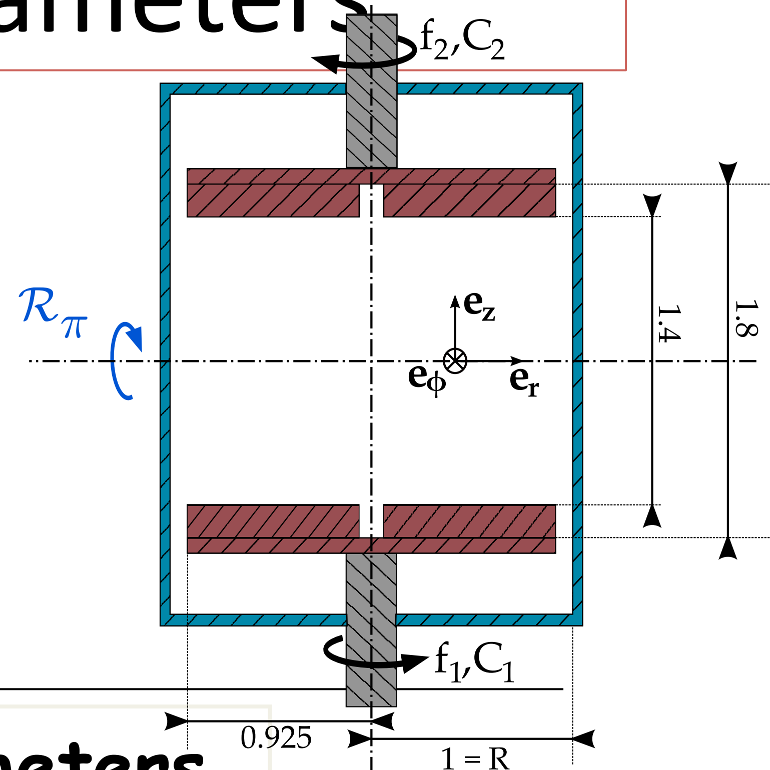








The control parameters



2 control parameters

Reynolds number =
Turbulence intensity

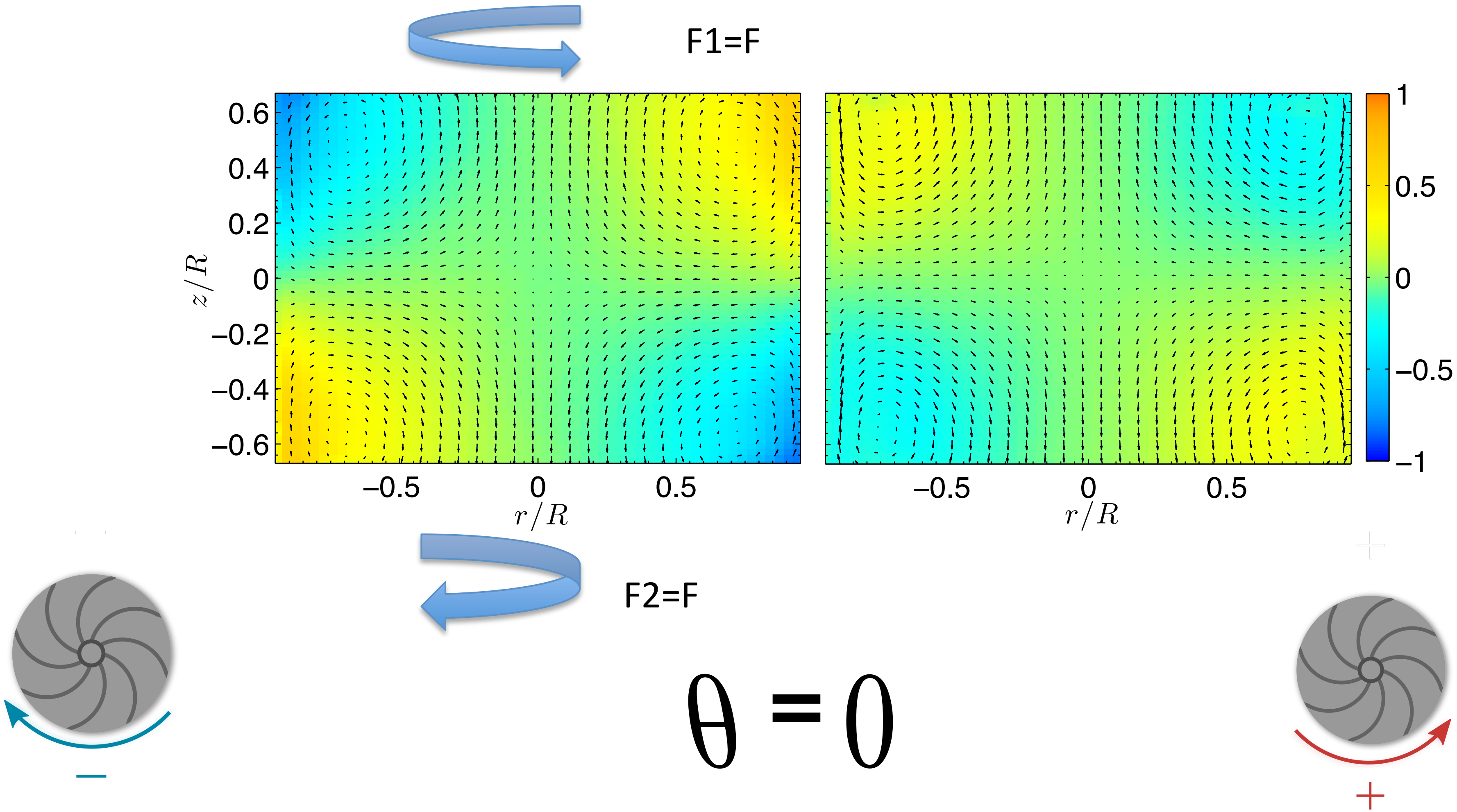
Rotation number =
System asymmetry

$$Re = \frac{2\pi f R^2}{\nu}$$

$$f = \frac{f_1 + f_2}{2}$$

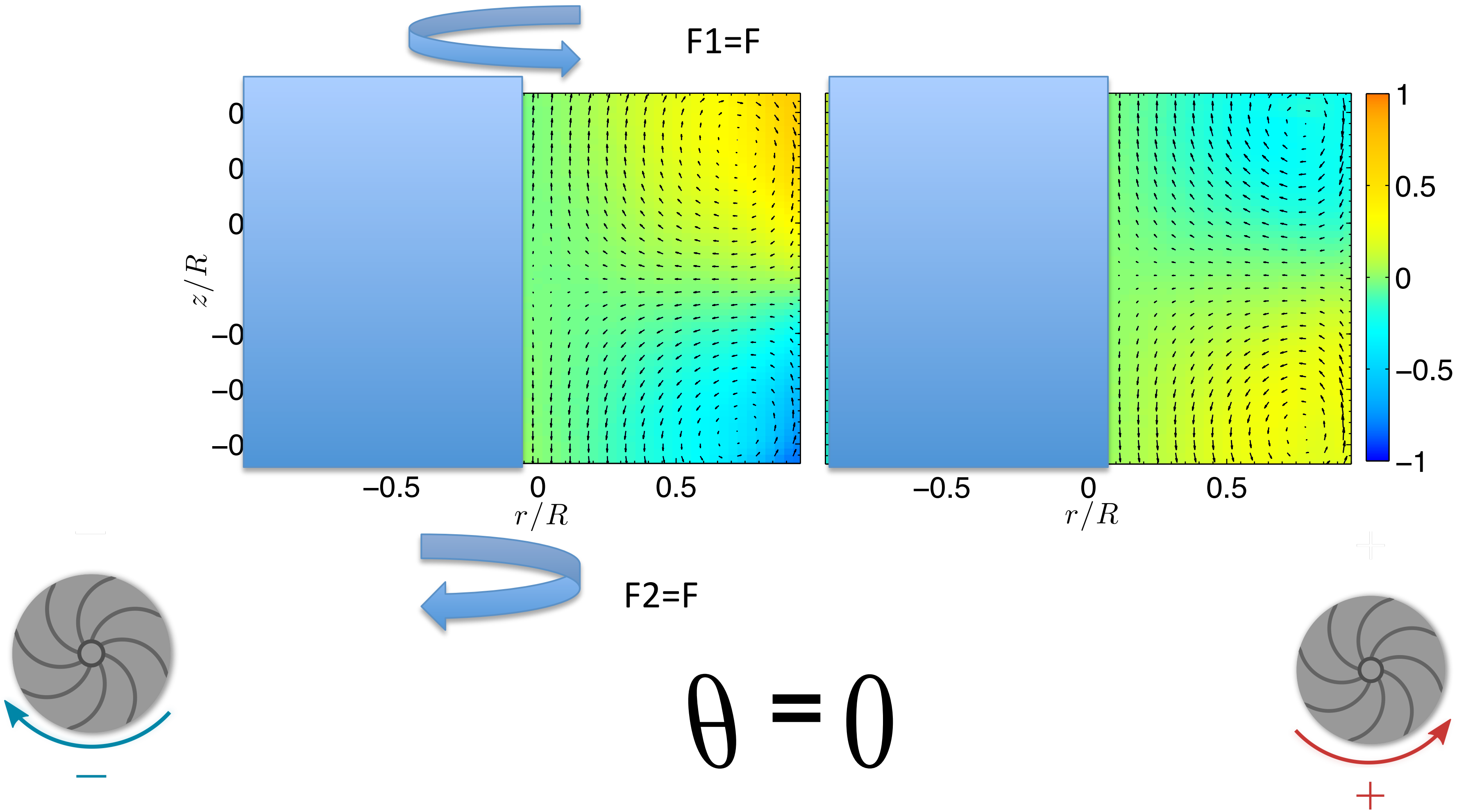
$$\theta = \frac{f_1 - f_2}{f_1 + f_2}$$

Universality of response to asymetry



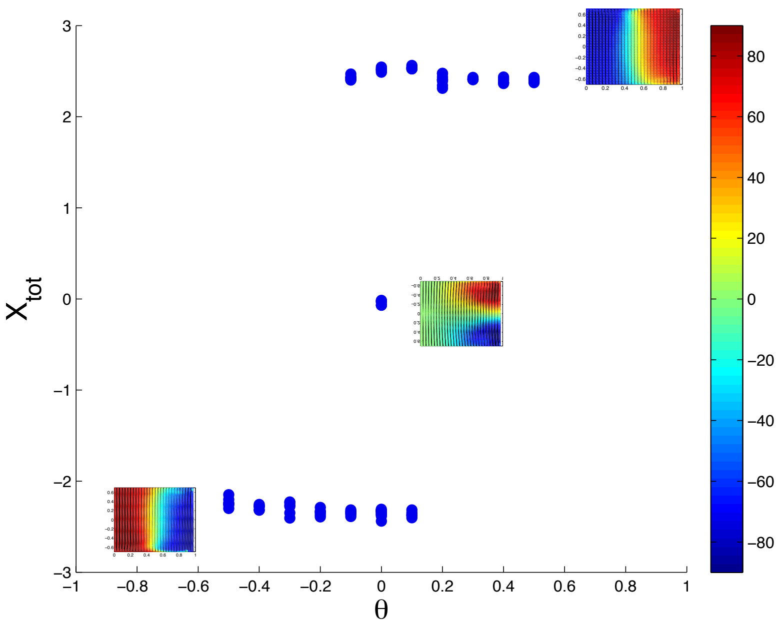
$$\theta = 0$$

Universality of response to asymetry

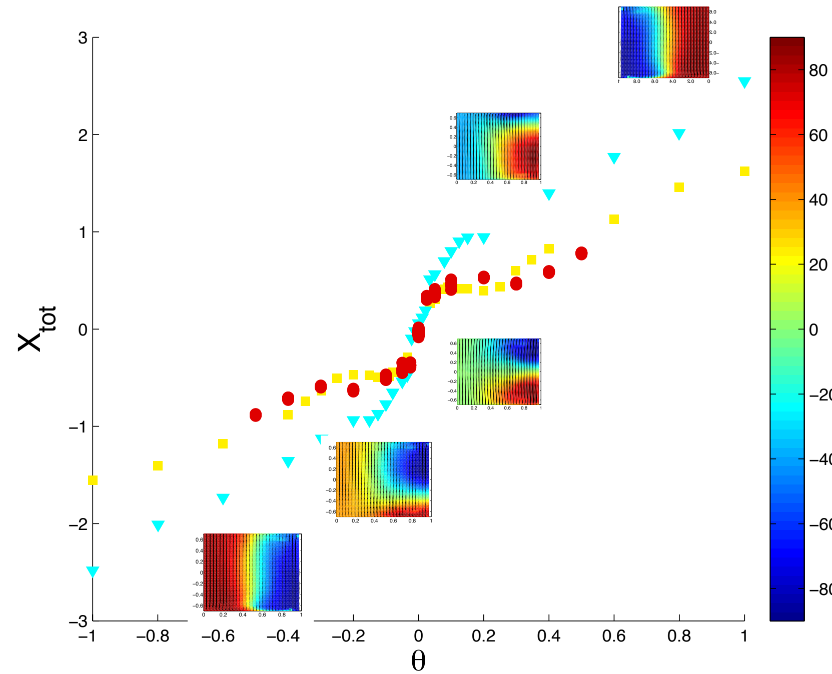


$$\theta = 0$$

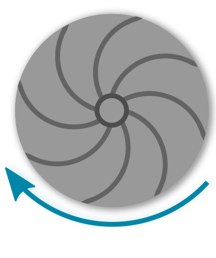
Universality of response to asymetry



Alpha<-53

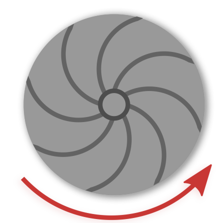


Alpha>-53

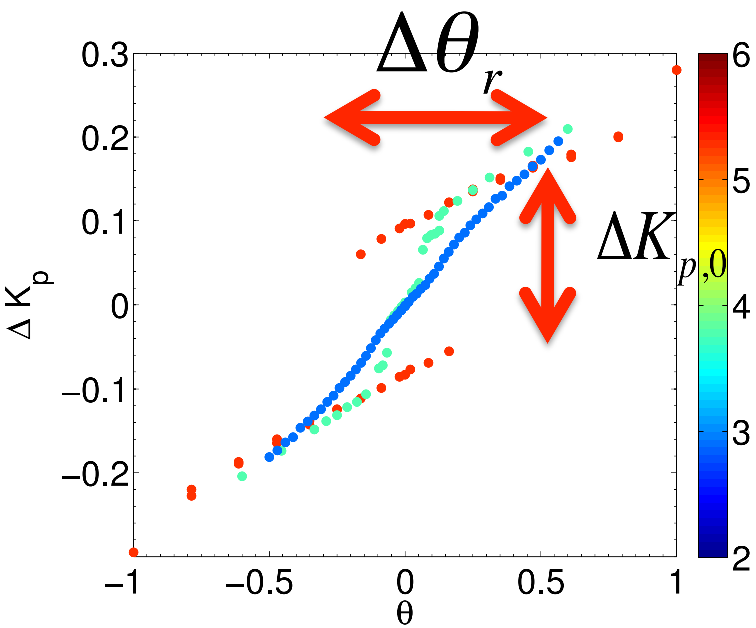


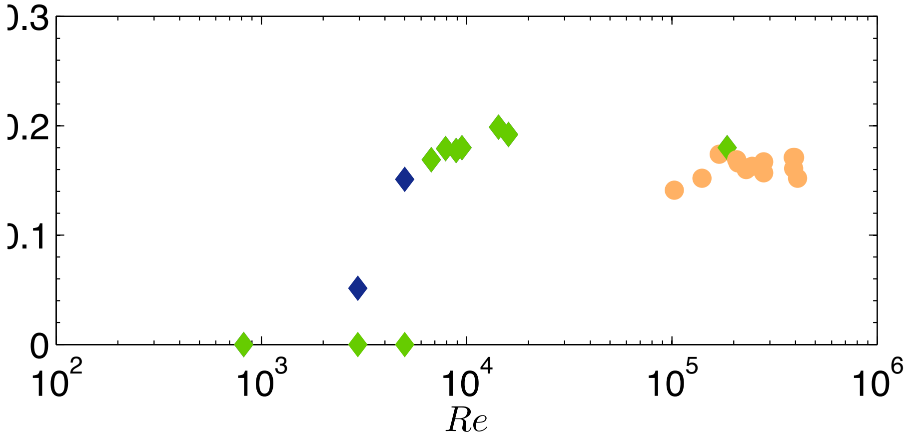
$$X_{tot} = \langle \omega_\theta / r \rangle$$

$$I = \langle r u_\theta \rangle$$

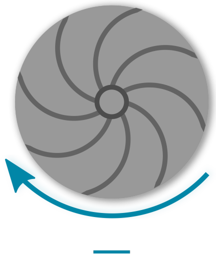


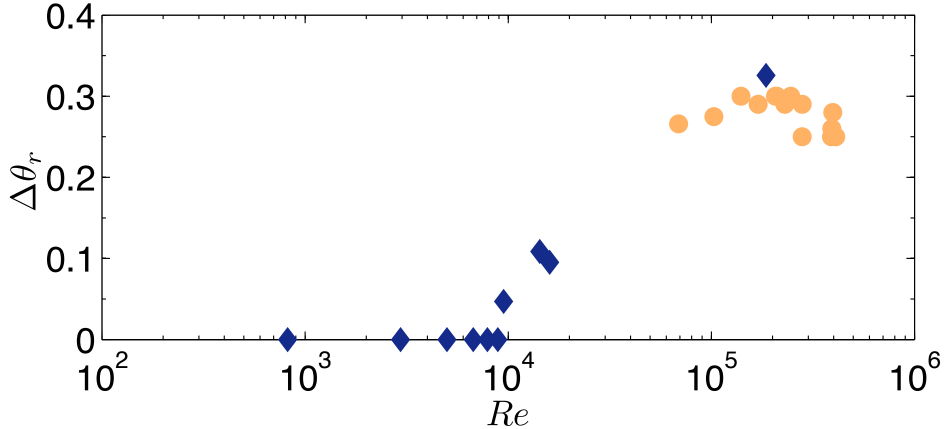
Scaling laws with Reynolds number





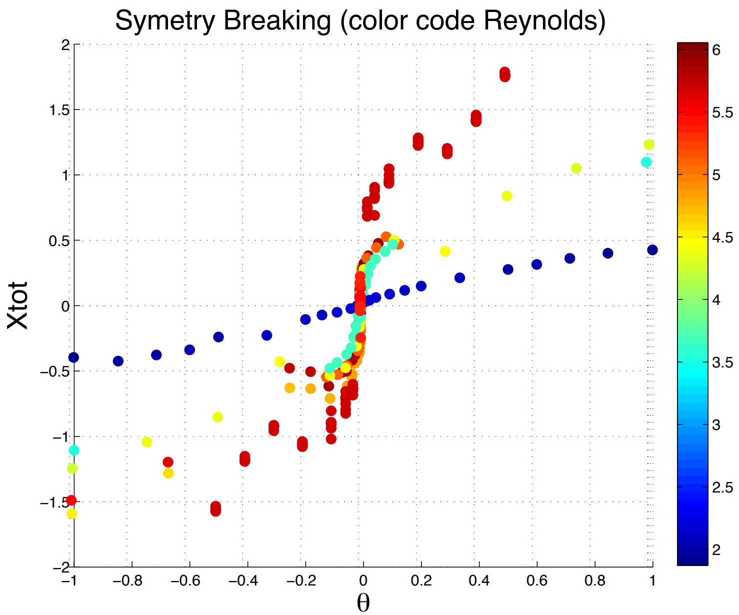
Alpha<-53

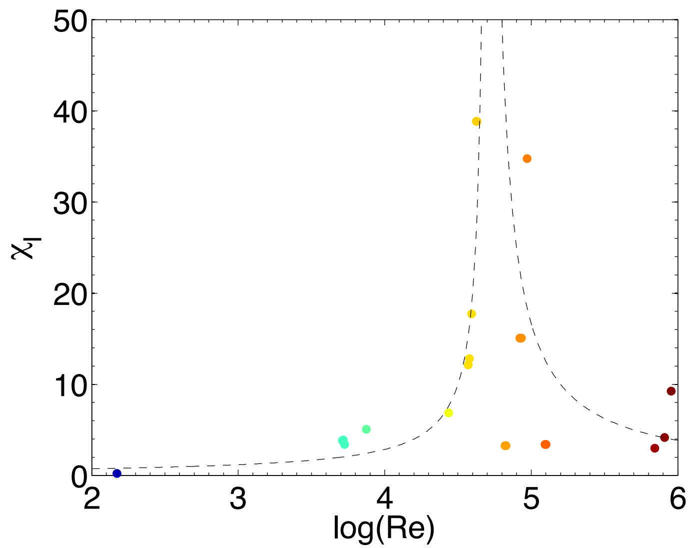


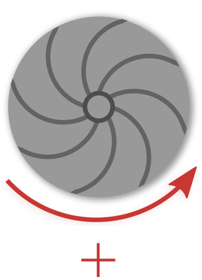


Why -53?

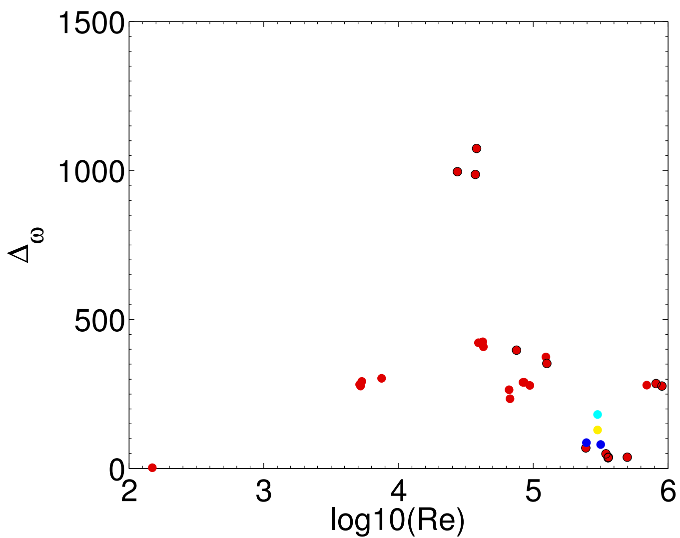
Scaling laws with Reynolds number(II)







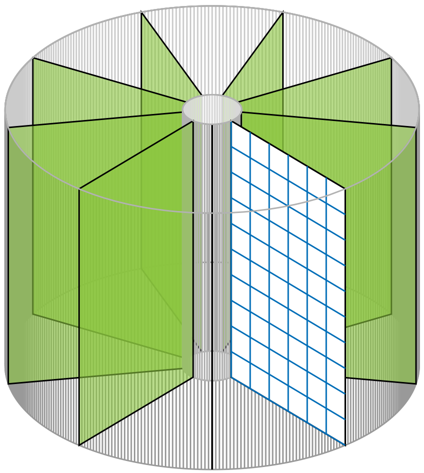
Alpha>-53



Why Re=40000?

Can we understand this with statistical mechanics argument

Axisymmetric turbulence



Invariant by rotation around vertical axis

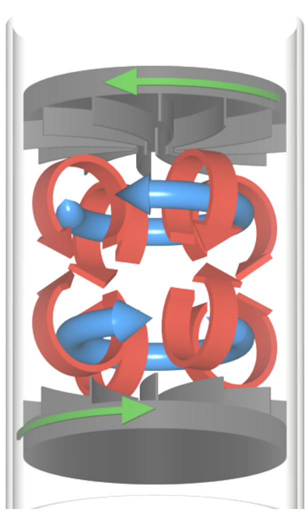
Can be described by only two fields

Angular momentum
(Toroidal Field)

$$\sigma(r,z) = r u_\theta$$

Azimuthal vorticity density
(Poloidal Field)

$$\xi(r,z) = \omega_\theta / r$$

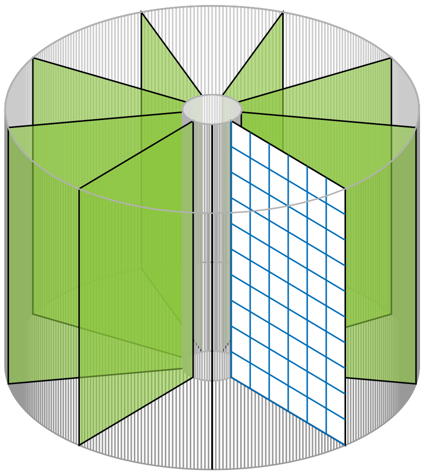


Useful quantity

$$\xi = -\Delta^* \psi$$

Stream function

Conservation laws



Energy

$$E = E_p + E_t = \int \xi \psi \, d\vec{x} + \int \frac{\sigma^2}{r^2} d\vec{x}$$

$$H_n = \int \xi \sigma^n \, d\vec{x}$$

Generalized Helicity

$$I_n = \int \sigma^n \, d\vec{x}$$

Casimir

Important ones

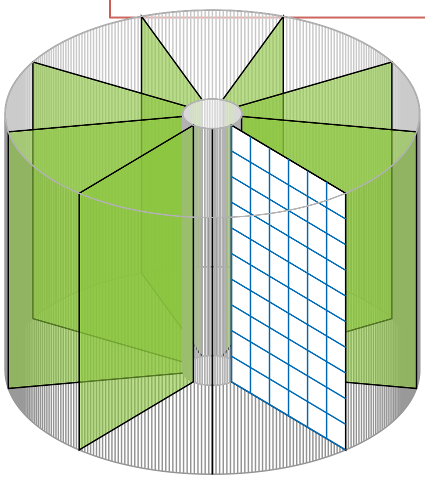
$$X_{tot} = \int \xi \, d\vec{x} = \langle \xi \rangle_x$$

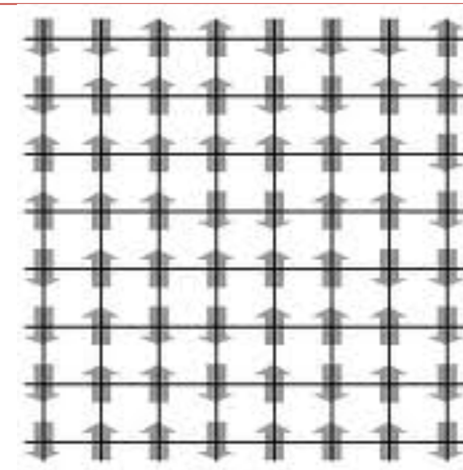
Circulation

$$I = \int \sigma \, d\vec{x} = \langle \sigma \rangle_x$$

Angular momentum

From Axi-Euler to Ising





$$E = \int \xi \Delta^{-1} \xi \, d\vec{x} + \int \frac{\sigma^2}{r^2} d\vec{x}$$

$$E = \sum_{x,y} J_{xy} \vec{S}_x \bullet \vec{S}_y$$

$$J_{xy} \Rightarrow \Delta^{-1} \quad and \; 1/r^2$$

$$\vec{S}_x \Rightarrow (\sigma(x), \xi(x))$$

$$\vec{M} = \left\langle \vec{S}_x \right\rangle_x \Rightarrow \left(I, X_{tot}\right)$$

$$kT = \beta^{-1} \Rightarrow \overline{u_\theta^2} - \overline{u_\theta}^2$$

Analogy

Long-range 2 component Ising

Thalabard PhD:

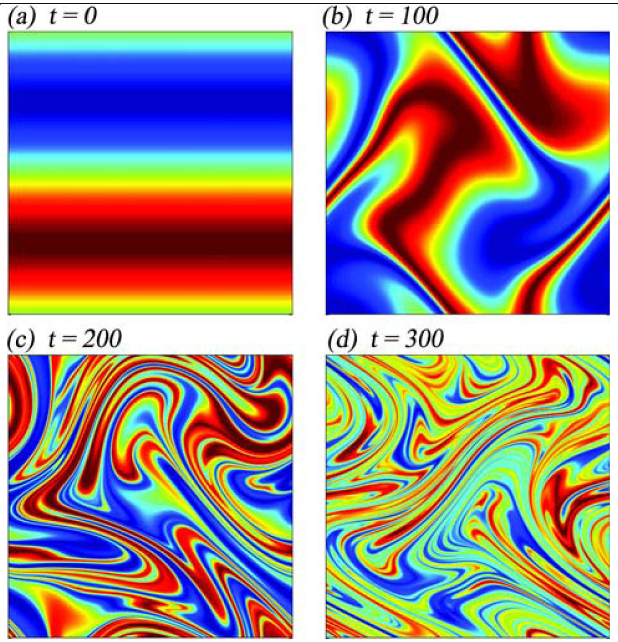
Statistical Mechanics ?

Mixing Property of Turbulence

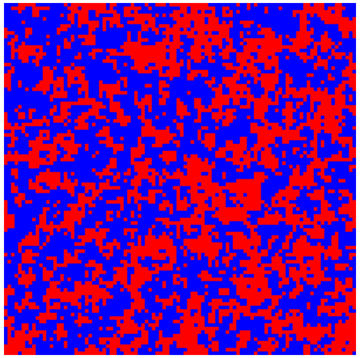
$\rho(\sigma)$ Probability distribution

$$S = -\int \rho \ln \rho \, dx$$

Mixing Entropy



Ising: energy conservation



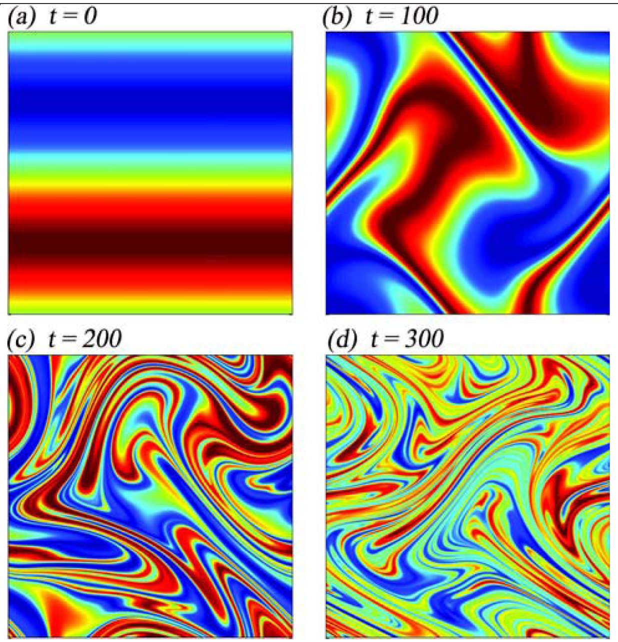
$$E = \sum_{x,y} J_{xy} \vec{S}_x \bullet \vec{S}_y$$

Statistical Mechanics

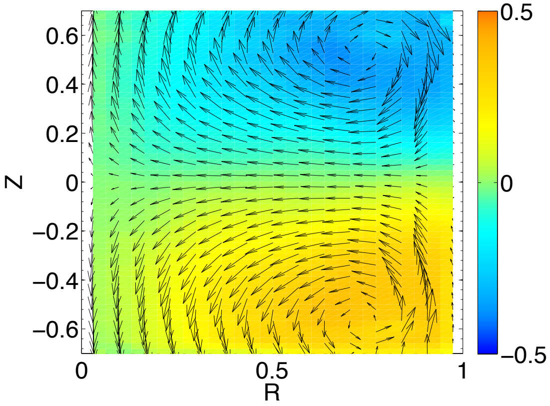
Mixing Property of Turbulence

$$\rho(\sigma)$$ Probability distribution

$$S = -\int \rho \ln \rho \, dx$$ Mixing Entropy



Von karman turbulence



Kinetic Energy $$E = \frac{1}{2}\int \xi \psi \, dx + \frac{1}{2}\int \frac{\sigma^2}{r^2} dx$$

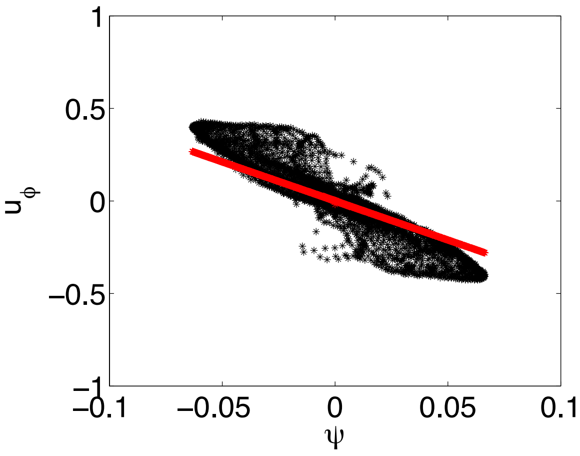
Helicity+Casimirs $$I = \int \sigma \, dx \quad H = \int \sigma \xi \, dx$$

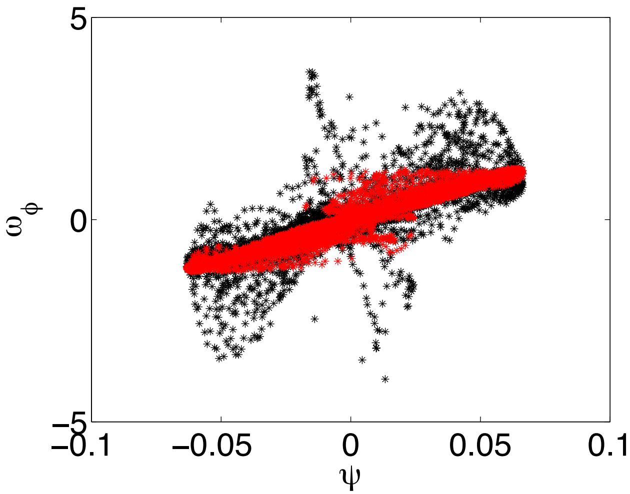
Leprovost *et al.*
Phys. Rev. E. **73** , 2006

Characterization of mean states

$$u_\theta = B\psi + A$$

$$\omega_\theta = Bu\theta + D\psi + Cr$$



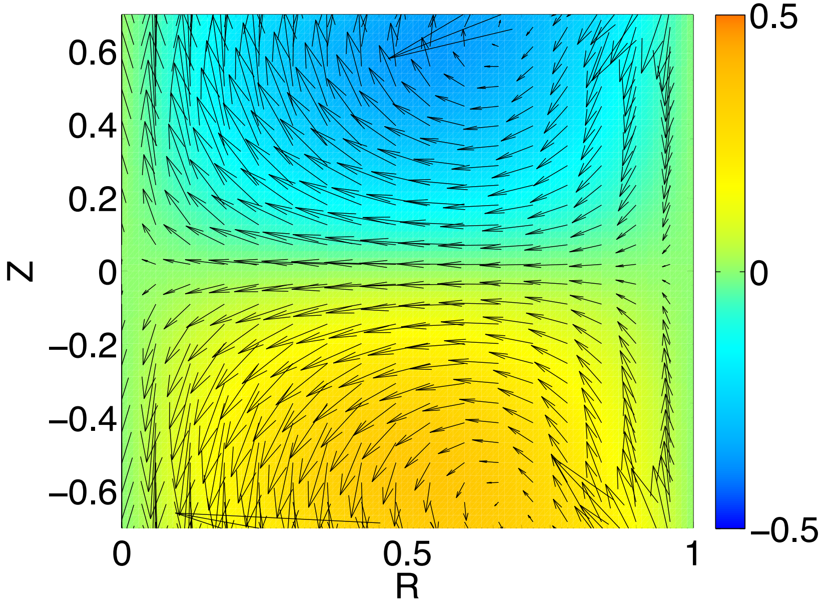


Rather well satisfied in the flow

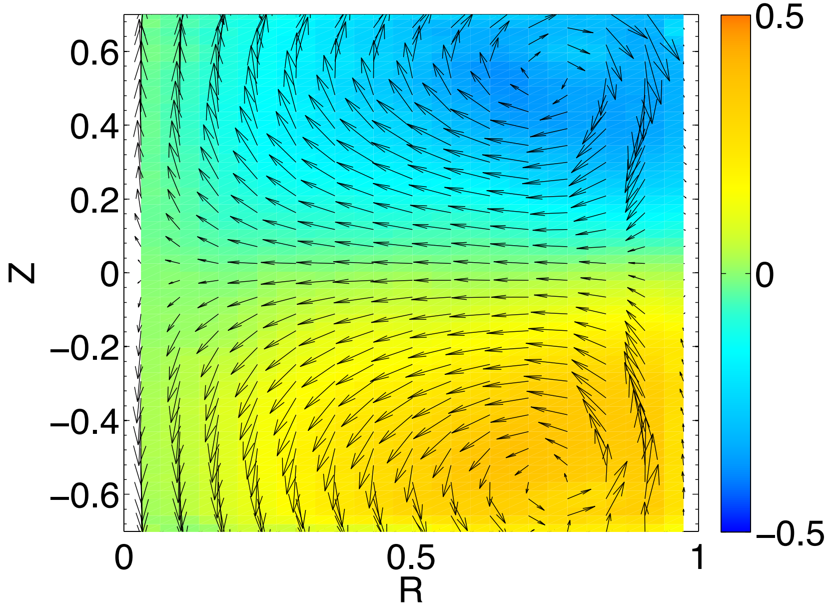
Characterization of mean states

$$u_\theta = B\psi + A$$

$$\omega_\theta = Bu\theta + D\psi + Cr$$



Stat Model

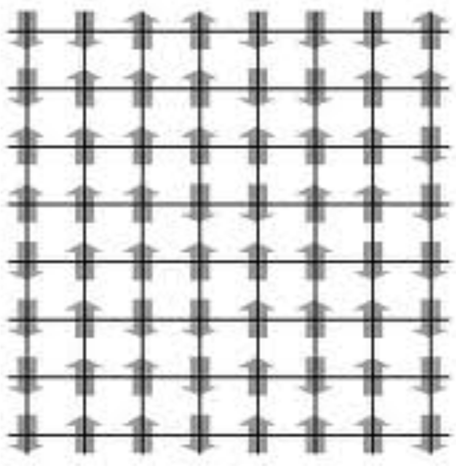


Experience

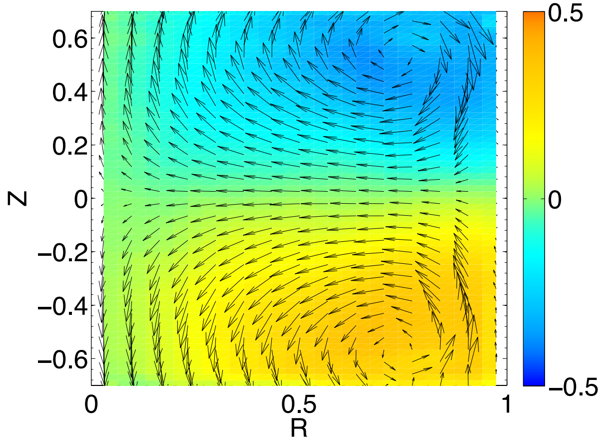
Turbulence vs Ising : OK

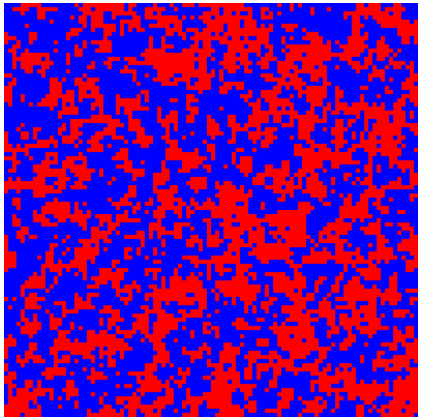
Von Karman

Ising



Disordered State

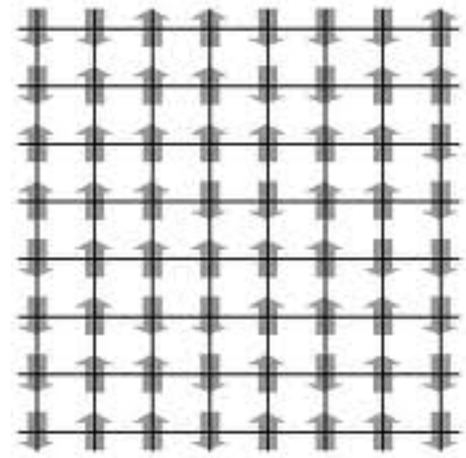




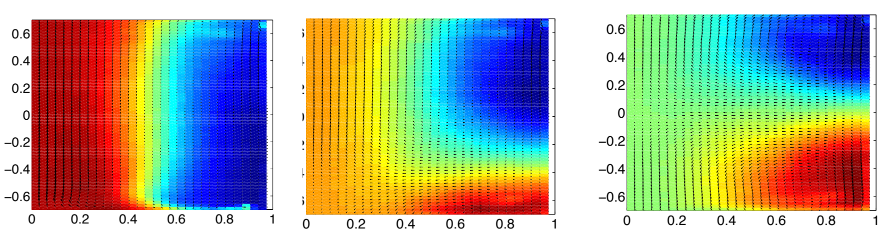
Turbulence vs Ising : OK

Von Karman

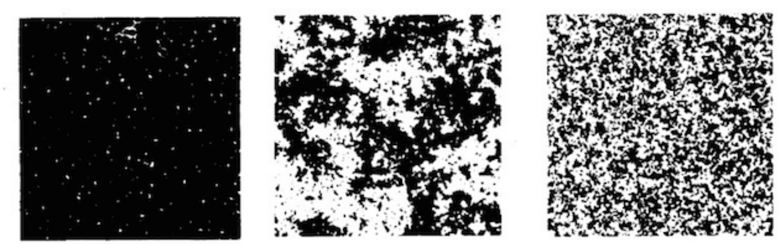
Ising



Transition from disordered state to ordered state



T>Tc Tc T<Tc



T<Tc Tc T>Tc

An Ising-turbulence

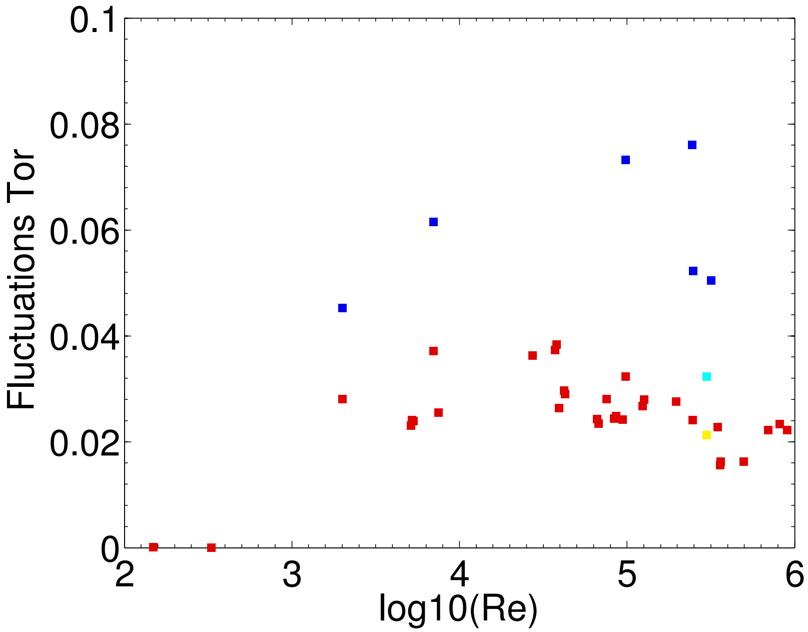
(Mean Field because long-range)

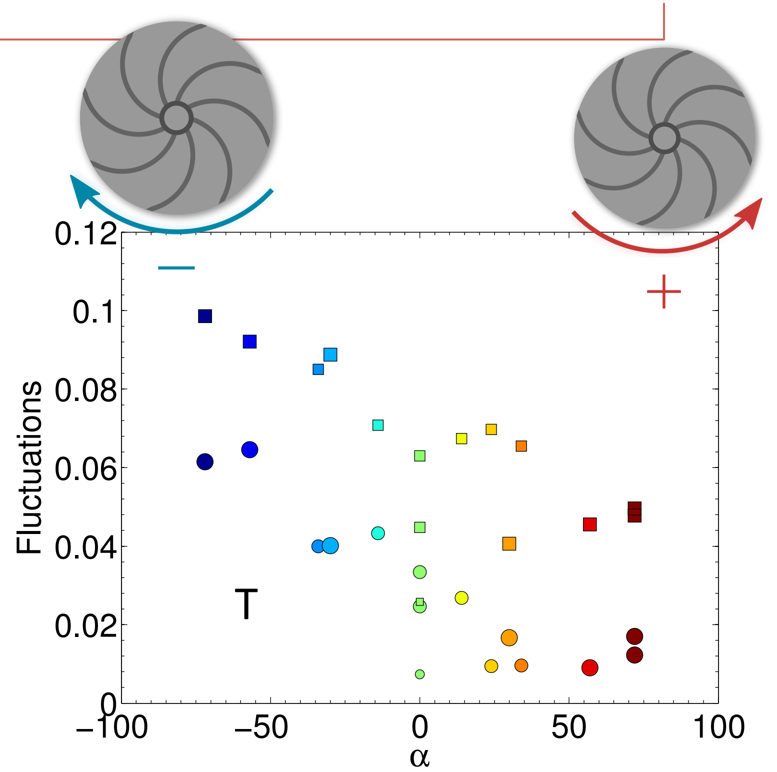
Quantity	Analog	Critical exponent	Hydrodynamic name
Spin	$(ru_\phi, \omega_\phi/r)$		Angular momentum
Magnetization	(I, X_{tot})	$1/2$	and circulation
Thermostat			Forcing and dissipation
Temperature	$(1/\beta_{micro}, 1/\beta_{cano})$		Toroidal fluctuations
Symmetry breaking field	θ		Asymmetry
Susceptibility	(χ_I, χ_X)	-1	
Spontaneous magnetization	$(\xi_+ - \xi_-, \Delta K_p)$	$1/2$	Torque asymmetry
Coercitif field	$\Delta\Theta$	$1/2$	
Specific heat	$E_{tor}/Fluc(E_{tor})$	0	inverse turbulence intensity

Table 3. Analogy between a two components spin system and the von Karman flow

Thalabard PhD: showed that axisymmetric NSE equivalent to a two component spin model

Temperature in VK

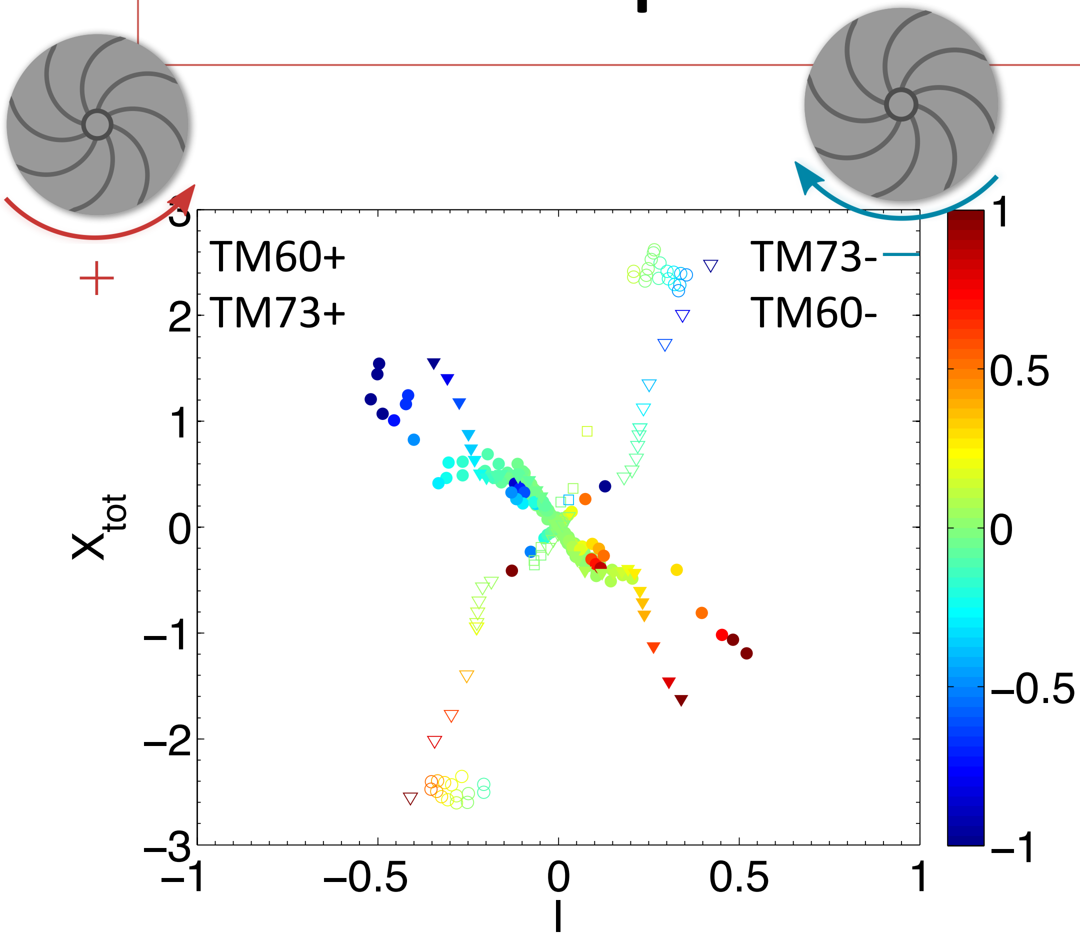




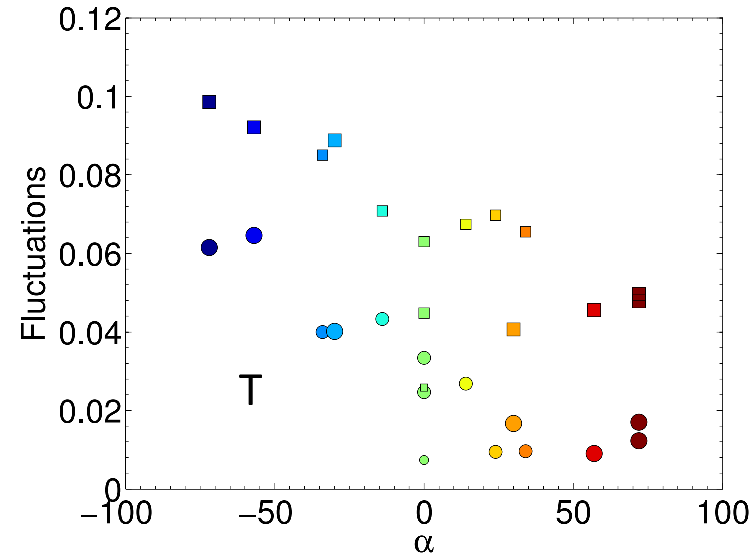
The Reynolds Changes the temperature

The angle alpha changes the temperature

Propeller's influence



The forcing polarizes the mean spin direction

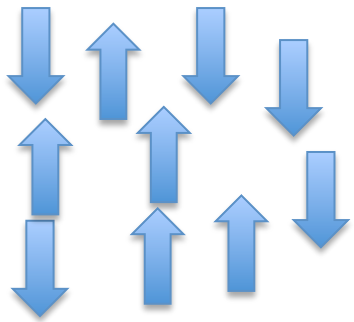


The angle alpha changes the temperature

Brisure de symetrie pour Ising

Diu *et al.* - Physique Statistique - Hermann

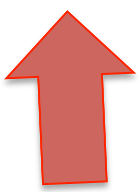
Systeme de spin

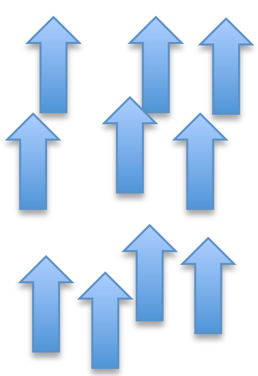


Symetrie S->-S

$$M = \sum S_i = 0$$

Champ h



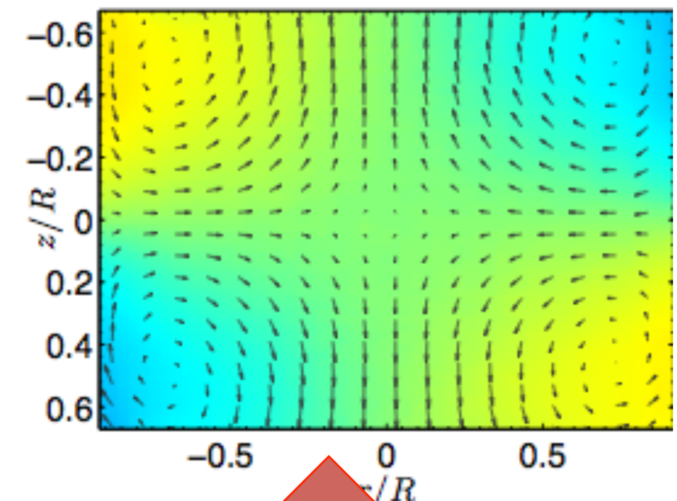


$$M \neq 0$$

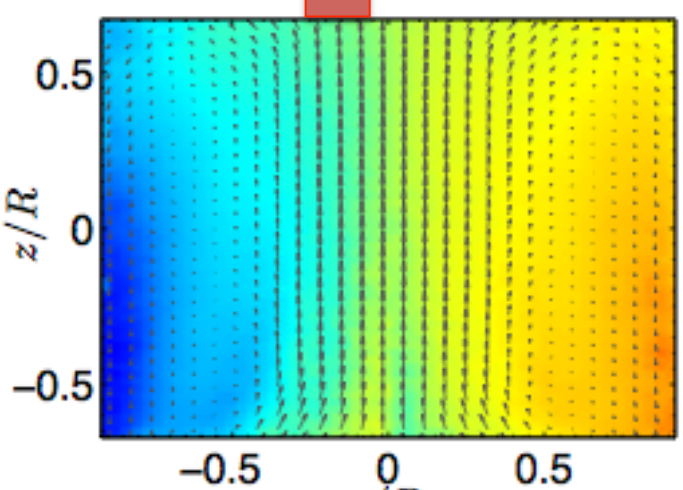
Brisure de symetrie von Karman

Cortet *et al.* –*Physical Review Letters* **105 214501 (2010)**

Champ turbulent moyen



Rotation **θ**



Symetrie ru->-ru

$$\langle I \rangle = \sum r \langle u_\theta \rangle = 0$$

$$\langle I \rangle = \sum r \langle u_\theta \rangle \neq 0$$

An Ising-turbulence

(Mean Field because long-range)

Quantity	Analog	Critical exponent	Hydrodynamic name
Spin	$(ru_\phi, \omega_\phi/r)$		Angular momentum
Magnetization	(I, X_{tot})	$1/2$	and circulation
Thermostat			Forcing and dissipation
Temperature	$(1/\beta_{micro}, 1/\beta_{cano})$		Toroidal fluctuations
Symmetry breaking field	θ		Asymmetry
Susceptibility	(χ_I, χ_X)	-1	
Spontaneous magnetization	$(\xi_+ - \xi_-, \Delta K_p)$	$1/2$	Torque asymmetry
Coercitif field	$\Delta\Theta$	$1/2$	
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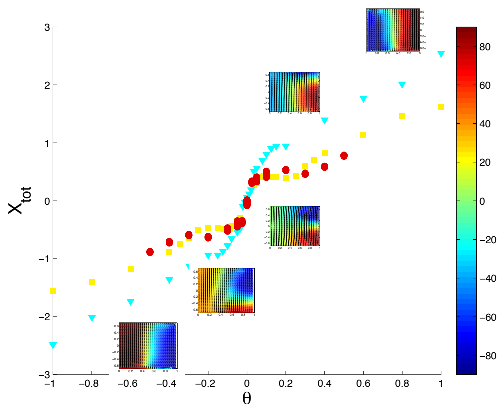
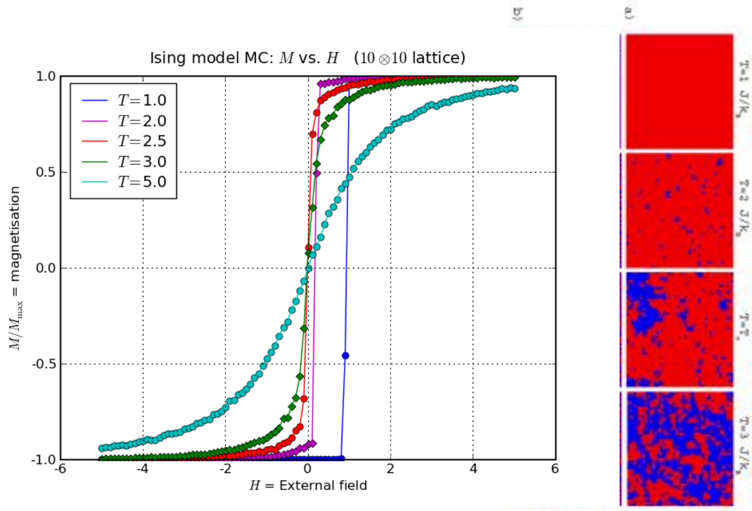
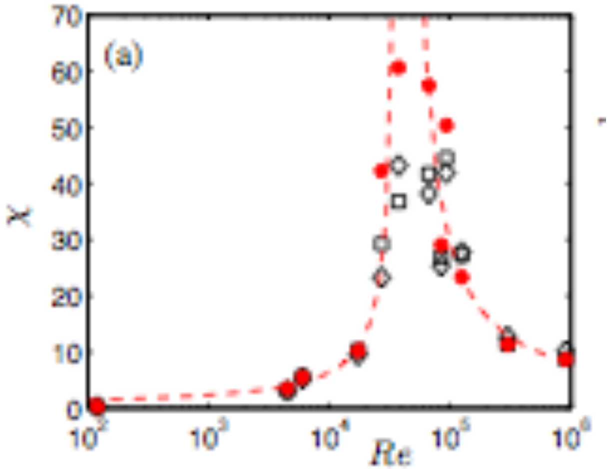
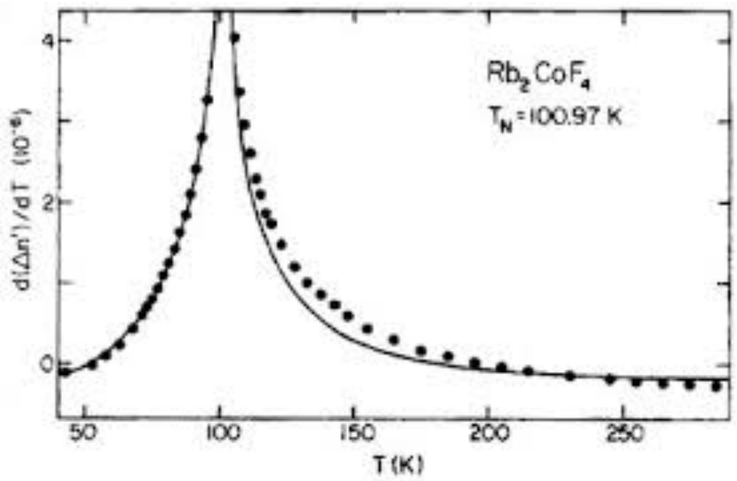
Thalabard PhD: showed that axisymmetric NSE equivalent to a two component spin model

Transition de phase

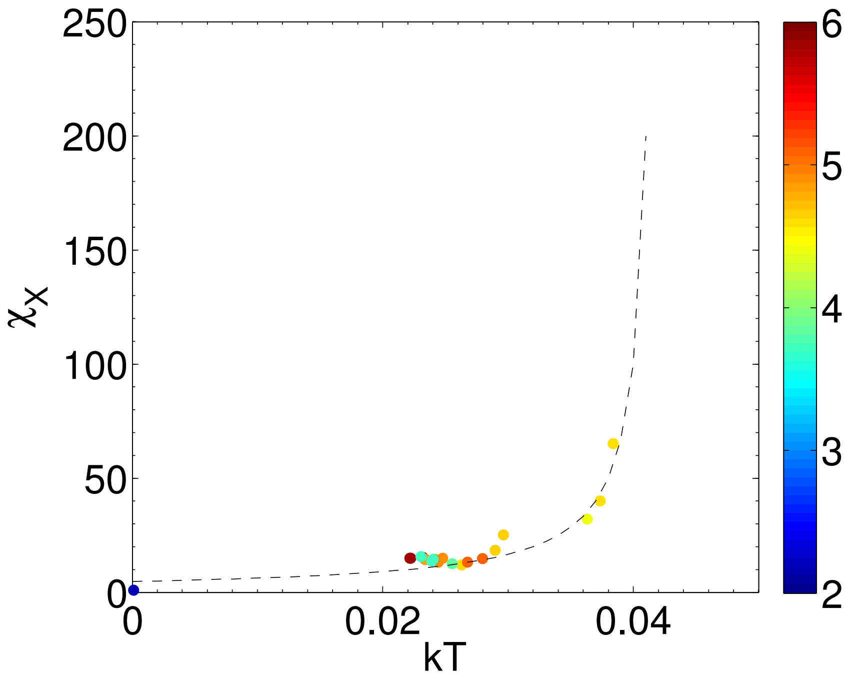


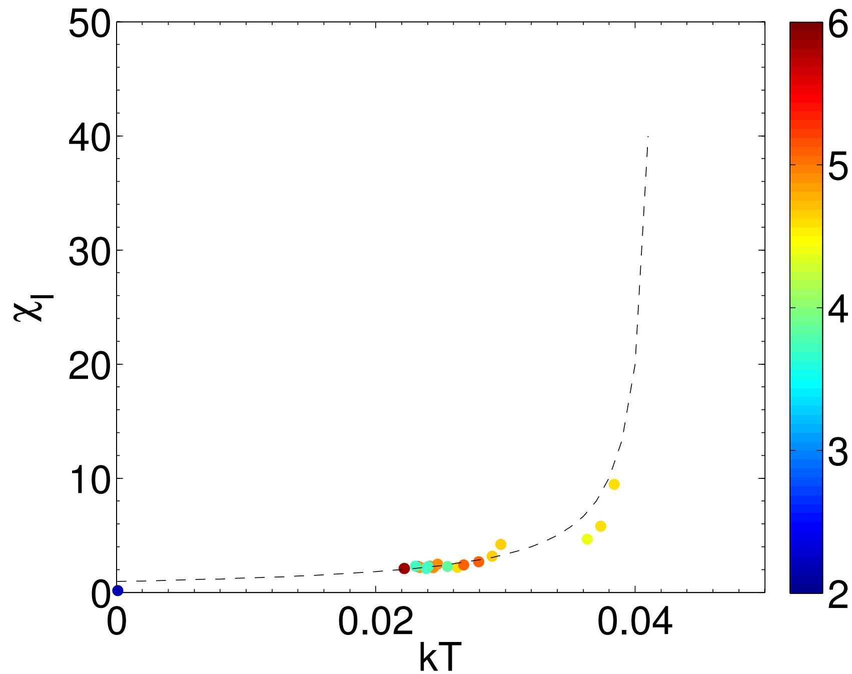






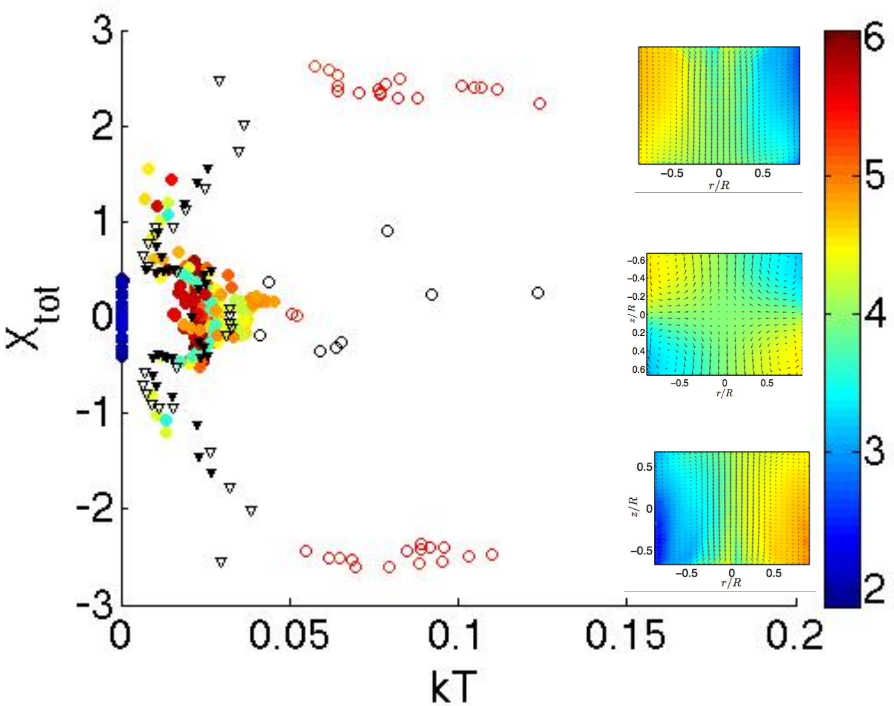
Phase transition (ii)

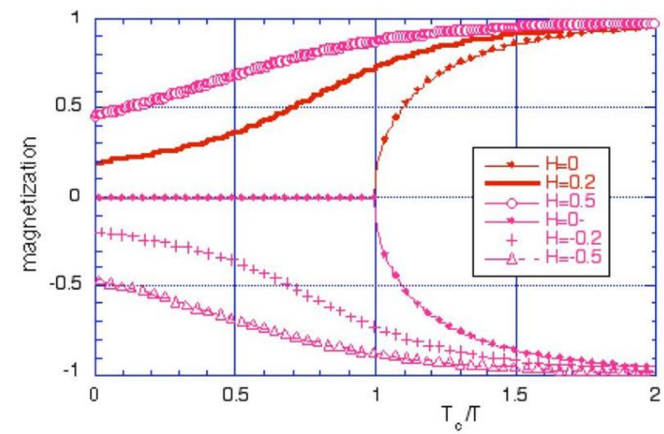




Diverging susceptibilities; exponent -1

Transition de phase (iii)





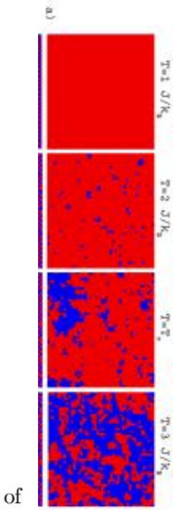
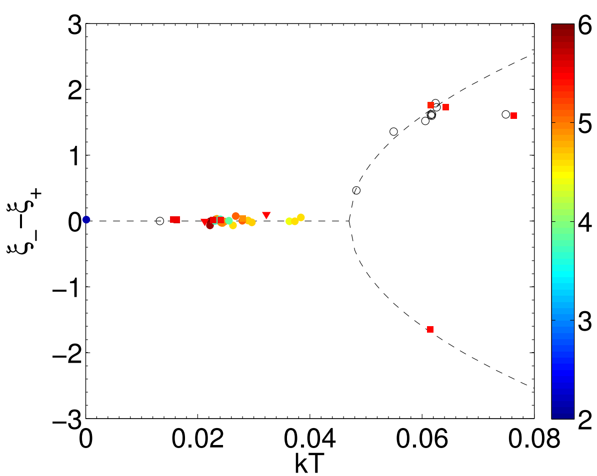
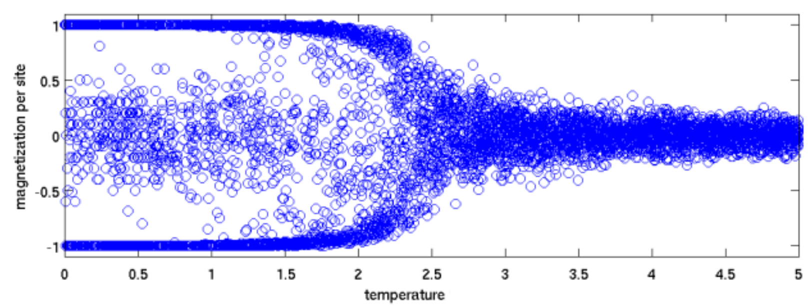
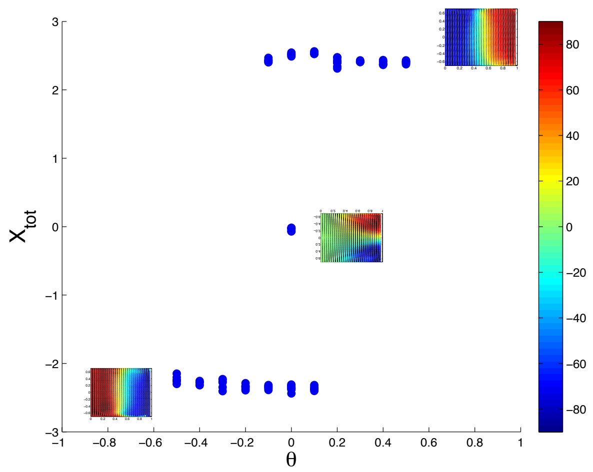


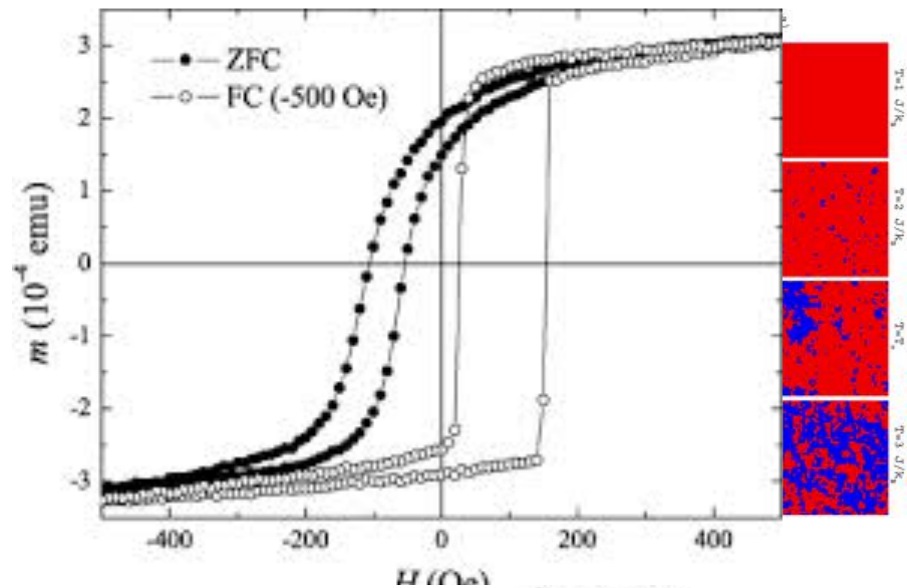
Figure 5: Magnetization, $<\sigma>$, versus temperature, $1/K$ for fixed values of the dimensionless magnetic field, here denoted as H.

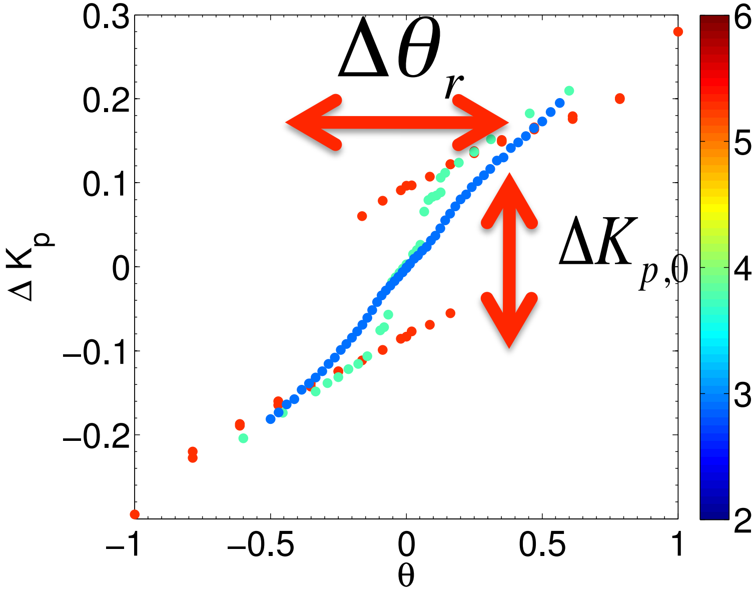


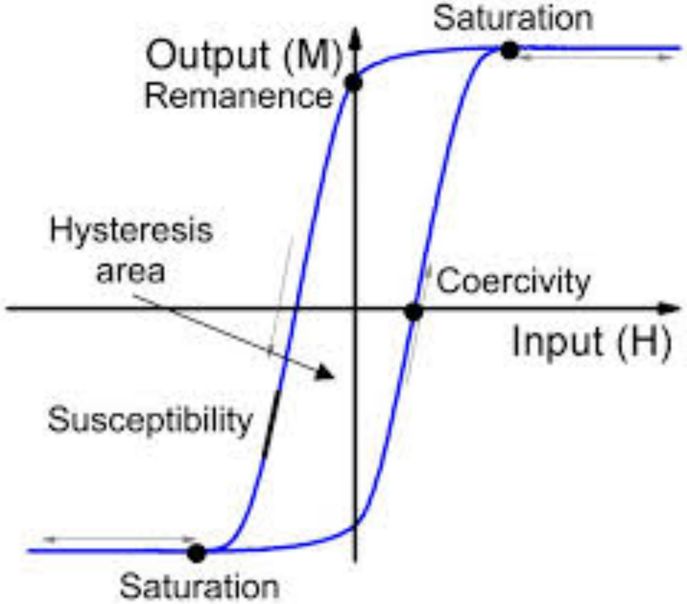


Transition de phase (ii)

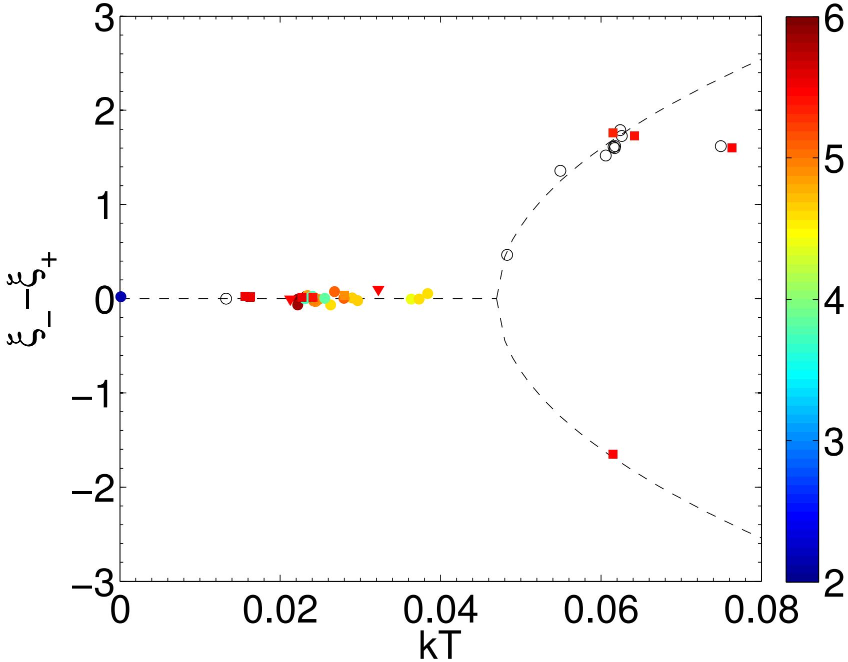


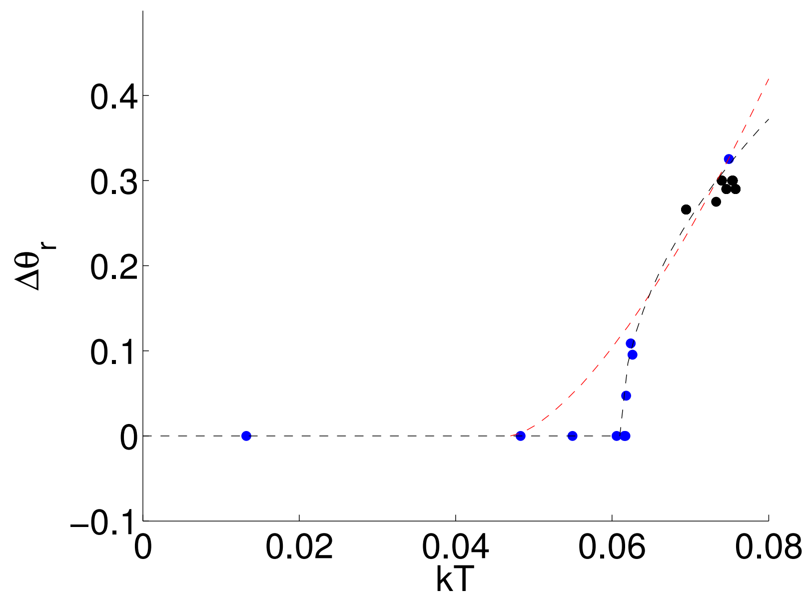






Phase transition (iii)

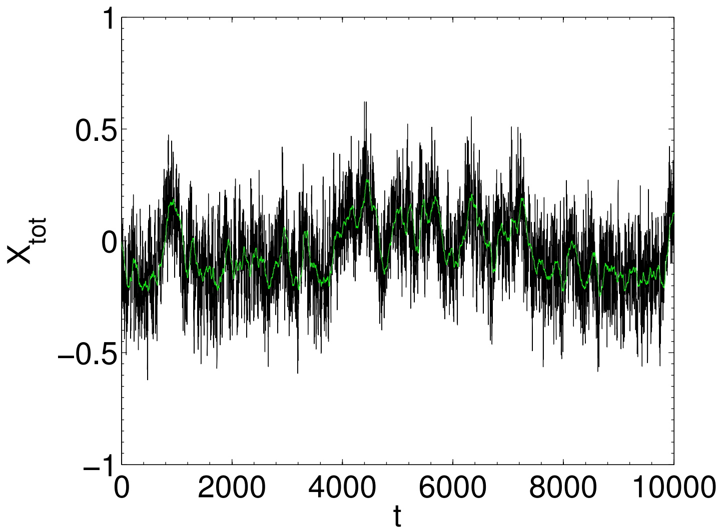


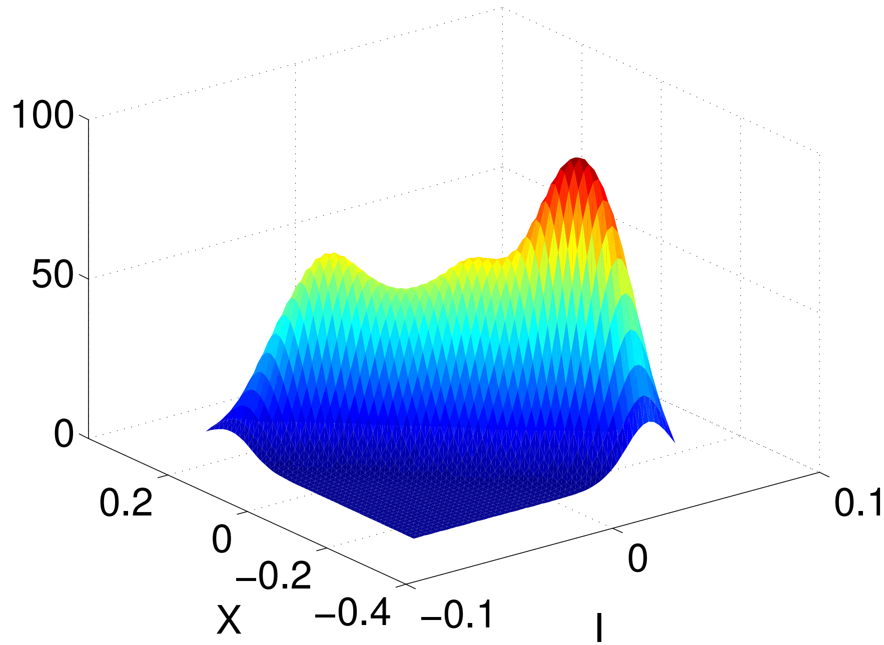


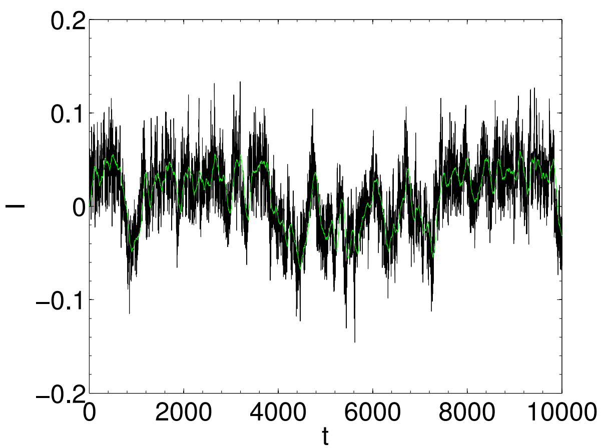
Mean Aimantation: sqrt(T-Tc)

Coercivity: sqrt(T-Tc)

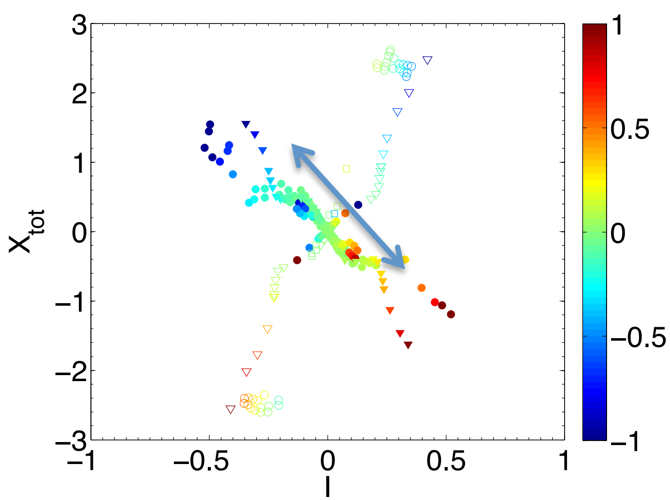
Goldstone Boson?



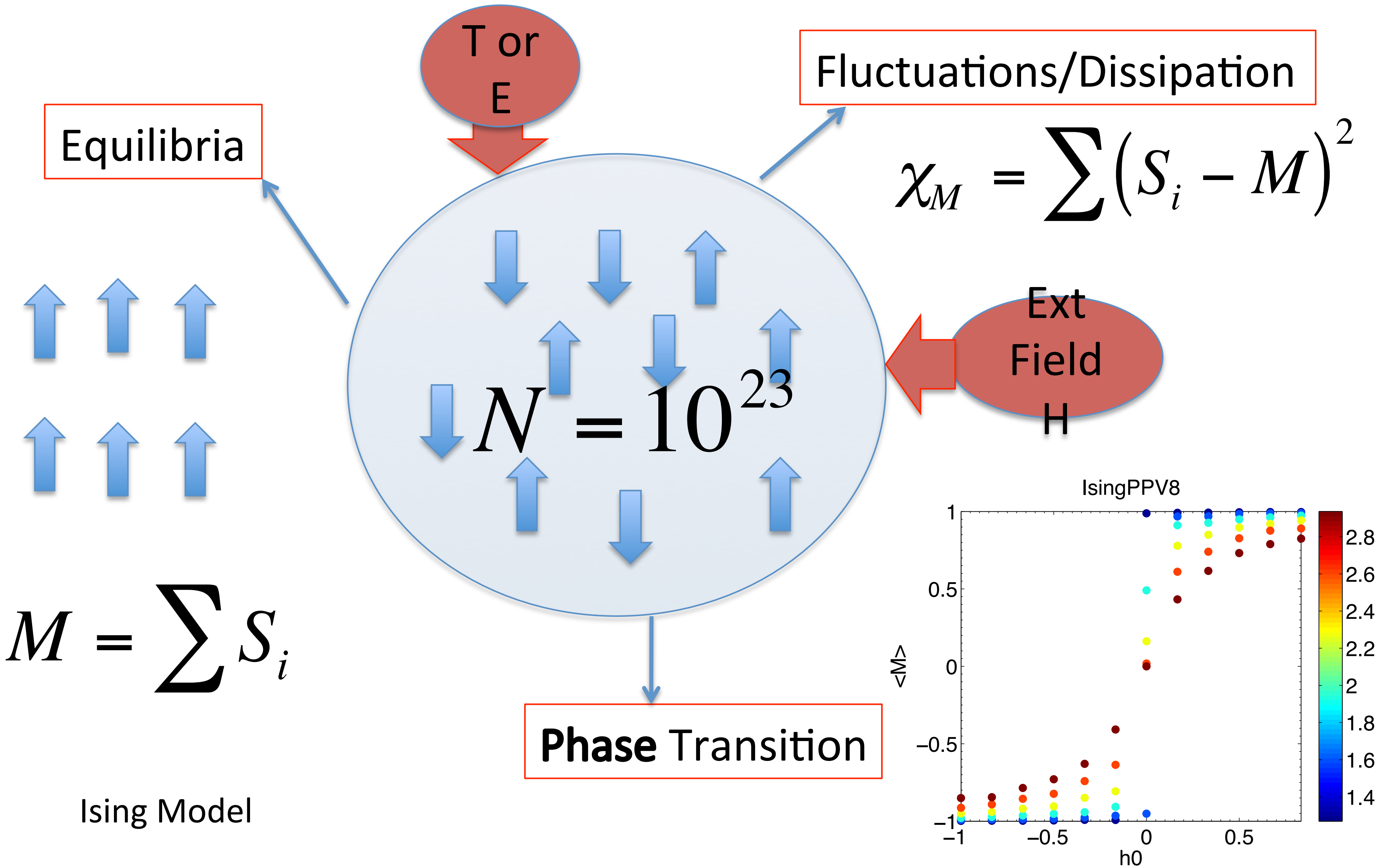




No free rotation
Wandering along
Polarization



Ferromagnetism

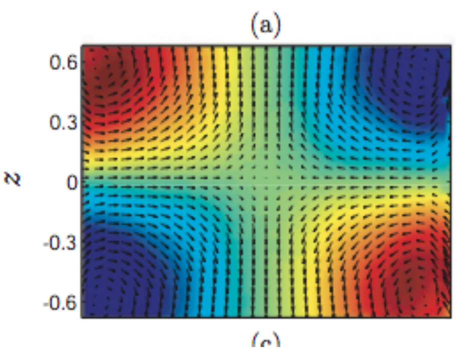


FerroTurbulence

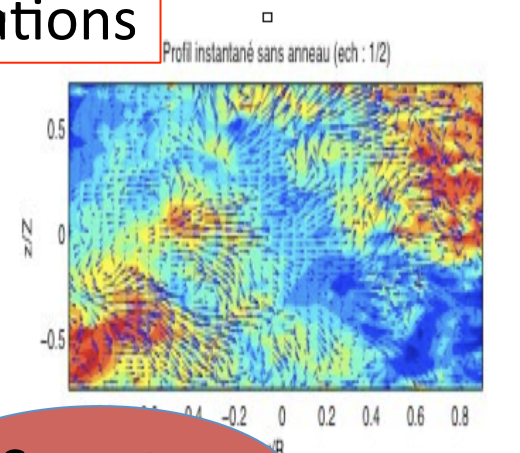
F, Re and Diss

$$N = \mathrm{Re}^{9/4}$$

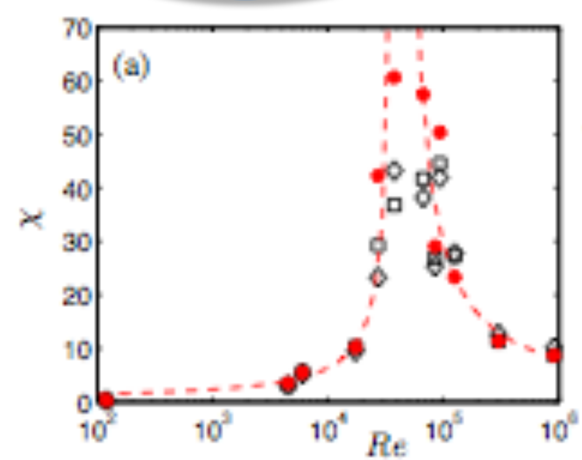
Stationary States



Fluctuations



Symetry Breaking $\theta \subset [-1,1]$



Bifurcations

Von Karman Experiment

What about ultra-high Reynolds number turbulence?

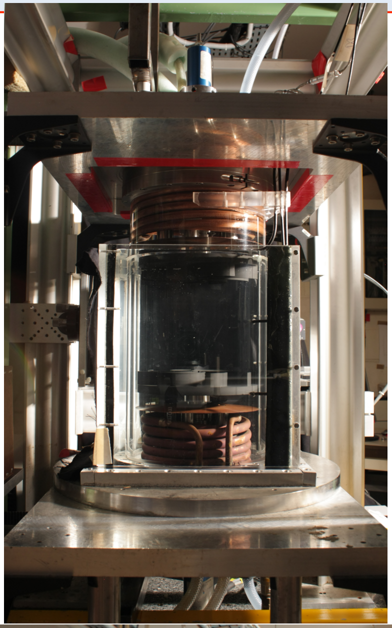
SHREK

SUPERFLUIDE À **H**AUT **R**EYNOLDS EN **E**COULEMENT DE VON **K**ÁRMÁN

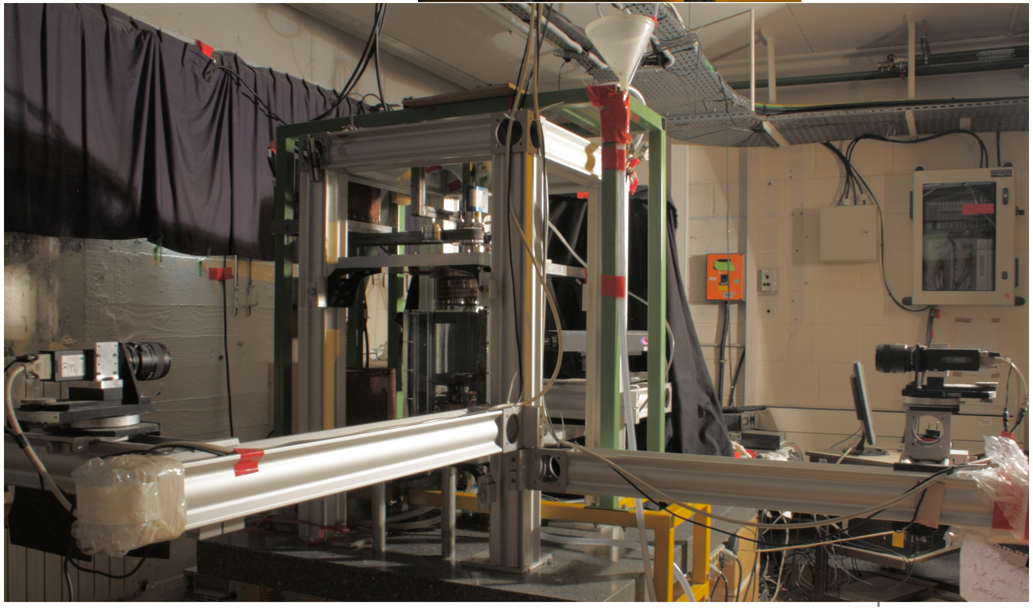
ENSL, CEA/SBT, CEA/SPEC, I. NEEL, LEGI, Luth

SHREK vs VKE

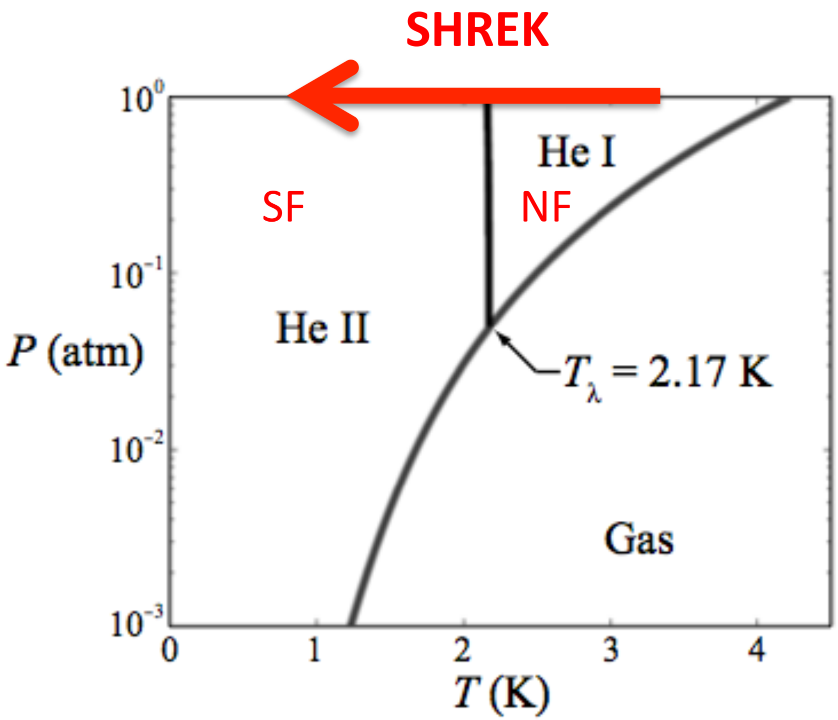
1.2 m

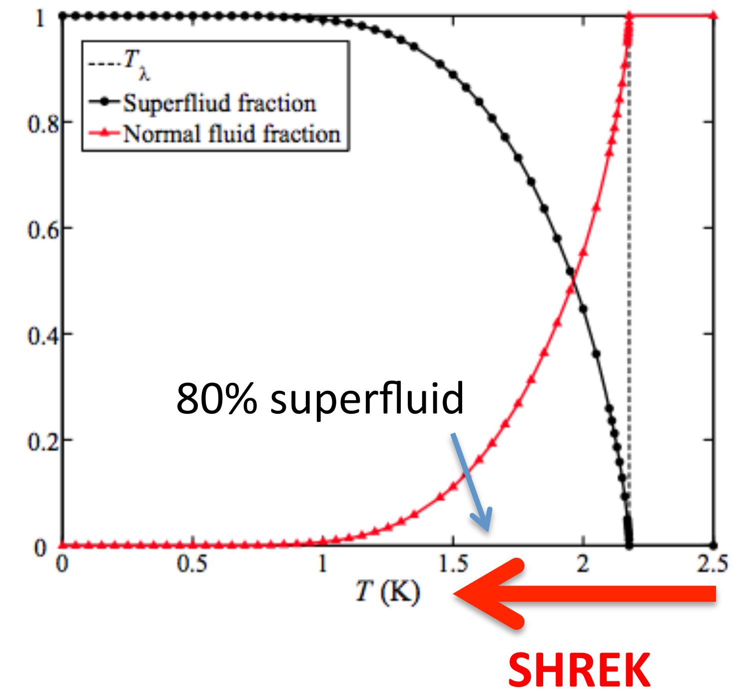


0.3 m



De Exploring the issue through Helium 4

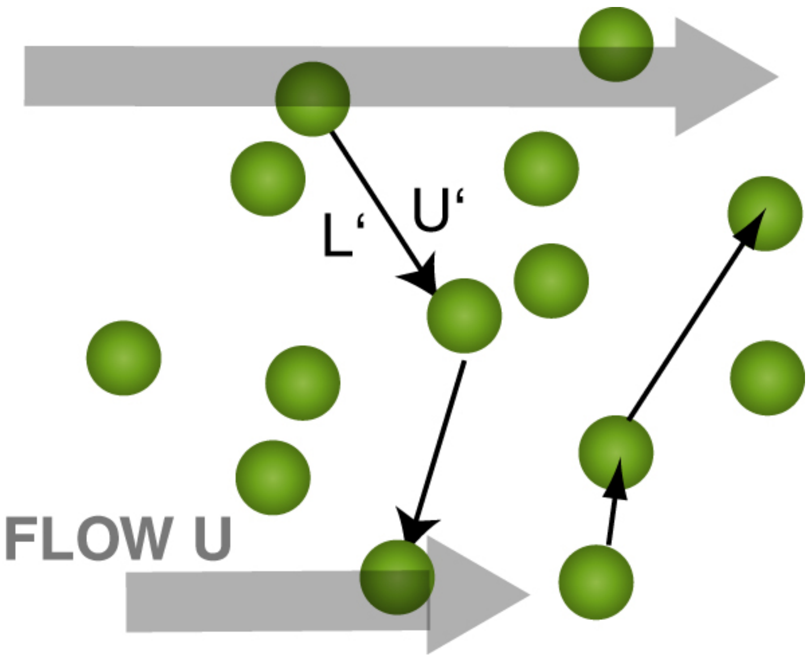




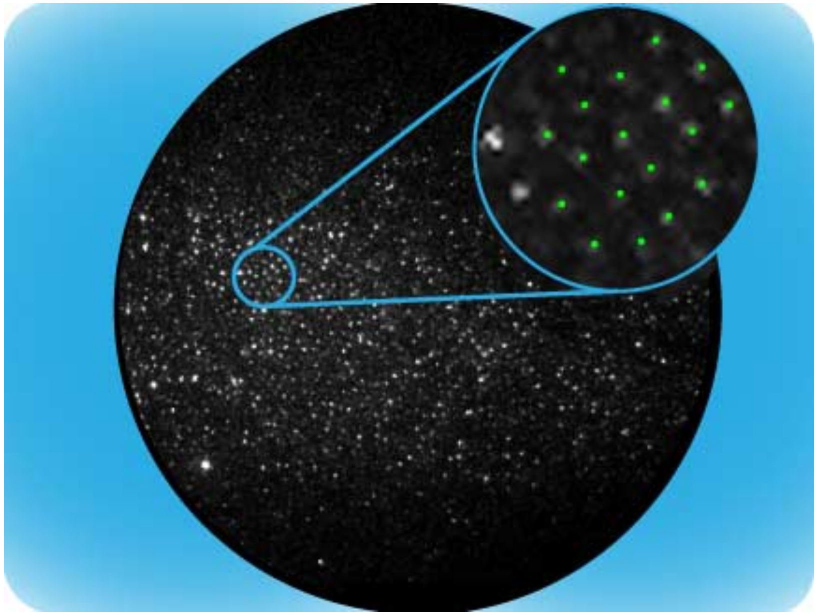
→ Helium 4 special properties are used

→ Technological issues: cooling, measurements

Fluid vs superfluid dissipation

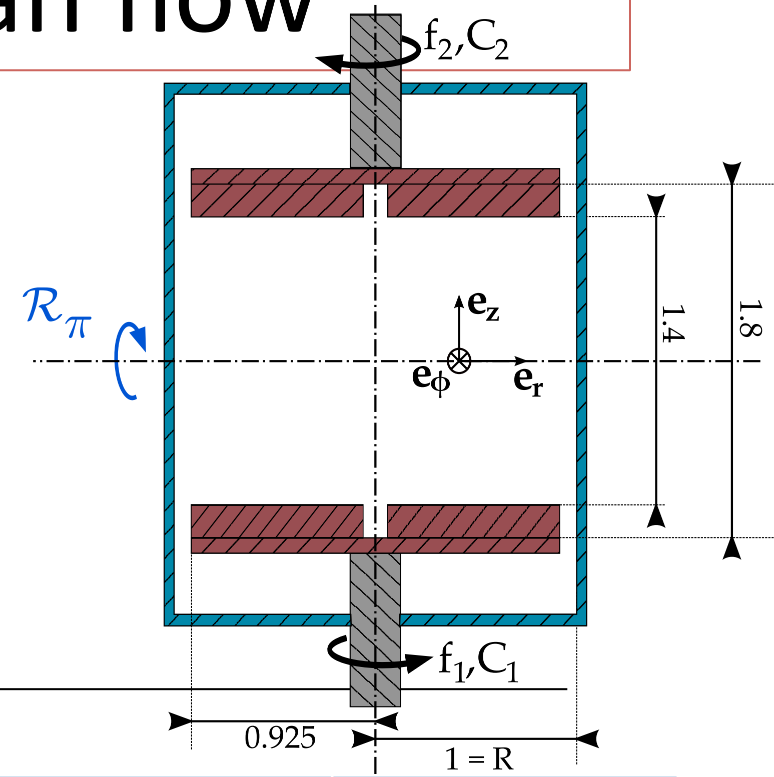


Fluid
dissipation=viscosity



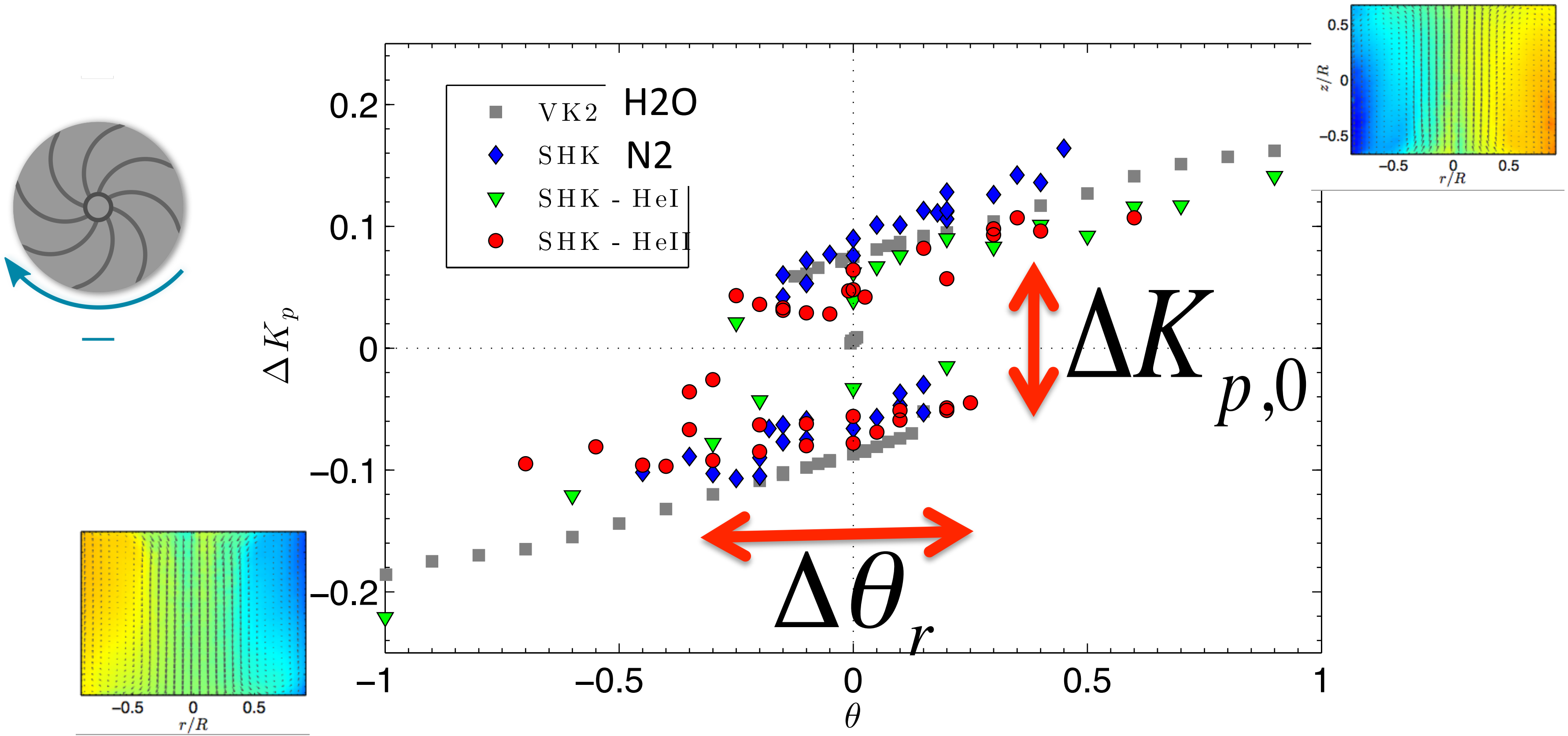
Superfluid
Dissipation=quantized vortices
reconnection

The von Karman flow



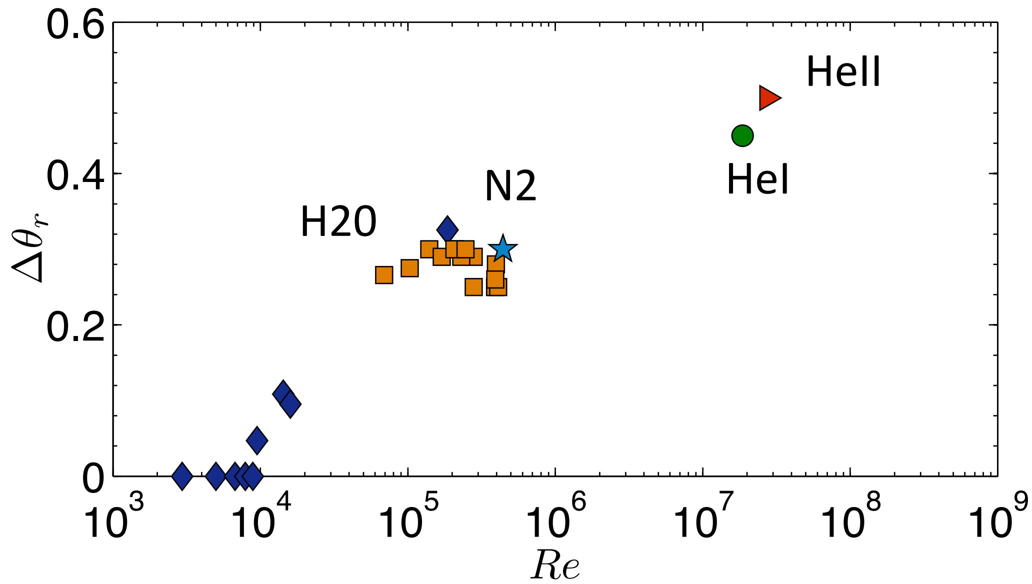
SetUp	Fluid	P(bars)	T(K)	Re
SHREK	HeI	1.1	2.62	10^8
SHREK	HeII	1.1	1.63	?
SHREK	N2	3.73	284	10^5
VKE VKE	H2O Glyc	1.8 1.8	300 300	10^5 10^2

Properties of the hysteresis cycle

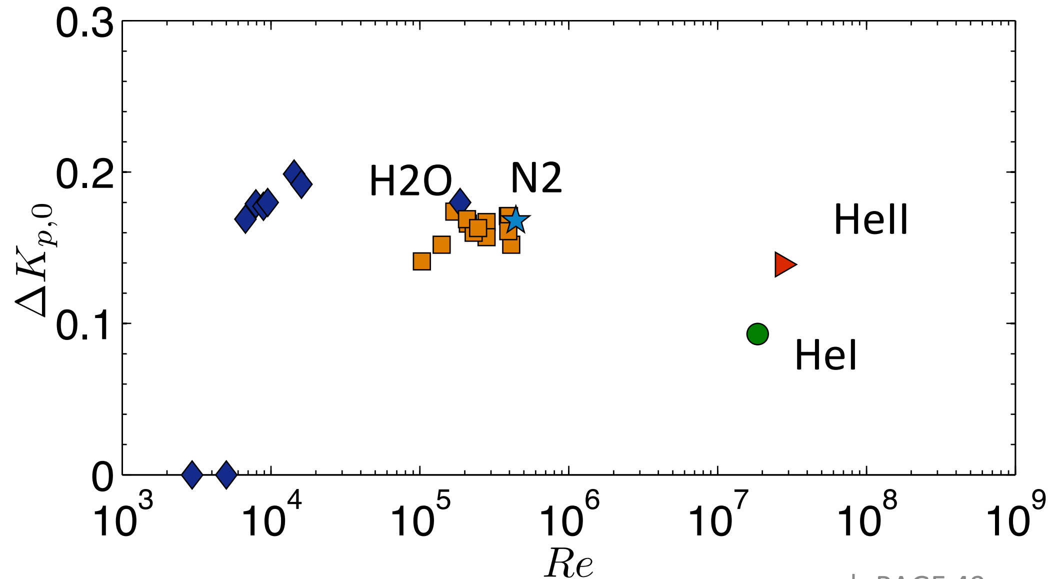


Same hysteresis cycle with Re dependent properties

Properties of the hysteresis cycle



Possibility to define an « effective viscosity » for HeII

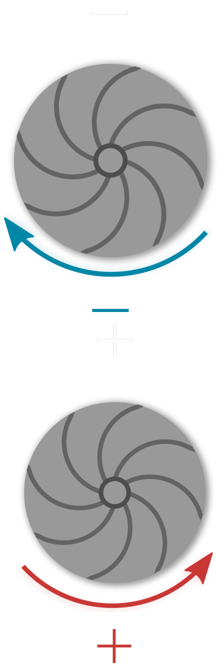


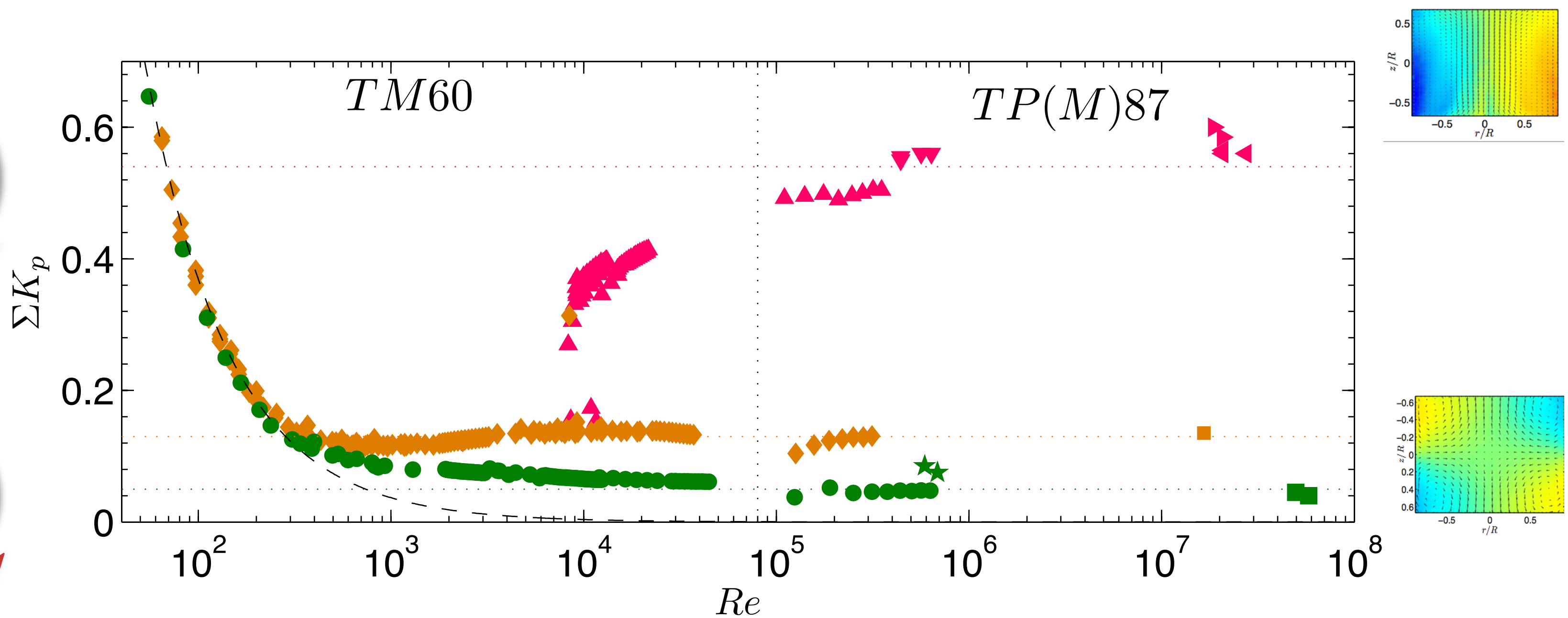
Effective viscosity of the order of

$$\nu_{eff} = 10^{-8} m^2 s^{-1}$$

Questions raised by our results

Dissipation for different symmetric Forcing

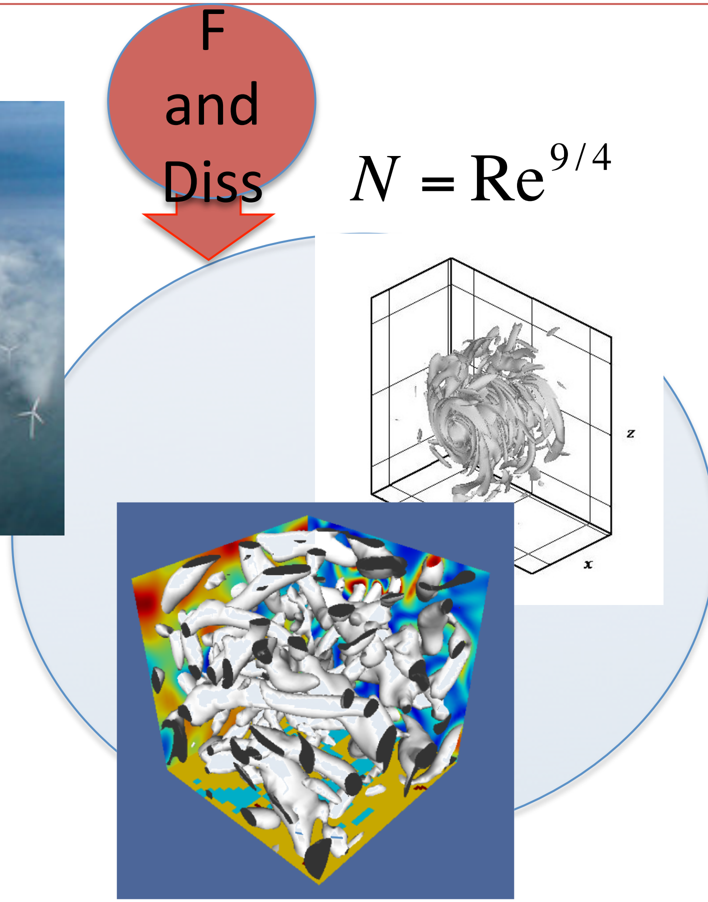




Connection between dissipation and statistical mecanics????
Temperature influence?

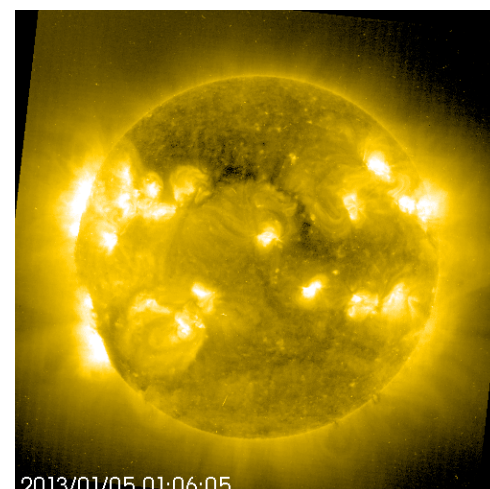
Can we describe these structures by statistical mechanics?

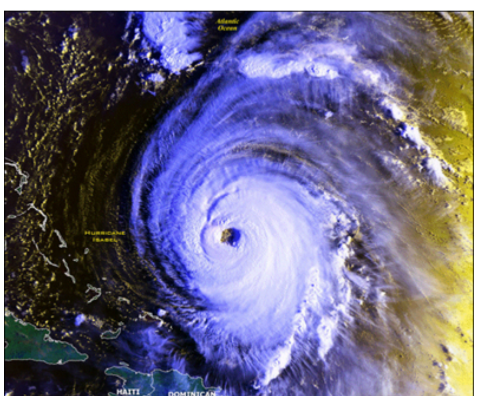
N=3 10^{20}



$$N = \mathrm{Re}^{9/4}$$

N=3 10^{20}





Out-of-equilibrium statistical physics!

N= 10^{27}

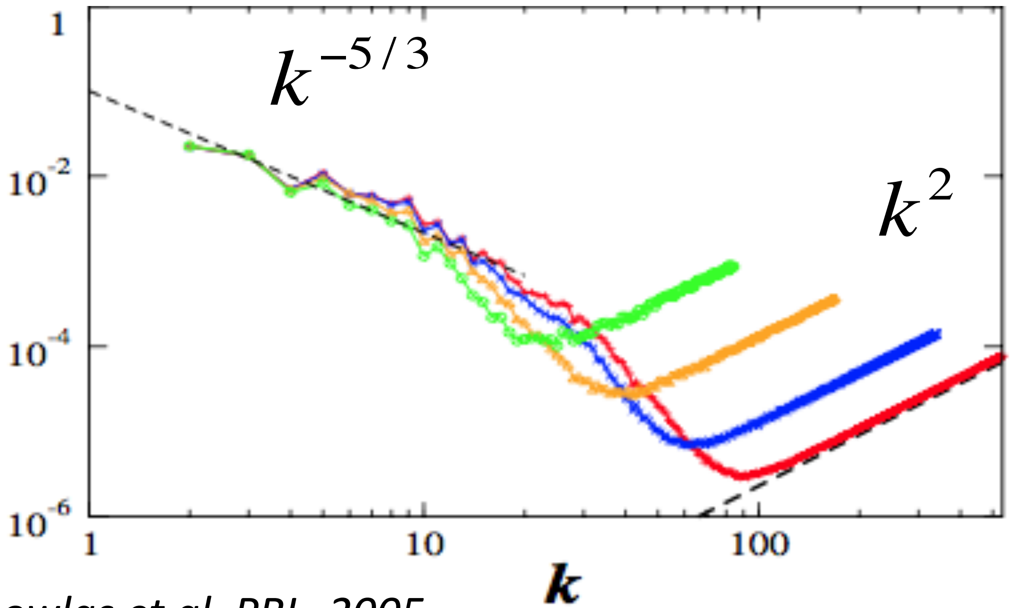
The UV catastrophy

« Classical thermodynamics »: energy equipartition between modes

$$E(k) = \vec{u}(k)\vec{u}(-k)k^{d-1} \propto k^{d-1}$$

UV Catastophy!

E(k)



Can we prevent it?

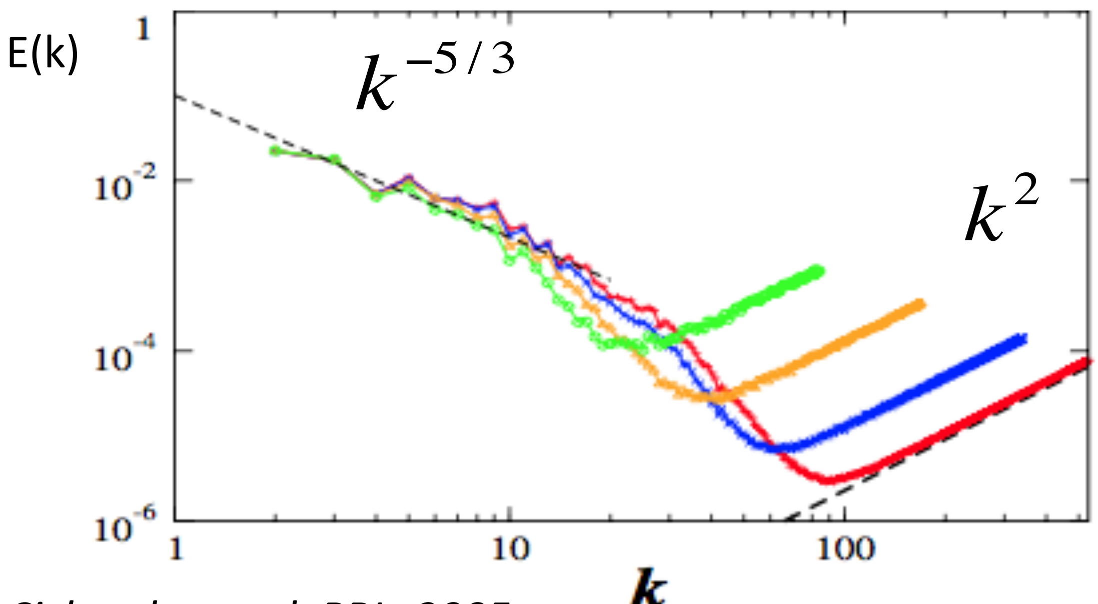
Cichowlas et al, PRL, 2005

The UV catastrophy (ii)

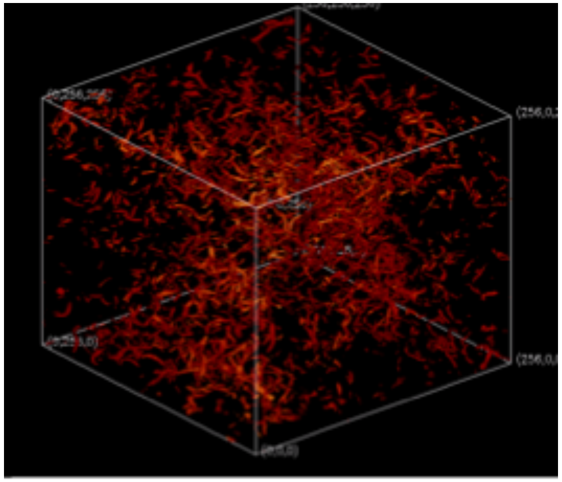
In 3D, vortex stretching

$$D_t \vec{\omega} = \vec{u} \bullet \nabla \vec{\omega}$$

Enstrophy not bounded!



Cichowlas et al, PRL, 2005



Unsolved problem!!!!

The UV catastrophy (iii)

In 2D: YES! We can

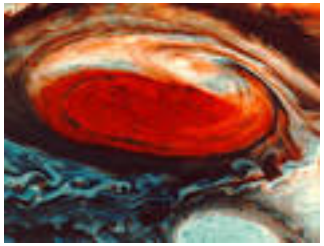
Conservation of additional invariants

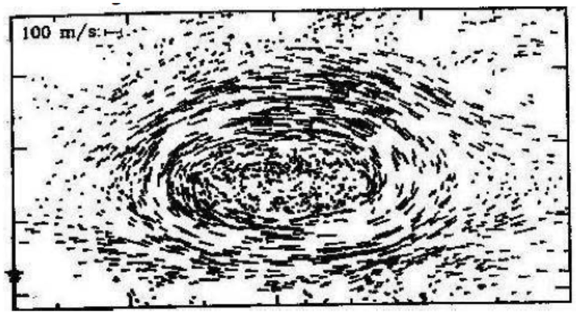
$$E = \int \vec{u} \bullet \vec{u} \, d\vec{x}$$

$$\Omega = \int \omega^2 \, d\vec{x}$$ Enstrophy

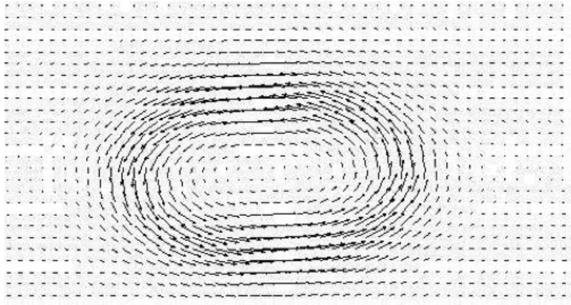
$$C_n = \int \omega^n \, d\vec{x}$$ Casimirs

Bonus: yes to Q2





Observation (Voyager)



Statistical Equilibrium

Velocity field of Jupiter's Great Red Spot

Bouchet and Sommeria, JFML, 2002

The UV catastrophy (iv)

What about 2D1/2=3D with symmetry?

If continuous symmetry:
Noether->Conservation of additional invariants

Casimirs

Exemple:

translation

$$C = \int p^n d\vec{x} \quad p = r \bullet u$$

rotation

$$C = \int \sigma^n d\vec{x} \quad \sigma = r \times u$$

« Axisymmetric turbulence »