Cryogenic Thermal Convection
Experimental Investigation

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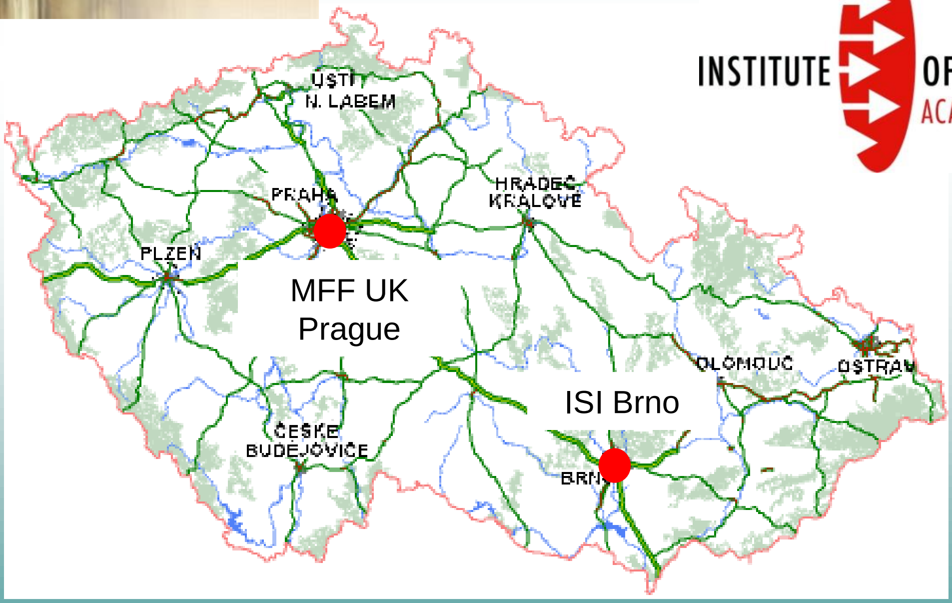
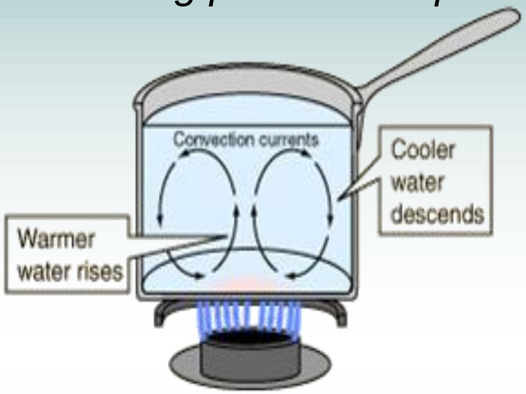
[2]Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, Praha, Czech Republic

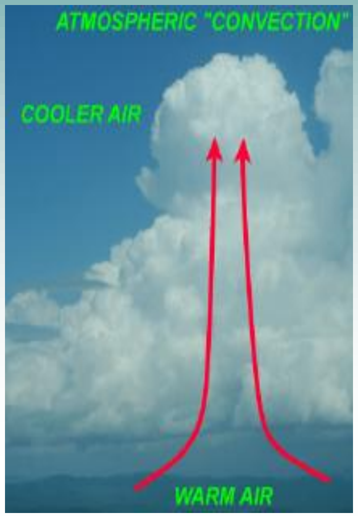


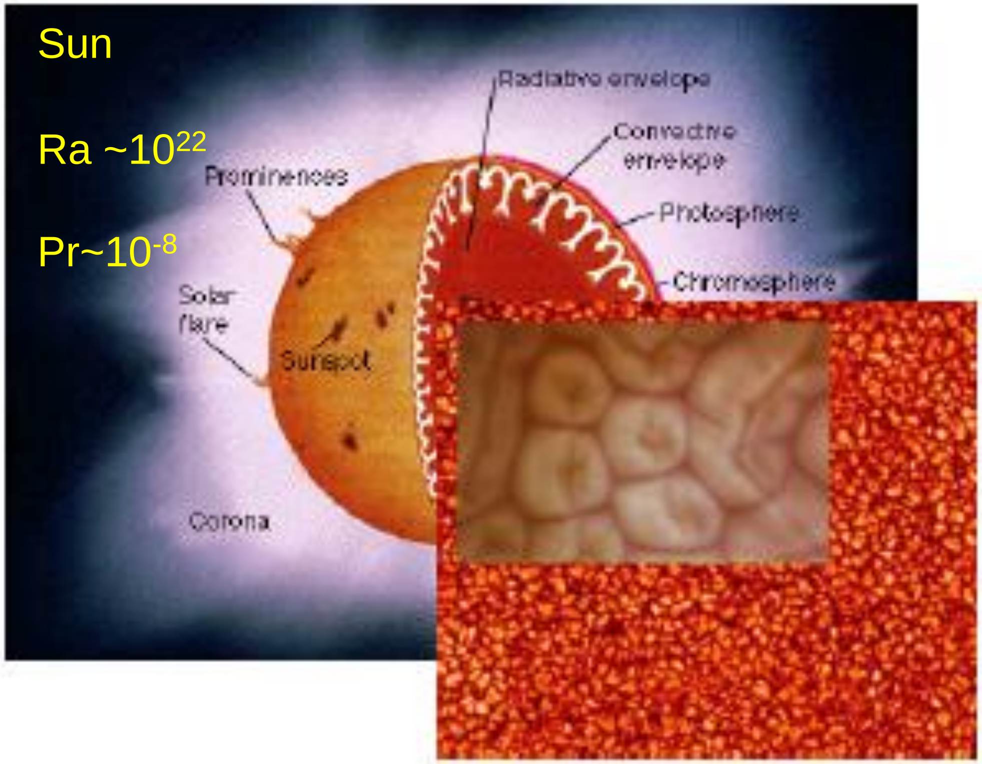
Les Houches, March 17, 2014

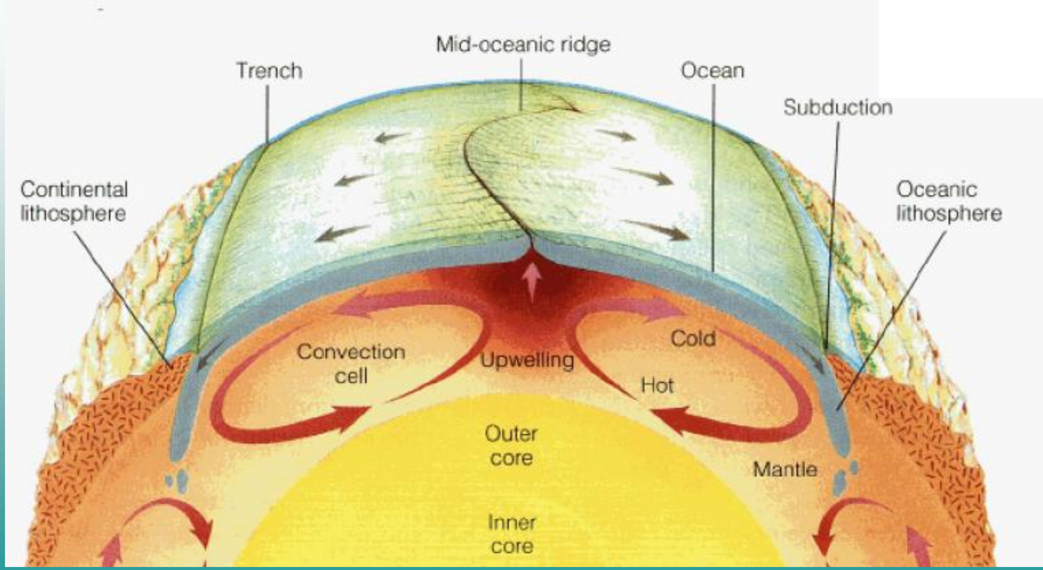
Examples of thermal convection: in everyday practice and at limiting values of the control parameters in Nature

Thermal convection transports and mixes heat from the bottom of a cooking pot to the top.







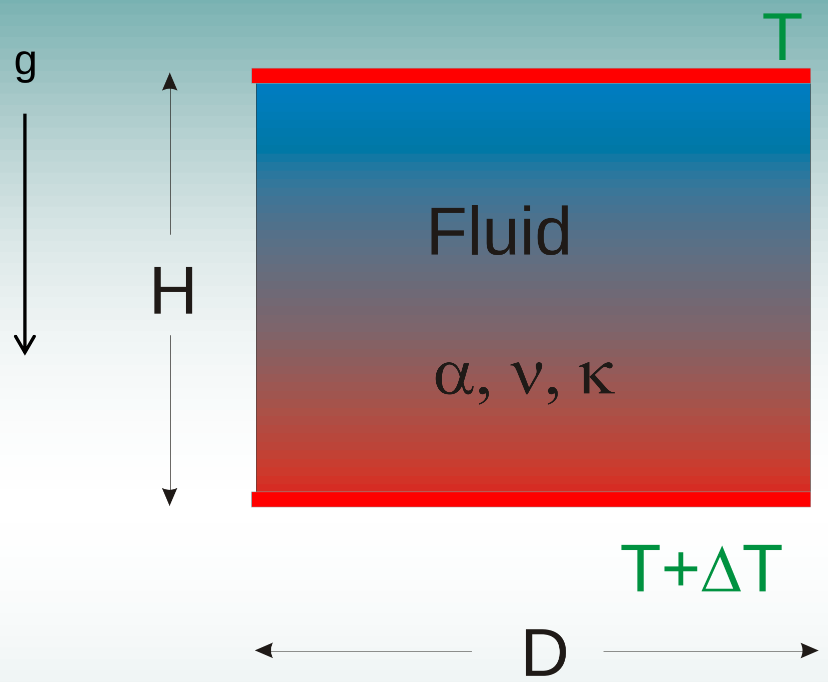


Mantle

Ra ~ 10^{7-9}

Pr~10^{23}

Rayleigh-Benard Convection - a simple model system



α fluid thermal expansion coefficient
ν fluid kinematic viscosity
κ fluid thermal diffusivity
λ fluid thermal conductivity

The convective heat transfer efficiency is described by the Nusselt number

$$\mathrm{Nu} = \frac{LH}{\lambda \Delta T}$$

H is the total convective heat flux density

Control parameters for convection

$$\mathrm{Ra} = \frac{g\alpha\Delta T H^3}{\nu\kappa}$$

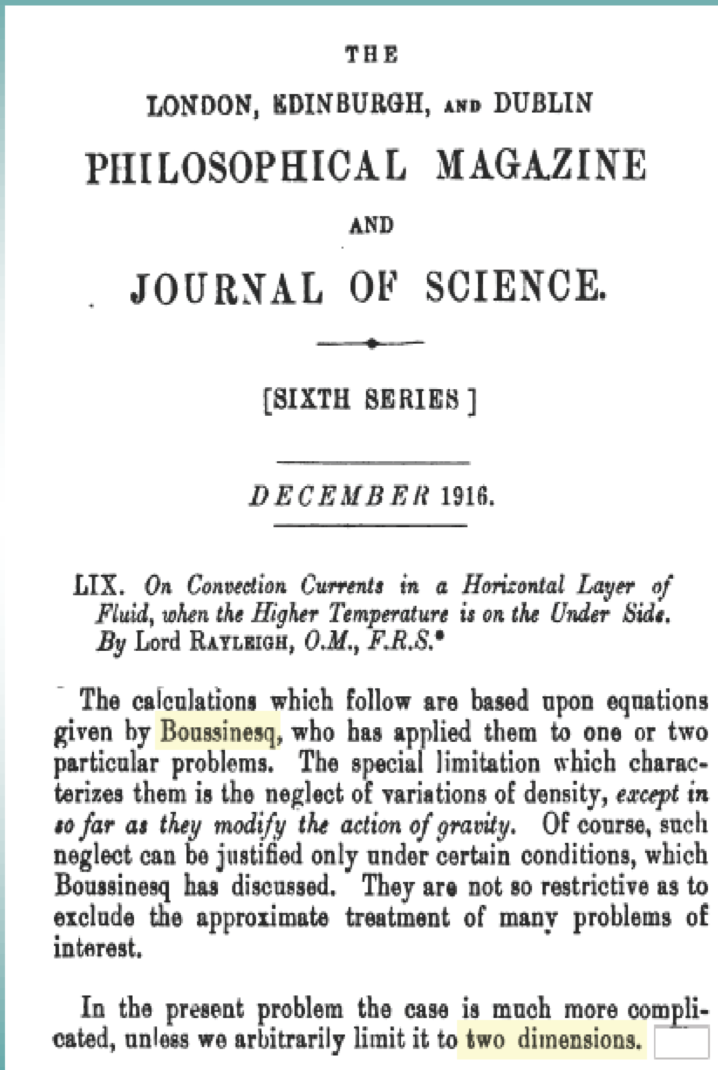
$$\mathrm{Pr} = \nu/\kappa$$

$$\Gamma = D/H$$

F. Chillà[1,a] and J. Schumacher[2,b] Eur. Phys. J. E (2012) 35: 58

New perspectives in turbulent Rayleigh-Bénard convection

RBC History



THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SIXTH SERIES.]

DECEMBER 1916.

LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.* *By* Lord RAYLEIGH, *O.M., F.R.S.**

The calculations which follow are based upon equations given by Boussinesq, who has applied them to one or two particular problems. The special limitation which characterizes them is the neglect of variations of density, *except in so far as they modify the action of gravity.* Of course, such neglect can be justified only under certain conditions, which Boussinesq has discussed. They are not so restrictive as to exclude the approximate treatment of many problems of interest.

In the present problem the case is much more complicated, unless we arbitrarily limit it to two dimensions.

Bénard 1900:

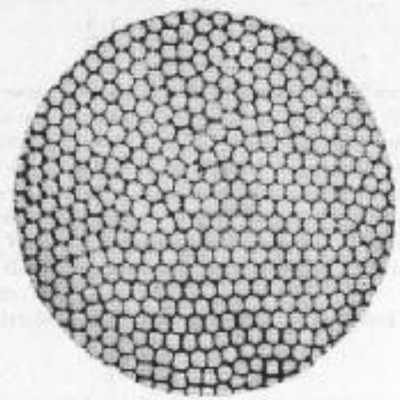
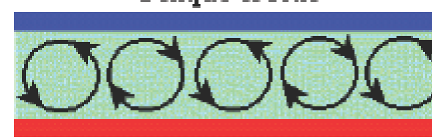


FIG. 1. Bénard cells in spermaceti. A reproduction of one of Bénard's original photographs.



Boussinesq equations:

$$\dot{T} + \vec{u} \cdot \vec{\nabla} T = \kappa \Delta T$$

$$\dot{\vec{u}} + \vec{u} \cdot \vec{\nabla}\, \vec{u} + \frac{1}{\rho} \vec{\nabla} p = \nu \Delta \vec{u} + g \alpha \hat{j} \left(T - T_0\right)$$

$$0 = \vec{\nabla} \cdot \vec{u}$$

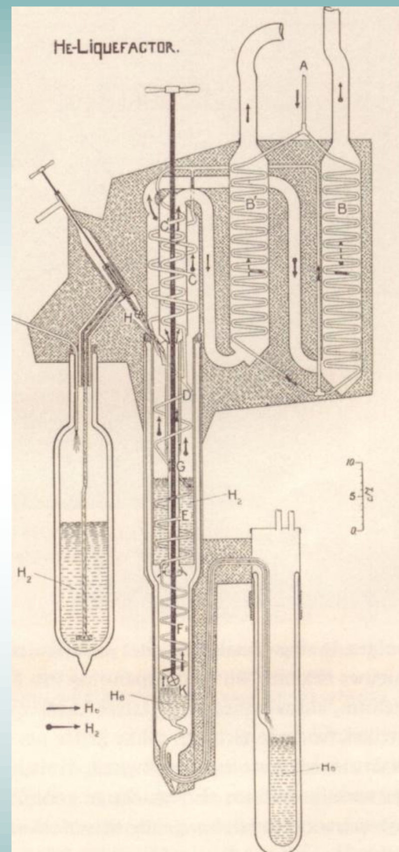
Cryogenic RB convection...

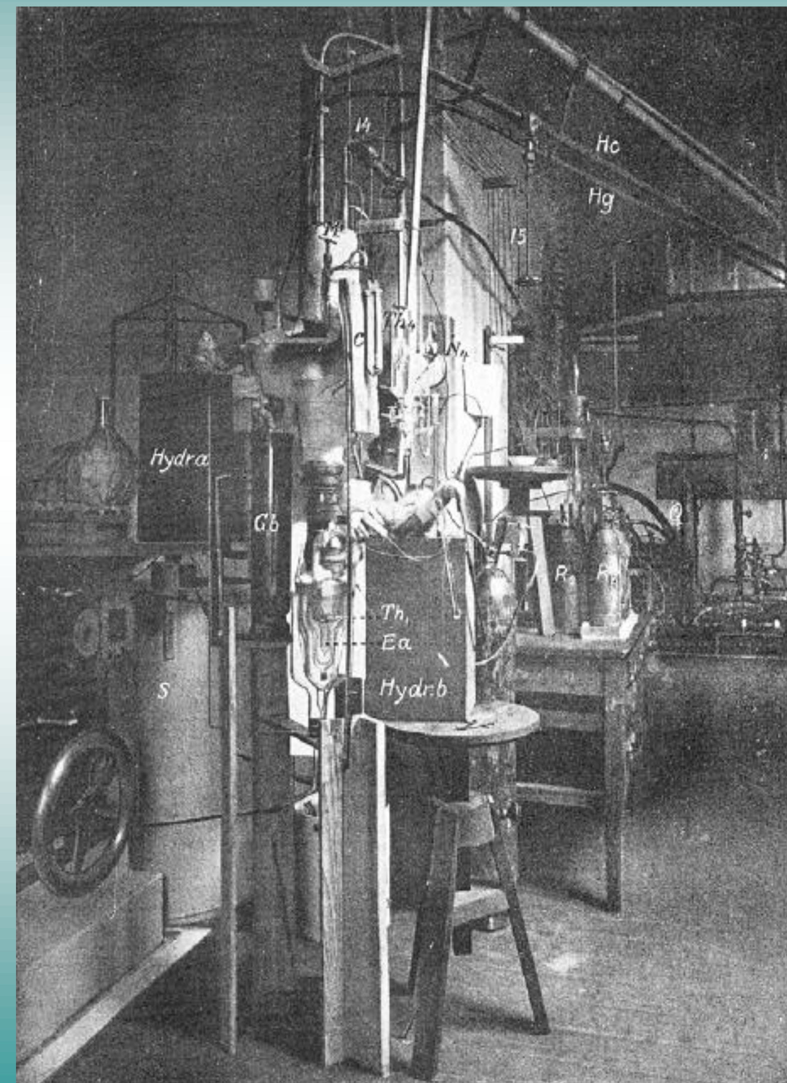
July 10., 1908
Helium liquified - 4,2 K
Beginning of low temperature physics

Heike Kamerlingh-Onnes

Nobel prize 1913

"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"





Cryogenic helium gas and normal liquid helium serve as outstanding working fluids for cryogenic fluid dynamics

Measures of turbulence intensity

Reynolds number

For isothermal flows

$$Re = \frac{UL}{\nu}$$

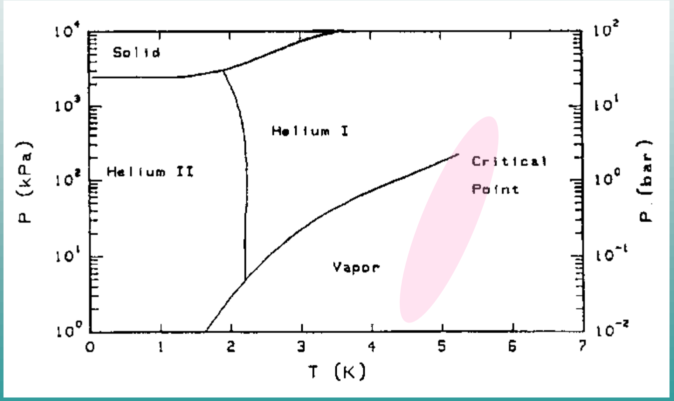
Rayleigh number

for thermally driven flows in a gravitational field

$$Ra = \frac{g\alpha\Delta T L^3}{\nu\kappa}$$

	Ra	Re
Sun	$10^{20\text{-}24}$	10^{13}
Ocean	$10^{21\text{-}27}$	10^{9}
Atmosphere	10^{17}	10^{9}
Navy (ship)		10^{9}
Aerospace (aircraft)		10^{8}-10^{9}

	T (p)	ν (cm^2/s)	$\alpha/\nu\kappa$
air	20 C	0,15	0,122
water	20 C	$1{,}004 \times 10^{-2}$	14,4
Normal 3He	above Tc	~ 1, olive oil	
Normal fluid of 3He B	around 0.6 Tc	~ 0.2, air	
Helium I	2,25 K (SVP)	$1{,}96 \times 10^{-4}$	$3{,}25 \times 10^{5}$
Helium II	1,8 K (SVP)	$9{,}01 \times 10^{-5}$	X
He-gas	5,5 K (2,8 bar)	$3{,}21 \times 10^{-4}$	$1{,}41 \times 10^{8}$



- **Cryogenic He** offers outstanding working fluids with known, **tuneable (in situ)** properties for the controlled, laboratory high Re and Ra turbulence experiments
- Clean cryogenic environment, deep cryogenic vacuum outside the cell (adiabatic sidewalls)
- Essentially no parazitic heat leaks
- Well-developed accurate thermometry

J. Fluid Mech. (1975), vol. 67, part 1, pp. 17–28 17
Printed in Great Britain

Free convection in low-temperature gaseous helium

By D. C. THRELFALL
Cavendish Laboratory, University of Cambridge

The experimental can was made from low-thermal-conductivity copper-nickel alloy of internal diameter **48.4 mm and thickness 0.2 mm.** The top Cu plate was 5 mm thick and the lower plate was **6 mm thick. They were 19.96 mm** apart

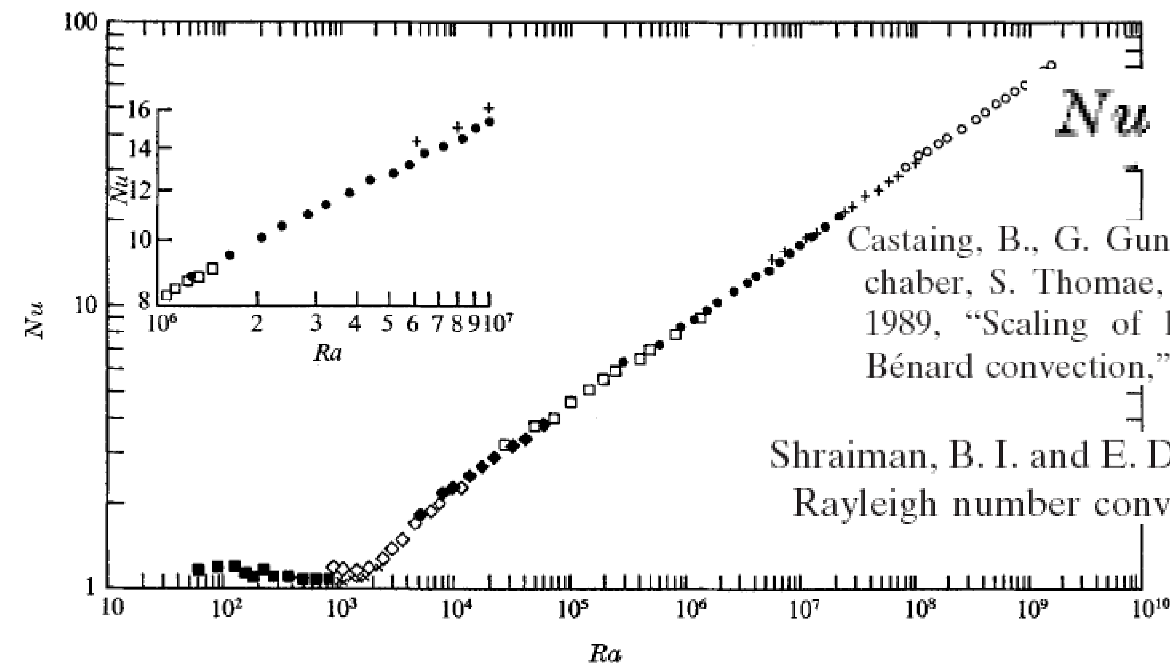


FIGURE 2. Summary of the heat-transfer experiments; $60 < Ra < 2 \times 10^9$. Above $Ra = 4 \times 10^5$, $Nu = 0{\cdot}173 Ra^{0{\cdot}2800}$; the inset shows the worst scatter in this region with all points plotted.

$$Nu = 0{\cdot}173 Ra^{0{\cdot}2800}$$

Castaing, B., G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X. Z. Wu, S. Zaleski, and G. Zanetti, 1989, "Scaling of hard thermal turbulence in Rayleigh-Bénard convection," J. Fluid Mech. **204**, 1–30.

Shraiman, B. I. and E. D. Siggia, 1990, "Heat transport in high-Rayleigh number convection," Phys. Rev. A **42**, 3650–3653.

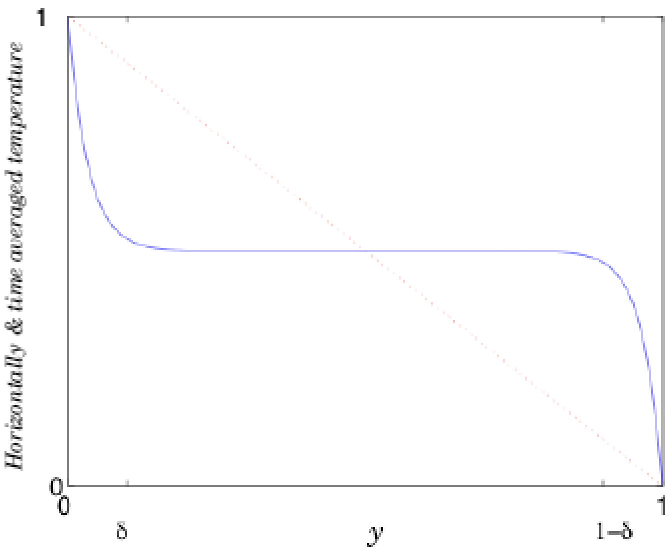
$$Nu \sim Ra^{2/7}$$

Challenge:

find Nu(Ra,Pr)

Malkus' argument:

For turbulent convection, the mean temperature profile should look like:



$$\mathrm{Nu} \sim \delta^{-1} \quad \ldots \quad \delta = f(\mathrm{Ra})$$

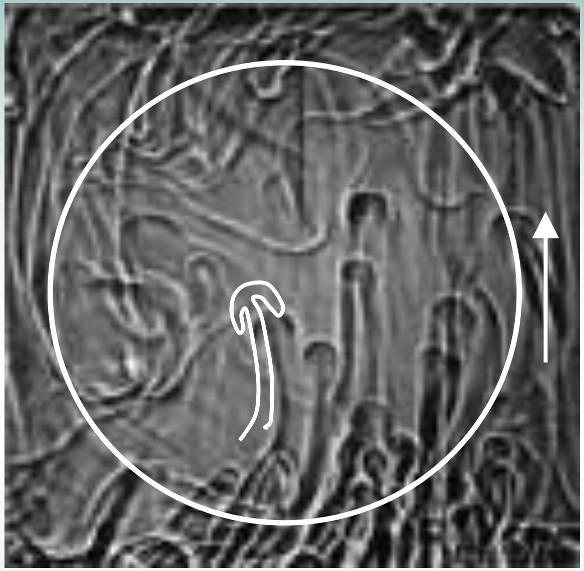
Assume boundary layer thickness is determined by a ***marginal stability condition***:

$$\mathrm{Ra}_c = \mathrm{Ra}_\delta = \frac{g\alpha \frac{1}{2}(T_{hot} - T_{cold})(\delta h)^3}{\nu\kappa} = \frac{1}{2}\,\frac{g\,\alpha(T_{hot} - T_{cold})\,h^3}{\nu\kappa} \times \delta^3 = \frac{1}{2}\,\mathrm{Ra}\,\delta^3$$

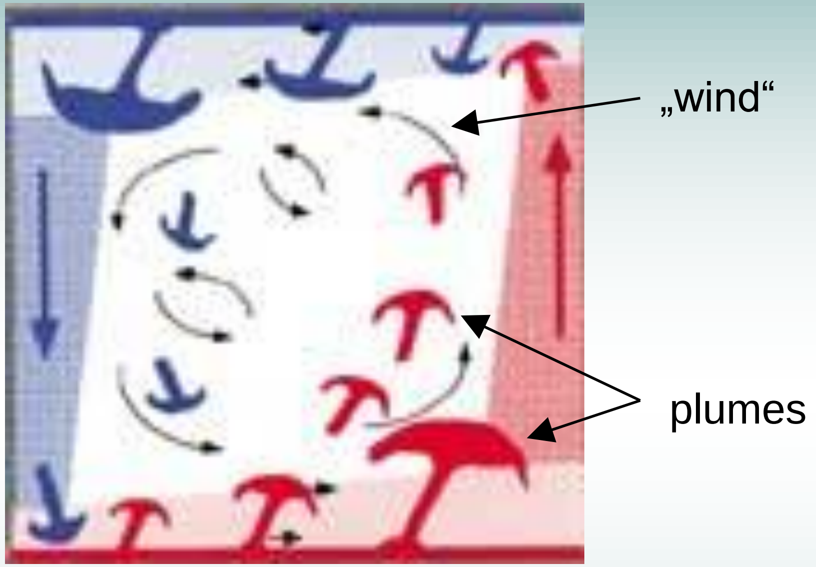
$$\Rightarrow \quad \delta \sim \mathrm{Ra}^{-1/3} \quad \Rightarrow \quad \mathrm{Nu} \sim \mathrm{Ra}^{1/3}$$ ***uniformly*** in Pr

Character of convection, „wind“ and plumes

Studied dependence Nu(Ra) relates to the character of convection inside the cell.



J. Zhang, S. Childress, A. Libchaber, Phys. Fluids 9, 1034 (1997), Ra~10^8



Leo P. Kadanoff, Physics Today, August 2001

Ra < 2000, typical conducting heat transfer, i.e. Nu = 1, for higher Ra: stable convection, oscillatory convection, chaos, transition to turbulence.

THE PHYSICS OF FLUIDS VOLUME 5, NUMBER 11 NOVEMBER 1962

Turbulent Thermal Convection at Arbitrary Prandtl Number

ROBERT H. KRAICHNAN
Courant Institute of Mathematical Sciences, New York University, New York
(Received May 24, 1962)

The mixing-length theory of turbulent thermal convection in a gravitationally unstable fluid is extended to yield the dependence of Nusselt number H/H_0 on both Prandtl number σ and Rayleigh number Ra. The analysis assumes a layer of Boussinesq fluid contained between infinite, horizontal, perfectly conducting, rigid plates. Also obtained is the dependence of mean temperature deviation $\bar{T}(z)$, rms temperature fluctuation $\tilde{\theta}(z)$, and rms velocity upon height z above the bottom plate. The theory gives $H/H_0 \propto \mathrm{Ra}^{1/3}$ (high σ), $H/H_0 \propto (\sigma\,\mathrm{Ra})^{1/3}$ (low σ), and $H/H_0 \sim 1$ (very low σ). The boundaries of the several σ ranges are determined. At one intermediate Prandtl number only, the behavior of $\bar{T}(z)$ and $\tilde{\theta}(z)$ reduces to that previously found by Priestley. At high σ, there is a range of z, outside the molecular conduction region, where $\bar{T}(z) \propto z^{-1}$, $\tilde{\theta}(z) \propto z^{-1}$. The results at very low σ reduce to those of Ledoux, Schwarzschild, and Spiegel. The dynamics are found to be importantly modified at extremely large Ra because of the stirring action of small-scale turbulence generated in shear boundary layers attached to the eddies of largest scale. The consequent corrected asymptotic law of heat transport at fixed σ is $H/H_0 \propto [\mathrm{Ra}/(\ln \mathrm{Ra})^3]^{1/2}$.

verified only if boundary layers are artificially removed from problem (Toschi & Lohse)

Postulated "*ultimate*" high-Ra scaling: Nu ~ $\mathrm{Ra}^{1/2}$

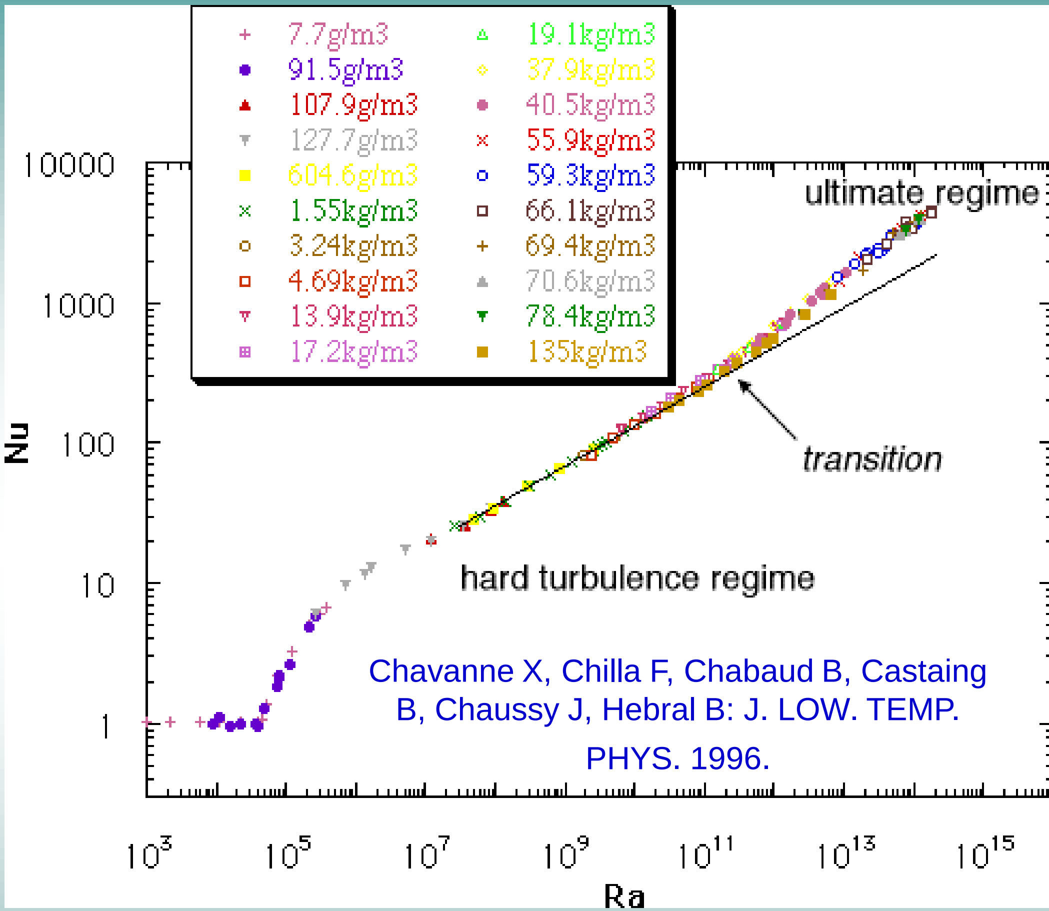
PRL **108**, 024502 (2012) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2012

Transition to the Ultimate State of Turbulent Rayleigh-Bénard Convection

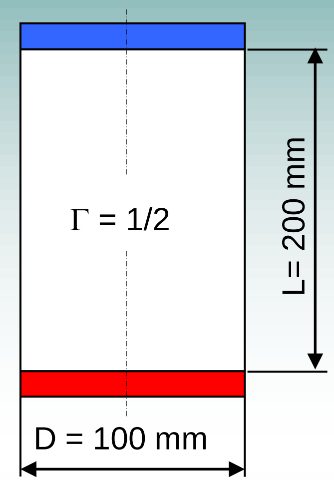
Xiaozhou He,[1] Denis Funfschilling,[2] Holger Nobach,[1] Eberhard Bodenschatz,[1,3,4] and Guenter Ahlers[5]

Cryogenic experiments with utilization of ^{4}He

Chavanne and co-authors (1996 – Grenoble)



CONVECTION CELL



Measured dependence:

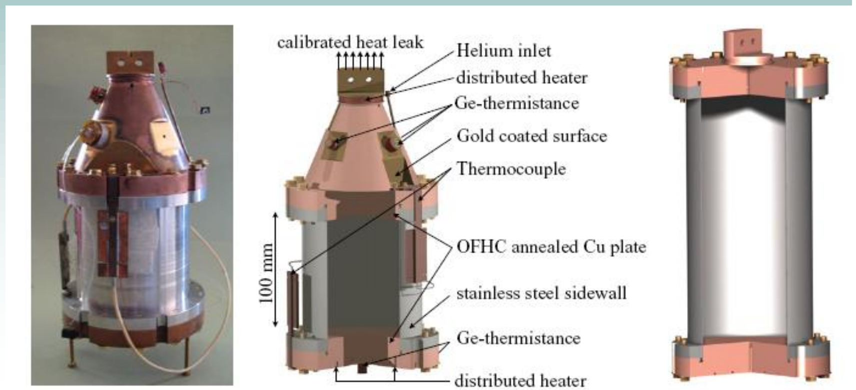
$$Nu \sim Ra^{2/7}$$

Transition to the power law with exponent 0.4 in the vicinity of Ra ~ 10^{11} - interpreted as transition to the Kraichnan regime with $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$.

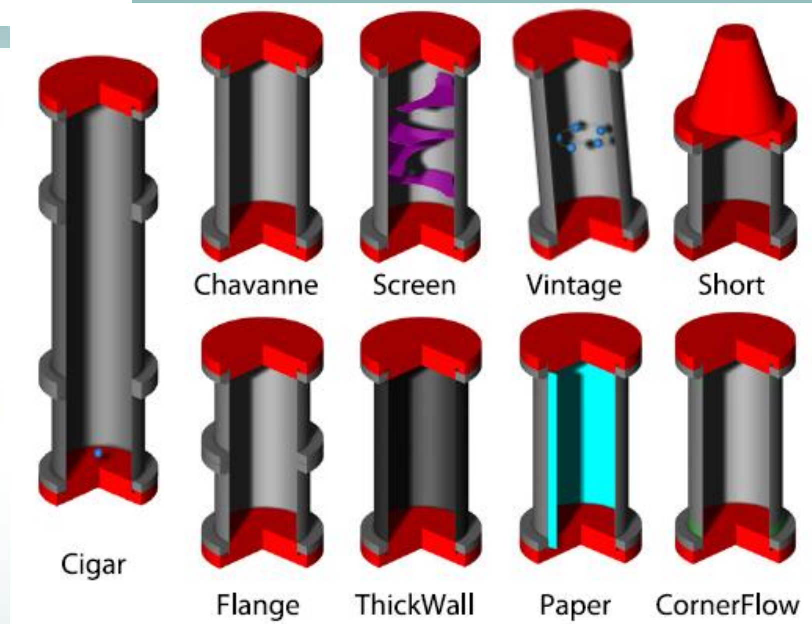
On the triggering of the Ultimate Regime of convection

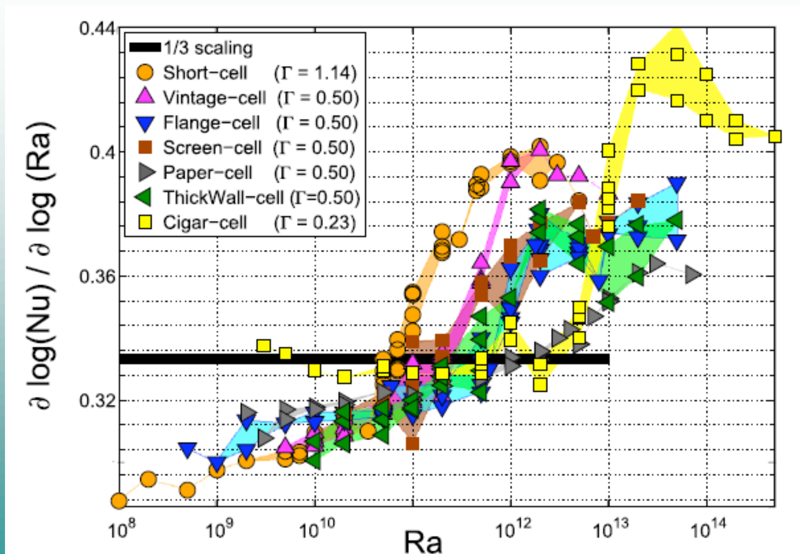
P-E Roche[1], **F Gauthier, R Kaiser and J Salort**
Institut Néel, CNRS/UJF, BP 166, F-38042 Grenoble cedex 9, France

New Journal of Physics **12** (2010) 085014 (26pp)



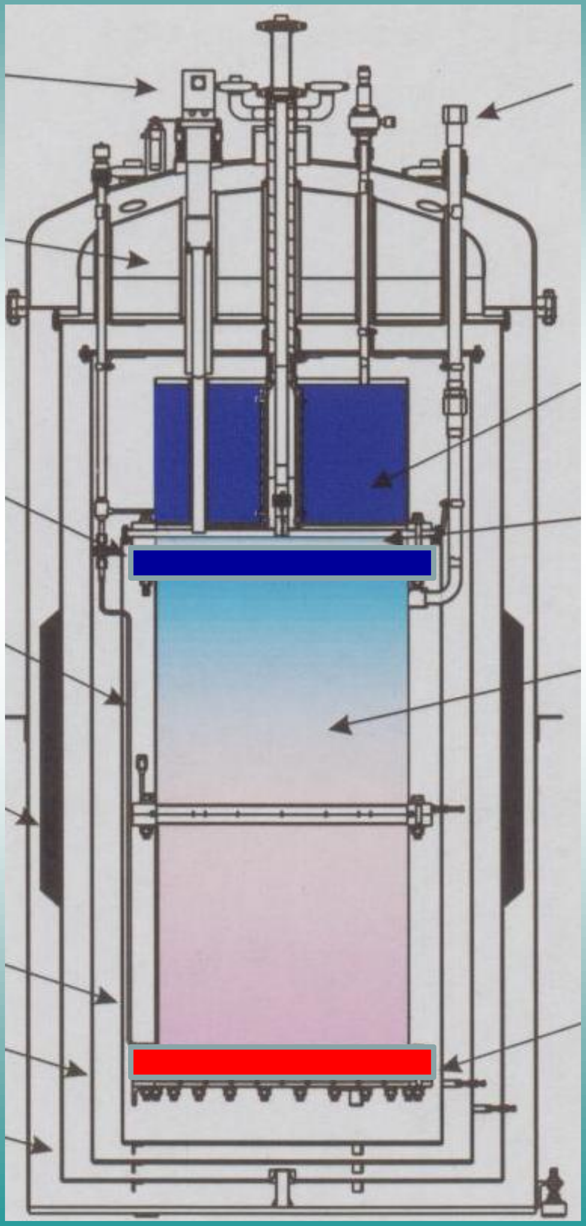
(a) (b) (c)

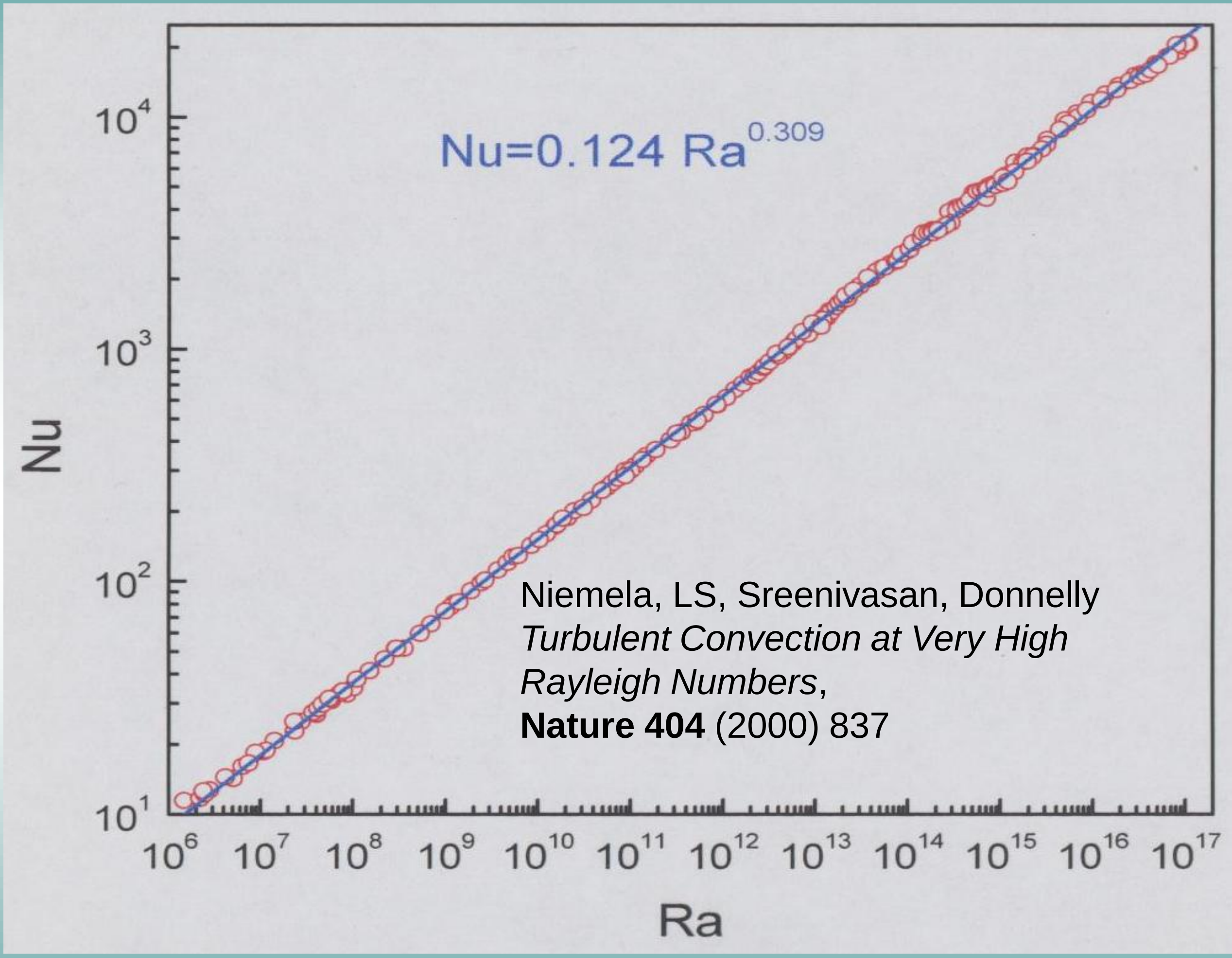




<<Grenoble regime>>

Oregon/Trieste Cryogenic turbulent convection cryostat





Niemela, LS, Sreenivasan, Donnelly
Turbulent Convection at Very High Rayleigh Numbers,
Nature 404 (2000) 837

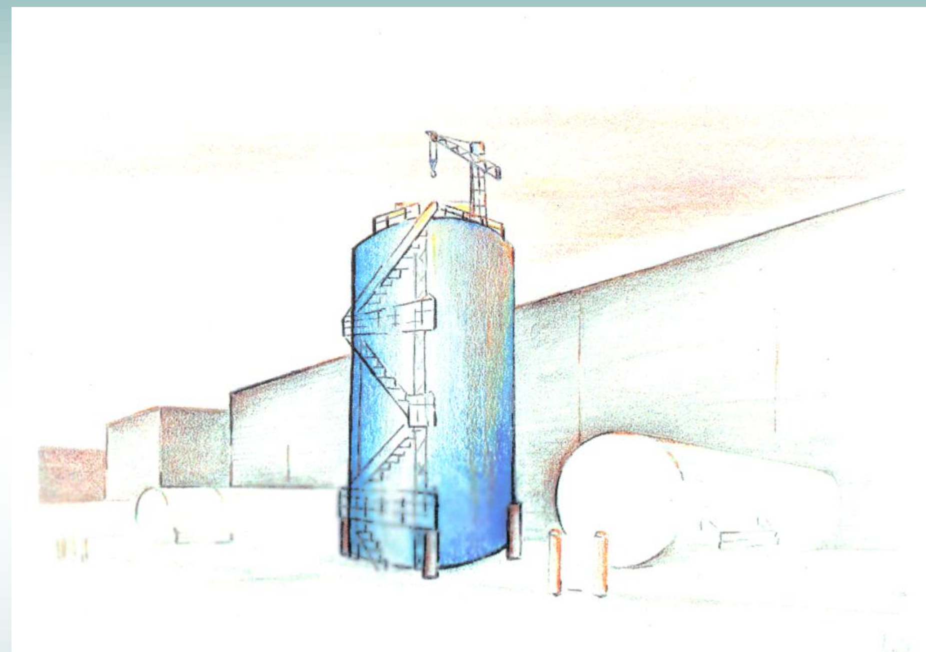
Heat transfer efficiency in cryogenic turbulent convection

A proposed experiment: a 10m high convection cell capable of $Ra \sim 10^{21}$ nearly comparable to that of the Sun.

Inside cell dimensions
D = 5m, L = 10m,
Max volume ~ 25,000 gallons of liquid helium equivalent

Outside dimensions
~7 m dia and ~20 m high

Refrigeration needed
< 200 W



RHIC, BNL

Huge accelerator facilities like CERN or BNL would have plenty of liquid helium on hand, used to cool superconducting magnets.

J. Fluid Mech. (2000), *vol.* 407, *pp.* 27–56. Printed in the United Kingdom

Scaling in thermal convection: a unifying theory

By SIEGFRIED GROSSMANN[1] AND DETLEF LOHSE[2]

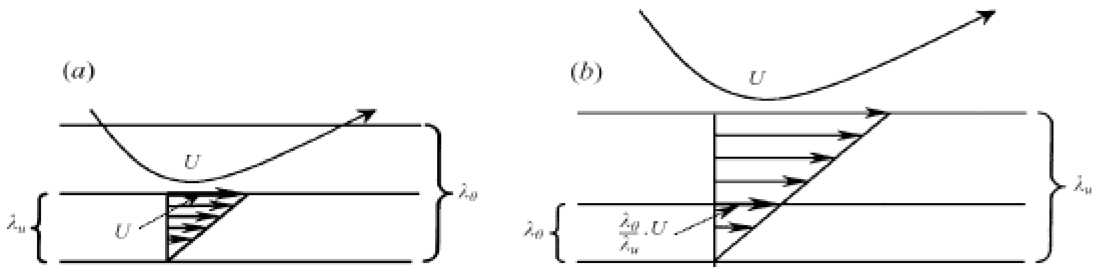


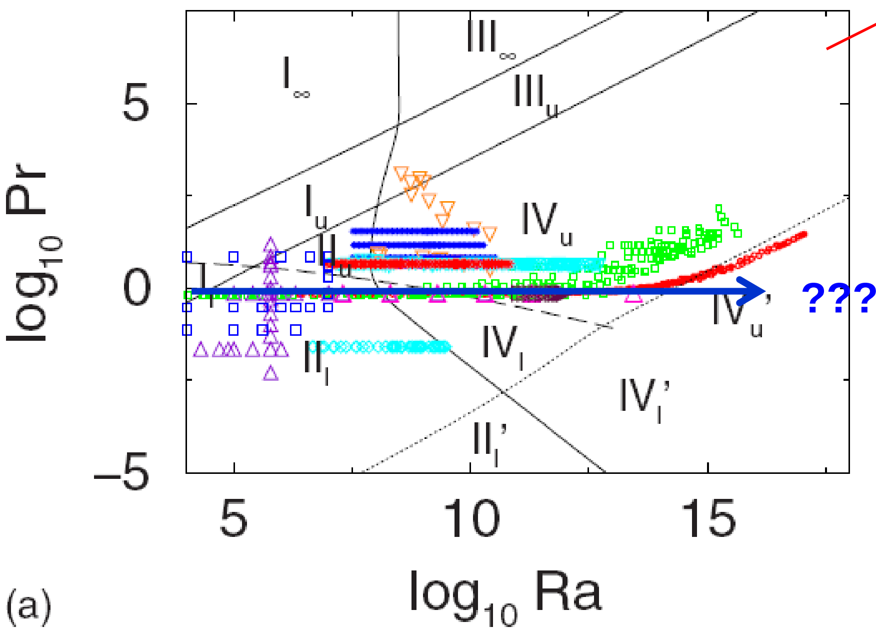
FIGURE 1. Sketch of the boundary layers, (*a*) for low *Pr* where $\lambda_u < \lambda_\theta$ and (*b*) for large *Pr* where $\lambda_u > \lambda_\theta$.

Latest update

Stevens R J A M, van der Poel E P, Grossmann S and Lohse D 2013 The unifying theory of scaling in thermal convection: the updated prefactors *J. Fluid Mech.* ***730,*** *295*

Review up to 2009

Ahlers G, Grossmann S and Lohse D 2009 Heat transfer and large scale dynamics in turbulent Rayleigh-B´enard convection *Rev. Mod. Phys.* **81 503–537**



Regime	Dominance of	BL	*Nu*
I_l	$\epsilon_{u,BL}, \epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.27 Ra^{1/4} Pr^{1/8}$
I_u		$\lambda_u > \lambda_\theta$	$0.33 Ra^{1/4} Pr^{-1/12}$
II_l	$\epsilon_{u,bulk}, \epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.97 Ra^{1/5} Pr^{1/5}$
(II_u)		$\lambda_u > \lambda_\theta$	$(\sim Ra^{1/5})$
III_l	$\epsilon_{u,BL}, \epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$6.43 \times 10^{-6} Ra^{2/3} Pr^{1/3}$
III_u		$\lambda_u > \lambda_\theta$	$3.43 \times 10^{-3} Ra^{3/7} Pr^{-1/7}$
IV_l	$\epsilon_{u,bulk}, \epsilon_{\theta,bulk}$	$\lambda_u <$	$4.43 \times 10^{-4} Ra^{1/2} Pr^{1/2}$
IV_u		$\lambda_u >$	$0.038 Ra^{1/3}$

Experimental, theoretical and numerical challenge:

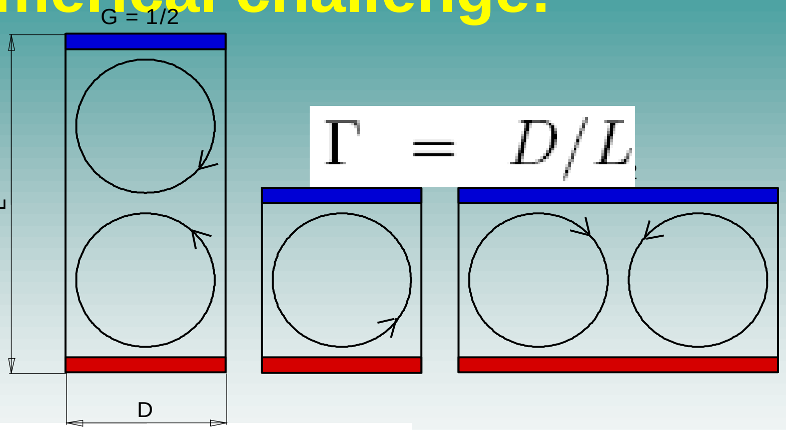
Find $\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}; \mathrm{Pr})$ scaling law

quantities $C^0_{exp} \ldots C^N_{exp}$

Experimentally: take care of experimental details

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}; \mathrm{Pr}; C^0_{exp} \ldots C^N_{exp})$$

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}; \mathrm{Pr}; \Gamma, C^1_{exp} \ldots C^N_{exp})$$



$$\Gamma = D/L$$

a first approximation in taking into account the shape of the RBC cell.

The remaining experimental parameters could be divided into two groups, which could loosely be called geometrical and physical. The first group includes the actual shape of the cell (e.g., rectangular or cylindrical), possible deviation of the plates from horizontal position or the surface roughness - i.e., the top and bottom plates are still treated as ideally conducting and the sidewall as ideally insulating. The second group includes the actual physical properties of the working fluid (which are known with limited accuracy and are deduced based on measurements of its temperature and pressure, within certain error bars) as well as those of the RBC cell, such as thickness, thermal conductivity and heat capacity of plates and walls, heat conductivity of electrical leads and, generally, the physical properties of the surrounding medium.

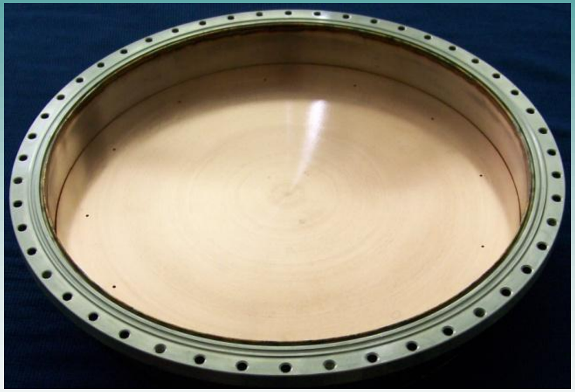
Serious warning: it is implicitly assumed that the working fluid can be treated as an Oberbeck-Boussinesq fluid with constant physical properties except its density which, moreover, is assumed to linearly depend on temperature

Experimental apparatus – ISI Brno, Czech Republic

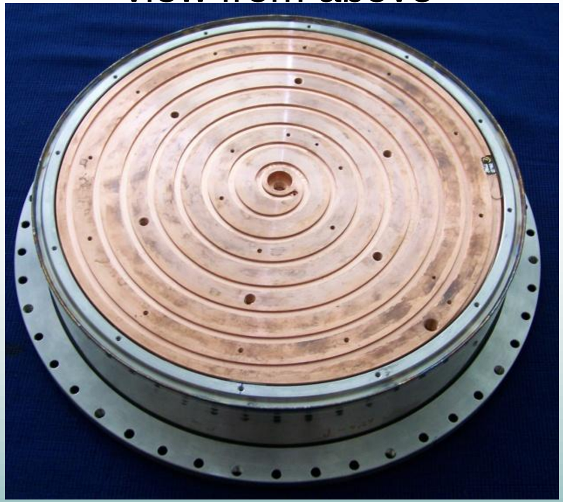
All details about ISI Brno apparatus (design, uncertainty of measurements etc.) find in:

P. Urban, P. Hanzelka, T. Kralik, V. Musilova, LS and A. Srnka, Rev. Sci. Instrum. **81**, 085103 (2010)

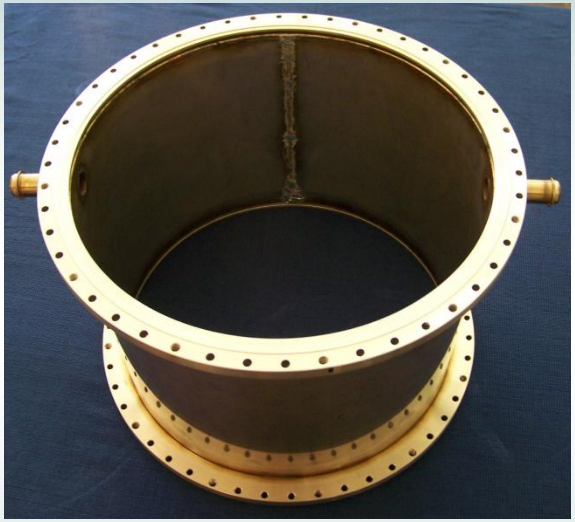
Convection cell - components



Bottom part of the cell - view from above



Bottom part of the cell - view from below



Central part of the cell

Top part of the cell

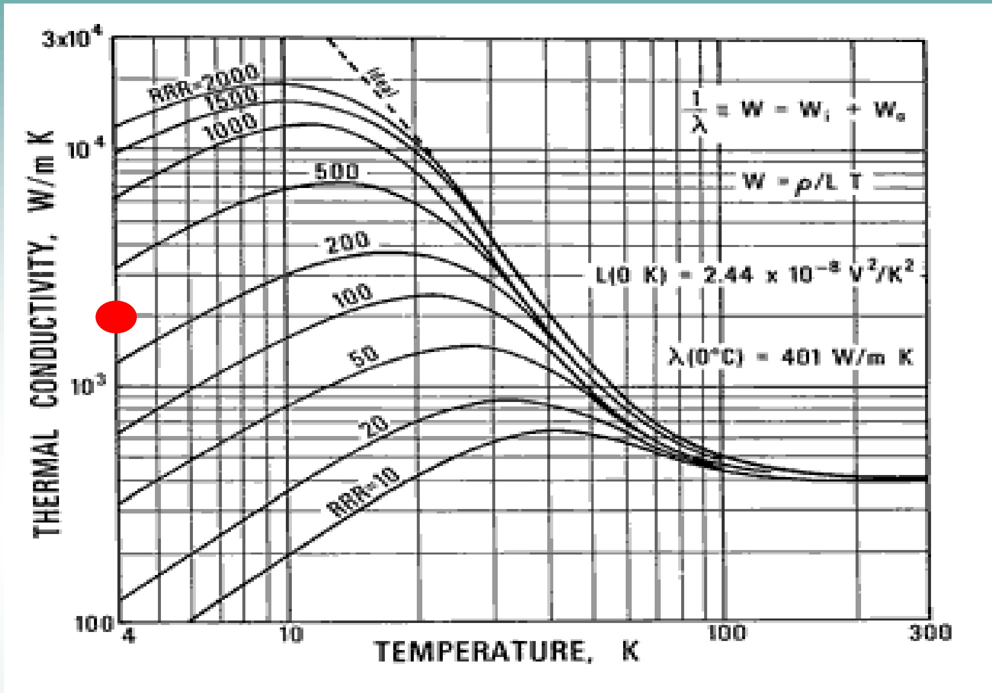
Copper plates (OFHC)

Electrical resistance heaters – glued within the grooves – heat power from 10 mW to 10 W.

Heaters ensure better than 1 mK temperature homogeneity (under the assumption that the heat is uniformly supplied or removed)

Roughness of the inner side, max. $R_a = 1.6\ \mu m$

Determination of the heat conductivity of the Cu plates



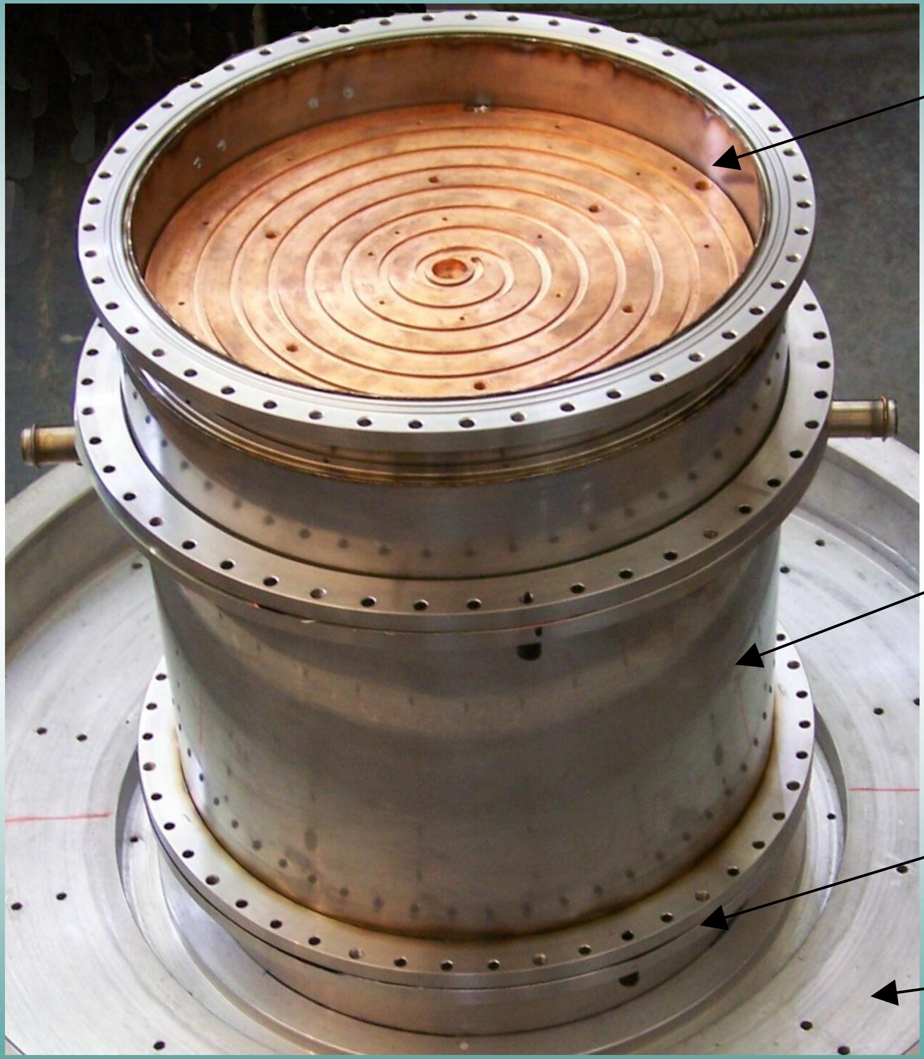
Measured RRR (Residual Resistivity Ratio), samples from Cu plates:

Unannealed sample RRR = 220

Annealed sample (650 °C/0,5 h.) RRR = 290

Corresponding heat conductivity (at 5 K) is 2 kW/m/K

Convection cell - assembly



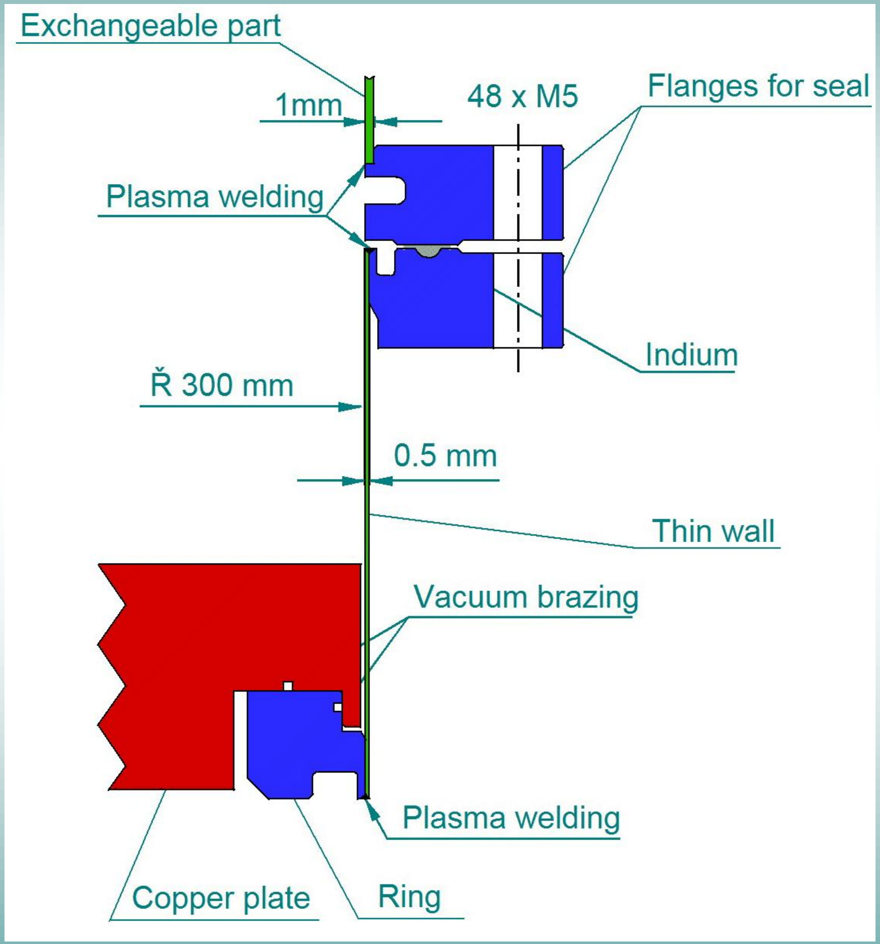
Top part with the portion of the heat exchange chamber

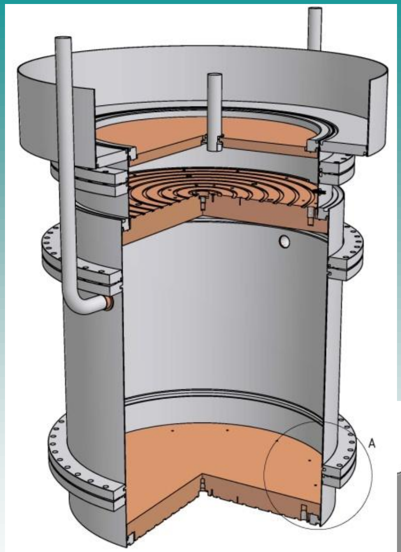
Exchangeable central part

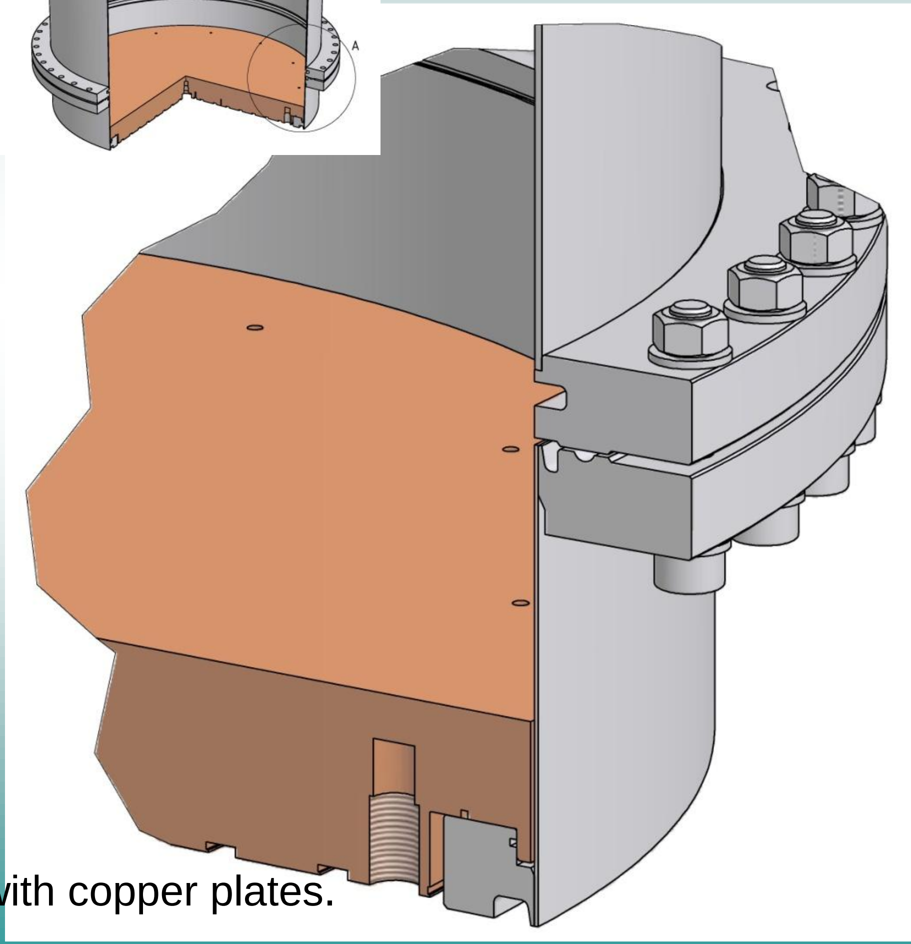
Bottom part

Assembling plate

Construction and technology - joints

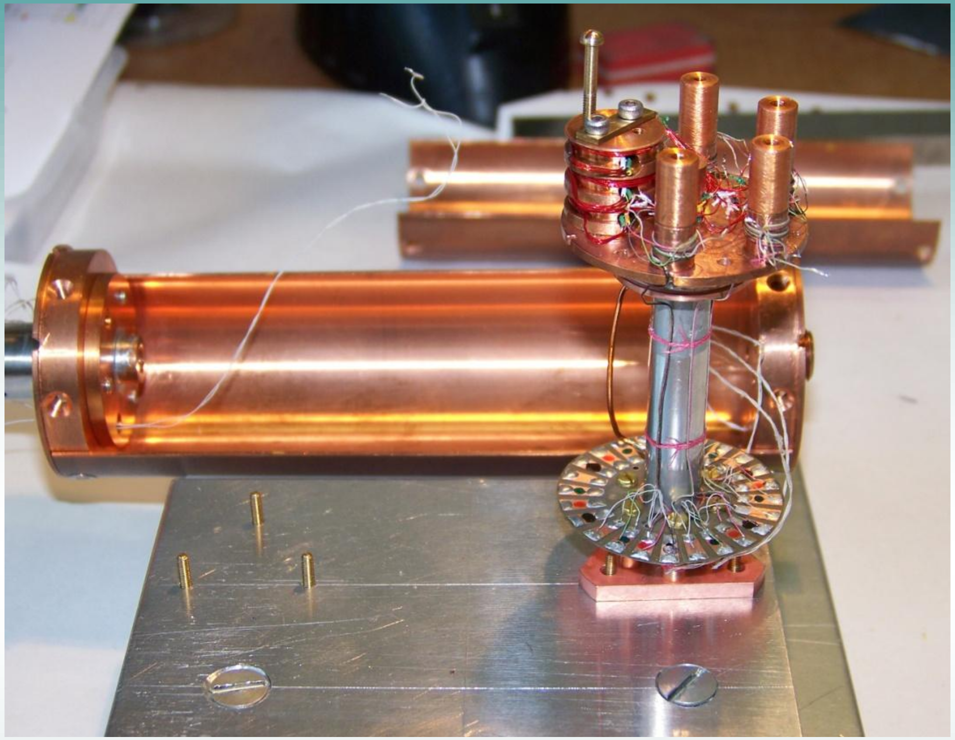


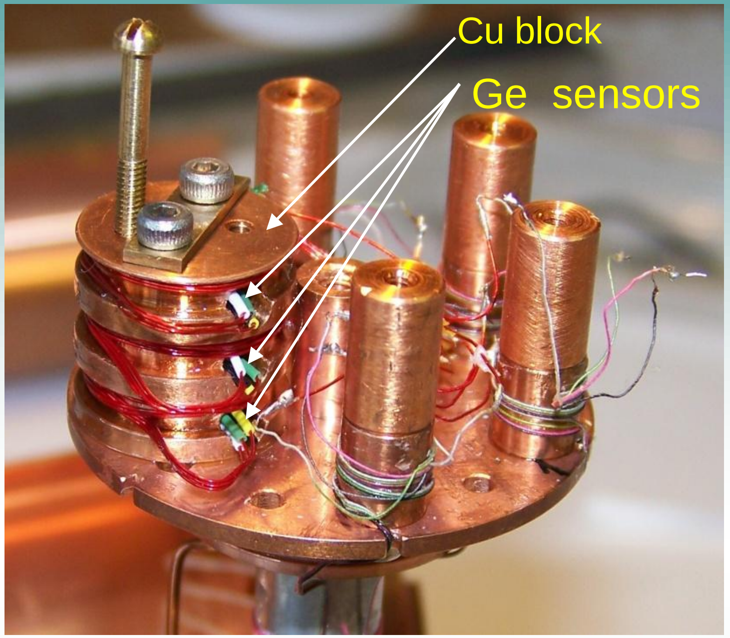




Thickness of the walls is 0.5 mm in indirect contact with copper plates.

Home-made calibration of Ge sensors



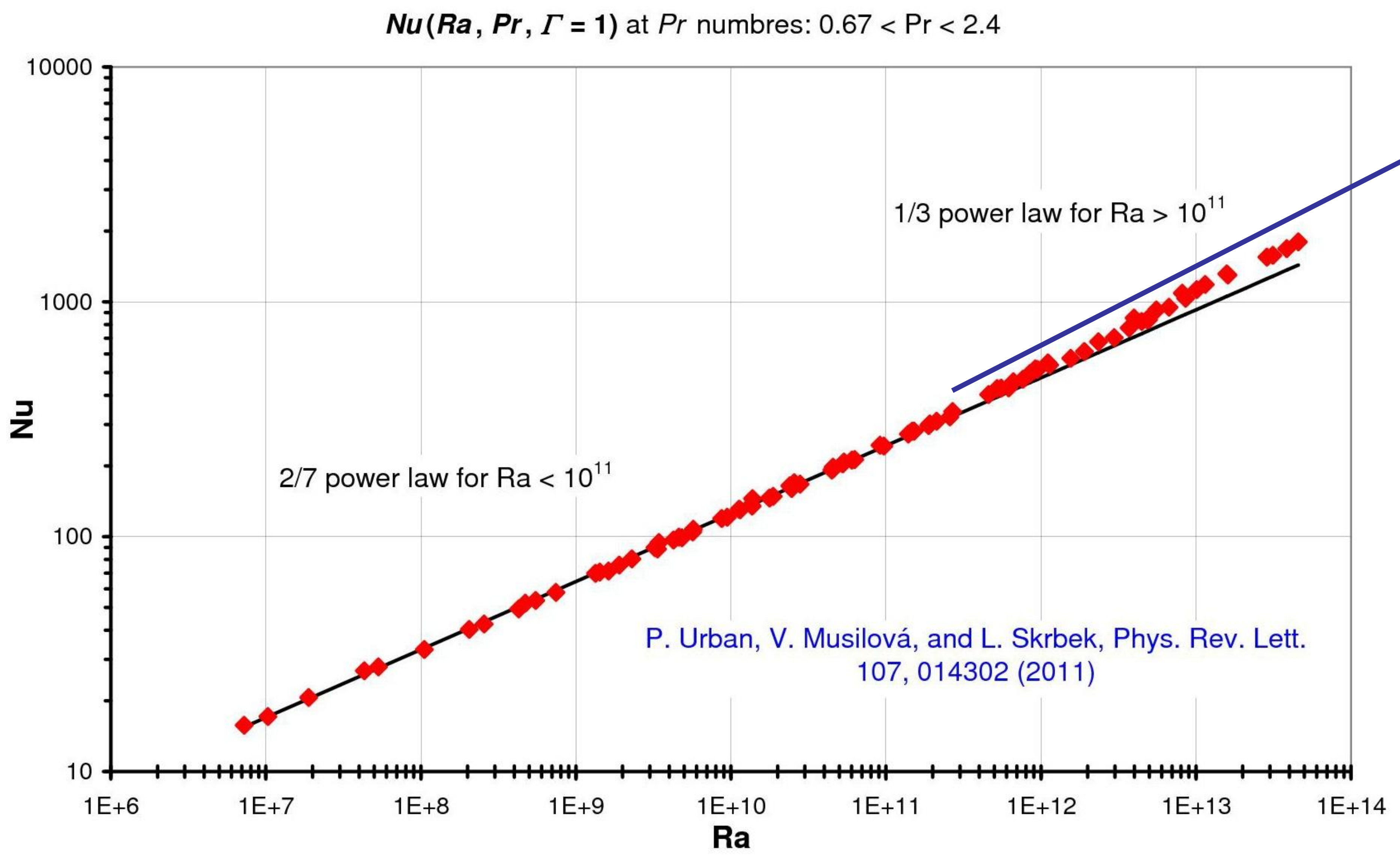


Readout via LakeShore 340 temperature controller,

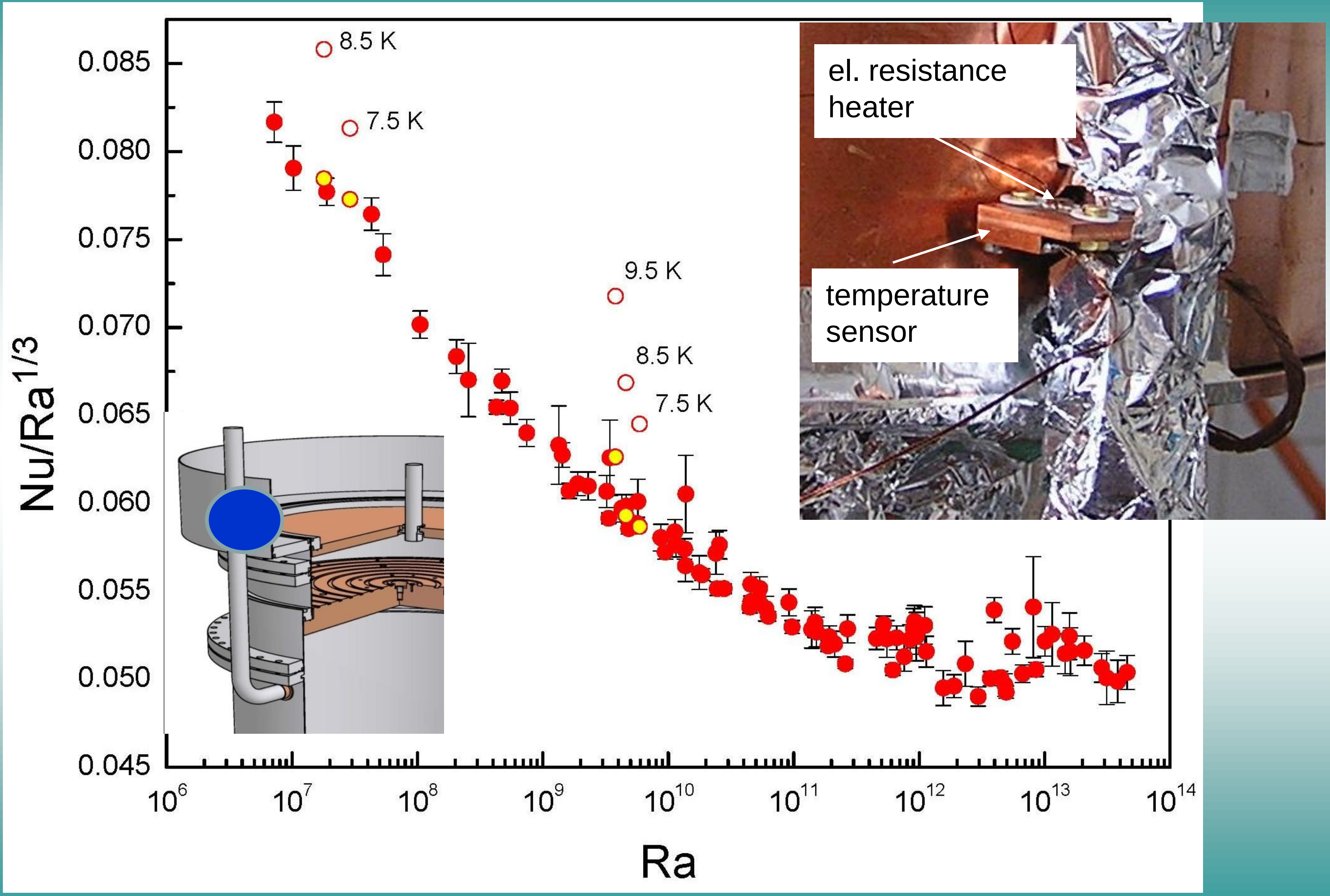
mutally ΔT +/- 1 mK within 4,2 K - 20 K.

The first Brno experiment – Nu(Ra) dependence

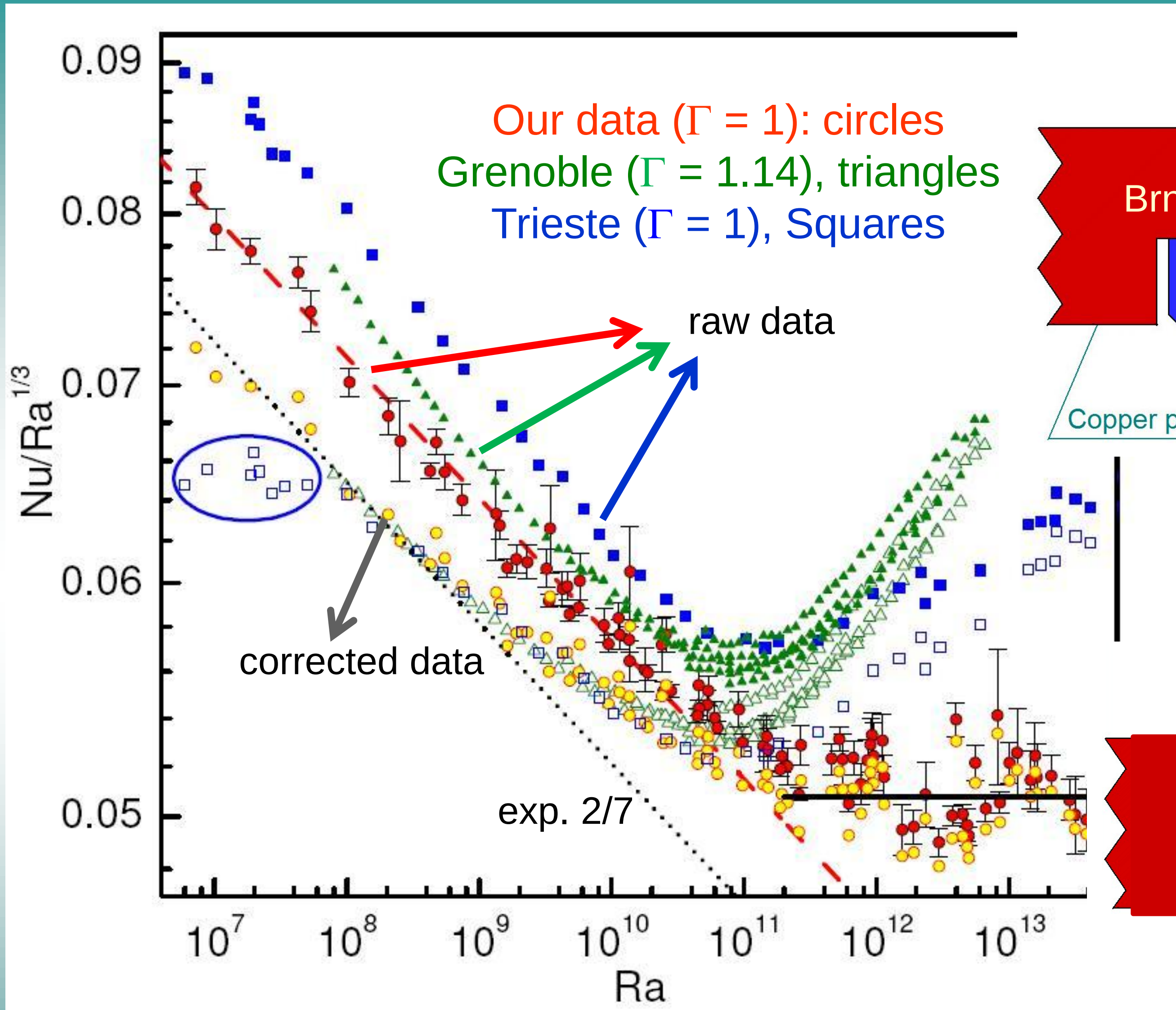
Range of Ra numbers: 7.2×10^6 to 4.6×10^{13}

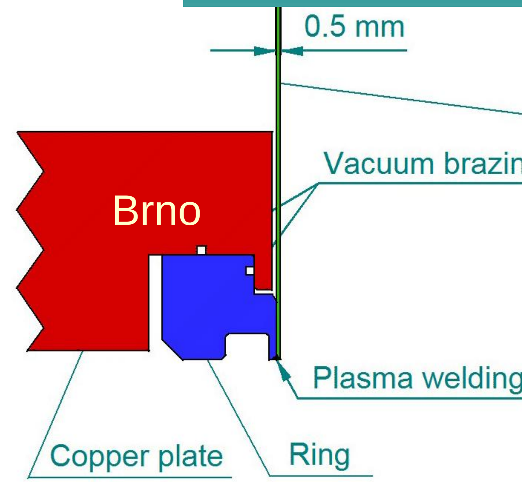


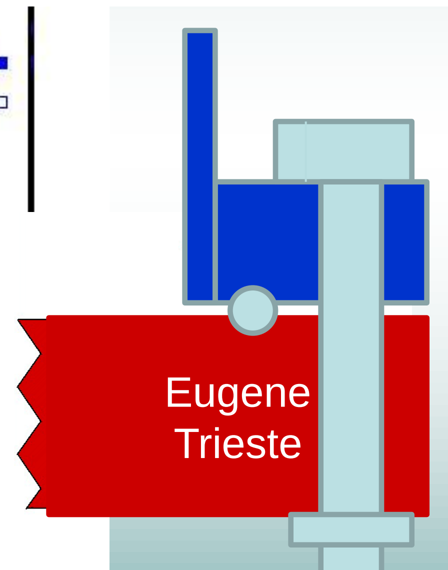
<< Chimney effect>>



Compensated Nu(Ra) dependence - cryogenic helium data







Trieste: J. J. Niemela, and K. R. Sreenivasan, JFM **481**, 355 (2003)

Grenoble: P.-E. Roche et al. New J. Phys. **12**, 085014 (2010)

Sidewall corrections

via the so called wall parameter defined by Roche et al.

Roche P -E, Castaing B, Chabaud B, Hebral B and Sommeria J, 2001 Side wall effects in Rayleigh Bénard experiments *European Physical Journal B* **24** 405-408

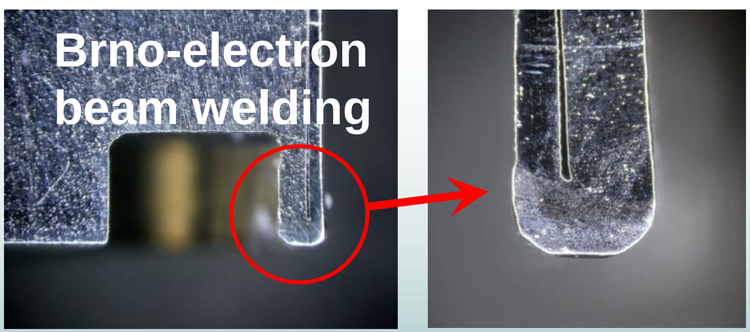
$$W = \frac{2\lambda_{\mathrm{w}}\delta}{\lambda R}$$

λ and λ_{w} heat conductivity of the working fluid and the sidewall

R cell radius

$$\mathrm{Nu}_{\mathrm{corr}} = \frac{\mathrm{Nu}_{\mathrm{raw}}}{1+f(W)} ;$$

$$f(W) = \frac{A^2}{\Gamma \mathrm{Nu}_{\mathrm{corr}}}\left(\sqrt{1+\frac{2W\Gamma\mathrm{Nu}_{\mathrm{corr}}}{A^2}}-1\right) \simeq A\sqrt{2}\sqrt{\frac{W}{\Gamma\mathrm{Nu}_{\mathrm{corr}}}}$$

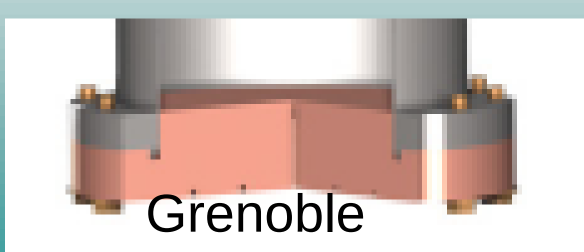


depending on actual value of λ

For our cell, $0.22 > W > 0.15$

Grenoble $0.66 > W > 0.35$

Oregon/Trieste $0.78 > W > 0.40$



based on the nominal thickness of the sidewall $\delta = 3$ mm

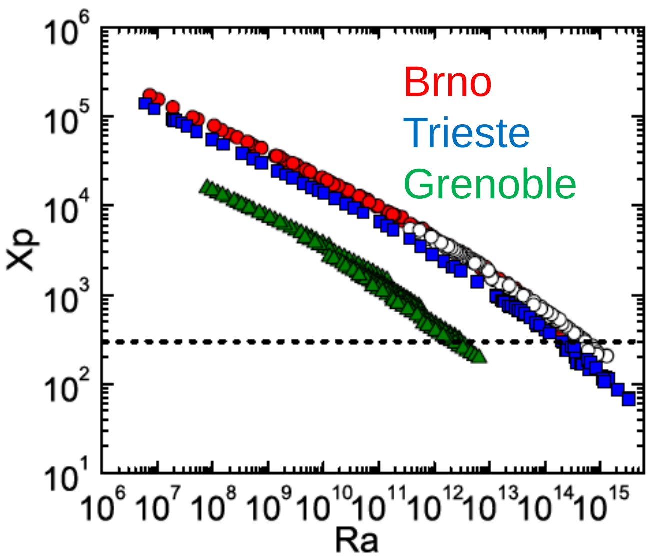
Remarks on corrections on finite heat conductivity and heat capacity of plates

A: Ge thermometers are placed vertically in the middle of top and bottom plates
temperature drop across the copper of height 2 × *(a=2)* = *a* → ***negligible correction***

B: Verzicco R 2004 Effect of nonperfect thermal sources in turbulent thermal convection. *Phys. Fluids* 16 1965-1979

parameter:

$$X_{\mathrm{p}} = \frac{R_{\mathrm{f}}}{R_{\mathrm{p}}} = \frac{L\lambda_{\mathrm{p}}}{a\mathrm{Nu}\lambda}$$

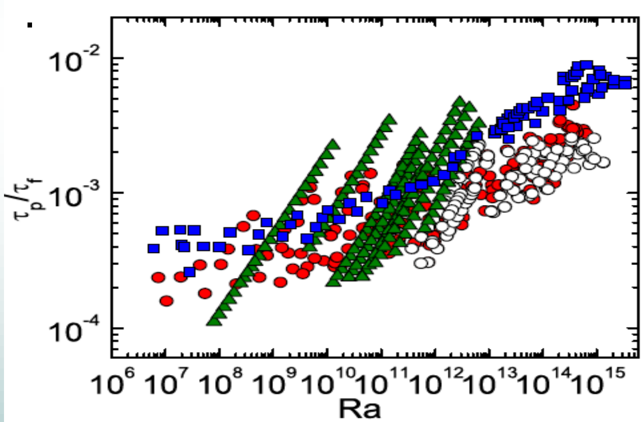


C: Is the plate is "fast enough" in order to supply enough heat for successive plumes ?

non-dimensional time between two successive emissions of plumes (Castaing et al., JFM (1989) ***204*** 1–30) $\tau_{\mathrm{f}} \approx \sqrt{\mathrm{RaPr}}/(4\mathrm{Nu}^2)$

Corresponding time for plates (Verzicco Phys. Fluids *(*2004) **16** 1965)

$$\tau_{\mathrm{p}} \approx \sqrt{\mathrm{RaPr}}\,(a/L)^2(\lambda/\lambda_{\mathrm{p}})$$



D: Chill`a F., Rastello M, Chaumat S and Castaing B 2004 Ultimate regime in Rayleigh-Benard convection: The role of plates *Phys. Fluids* ***16*** 2452–2456

Their – more strict - <thin plate condition> negligible "plates effect" in RBC experiments with cryogenic helium up to about Ra ≈ 1E12

No plates corrections have been applied to our data

The second ISI Brno experiment –

Nu(Ra) and Non Oberbeck–Boussinesq (NOB) fluid

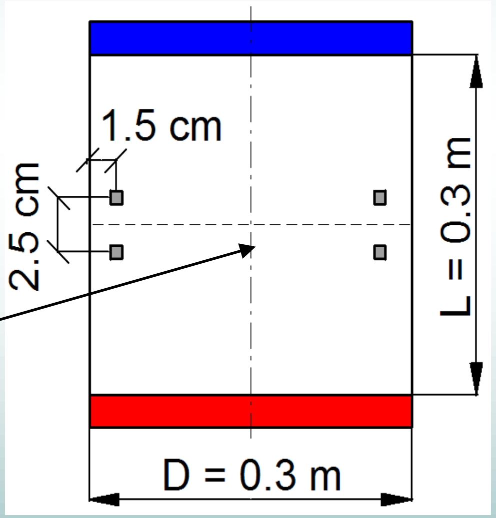
The Oberbeck-Boussinesq approximation is never exactly valid in practice: especially for working fluids in the vicinity of their critical point, where the relevant fluid properties (α, ν, κ, λ) might significantly vary over ΔT

$$\Rightarrow$$

asymmetry in boundary layers and appreciable change the mean temperature T_c of the **turbulent core** evaluated from the temperatures of the top and bottom plates.

$$T_c \neq T_m = \frac{T_B + T_T}{2}$$

T_c temperature of the fluid measured **in turbulent core** by four TTR-Ge sensors



T_T … top plate temperature

T_B … bottom plate temperature

- Nu(Ra) dependence was measured within the range of Ra numbers $10^{12} < Ra < 4.6 \times 10^{15}$.
- Nu(Ra) was evaluated with use of both T_m and T_{mp} temperatures

Effect of NOB conditions on the heat transfer efficiency at high Ra

OB conditions are never fully satisfied in practice......
Vrious experimental criteria have been suggested, such as $\alpha\Delta T < 0.1(0.2)$

In the vicinity of the critical point, the physical properties of helium gas (as well as of other gases) vary with temperature more rapidly than far away from it and this might influence the deduced heat transfer efficiency significantly, especially in laboratory experiments aiming to reach high Ra

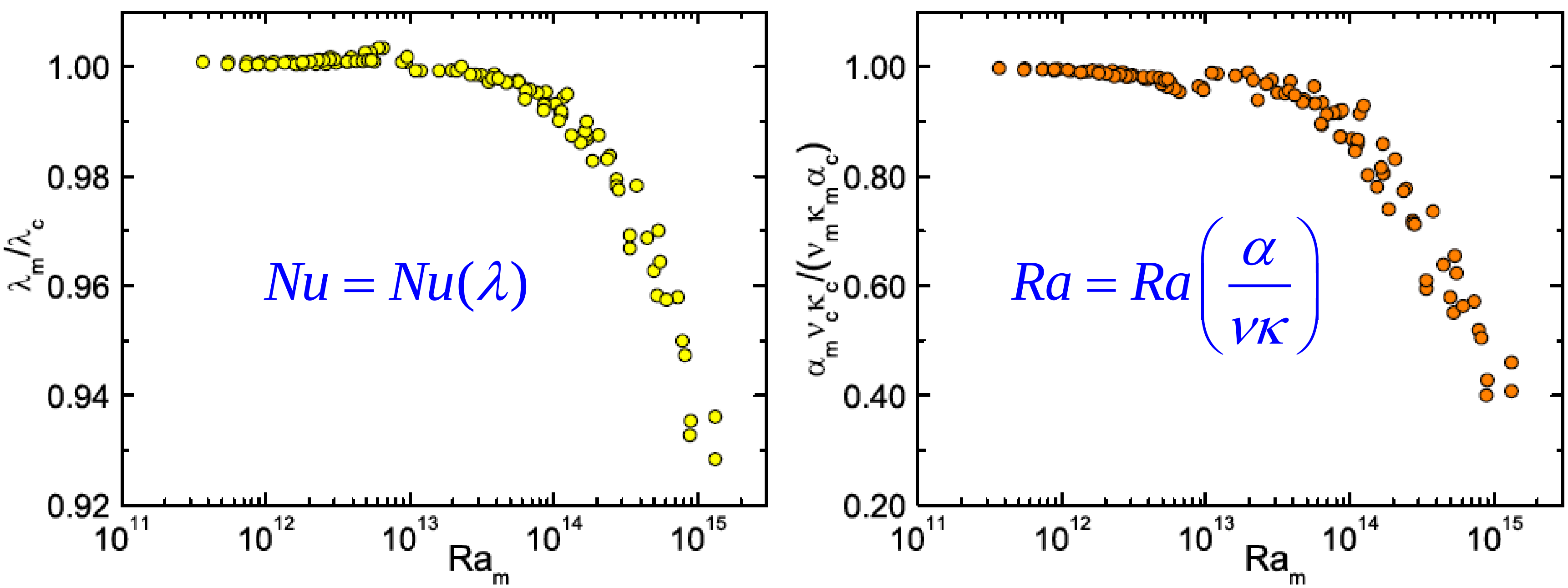
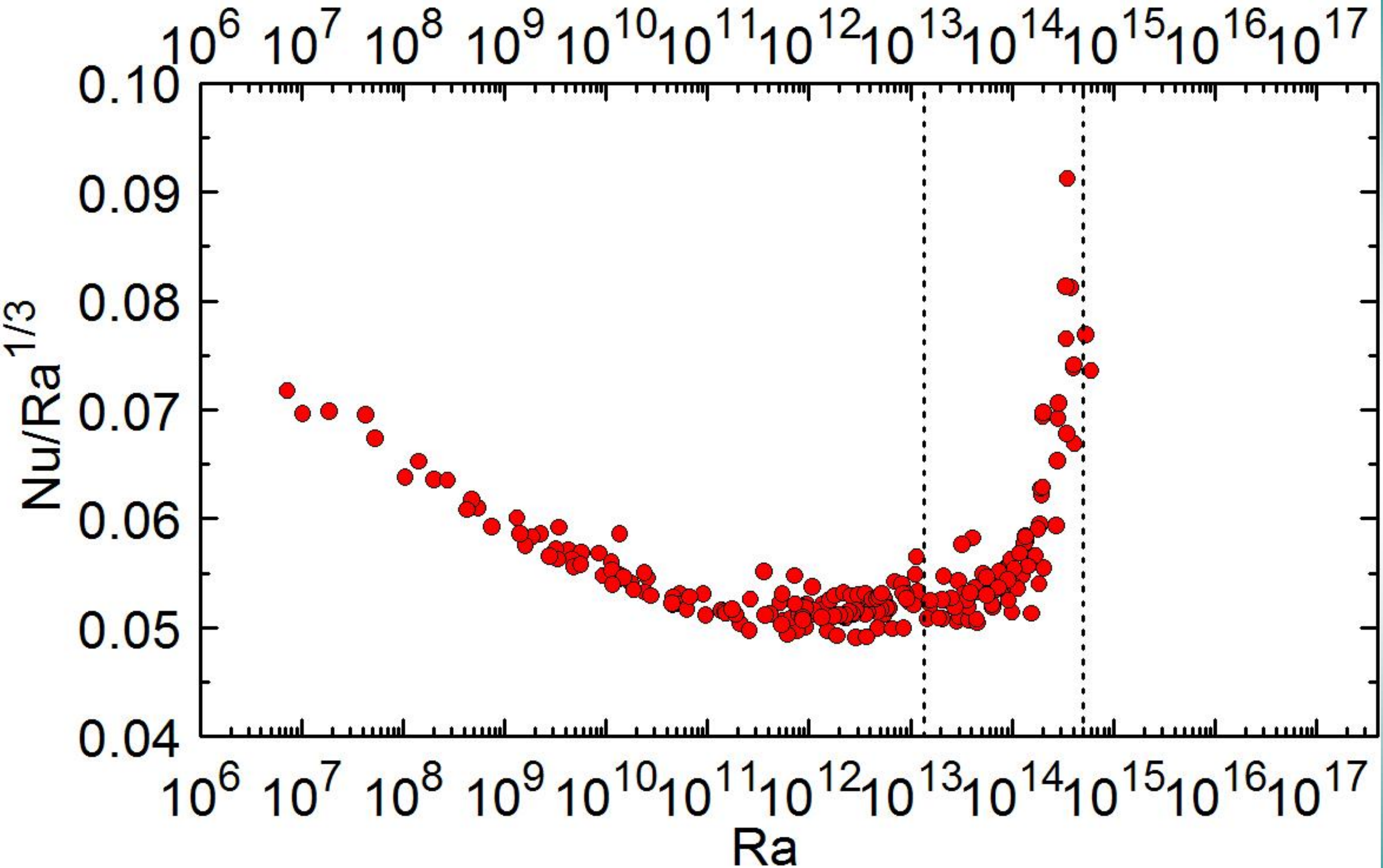
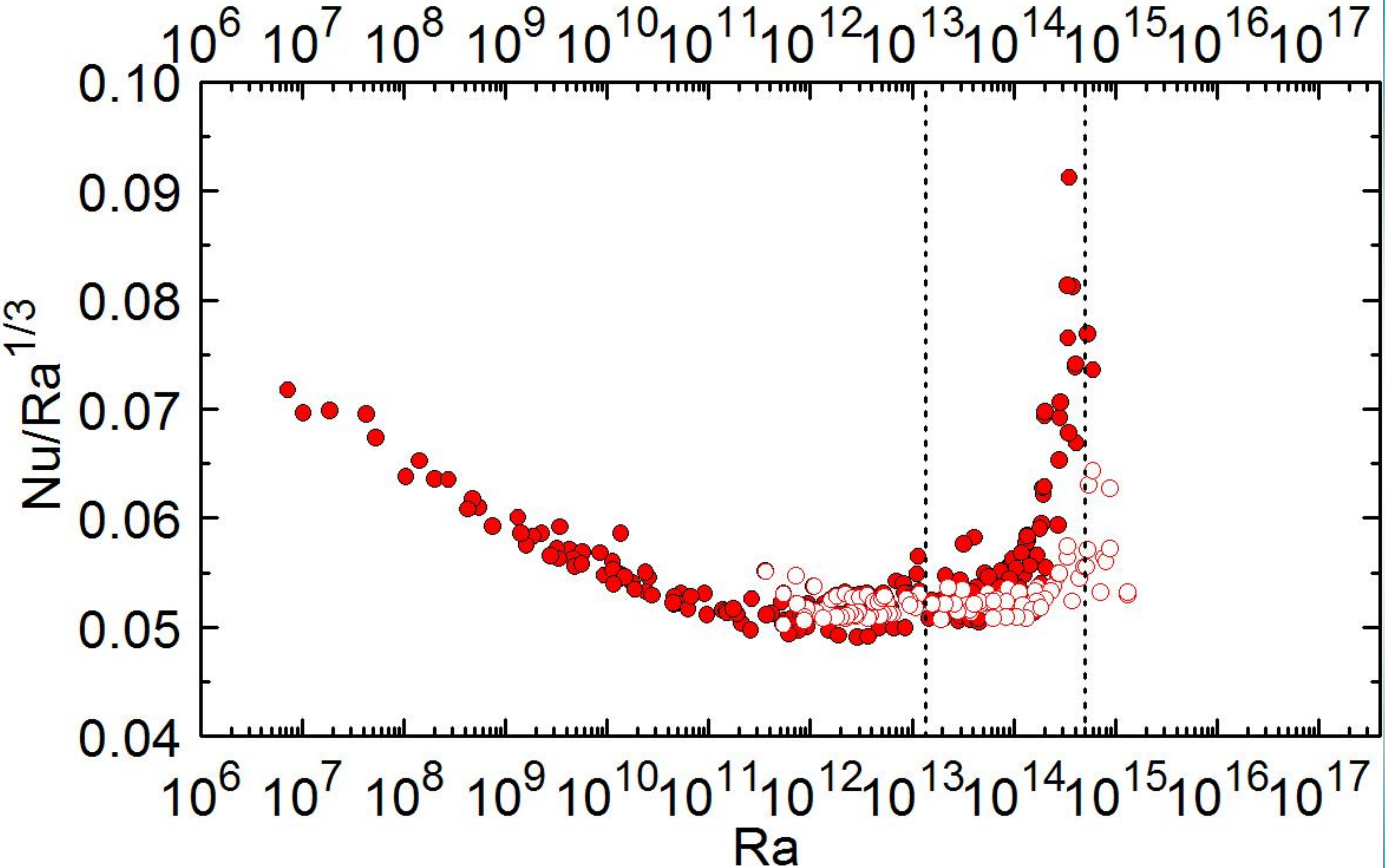


Figure 7. Ratios $\lambda_m/\lambda_c = Nu_c/Nu_m$ and $\alpha_m\nu_c\kappa_c/(\nu_m\kappa_m\alpha_c) = Ra_m/Ra_c$ evaluated for the working fluid – cryogenic helium gas – at T_m and T_c, plotted versus Ra_m.

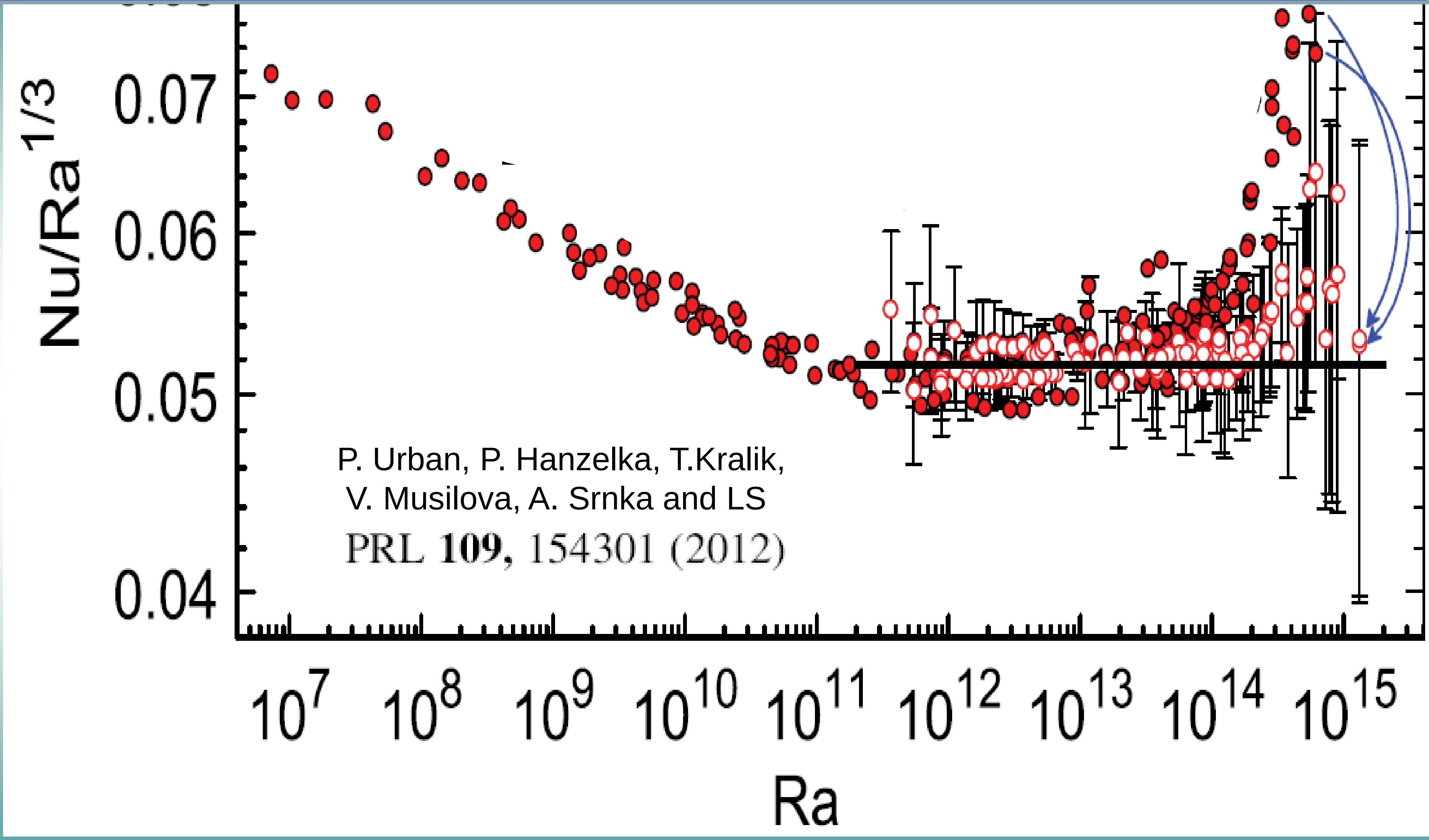


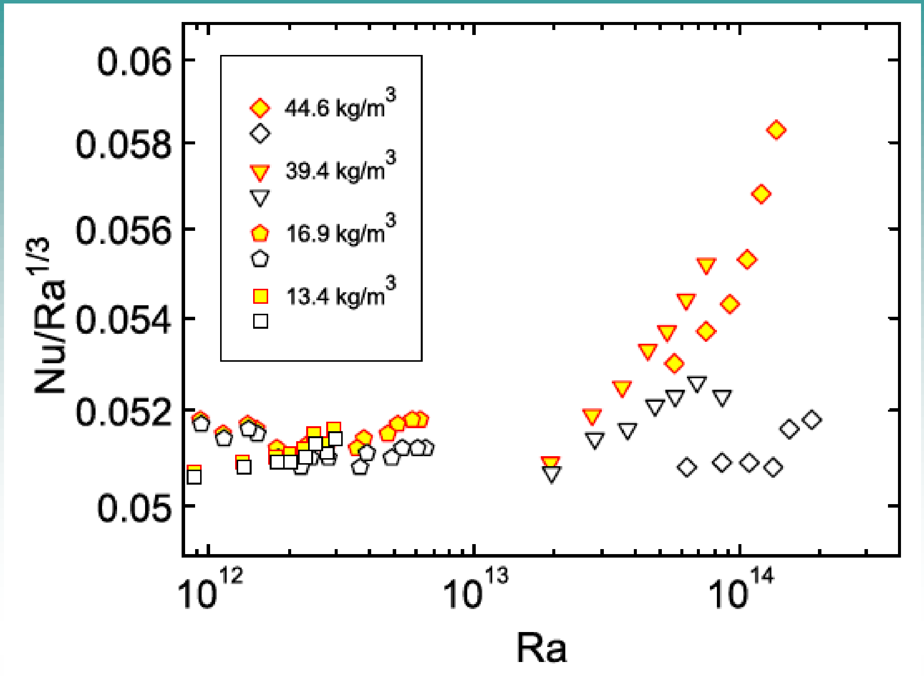
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17
0.10
0.09
0.08
0.07
0.06
0.05
0.04
Nu/Ra^1/3
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17
Ra

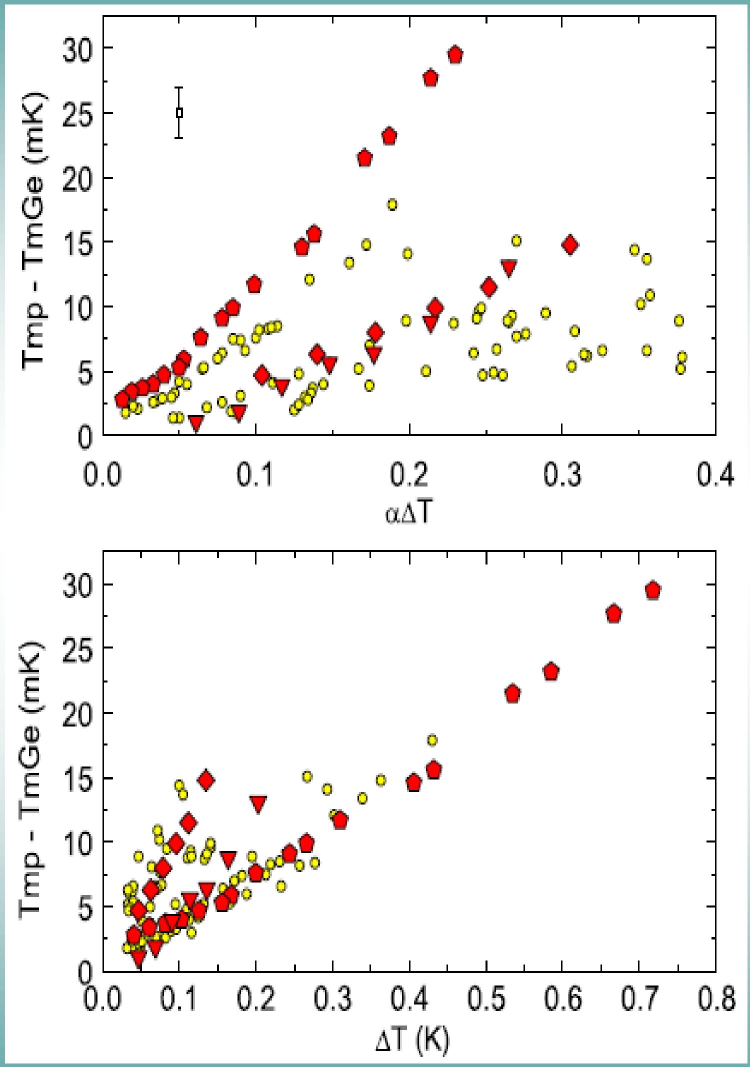


Nu/Ra^1/3
Ra
0.10
0.09
0.08
0.07
0.06
0.05
0.04
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17

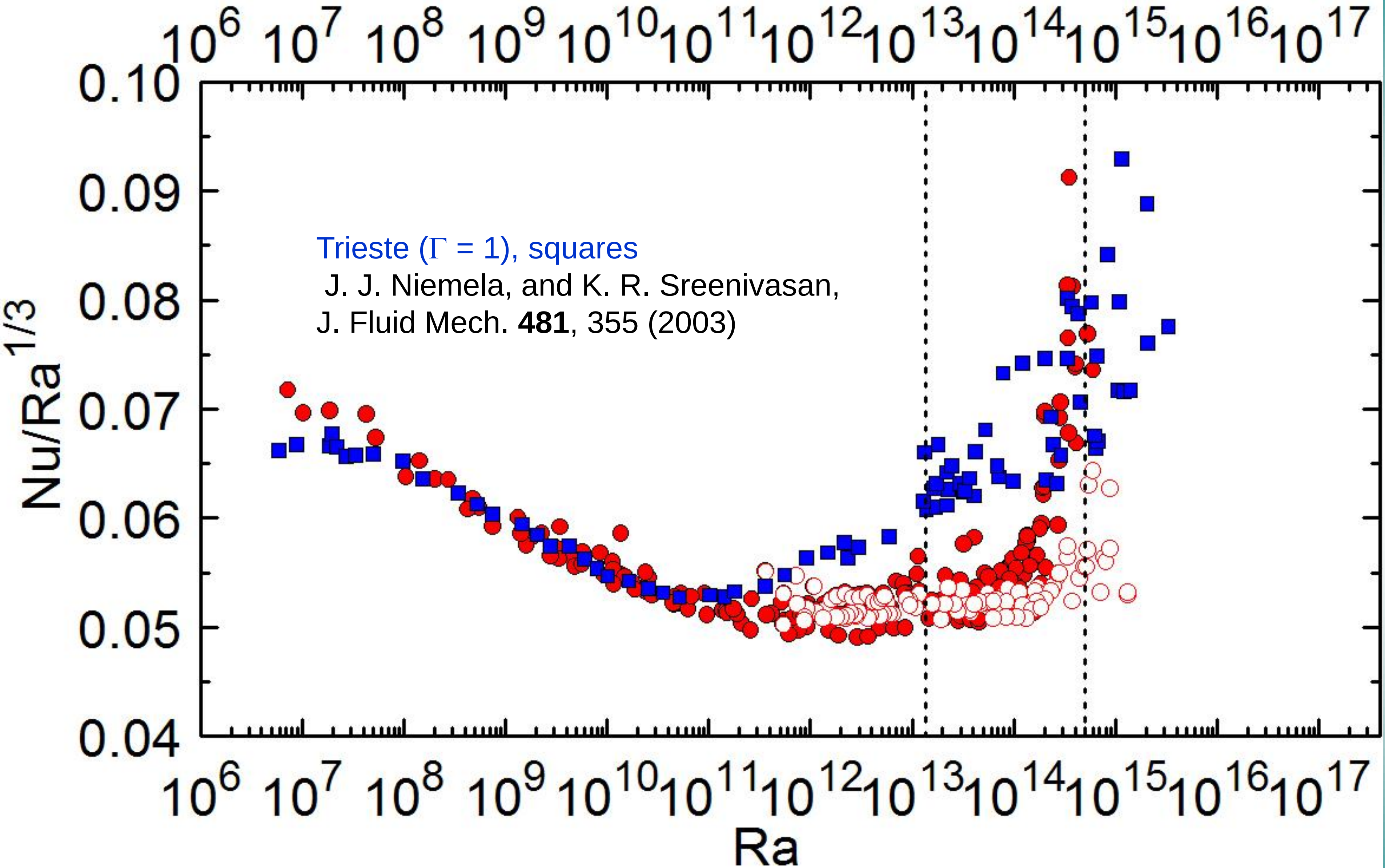
Effect of boundary layer asymmetry on Nu(Ra) scaling



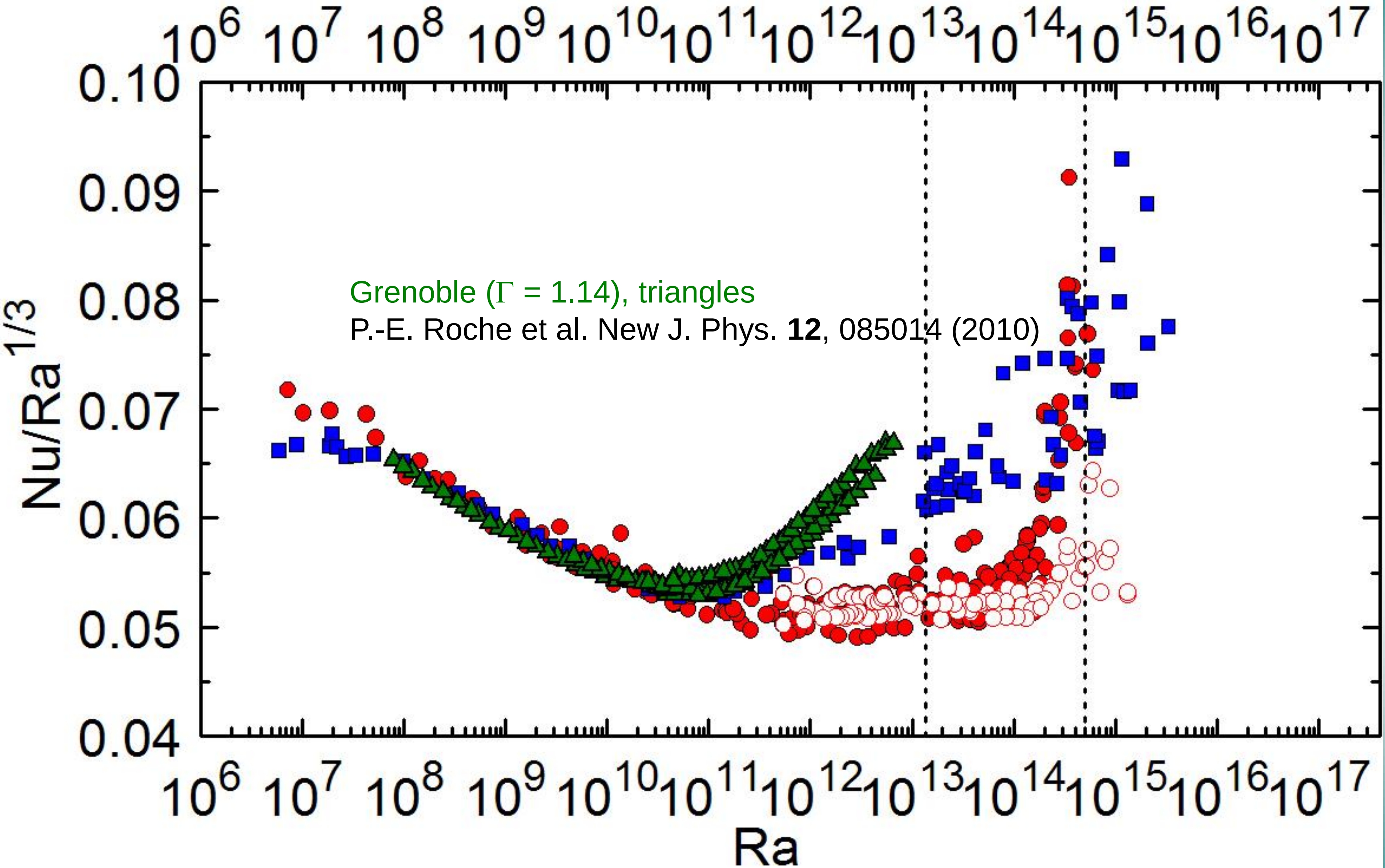




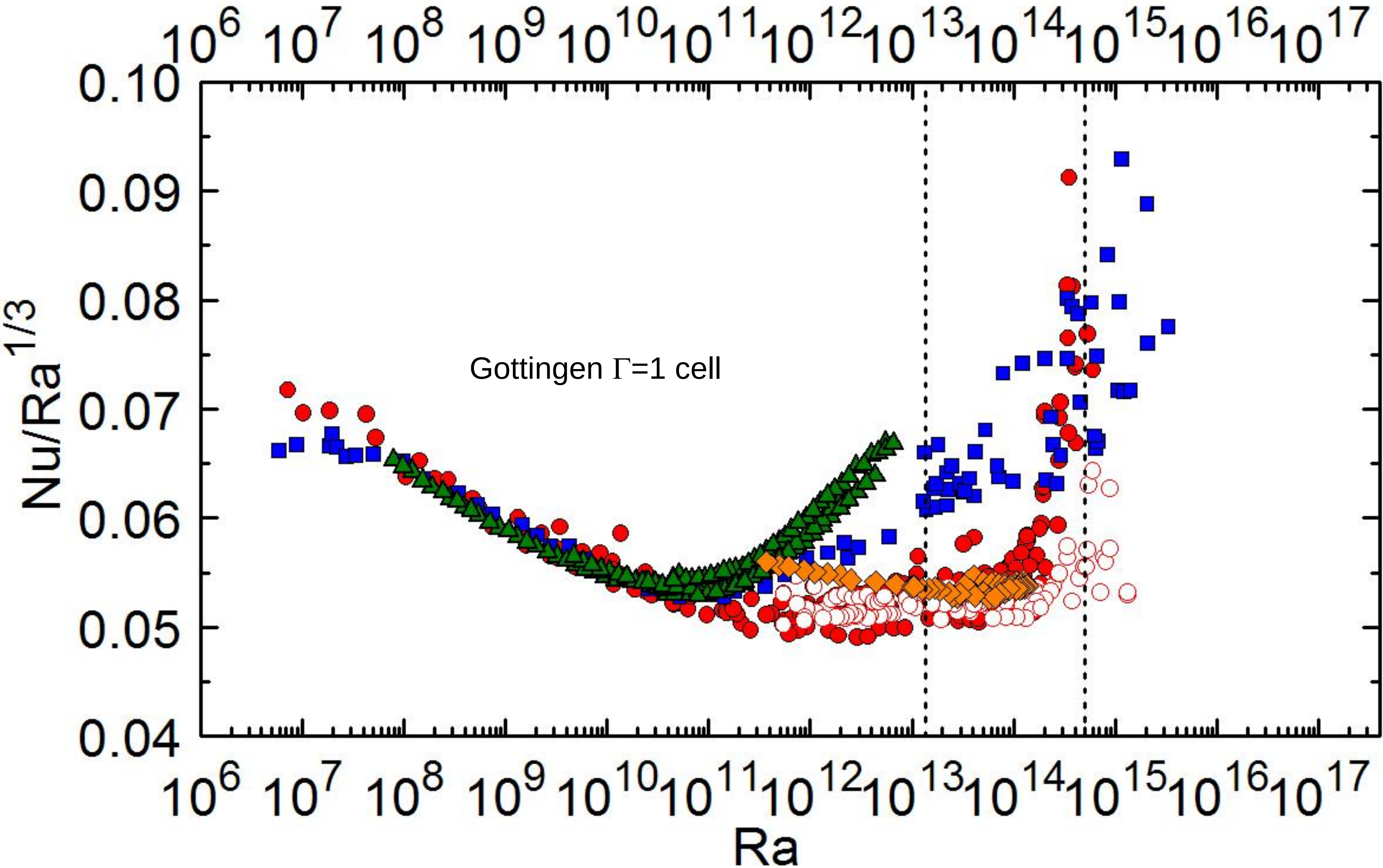
Measurements using three different densities of the working fluid as indicated and evaluated based on Tm (filled symbols) and Tc (open symbols). Note the significant difference in the observed Nu=Nu(Ra) scaling for Ra > 1E13.



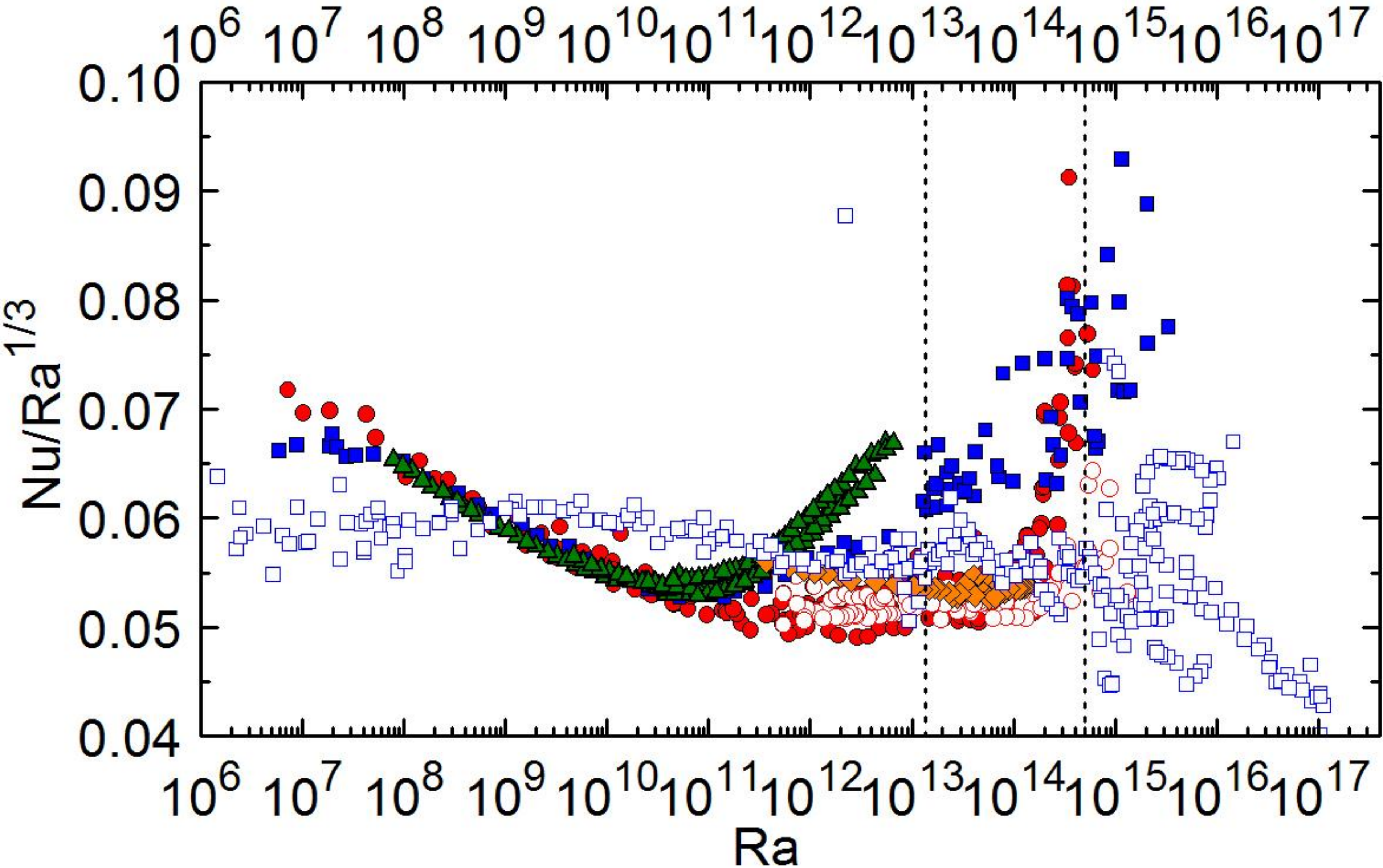
Trieste (Γ = 1), squares
J. J. Niemela, and K. R. Sreenivasan,
J. Fluid Mech. **481**, 355 (2003)
$Nu/Ra^{1/3}$
Ra
0.10
0.09
0.08
0.07
0.06
0.05
0.04
10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17}



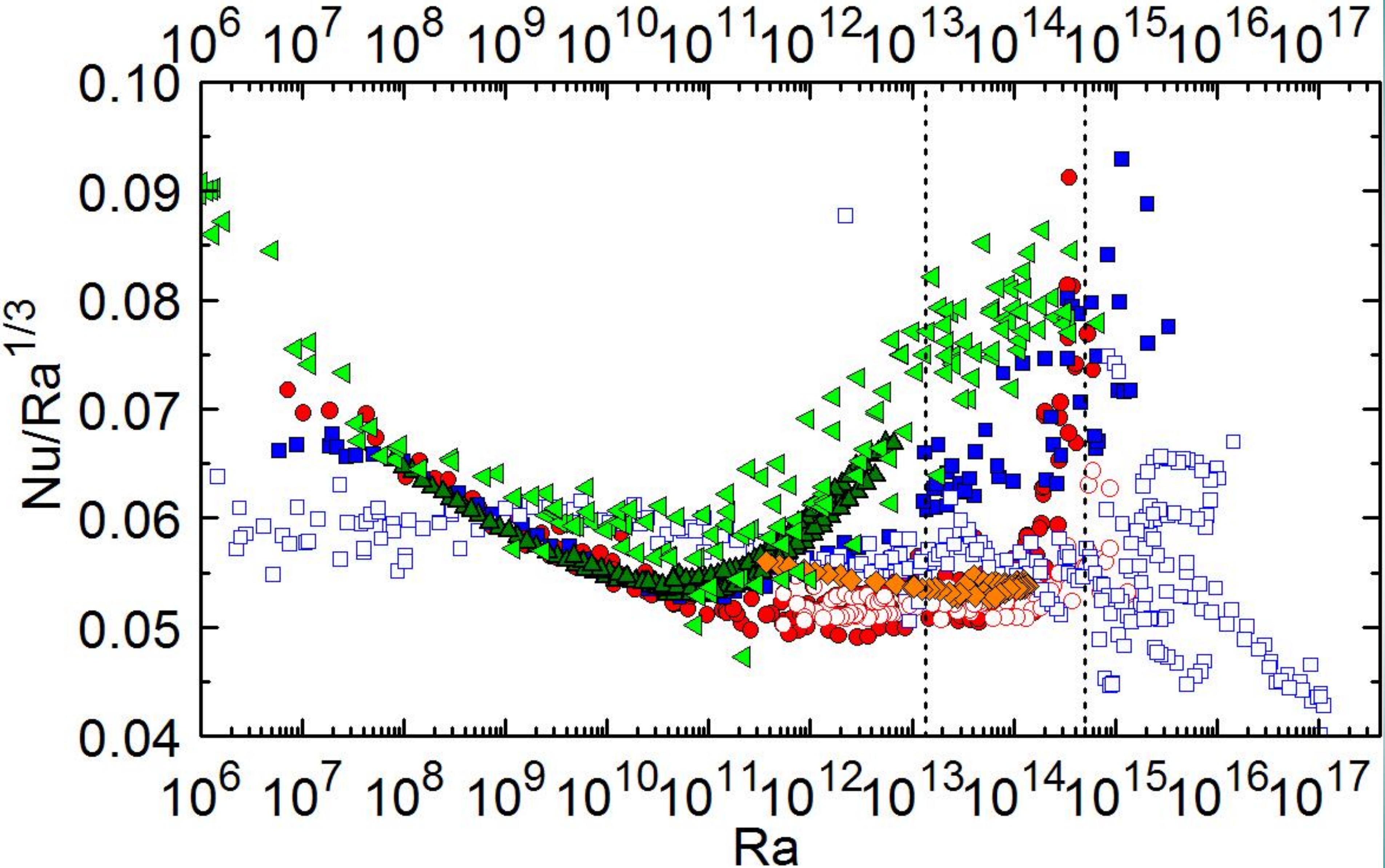
Grenoble (Γ = 1.14), triangles
P.-E. Roche et al. New J. Phys. **12**, 085014 (2010)
Nu/Ra^1/3
Ra
0.10
0.09
0.08
0.07
0.06
0.05
0.04
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17



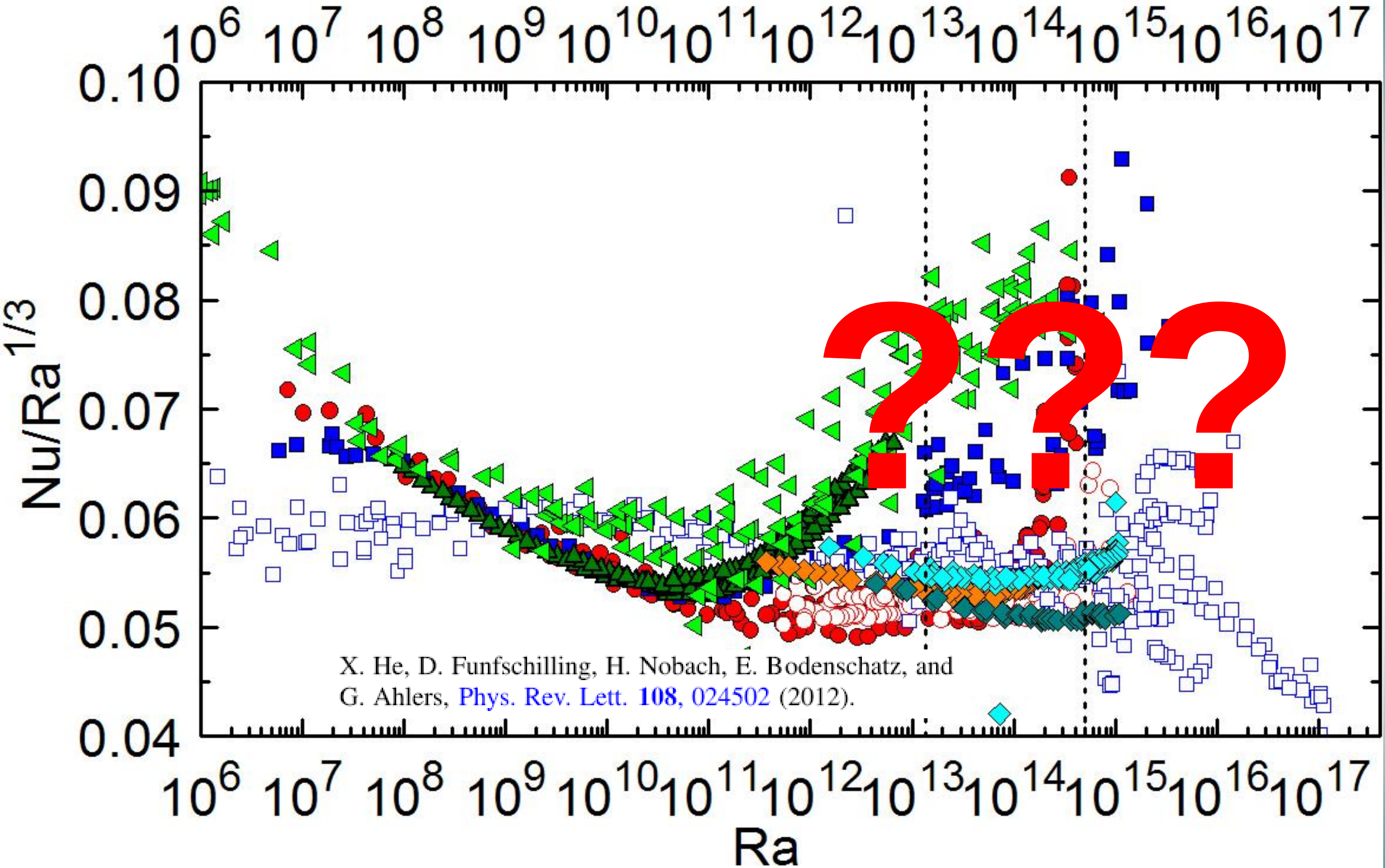
Gottingen Γ=1 cell
Nu/Ra^1/3
Ra
0.10
0.09
0.08
0.07
0.06
0.05
0.04
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17



10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17
0.10
0.09
0.08
0.07
0.06
0.05
0.04
Nu/Ra^1/3
Ra



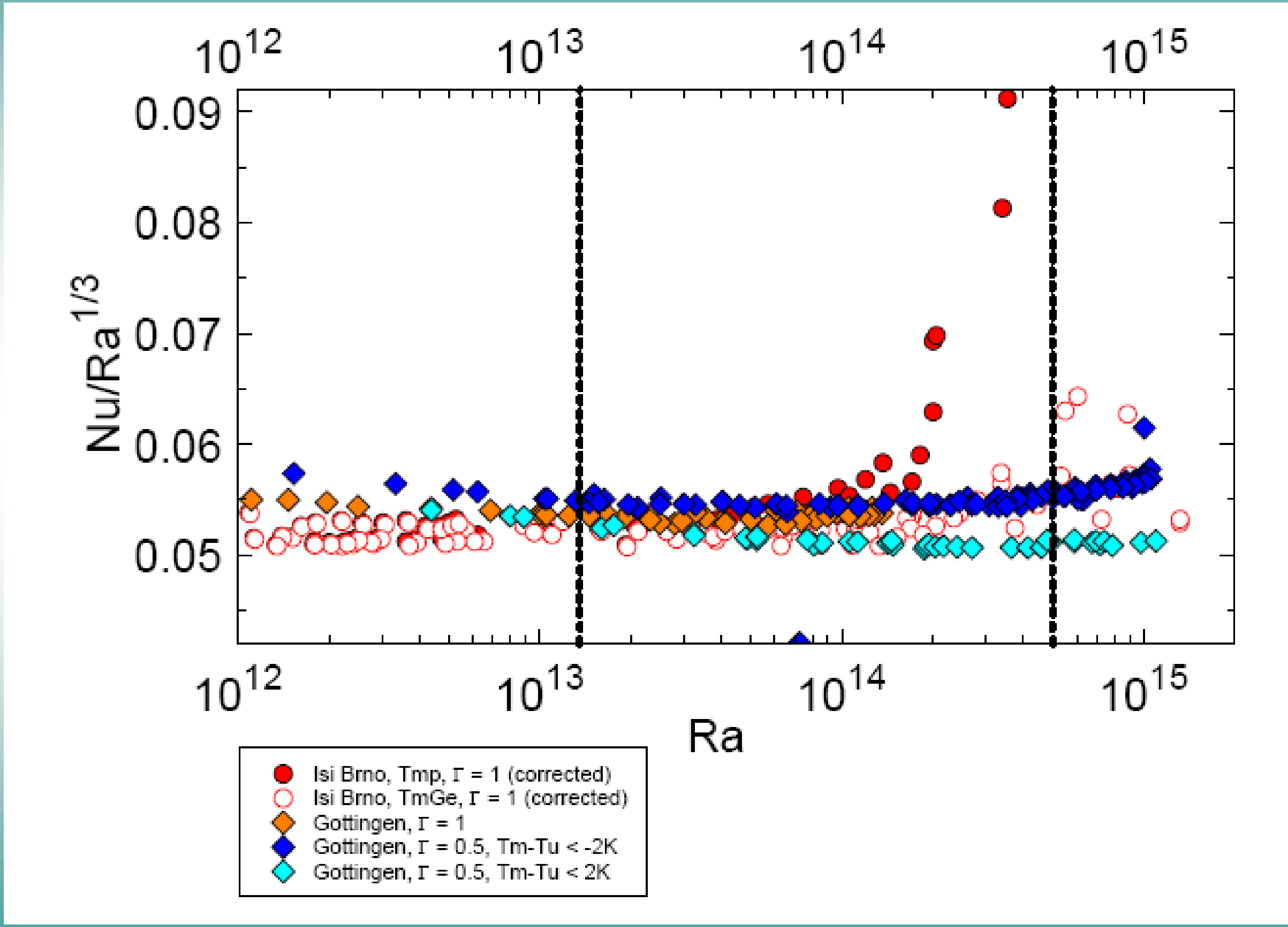
0.10
0.09
0.08
0.07
0.06
0.05
0.04
Nu/Ra^1/3
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17
Ra



0.10
0.09
0.08
0.07
0.06
0.05
0.04
Nu/Ra 1/3
10 6 10 7 10 8 10 9 10 10 10 11 10 12 10 13 10 14 10 15 10 16 10 17
Ra
???
X. He, D. Funfschilling, H. Nobach, E. Bodenschatz, and
G. Ahlers, Phys. Rev. Lett. 108, 024502 (2012).

X. He, D. Funfschilling, H. Nobach, E. Bodenschatz, and G. Ahlers, Phys. Rev. Lett. **108**, 024502 (2012).

Comparison of our and Gottingen heat transfer efficiency data



Why do the high Ra data from various experiments so badly disagree ???

In particular – has the transition to the ultimate regime been observed ???

In order to answer these questions, we believe more

thorough analysis of NOB effects is due

The compensated Nu as a function of Ra, based on the fluid properties evaluated for our data at four different temperatures: *Tm, Tc, Tb and Tt.*

This indicates the absolute gate where the Nu=Nu(Ra) data might lay

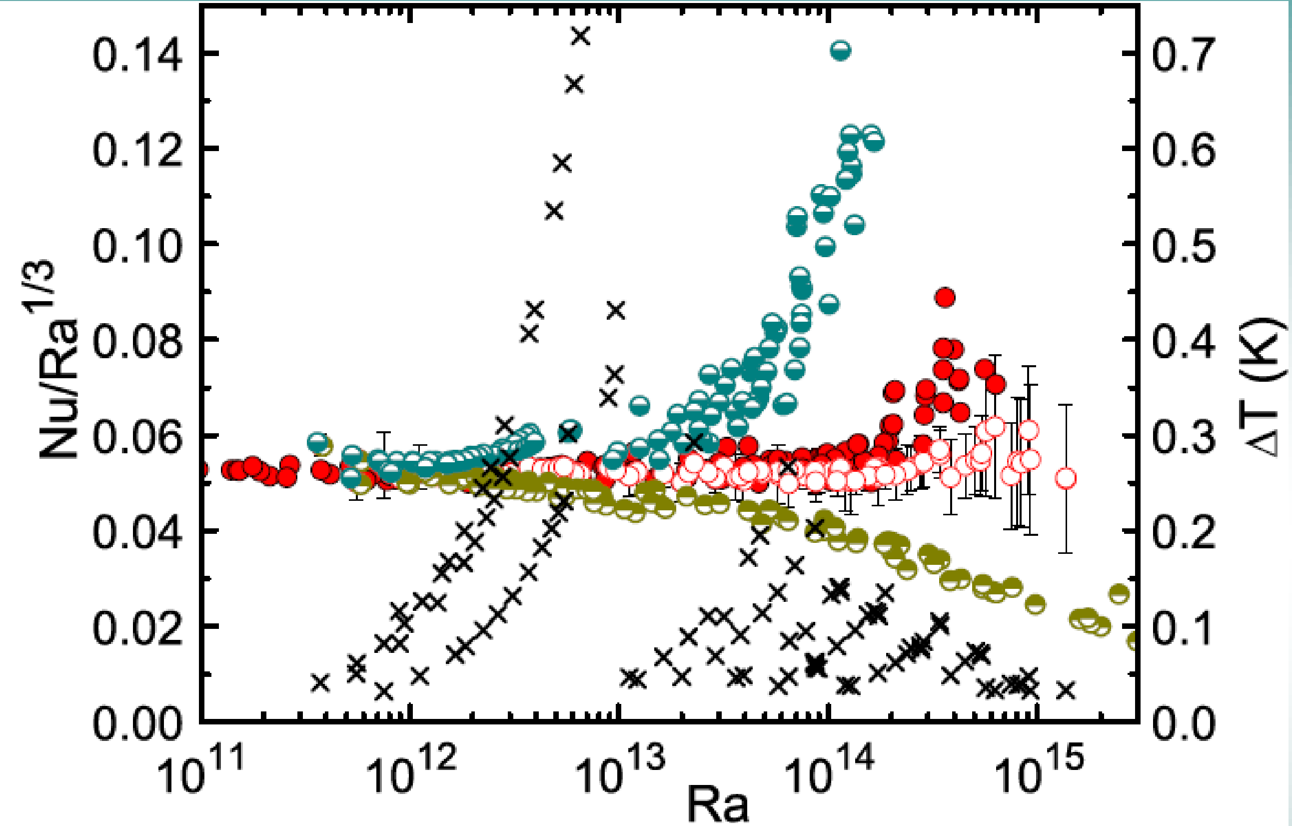
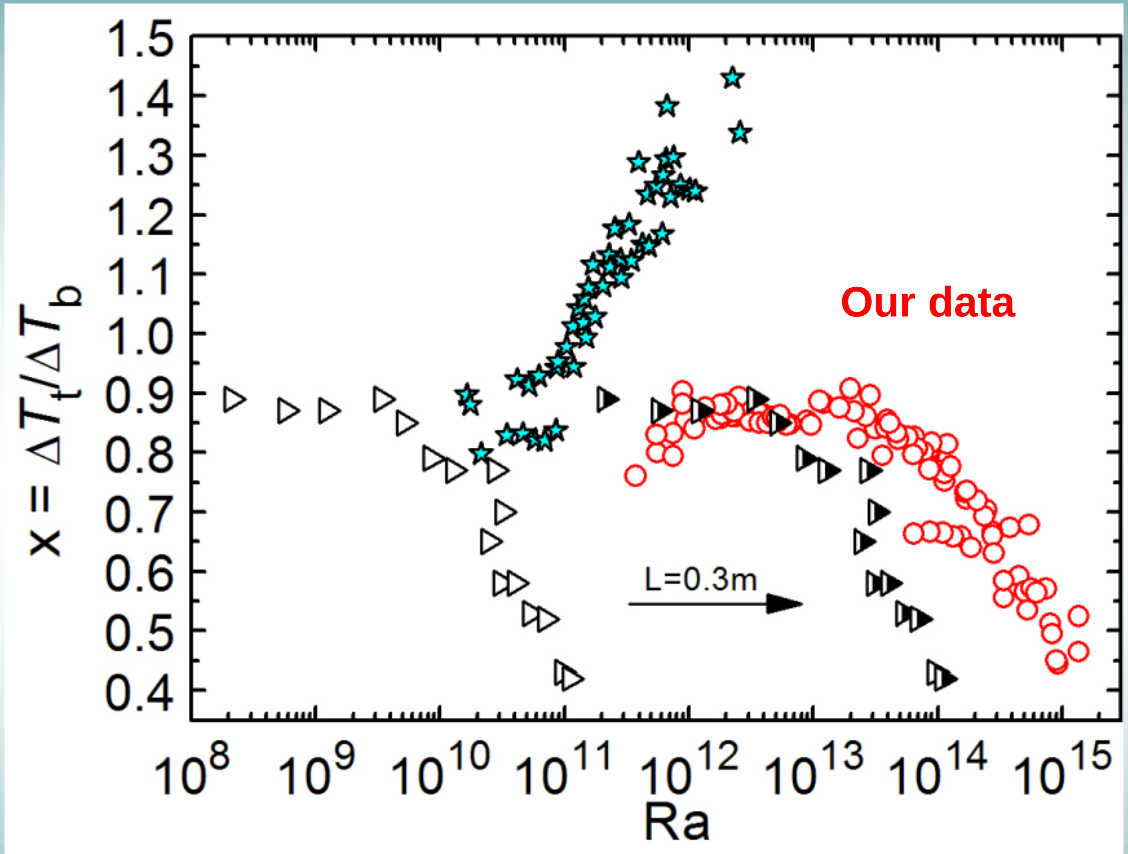


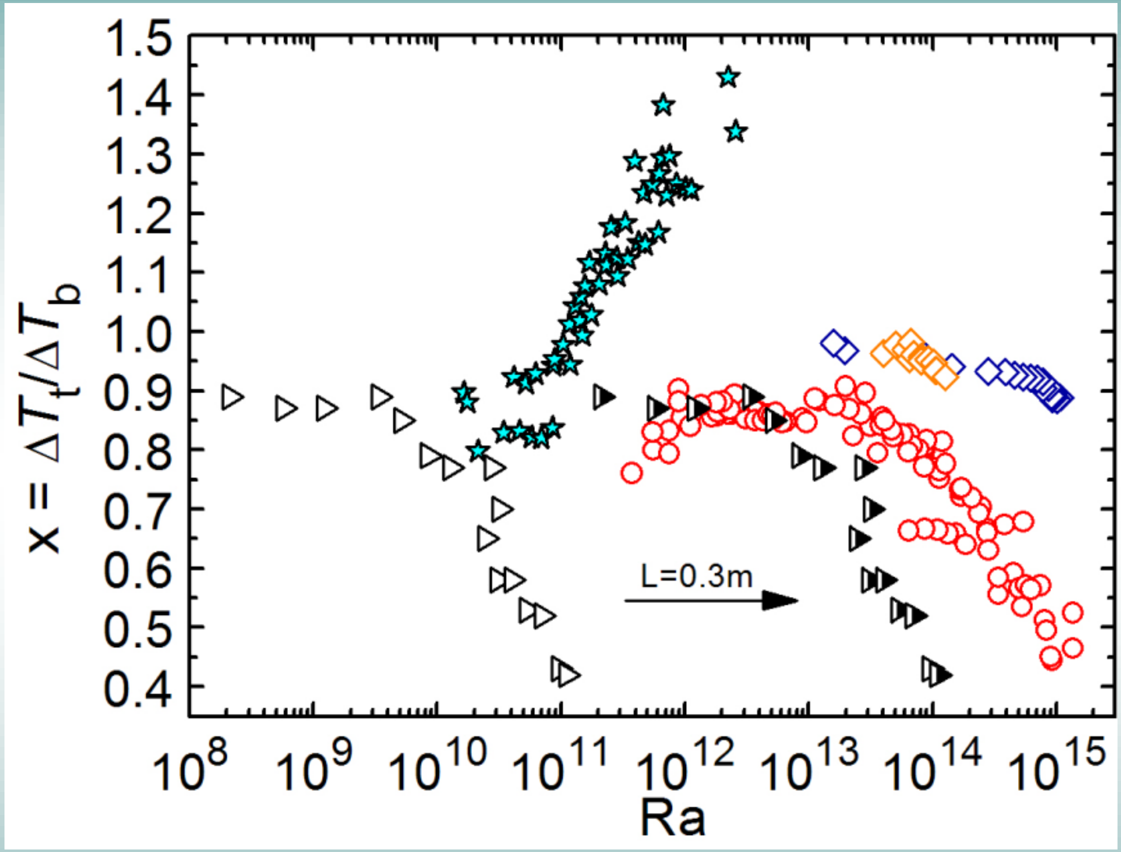
Figure 6. The compensated $\mathrm{Nu}/\mathrm{Ra}^{1/3}$ plots versus Ra, where Nu and Ra are not corrected and evaluated based on four different temperatures of the working fluid: T_m (red filled black circles; representing $\mathrm{Nu}_m/\mathrm{Ra}_m^{1/3}$ versus Ra_m), T_c (white filled red circles; representing $\mathrm{Nu}_c/\mathrm{Ra}_c^{1/3}$ versus Ra_c), T_t (olive circles with white filled bottom half, representing $\mathrm{Nu}_b/\mathrm{Ra}_b^{1/3}$ versus Ra_b) and T_b (dark blue circles with white filled top half, representing $\mathrm{Nu}_t/\mathrm{Ra}_t^{1/3}$ versus Ra_t). The crosses stand for the measured ΔT corresponding to each $\mathrm{Nu}_c/\mathrm{Ra}_c^{1/3}$ data point, plotted versus Ra_c.

Wu X Z and Libchaber A 1991 Non-Boussinesq effects in free thermal convection *Phys. Rev. A* **43** 2833–2839

$$x = \Delta T_t / \Delta T_b = (T_c - T_t)/(T_b - T_c)$$

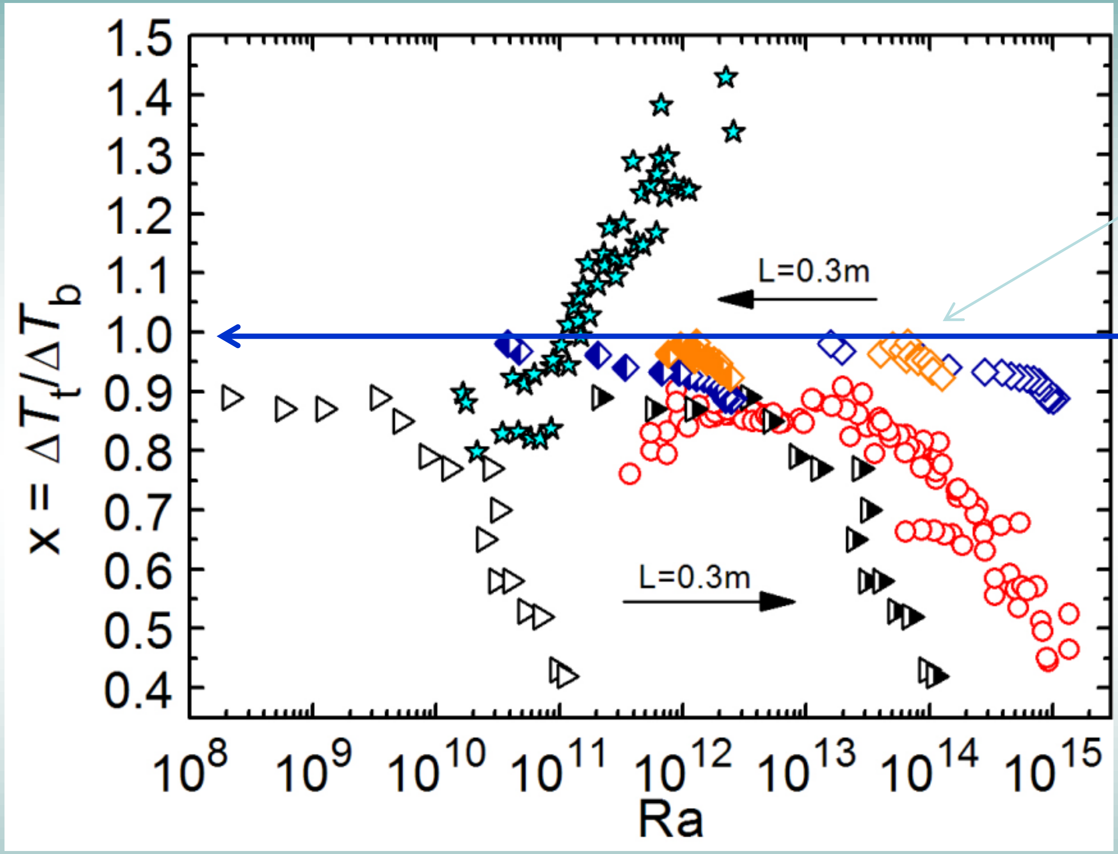


Tisserand J -C, Creyssels M, Gasteuil Y, Pabiou H, Gibert M, Castaing B and Chillà F 2011 Comparison between rough and smooth plates within the same RayleighBénard cell *Phys. Fluids* **23** 015105



$x = \Delta T_t / \Delta T_b$
Ra
L=0.3m

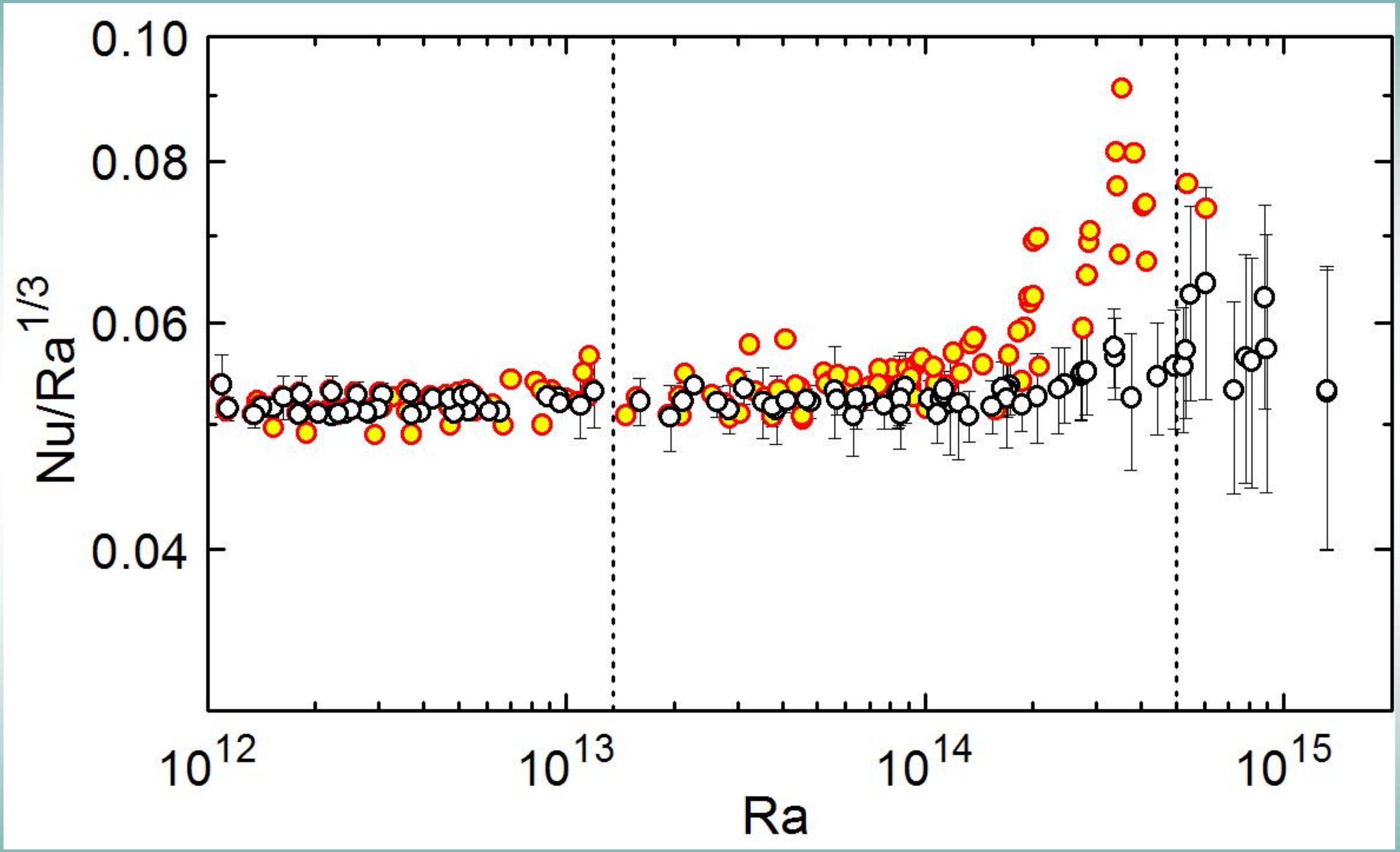
On asymmetry of boundary layers in various RBC experiments

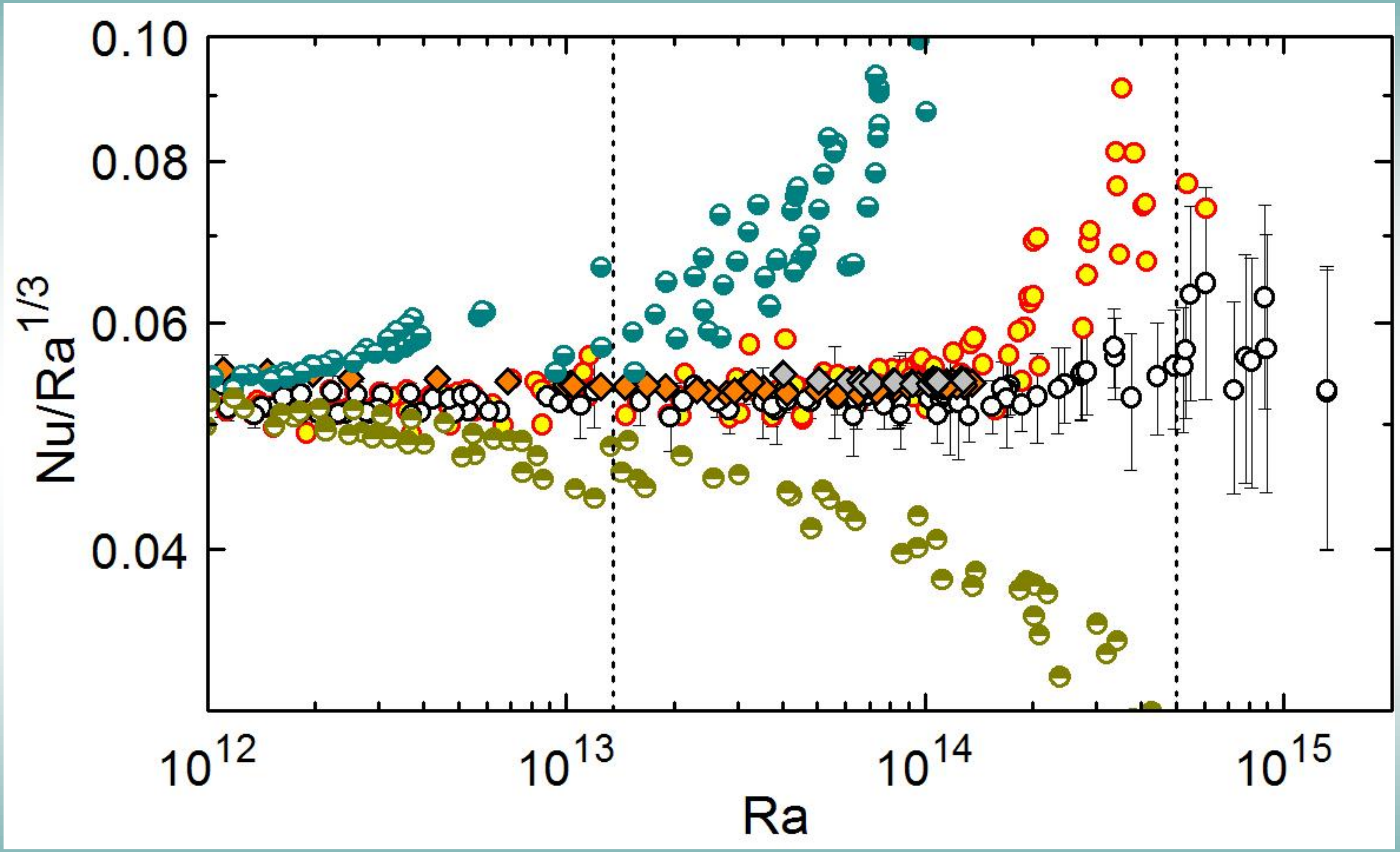


Gottingen SF6

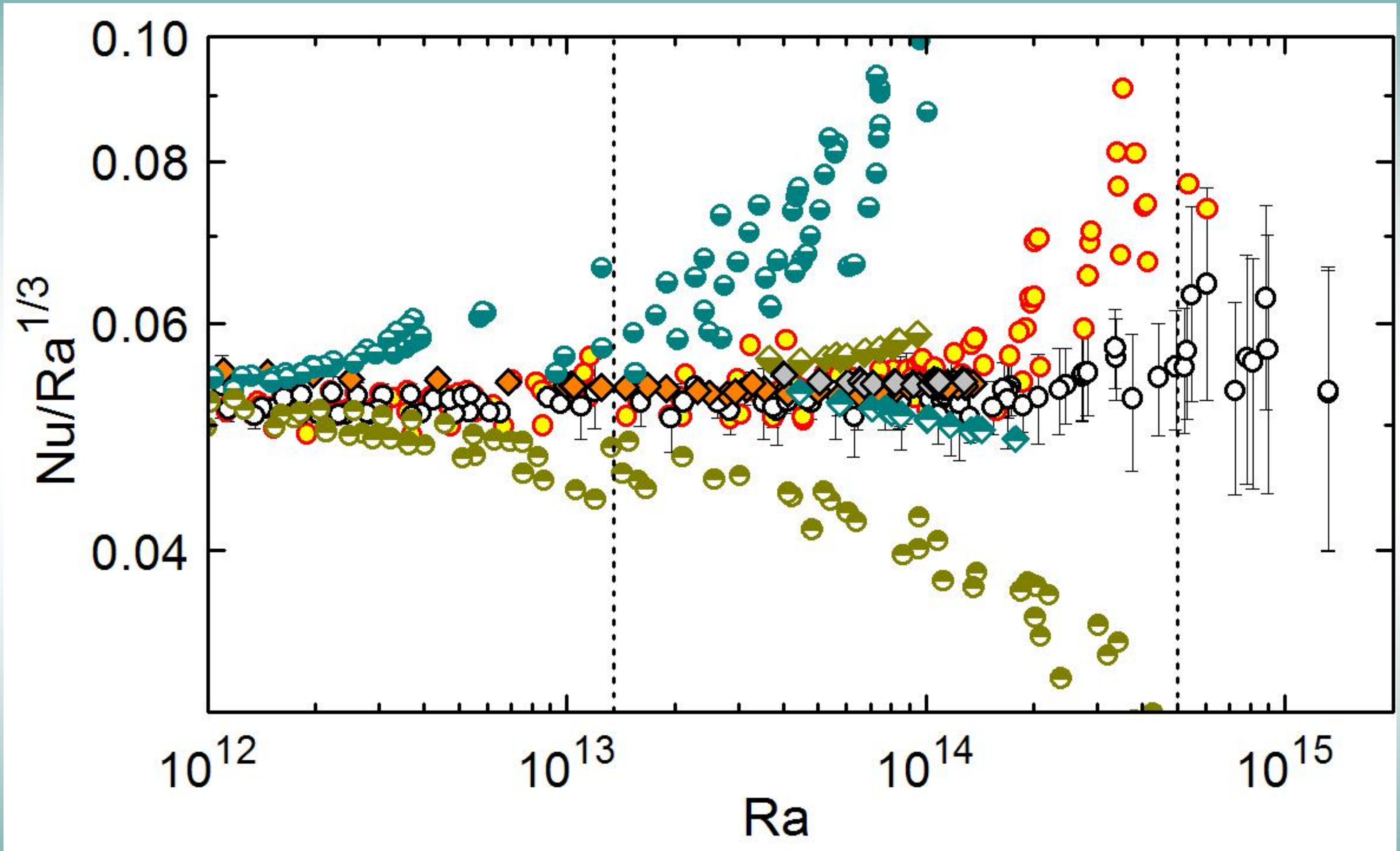
Γ=1 data

Γ=1/2 data


0.10
0.08
0.06
0.04
Nu/Ra^1/3
10^12
10^13
10^14
10^15
Ra



0.10
0.08
0.06
0.04
Nu/Ra^1/3
10^12
10^13
10^14
10^15
Ra

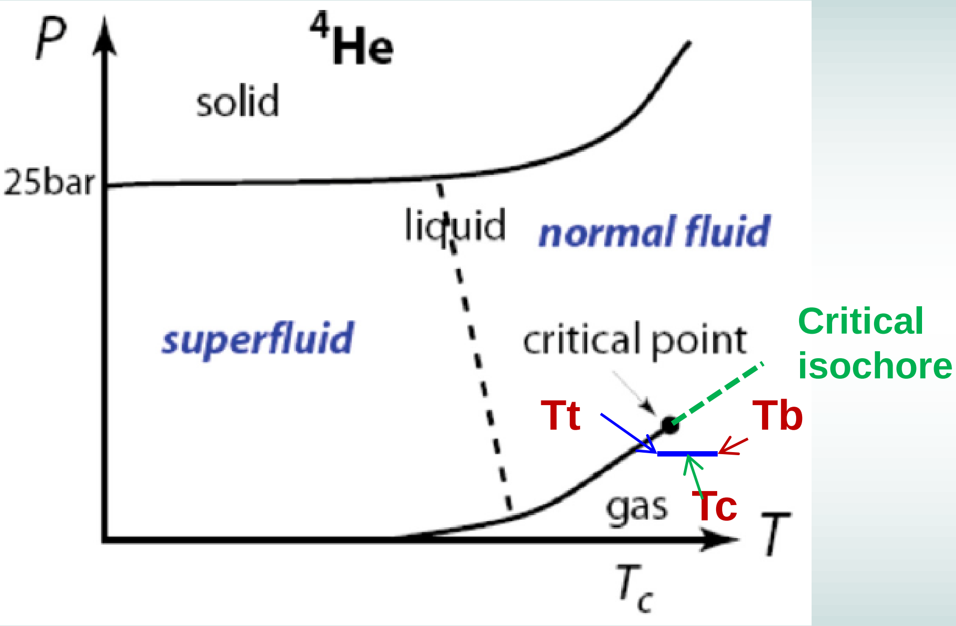


0.10
0.08
0.06
0.04
$Nu/Ra^{1/3}$
10^{12}
10^{13}
10^{14}
10^{15}
Ra

Next step – calculate the Effective heat transfer efficiency at high Ra

we directly measure the pressure *and three temperatures Tb; Tt and Tc simultaneously; Tc - an average of individual readings* of the small *in situ calibrated Ge sensors.*

most of the RBC cell, the bulk, is very close to *Tc*



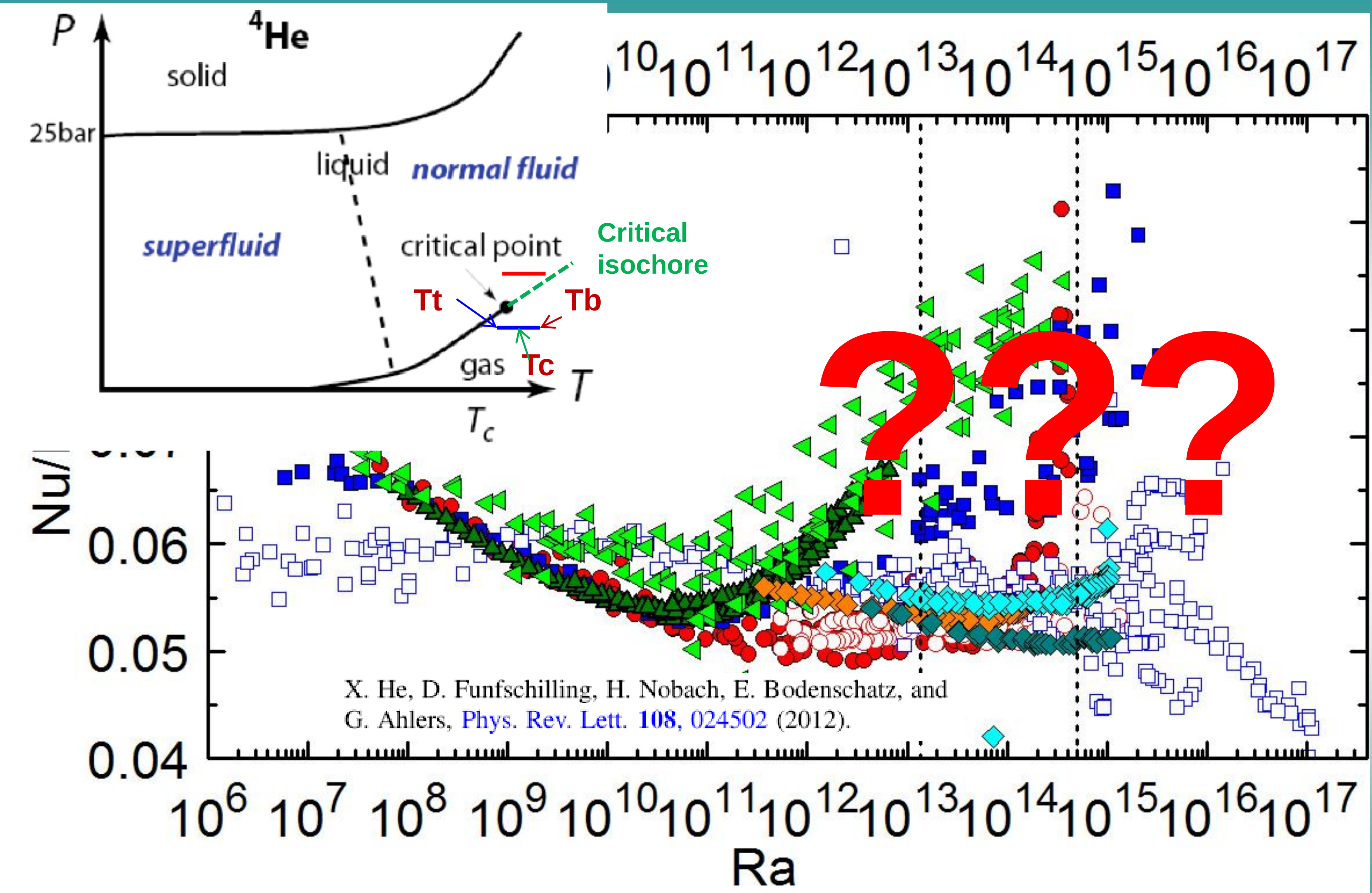
Below the saturated vapour curve and/or critical isochore), the conditions in the bottom (hotter) part of the RBC cell are substantially more OB than in the upper (colder) part

Assuming that the top and bottom boundary layers are independent, we replace the top half of the RBC cell by an inverse bottom half

In other words, we ignore the NOB top part of the RBC cell and construct an effective, fully symmetric RBC flow, where the temperature difference

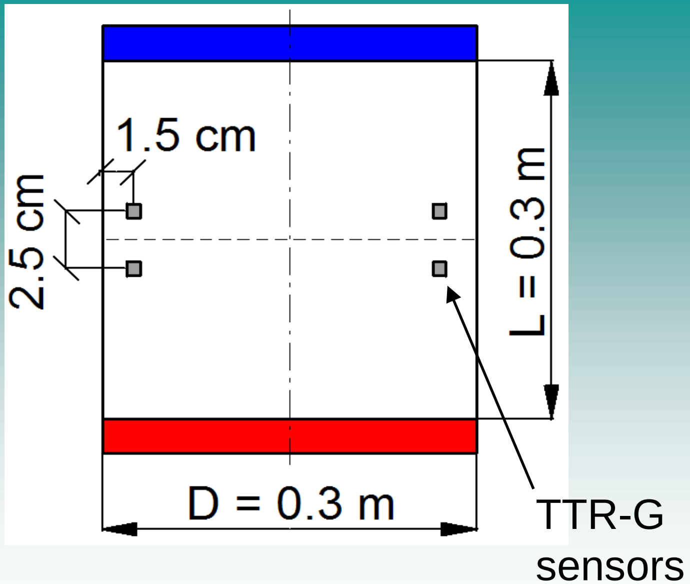
$$\Delta T_{\text{eff}} = 2(T_{\text{b}} - T_{\text{c}})$$

It is plausible to expect that this artificially constructed RBC flow will match the ideal OB conditions much more closely


P
4He
solid
25bar
liquid
normal fluid
superfluid
critical point
Critical isochore
Tt
Tb
Tc
gas
T
Tc
Nu/
0.06
0.05
0.04
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17
Ra
???
X. He, D. Funfschilling, H. Nobach, E. Bodenschatz, and G. Ahlers, Phys. Rev. Lett. 108, 024502 (2012).

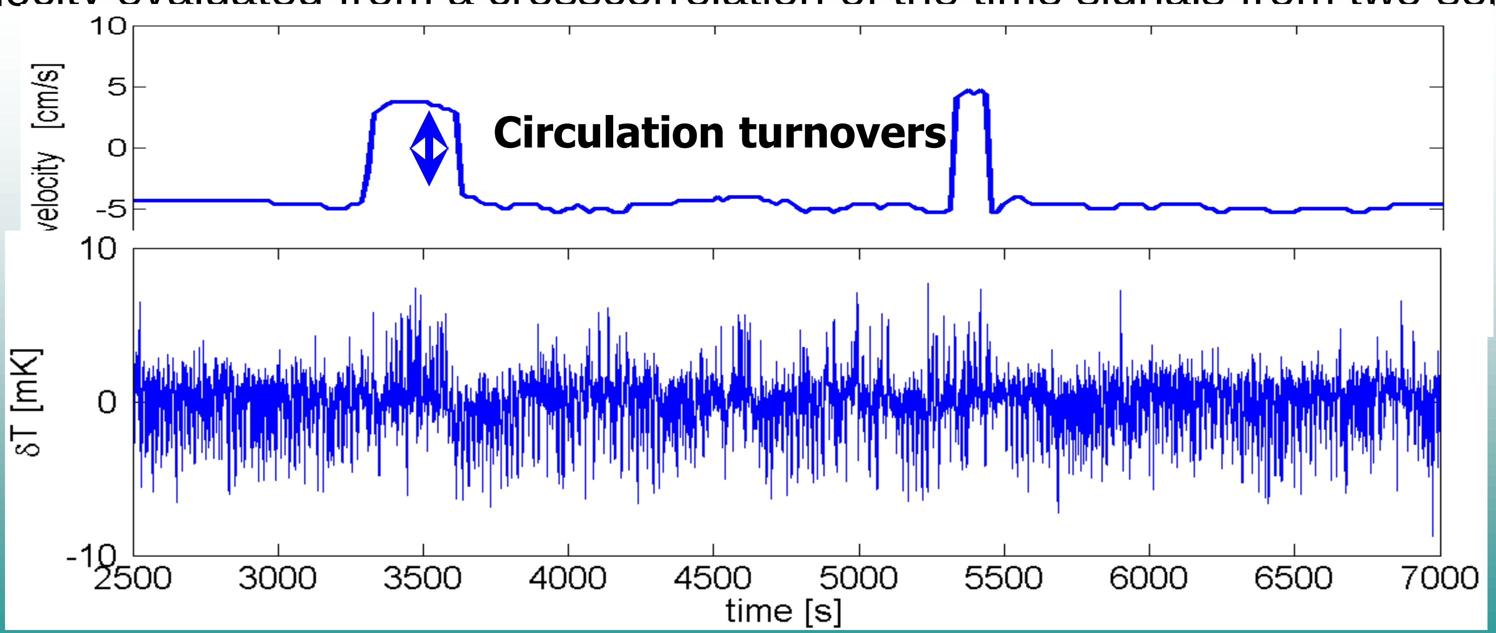
•Four small Ge sensors TTR-G (Institute of Semiconductor Physics, Kiev, Ukraine)
Electrical resistance and sensitivity of about 6 kΩ and 10^3 Ω/K respectively (at 5 K)
calibrated against Ge temperature sensors Lake Shore GR-200A-1500-1.4B imbeded in the plates (4 mK uncertainty) within ~ 1 mK

- **local temperature fluctuations**



Observation of LSC   Ra = 4.8×10^{11}, Nu = 415, Pr = 0.87

Velocity evaluated from a crosscorrelation of the time signals from two sensors



In analogy with

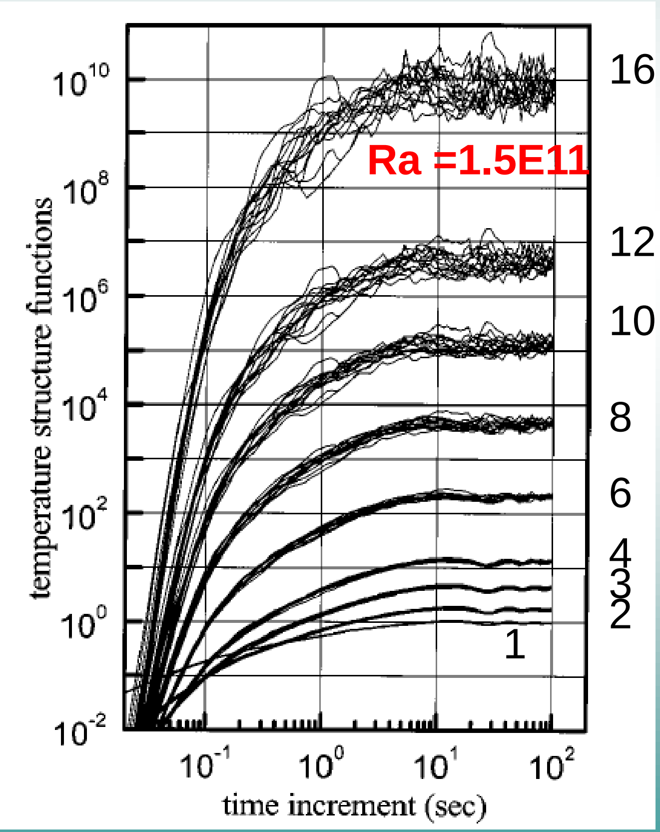
PHYSICAL REVIEW E **66**, 036303 (2002)

Temperature structure functions in the Bolgiano regime of thermal convection

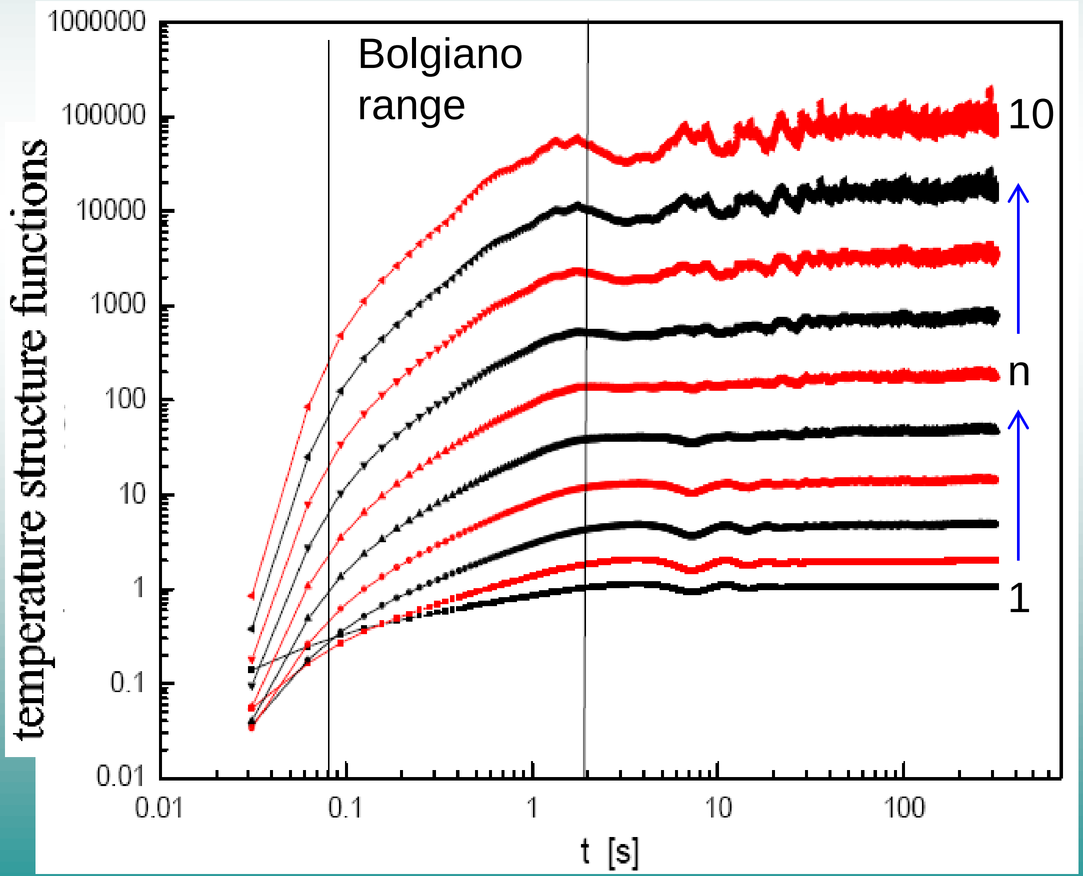
L. Skrbek,[1,2] J. J. Niemela,[1] K. R. Sreenivasan,[3] and R. J. Donnelly[1]

$$L_B = \frac{\varepsilon^{5/4}}{N^{3/4}(g\alpha)^{3/2}} \sim \frac{\mathrm{Nu}^{1/2} H}{(\mathrm{Ra}\,\mathrm{Pr})^{1/4}}$$

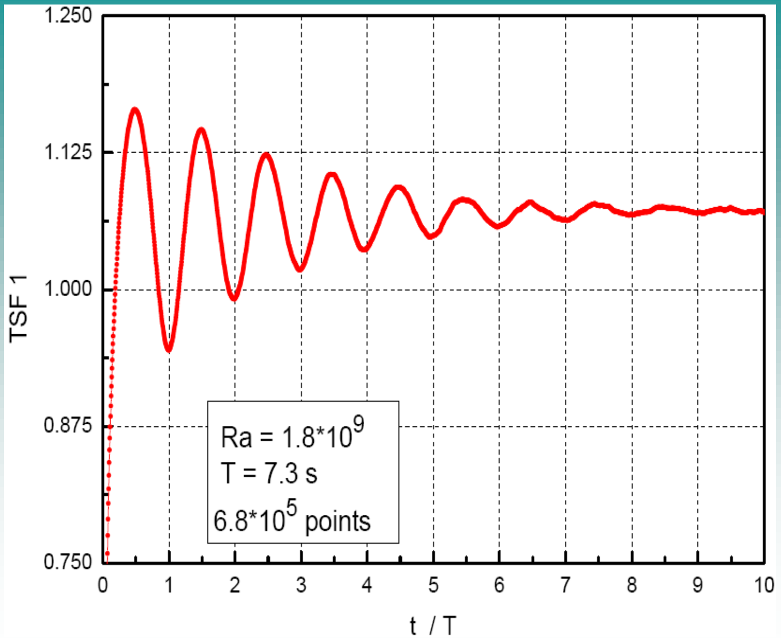
$$S_n^t(m\Delta t) = \langle |\theta(t_0 + m\Delta t) - \theta(t_0)|^n \rangle$$

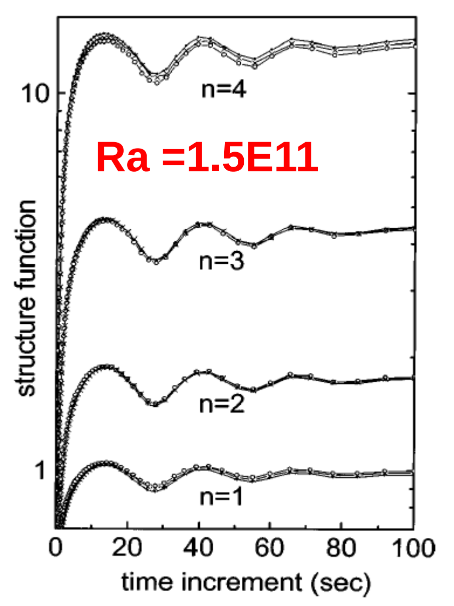


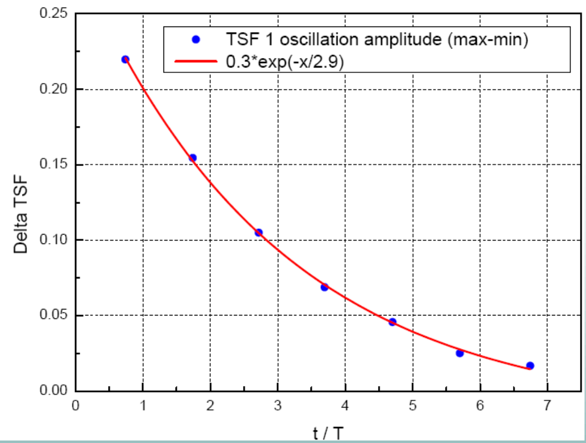
Structure functions order 1-10 Ra=1.8E9



Ra	Nu	Pr	LB [mm]
9,38E+07	32,19542	0,679	19,0545
3,49E+08	46,55939	0,68	16,5002
8,33E+08	60,18225	0,681	15,0798
1,76E+09	75,00998	0,684	13,9566
5,57E+09	105,763	0,692	12,3787
8,22E+09	120,624	0,693	11,9938
1,42E+10	140,3023	0,7	11,2549
4,20E+10	195,6444	0,717	10,0713
9,04E+10	247,7457	0,734	9,30231
2,19E+11	328,5279	0,771	8,47902
4,73E+11	421,1553	0,847	7,73726
2,51E+12	814,9216	1,358	6,30003

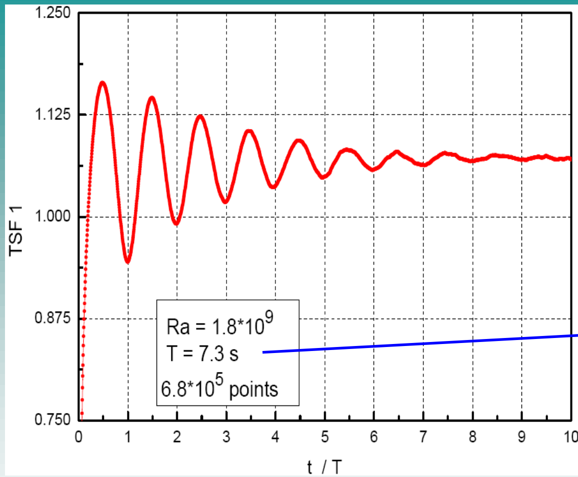




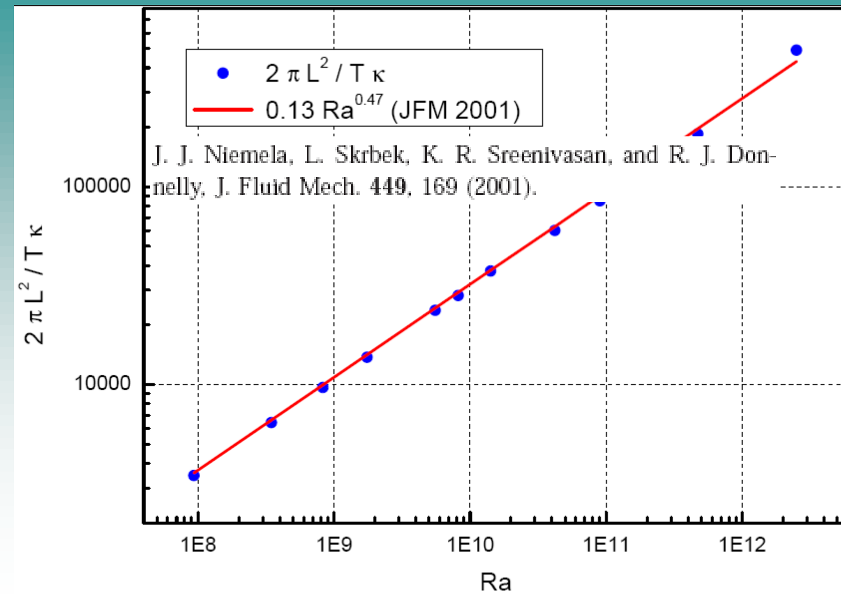


There is a surprisingly long characteristic decay time of order 3T

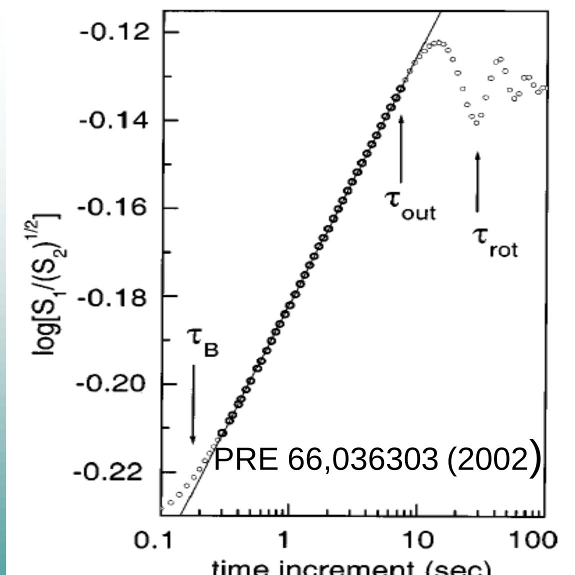
If diffusion process, then $\tau \sim L^2 / \nu_{eff} \Rightarrow \nu_{eff} \approx 40 \ cm^2 / s$



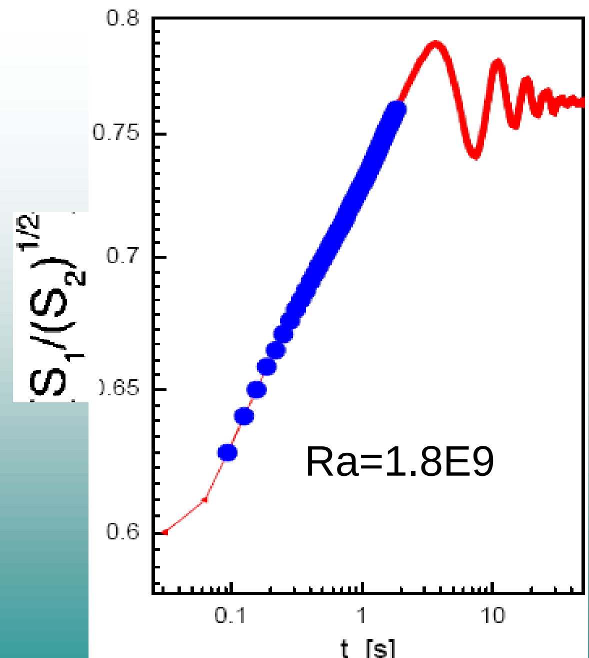
Non-dimensional frequency



Extended self similarity results



E. S. C. Ching, Phys. Rev. E **61**, R33 (2000)



Extended self similarity results – temperature structure functions

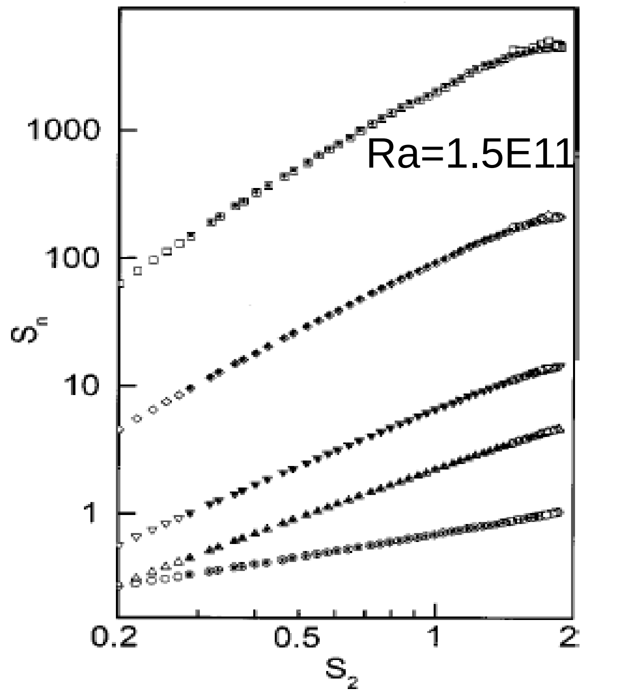
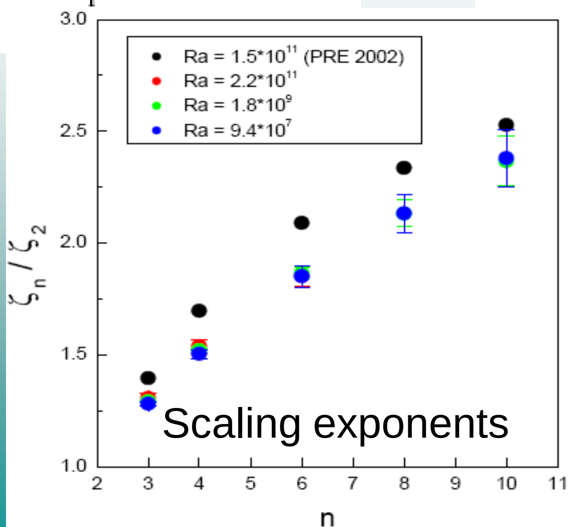
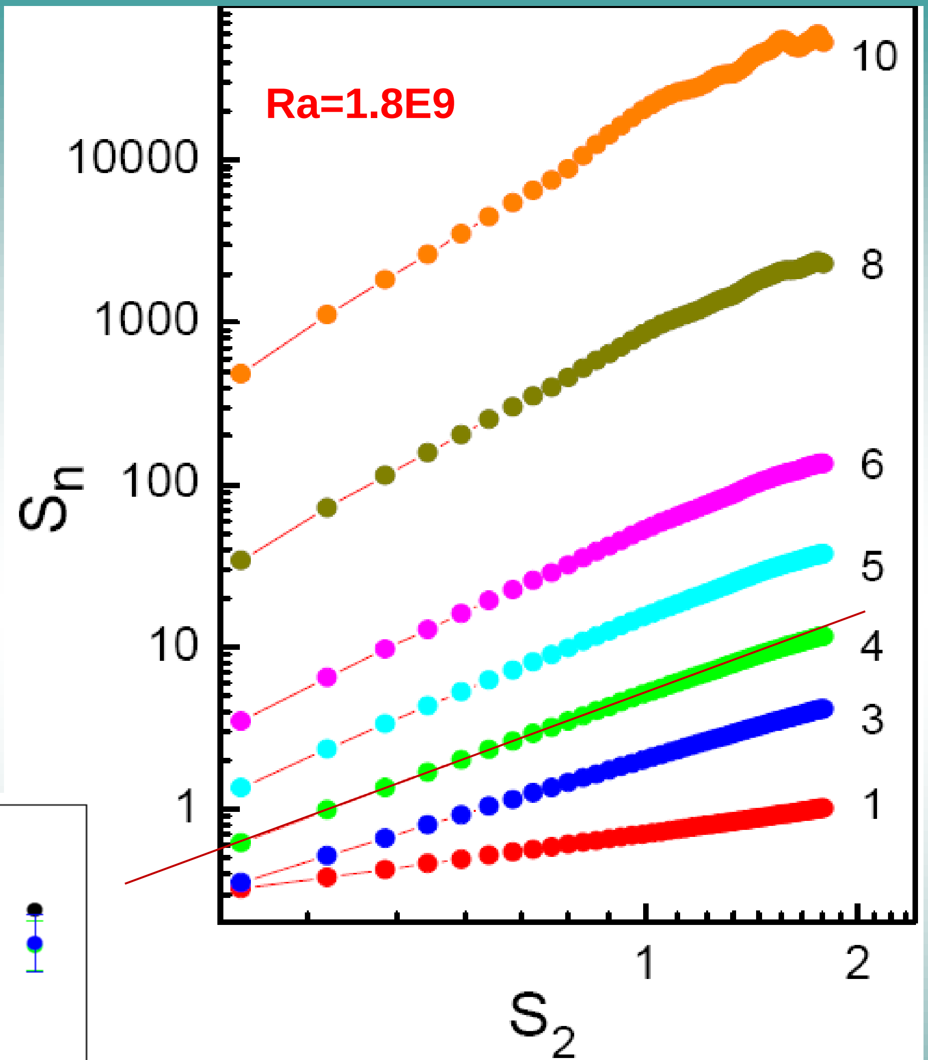


FIG. 6. Temperature structure functions of order 1, 3, 4, 6, and 8 (order increasing upwards) for 26 data files, plotted against the second-order structure function S_2. The data points within the Bolgiano range are highlighted.

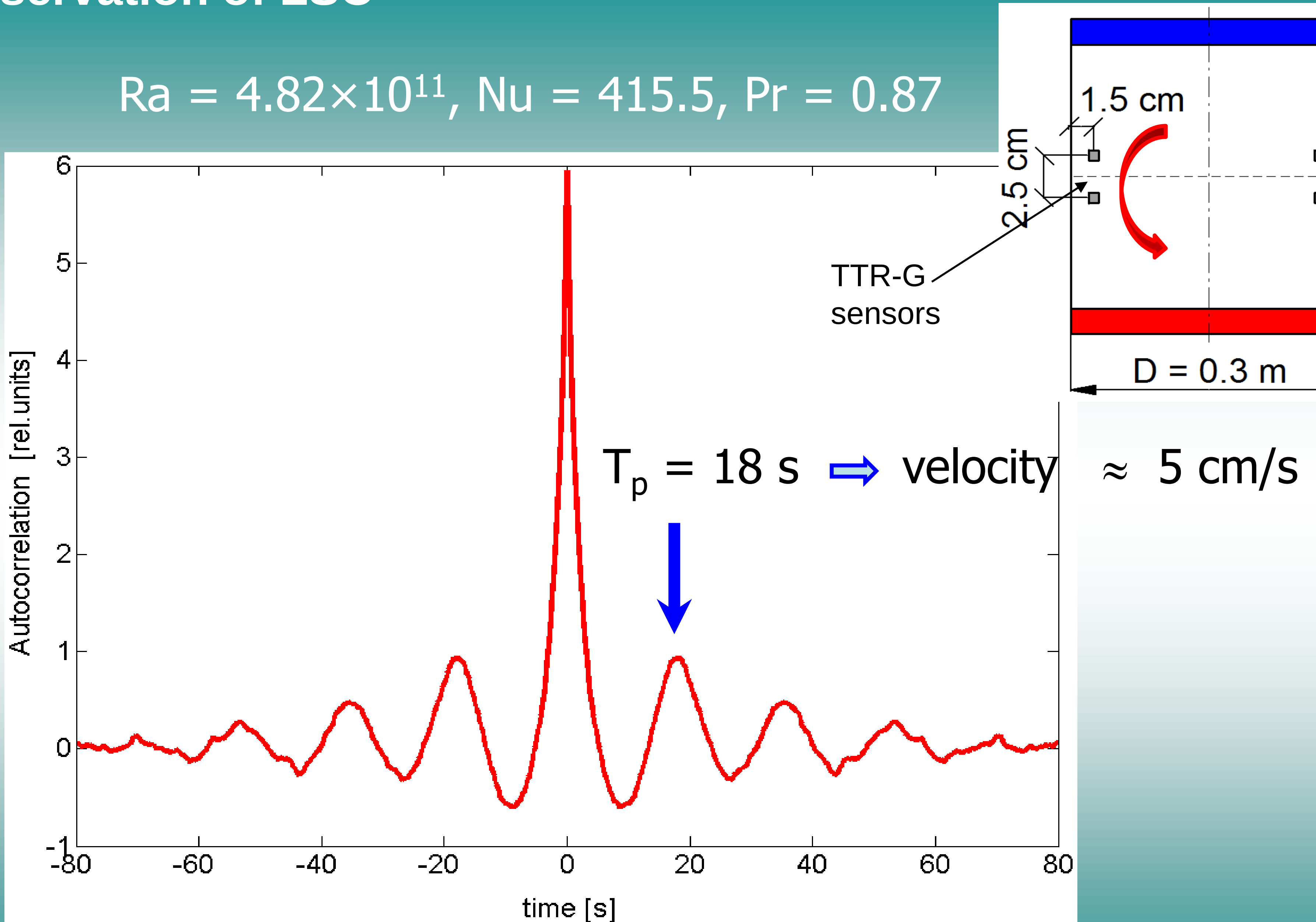




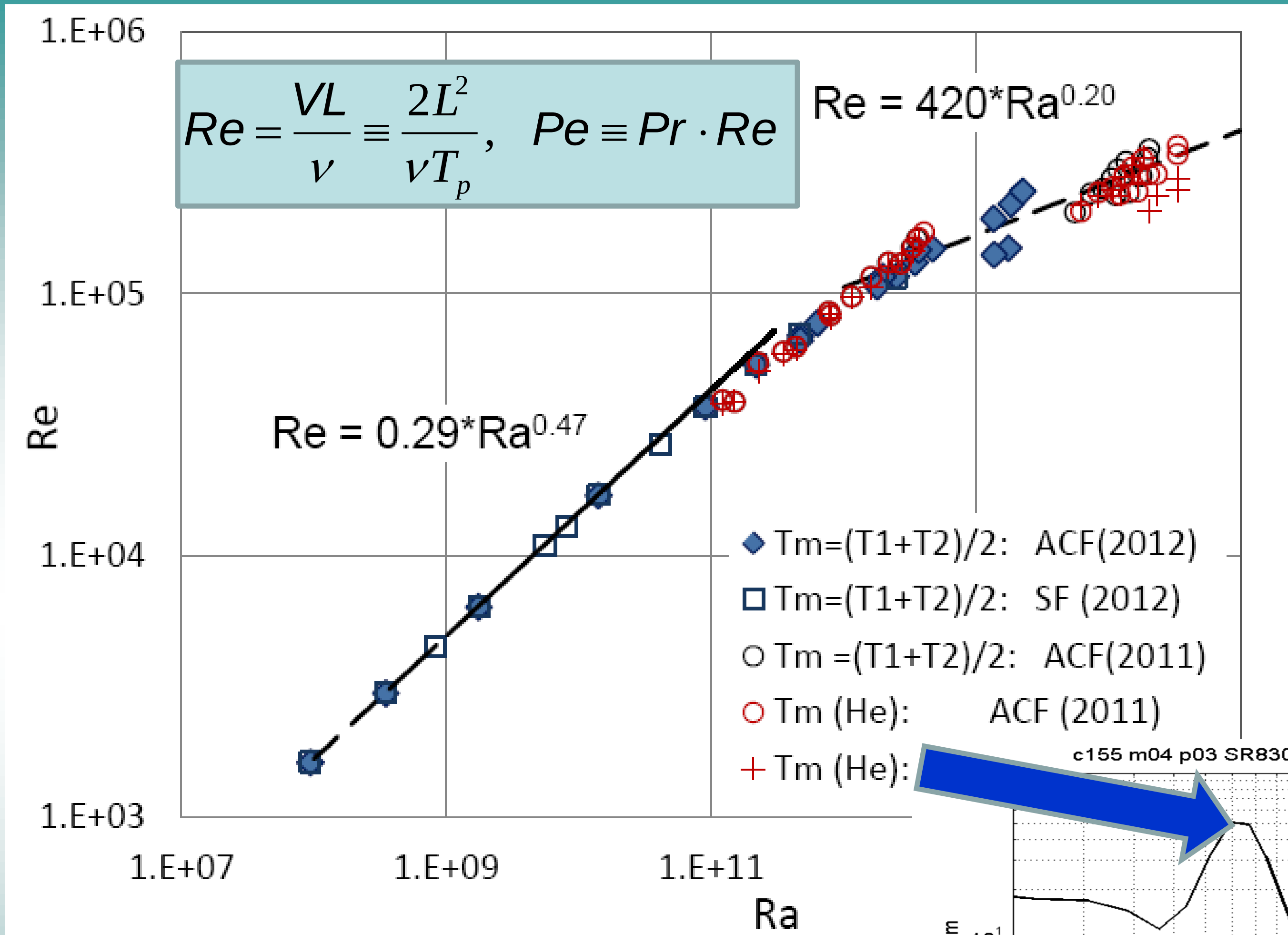
Our results shown over the expected Bolgiano range

Observation of LSC

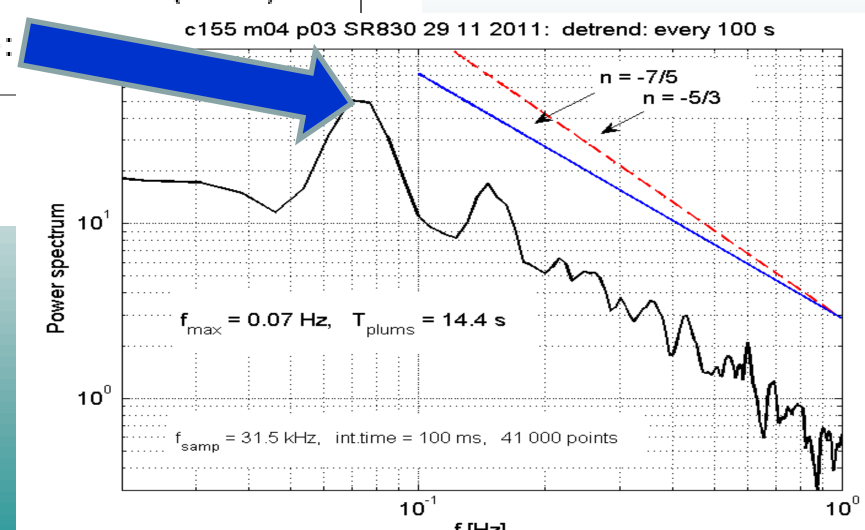
Ra = 4.82×10^{11}, Nu = 415.5, Pr = 0.87



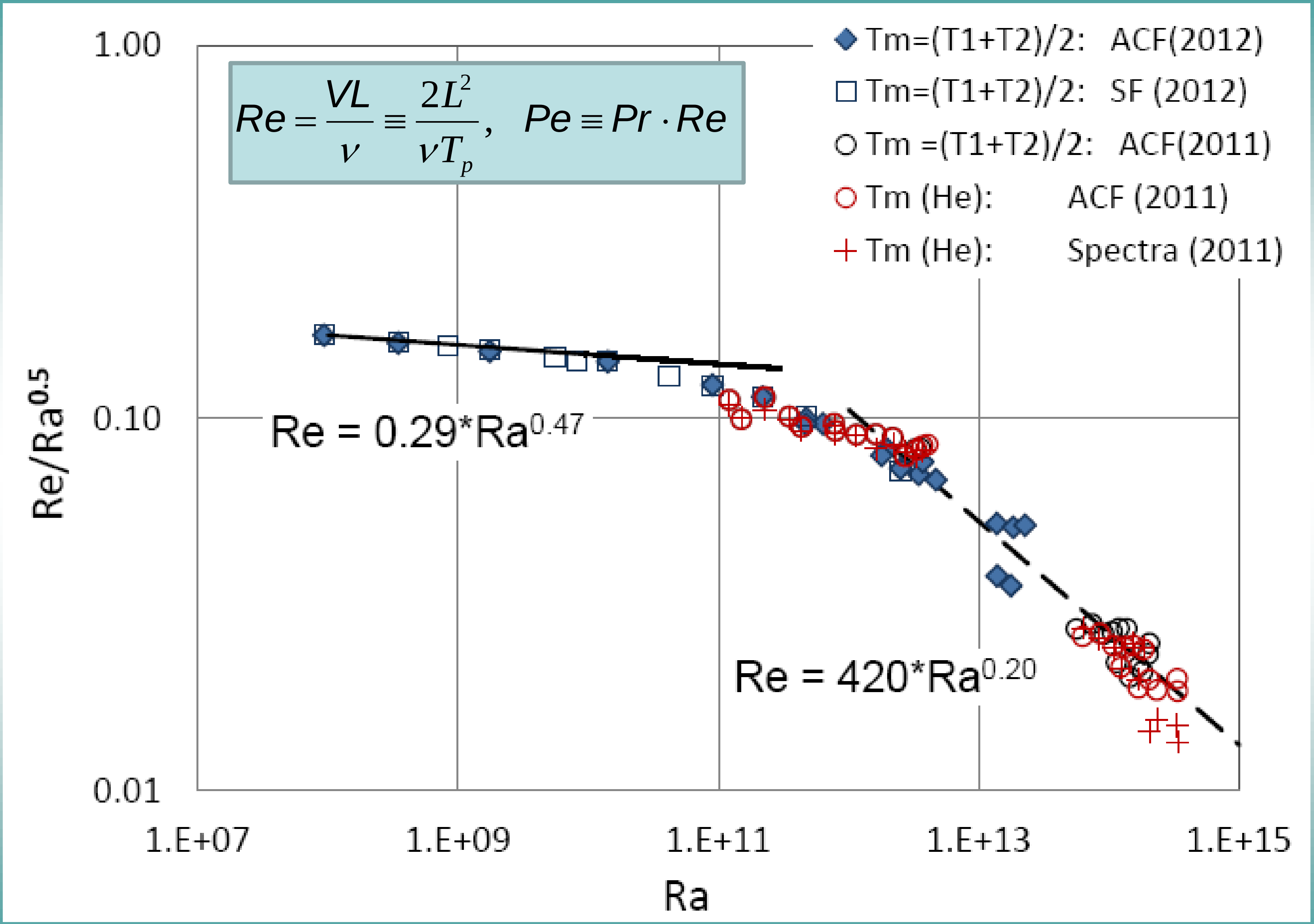
T_p = 18 s ⇨ velocity ≈ 5 cm/s



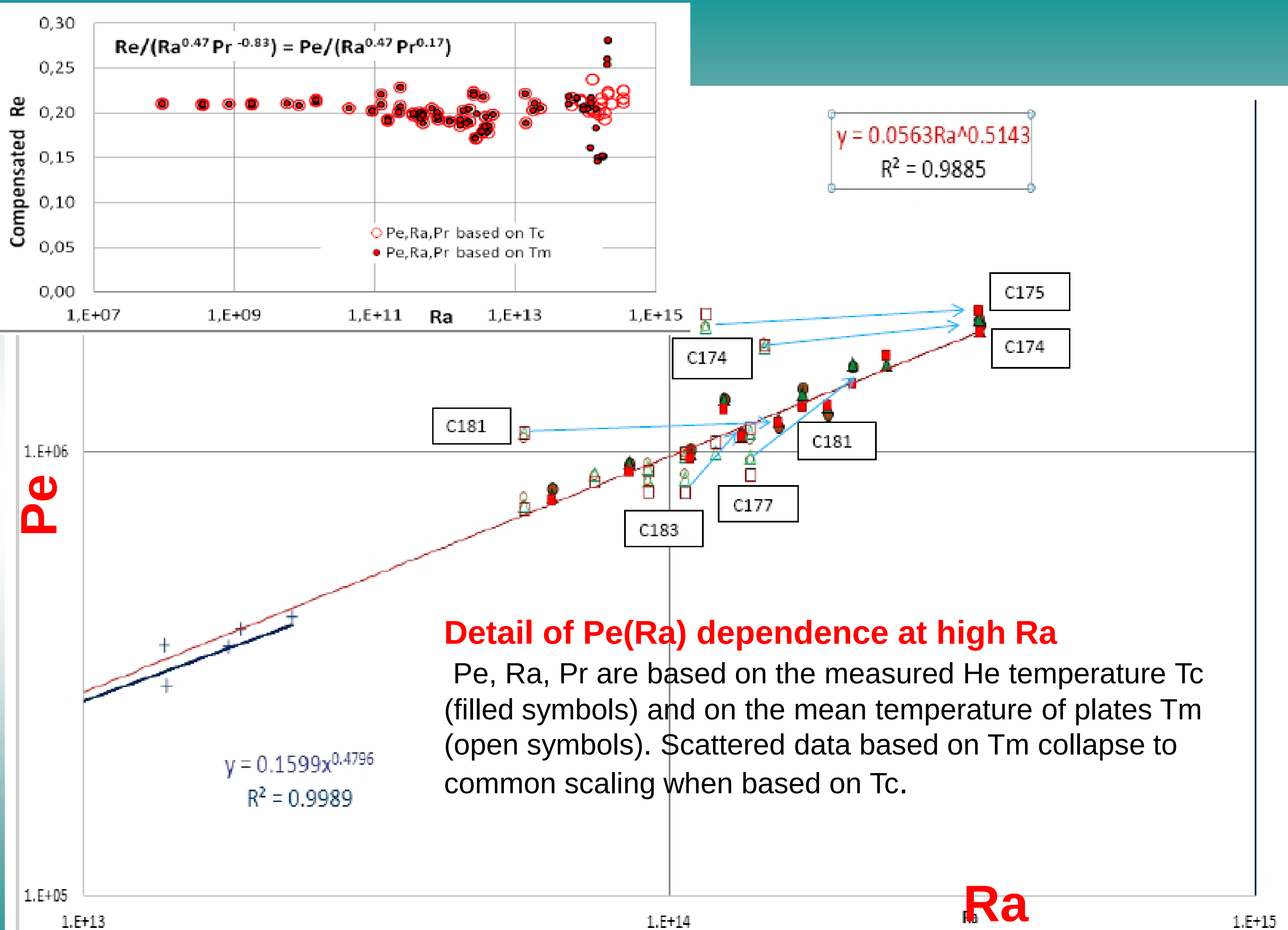
$$Re = \frac{VL}{\nu} \equiv \frac{2L^2}{\nu T_p}, \quad Pe \equiv Pr \cdot Re$$
Re = 420*Ra^0.20
Re = 0.29*Ra^0.47
Tm=(T1+T2)/2: ACF(2012)
Tm=(T1+T2)/2: SF (2012)
Tm =(T1+T2)/2: ACF(2011)
Tm (He): ACF (2011)
Tm (He):
1.E+06
1.E+05
1.E+04
1.E+03
1.E+07
1.E+09
1.E+11
Re
Ra



c155 m04 p03 SR830 29 11 2011: detrend: every 100 s
n = -7/5
n = -5/3
f_{max} = 0.07 Hz, T_{plums} = 14.4 s
f_{samp} = 31.5 kHz, int.time = 100 ms, 41 000 points
Power spectrum
f [Hz]
10^1
10^0
10^{-1}
10^0



$Re = \frac{VL}{\nu} \equiv \frac{2L^2}{\nu T_p}, \quad Pe \equiv Pr \cdot Re$
Tm=(T1+T2)/2: ACF(2012)
Tm=(T1+T2)/2: SF (2012)
Tm =(T1+T2)/2: ACF(2011)
Tm (He): ACF (2011)
Tm (He): Spectra (2011)
Re = 0.29*Ra$^{0.47}$
Re = 420*Ra$^{0.20}$
1.00
0.10
0.01
Re/Ra$^{0.5}$
1.E+07
1.E+09
1.E+11
1.E+13
1.E+15
Ra



Detail of Pe(Ra) dependence at high Ra

Pe, Ra, Pr are based on the measured He temperature Tc (filled symbols) and on the mean temperature of plates Tm (open symbols). Scattered data based on Tm collapse to common scaling when based on Tc.

Anomalous heat transport in two-phase convection of cryogenic helium

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Katepalli R. Sreenivasan, Ladislav Skrbek

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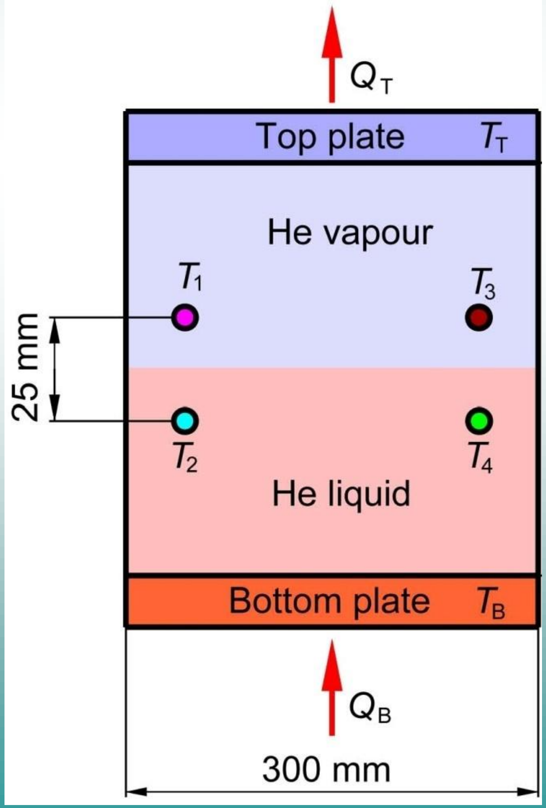
Department of Physics + Courant Institute of Math. Sciences, New York University, USA

Two-phase convection

- Transfer of heat through liquid and vapour layers
- Expected increase in efficiency due to phase transitions occurring
- Liquid level can be positioned to anywhere within the cell



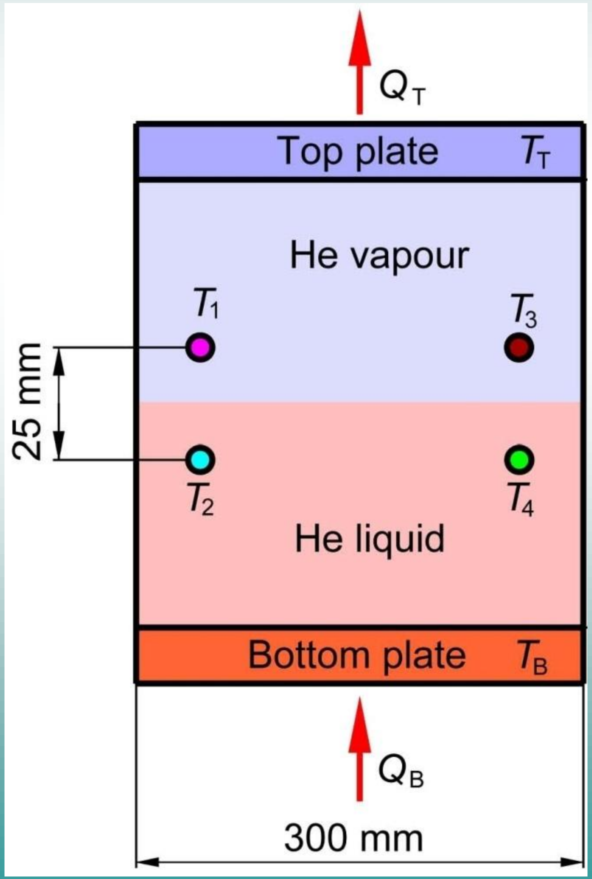
Non-equilibrium, irreversible process taking place in an open system!

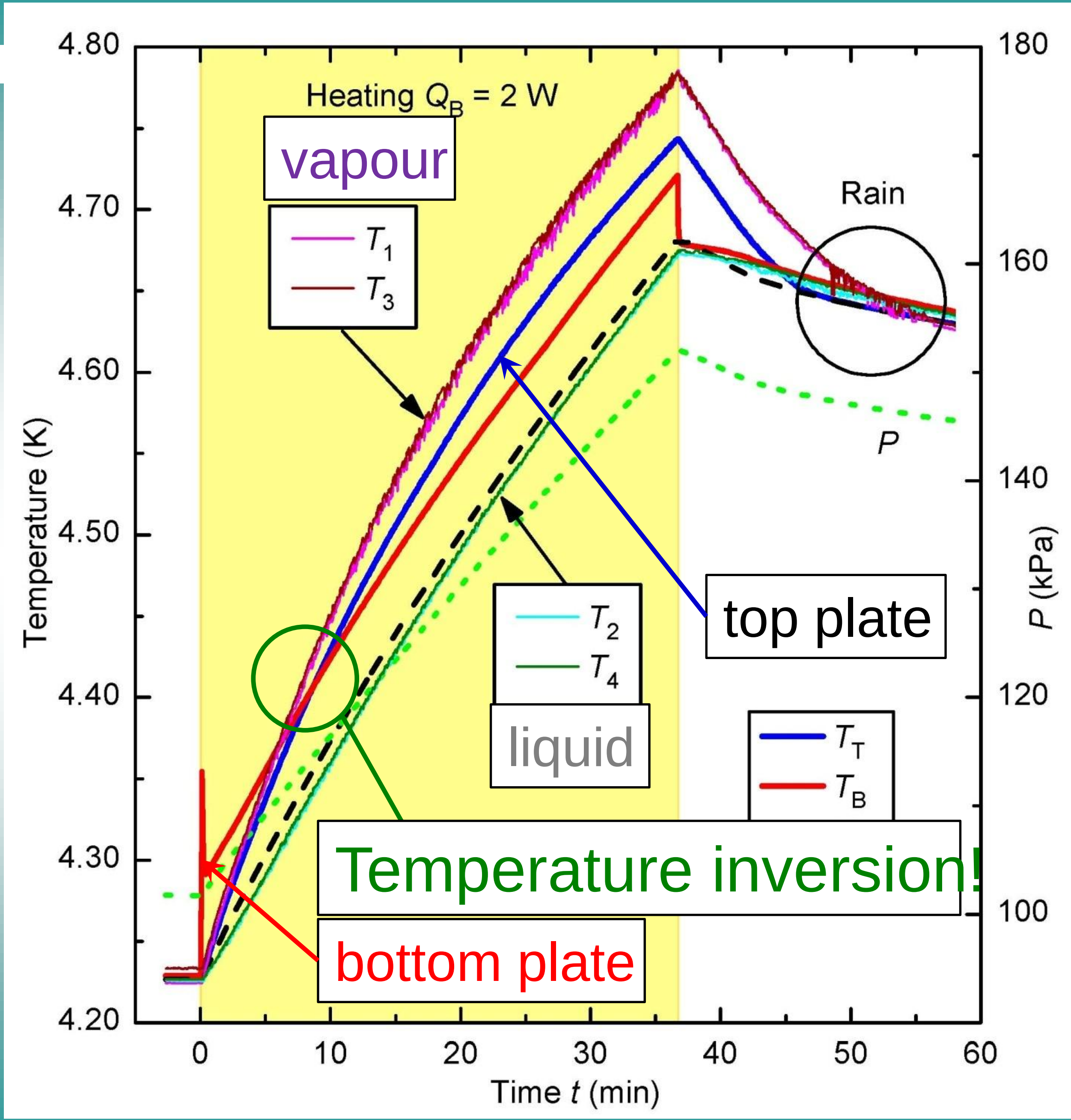
The heat input at the bottom plate is partly absorbed by the system (temperatures rise, liquid evaporates) and partly transmitted to the He bath through the top plate.

We monitor the temperatures of the two plates (T_T, T_B), and of 4 small Ge sensors in the cell interior ($T_1 \ldots T_4$), as well as the pressure inside the cell.

Hot body A thermally connected to a cold body B → heat flows from A to B.

Here we describe the opposite case!





Laws of Thermodynamics…?

2nd Law (R. Clausius): *"No process is possible whose sole result is the absorption of heat from a body of lower temperature to a body of higher temperature."*

Clausius R (1850) Über die bewegende Kraft der Wärme. Annalen der Physik 79:368–397, 500–524; trans (1851) [On the moving force of heat, and the laws regarding the nature of heat itself which are deducible therefrom] Phil. Mag., 4th Series, 2(VII):1–21, 102–119.

Valid only for closed systems → no conflict.

A mechanism of energy transport from the bottom plate to the He vapour must exist that bypasses the liquid phase, or the liquid must exert mechanical work upon the vapour.

→ Nucleate/film boiling (& thermal expansion of the liquid)

Heat transfer through liquid layers

1. Forced convection
   - Low heat flux $\boldsymbol{q}$, low temperature difference $\boldsymbol{\Delta T}$
   - $\boldsymbol{q}$ is linear with $\boldsymbol{\Delta T}$

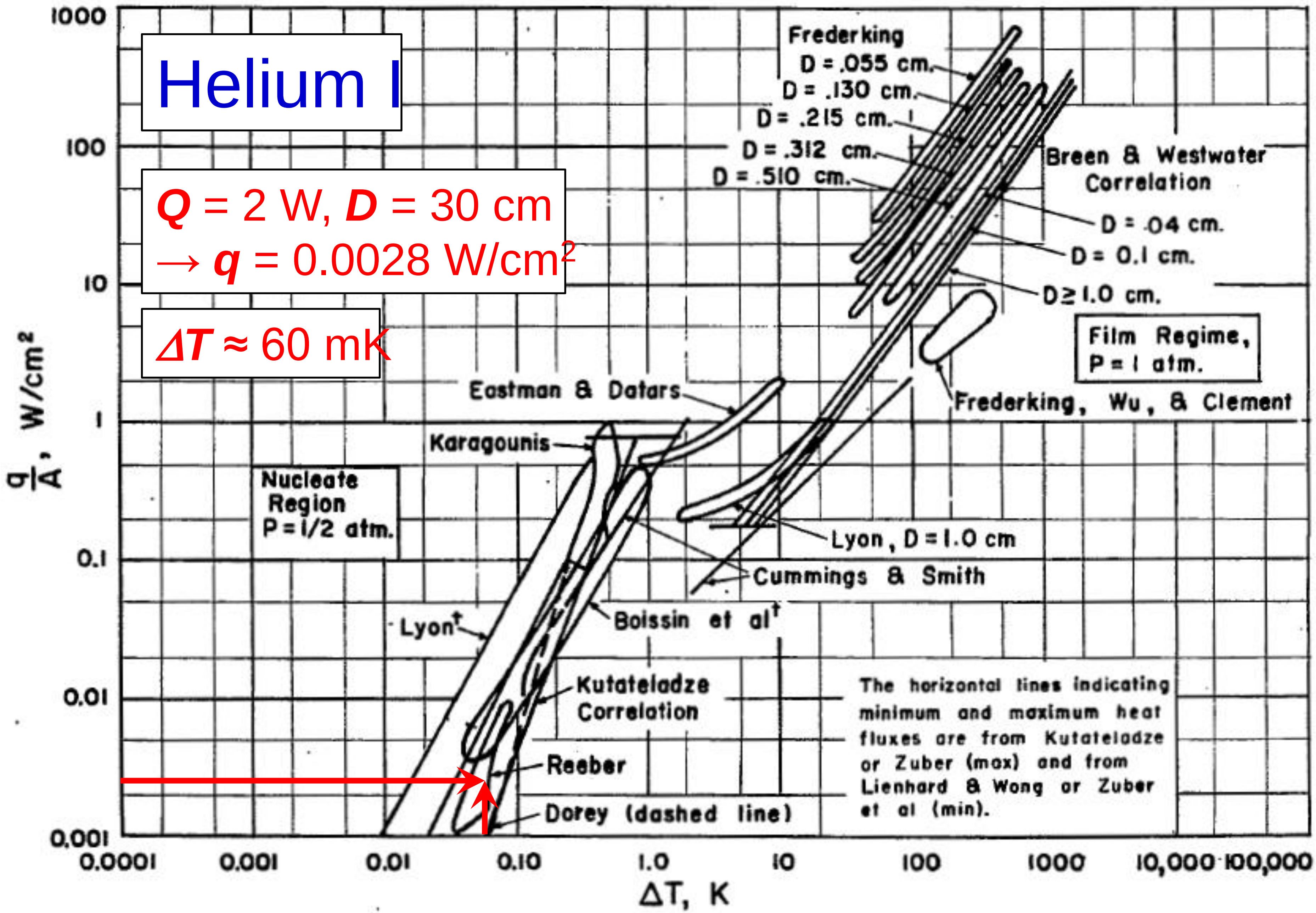
2. Nucleate boiling
   - Bubbles form on surface defects, then rise in the liquid
   - Hysteretic behaviour (overheated liquid layer)
   - Effective heat transfer, strong cooling of the solid
   - $\boldsymbol{q}$ non-linear with $\boldsymbol{\Delta T}$: $\boldsymbol{q} \approx \boldsymbol{c}\ (\Delta T)^n;\ \boldsymbol{n} \approx 1.5 \ldots 3$

3. Film boiling
   - Solid wall separated from liquid by a layer of vapour
   - Highest $\boldsymbol{q}$ and especially $\boldsymbol{\Delta T}$, no direct contact with liquid
   - Also hysteretic and non-linear: $\boldsymbol{q} \approx \boldsymbol{c'}\ (\Delta T)^m;\ \boldsymbol{m} \approx 1.3 \ldots 1.5$

Helium I

Q = 2 W, D = 30 cm
→ q = 0.0028 W/cm^2

ΔT ≈ 60 mK



Helium I

Nucleate boiling

Q = 2 W, D = 30 cm
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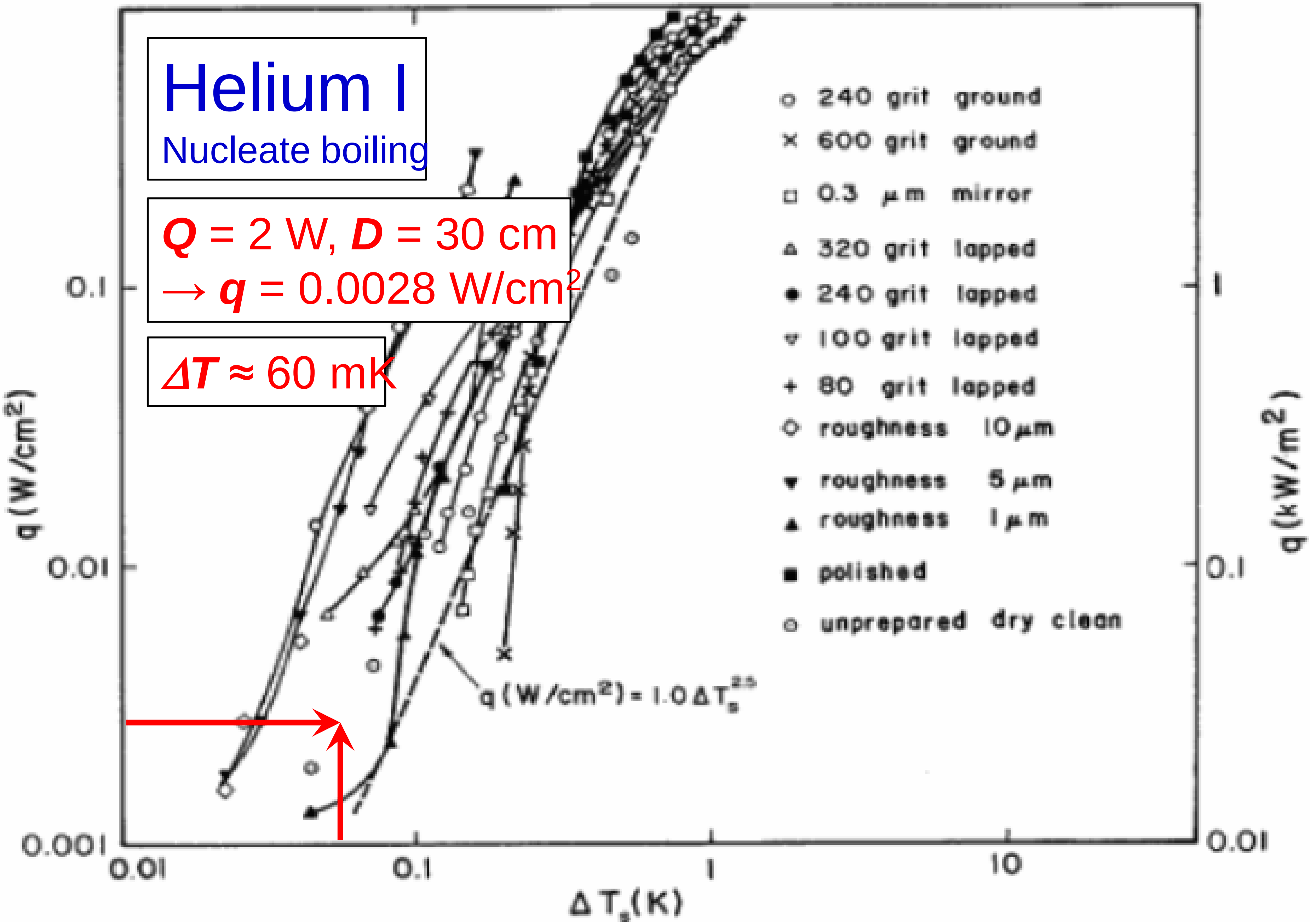
$\Delta T \approx$ 60 mK



Van Sciver S.W. ,Helium Cryogenics (Intl. cryogenics monograph series), Plenum Press, New York, 1986.

Can this system be modeled? How?

Interior of the cell

→ four subsystems (bottom plate, liquid, vapour, top plate)
→ each subsystem in near-equilibrium internally (well-defined T_i)
→ the entire cell in mechanical equilibrium (well-defined p)
→ temperature discontinuities on interfaces (timescale separation)

Heat flows

→ contact heat exchange at boundary XY: $q_{XY} = h_{XY}\,(T_Y - T_X)$; h_{XY} - model parameters
→ nucleate boiling: $q_B \approx c\,(\Delta T)^{2.5}$; $c \approx 4.0\ \mathrm{W\,cm^{-2}\,K^{-2.5}}$ (model parameter)
→ optional heat exchange between newly formed bubbles and the liquid

Phase transitions

→ rate of boiling from q_B, use ΔH for latent heat (numerical energy conservation)
→ rate of evaporation/condensation on liquid level: $\Delta N_1 = c_1\,(T_L - T_{sat})$
→ rate of condensation on the top plate: $\Delta N_2 = c_2\,(T_{sat} - T_{TP})$; if $T_{sat} > T_{TP}$

Together with conservation laws (mass, energy, cell volume) and precise values of physical properties of He&Cu allows numerical integration.

Can this system be modeled? How?

Physical properties of helium and copper

→ Properties of He taken from XHEPAK software [1], pre-processed for numerical calculations with a smoothing/fitting algorithm based on high order 2D polynomials (weighting in favour of data close to the saturated vapour curve)
→ precise low temperature heat capacity of Cu taken from [2]

What the model neglects (or does not explicitly include)

→ finite ***T*** gradients at interfaces, surface tension effects
→ the “real physics” of boiling, evaporation, condensation and heat transfer at boundaries (such as ***T***-dependent Kapitza resistance between Cu and He)
→ hysteretic effects in the onset of nucleate boiling
→ variations of pressure in the cell due to hydrostatic effects (gravity)
→ finite energy of fluid motion (liquid, vapour), effects of turbulence or viscosity

[1] McCarty R.D., Thermophysical Properties of Helium-4 from 2 to 1500 K with Pressures to 1000 Atmospheres. Technical Note 631, National Bureau of Standards (1972) ;
Arp V.D., McCarty R.D. The properties of critical helium gas. Tech Rep, Univ. of Oregon (1998).

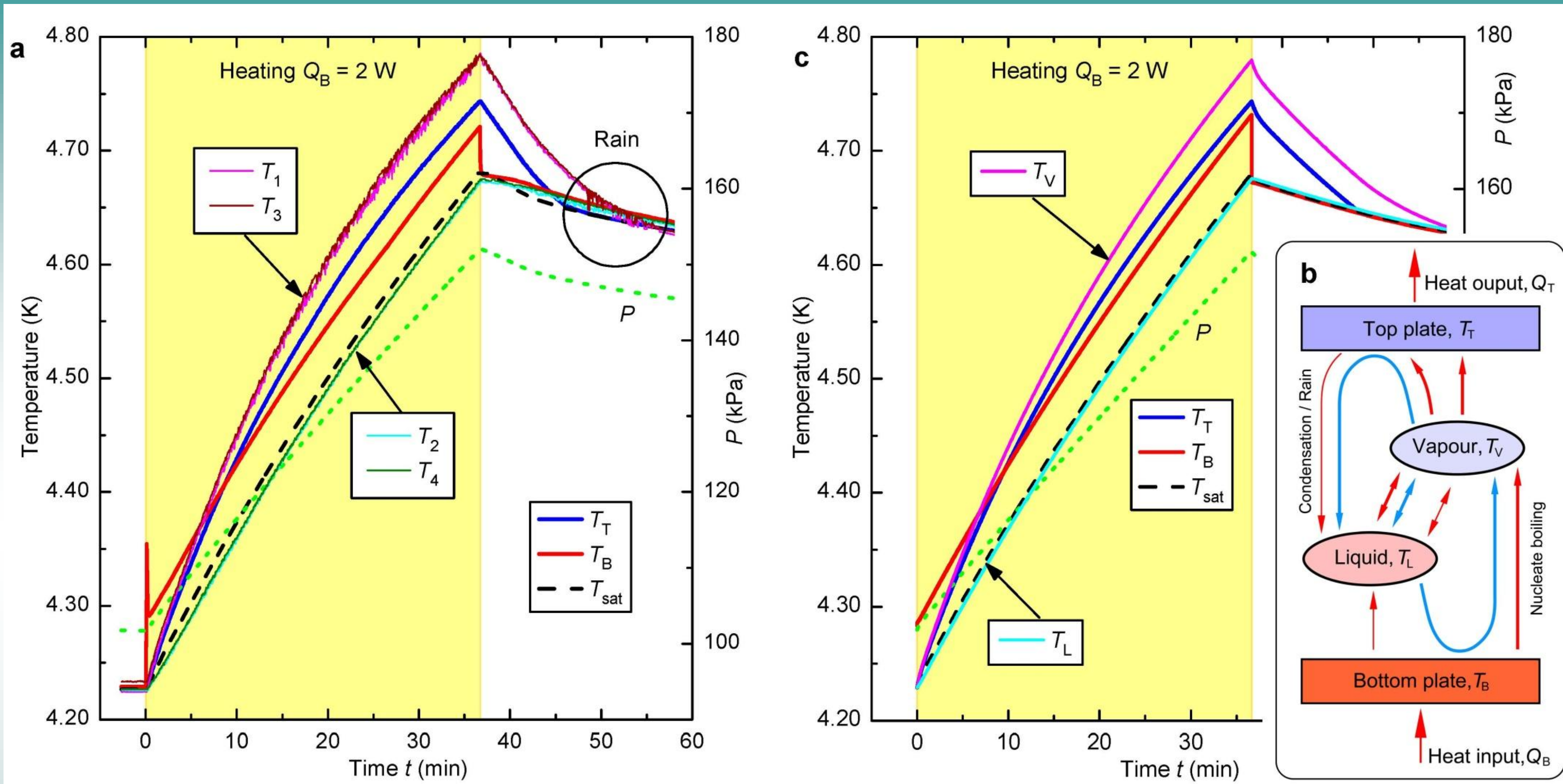
[2] White G.K., Collocott S.G., Heat capacity of reference materials: Cu and W. J Phys Chem Ref Data 13:4:1251-1257 (1984) .

Iteration procedure – one step

1. Get physical properties of He (and Cu) from values of T, p.
2. Calculate all heat fluxes independent of phase transition rates + Q_{boil}.
3. Calculate all phase transition rates (incl. boiling at lower plate).
4. Calculate additional heat flows due to phase transitions.
5. Get the new values of liquid and vapour masses and intermediate values of energies and densities to be used in step 6.

   Changes in energy and volume (density) are not known precisely due to the shifting liquid level and the associated mechanical work.
6. Formulate a closed set of 5 equations for vapour and liquid energies, densities and the change in volumes, incorporating the mechanical work between vapour and liquid. This implicitly includes heat expansion because energies and densities will be allowed to vary.
7. These equations are solved using an iterative algorithm (zero finder for non-linear systems). New values of T, p are found as well as new values of energy and density → next step.

Experiment Numerical simulation



Anomalous heat transport and condensation in convection of cryogenic helium

Proc. Nat. Acad. Sci. of the USA
110 8036-8039

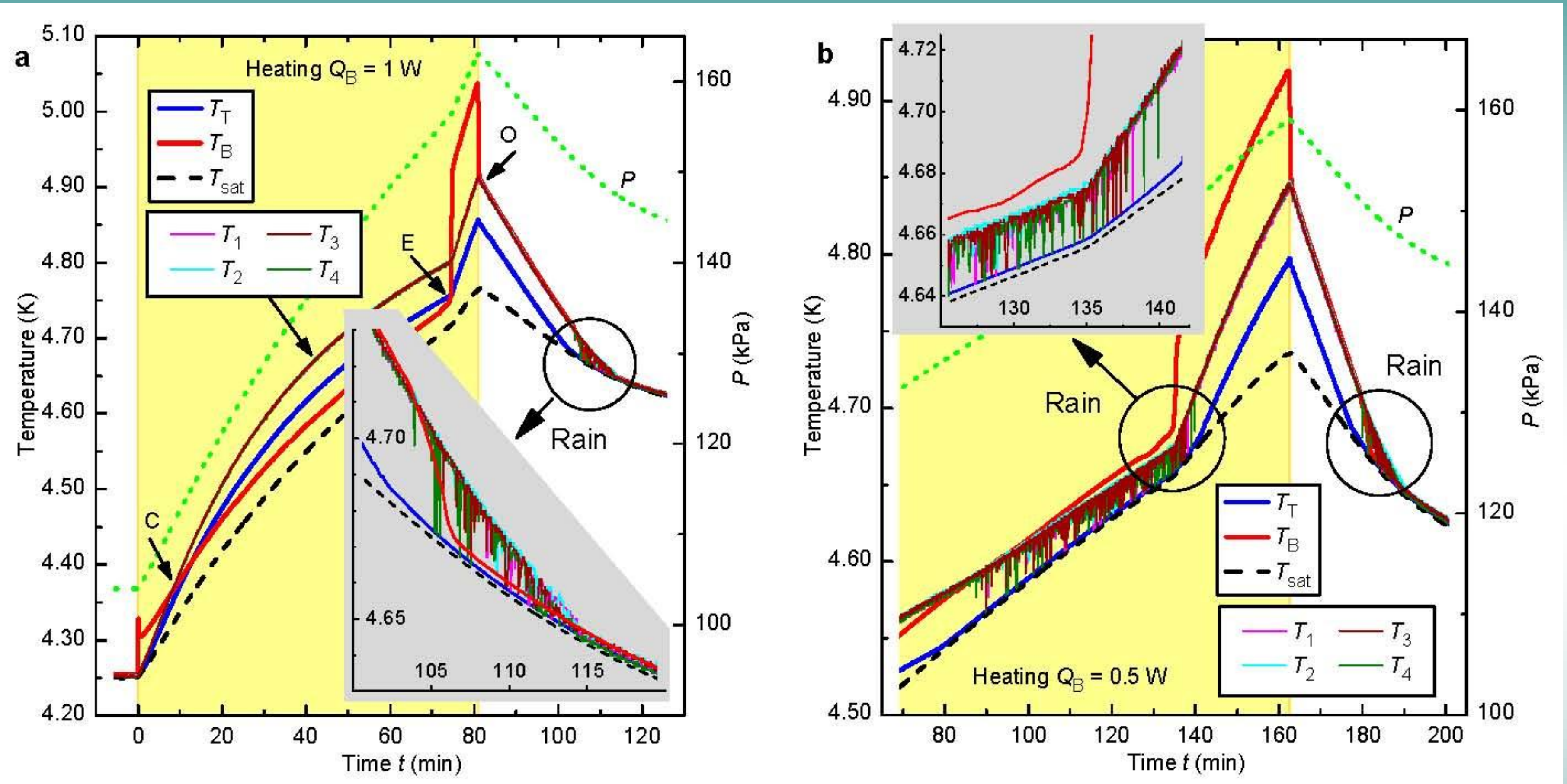
Pavel Urban[a], David Schmoranzer[b], Pavel Hanzelka[a], Katepalli R. Sreenivasan[c,d,1], and Ladislav Skrbek[b]

[a]Institute of Scientific Instruments, Academy of Sciences of the Czech Republic, 61242 Brno, Czech Republic; [b]Faculty of Mathematics and Physics, Charles University, 12116 Prague, Czech Republic; [c]Department of Physics, New York University, New York, NY 10003; and [d]Courant Institute of Mathematical Sciences, New York University, New York, NY 10012

Anomalous heat transport and weather formation in cryogenic ^4He Helium– rain

Ideas taken from

Zhong J-Q, Funfschilling D, Ahlers G (2009) Enhanced heat transport by turbulent two-phase Rayleigh-Bénard convection. *Phys Rev Lett* 102(12):124501.



a) and b) Average density of helium in the RB cell 25.7 kg/m³ (initially ~ 2.4 cm of liquid).

The short uncorrelated downward departures in temperature readings T_1 ... T_4 towards the temperature corresponding to saturated vapour ($\boldsymbol{T_{sat}}$) at pressure $\boldsymbol{p}$ indicate rain in the RB cell. The insets show the observed rain of helium droplets in greater detail.

Conclusions

• **Experimental study of scaling law Nu(Ra), statistical properties and Large Scale Circulation up to Ra = 10^{15} in a cryogenic cylindrical Γ = 1 cell 0.3 m in diameter**

• Subject to appropriate wall correction, the aspect ratio $\Gamma \cong 1$ Trieste, Grenoble and our cryogenic data give identical scaling law for Ra < 10^{11}

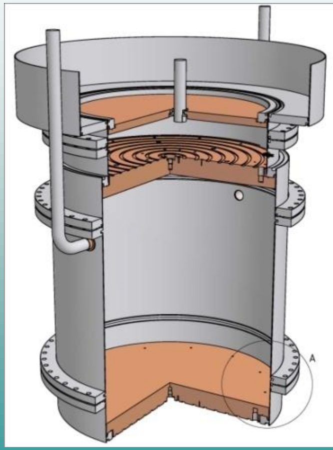
• For 10^{12} < Ra < 10^{15} within the experimental error Nu ~ $Ra^{1/3}$ if the mean temperature of the working fluid - cryogenic helium gas - is measured directly and corrections due to sidewall effect, adiabatic temperature gradient, and "chimney effects" are taken into account

• Non Oberbeck–Boussinesq (NOB) effects lead to significant changes in Nu(Ra) scaling at very high Ra, effective Nu=Nu(Ra) can be evaluated

•Generally, we see the transition to „ultimate Nu(Ra) scaling" as an open issue

• Observation of the coherent structures in the Large Scale Circulation by autocorrelation of time signals. Péclet number follows approximately dependence Pe ~ $Ra^{1/2}$ as previously observed for Ra ≤ 10^{13}. Coherent structures observed above Ra = 10^{13}

•Detailed report on temperature structure functions, their extended self-similarity properties, scaling exponents in the Bogiano range of scales, energy spectra etc. Is under preparation and will be be published elsewhere



Conclusions continued

- Two-phase convection shows inversion in the vertical temperature profile, a highly counter-intuitive and puzzling result
- This phenomenon is explained by the mechanism of nucleate boiling which allows direct transfer of energy from the bottom plate to the vapour phase, bypassing the liquid (heat + pressurization)
- Thermal expansion of the liquid contributes to the same effect
- A simplified model reproduces the experimental results rather well
- Helium rain is clearly observed in the experiment and confirmed by the model, an evaporation/condensation cycle exists temporarily
- Future: test in superfluid He II (no nucleate boiling unless severely overheated), test in a mixture of N_2 and He (~ air & water vapour)