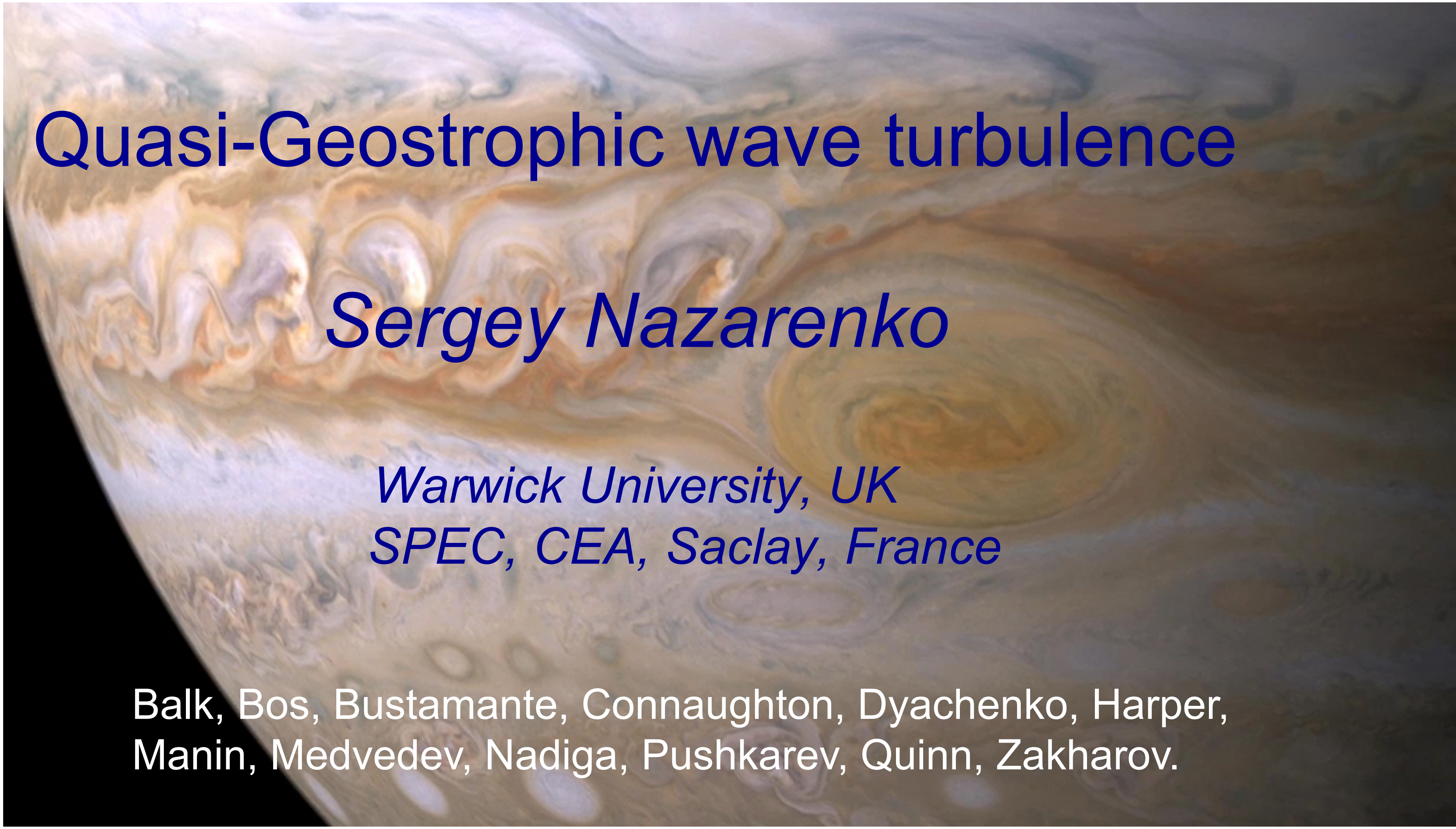


Quasi-Geostrophic wave turbulence

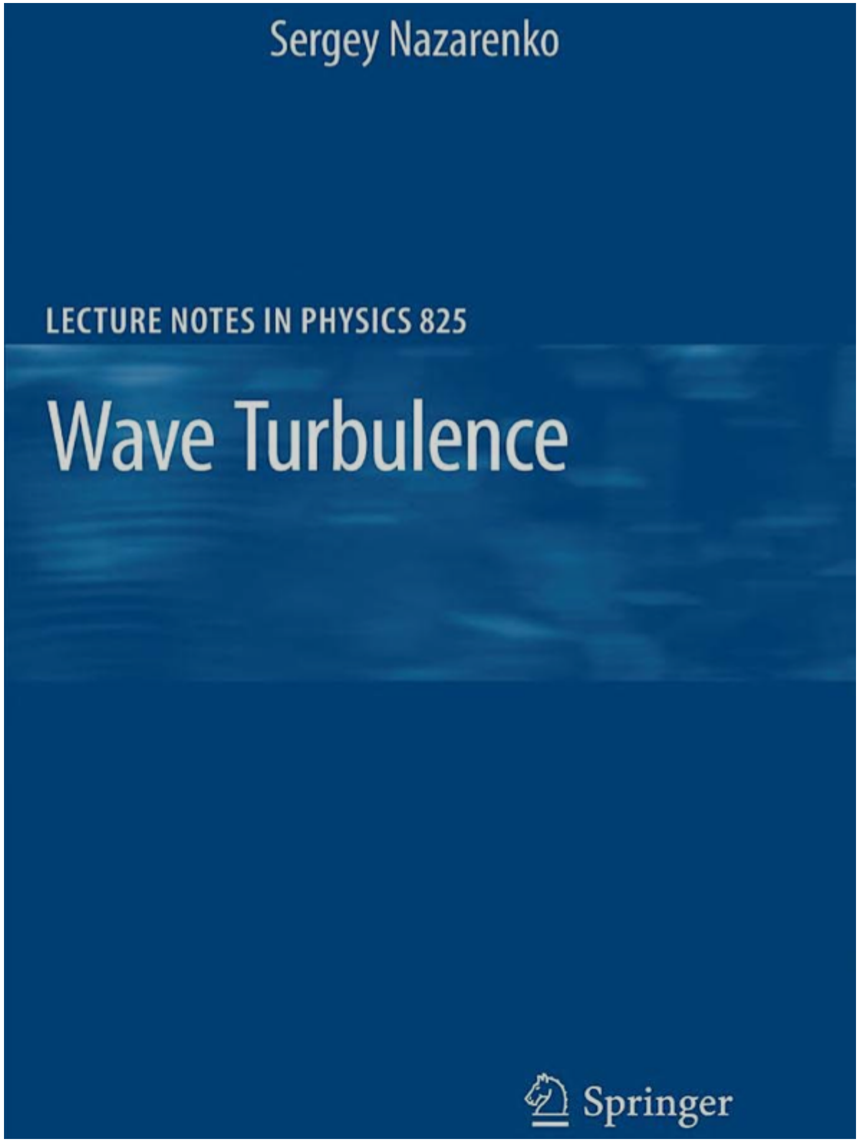
Sergey Nazarenko

Warwick University, UK
SPEC, CEA, Saclay, France

Balk, Bos, Bustamante, Connaughton, Dyachenko, Harper, Manin, Medvedev, Nadiga, Pushkarev, Quinn, Zakharov.

New Challenges in Turbulence Research III
Les Houches, 16 March 2014

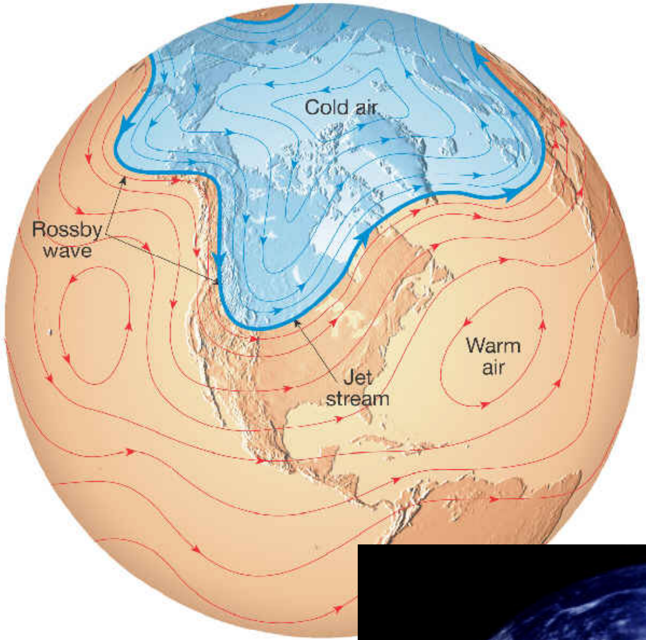
Chapter in the book:

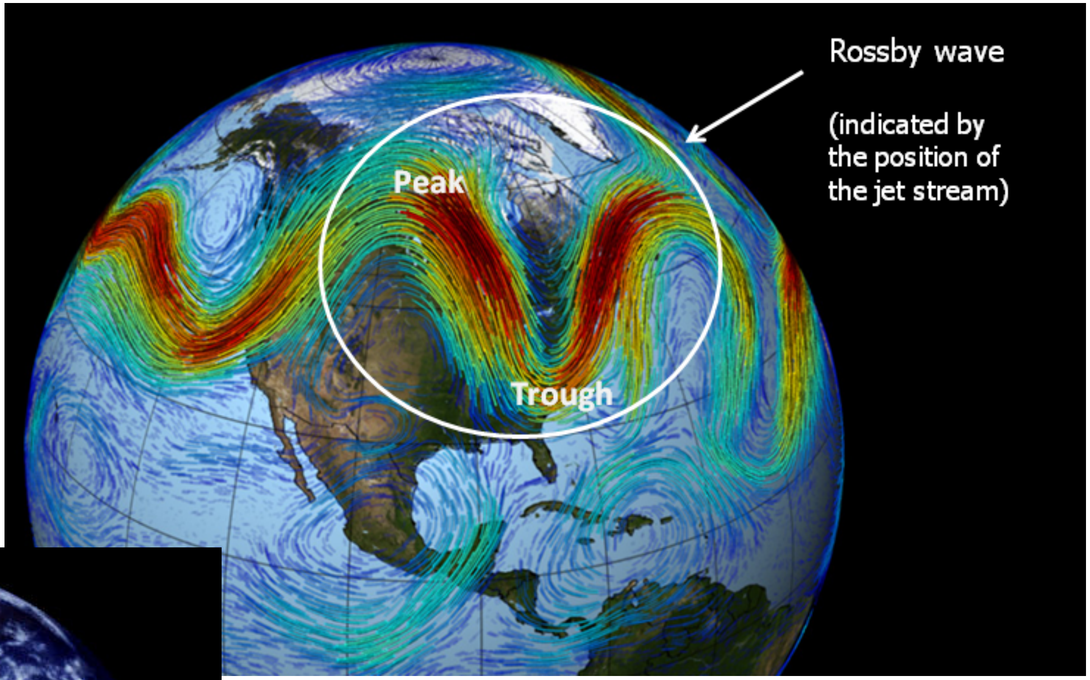


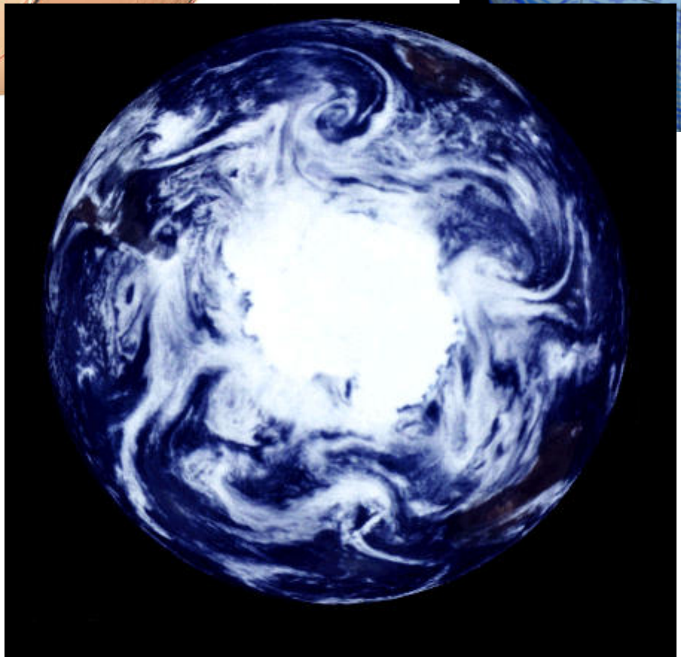
Review in preparation for Physics Reports

Contributions to ``Zonal Jets and Eddies”
Book edited by B. Galperin

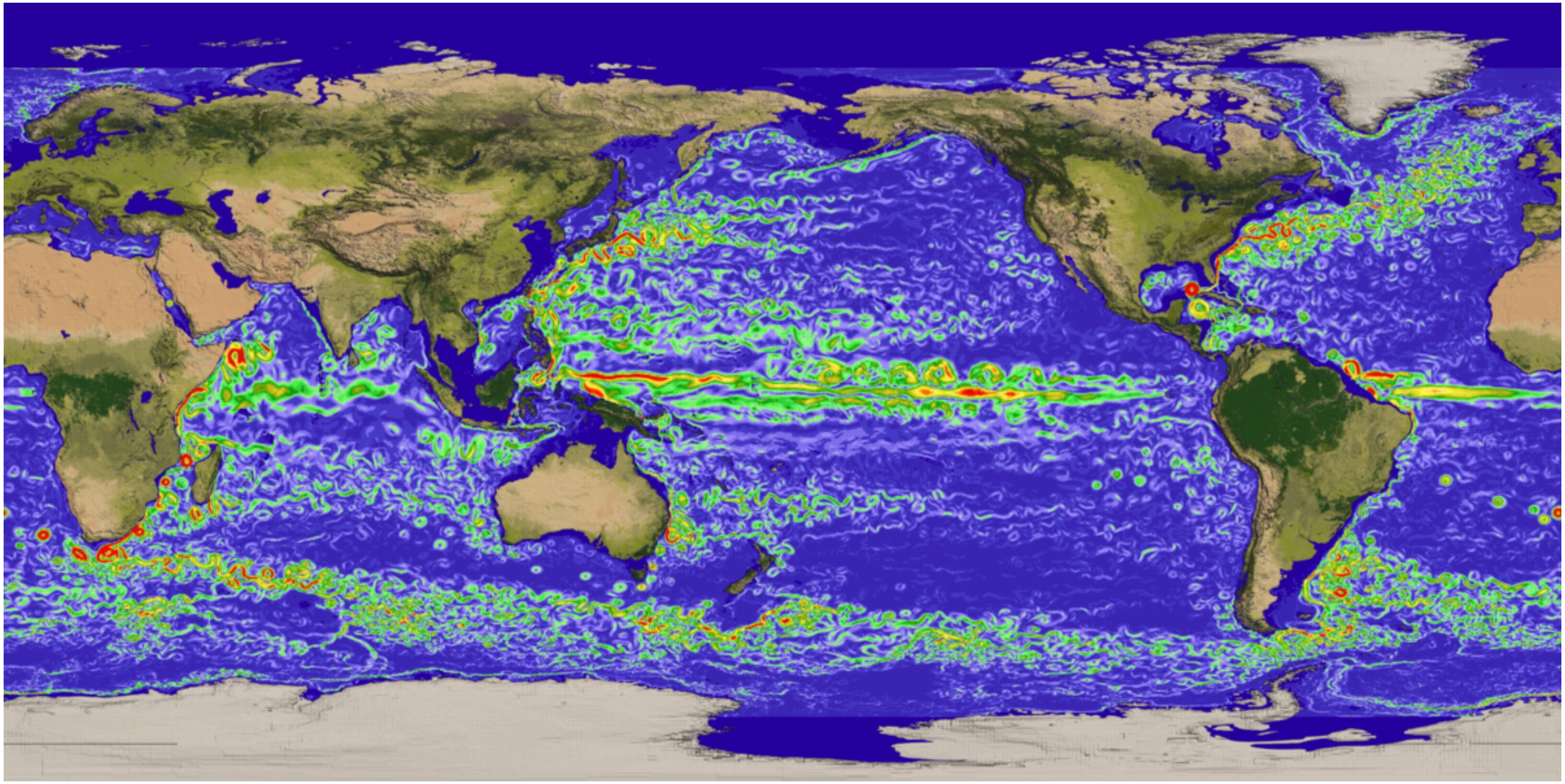
Rossby waves in Earth's atmosphere





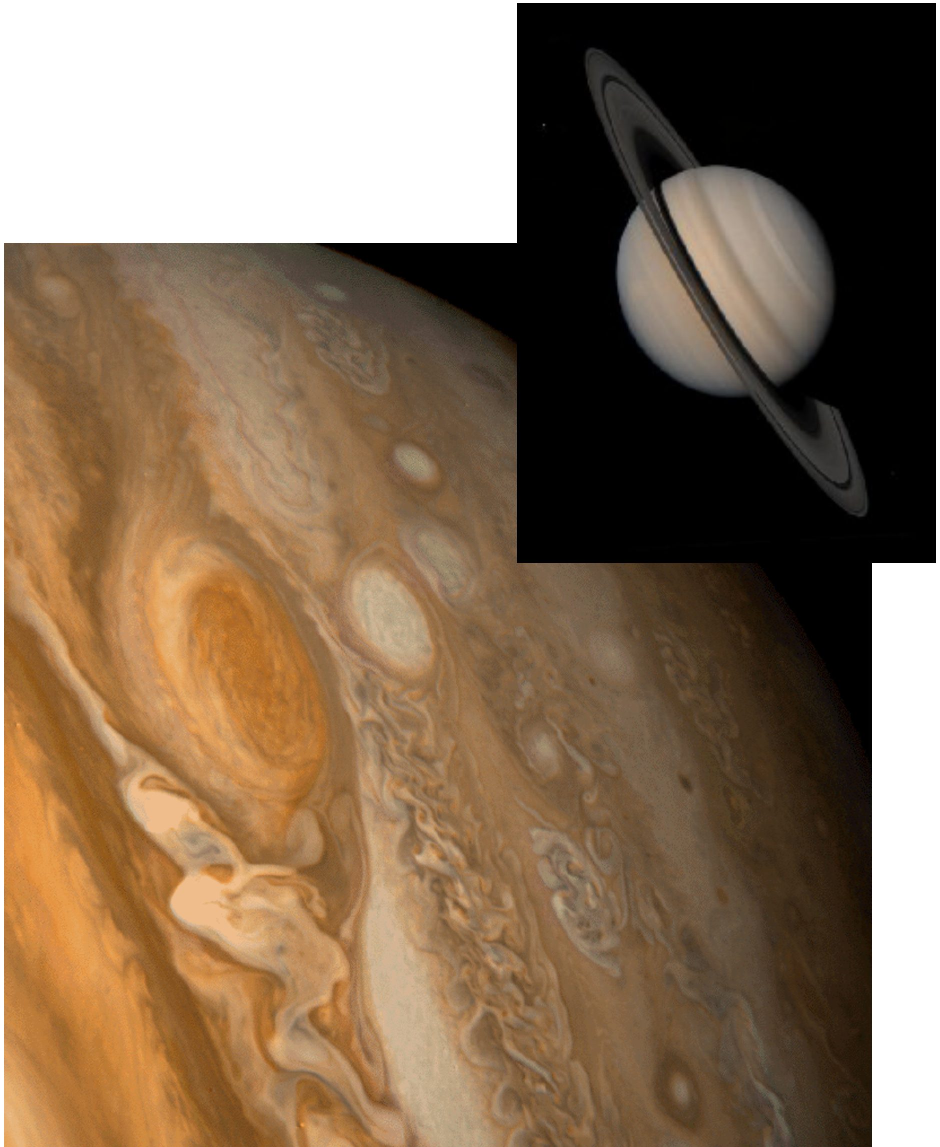


Zonal Jets in Earth's Oceans

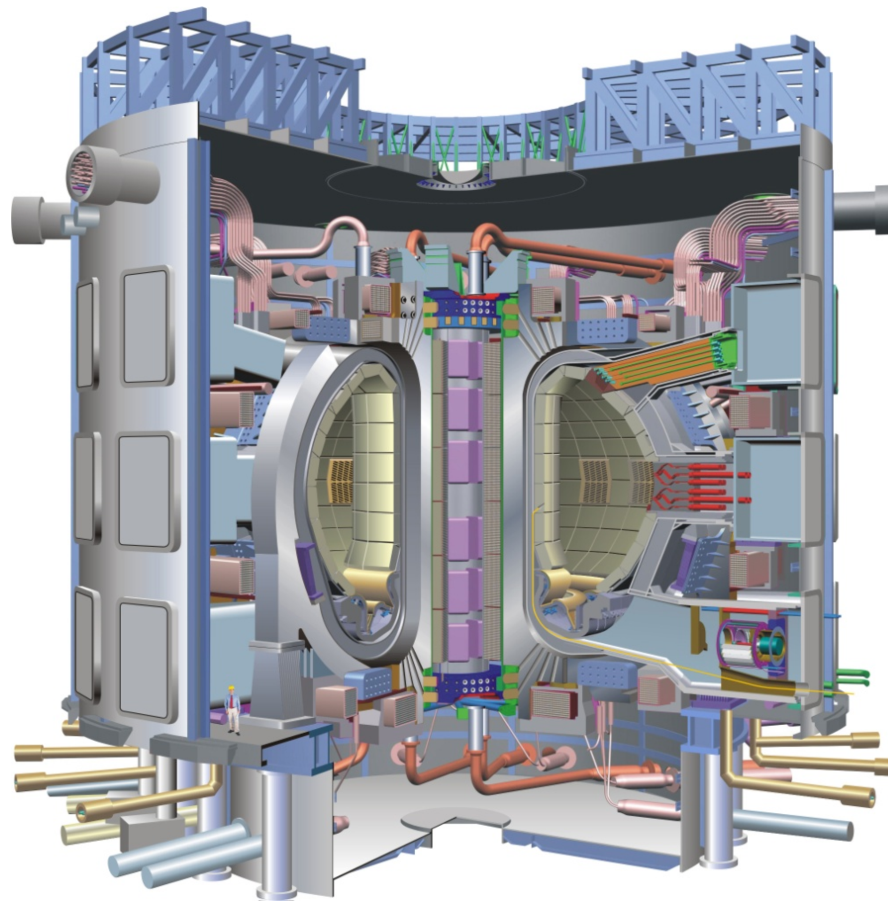


Eddy-resolving simulation of Earths oceans (Earth Simulator Center/JAMSTEC)

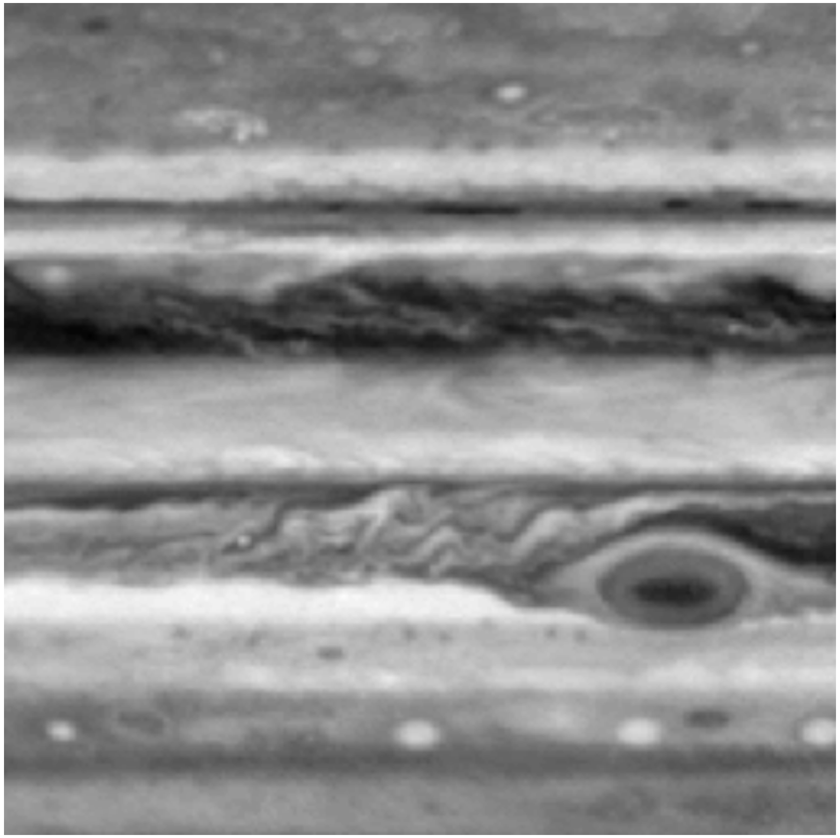
Rossby waves in atmospheres of giant rotating planets



Drift waves in fusion devices



Self-regulating in Rossby wave turbulence



- Rossby wave turbulence generates zonal flows
- Zonal flows suppress waves
- Hence transport barriers

Charney-Hasegawa-Mima equation

$$\frac{\partial}{\partial t}\left(\rho^2\nabla^2\psi-\psi\right)-\beta\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}=0$$

- ψ – streamfunction
- ρ – Deformation radius
- β – PV gradient
- x – east-west
- y – south-north

2D Euler equation limit

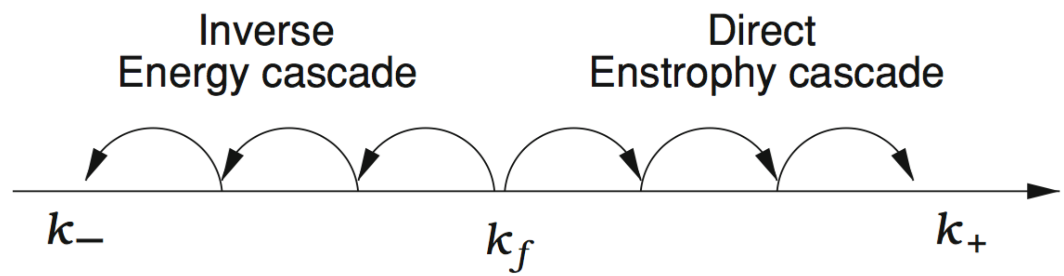
$$\frac{\partial}{\partial t}\left(\rho^2\nabla^2\psi \cancel{-\psi}\right) \cancel{-\beta\frac{\partial\psi}{\partial x}} + \frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x} = 0$$

$$E(\vec{k}) = \int\langle\vec{u}(\vec{x}+\vec{r})\cdot\vec{u}(\vec{x})\rangle e^{-i\vec{k}\cdot\vec{x}}d\vec{r}$$ - energy spectrum

$$\langle u^2\rangle = \int E(\vec{k})d\vec{k}$$ - energy

$$\langle(\nabla\times\vec{u})^2\rangle = \int k^2E(\vec{k})d\vec{k}$$ -enstrophy

Dual cascade behavior:



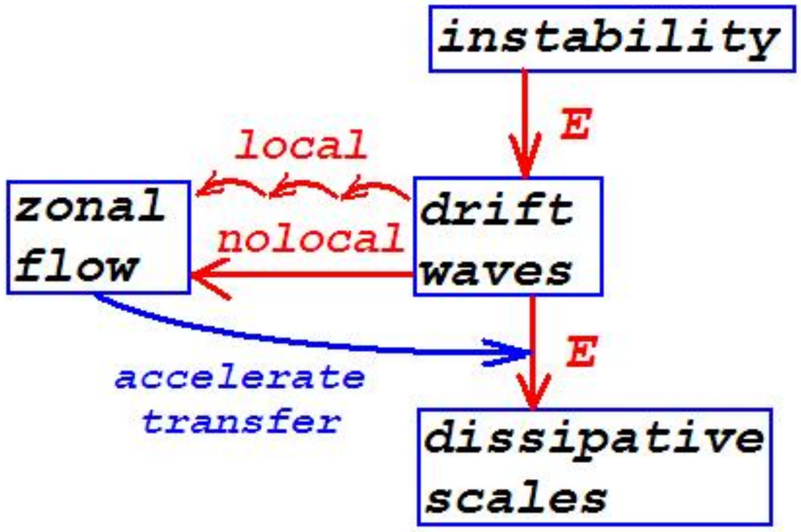
Extra quadratic invariant on β-plane

- *Balk, Nazarenko & Zakharov (1990)*
- *Adiabatic for the original β-plane equation: requires small nonlinearity and possibly random phases.*
- *For case $k\rho \gg 1$:*

$$\Phi = \int \frac{k_x^2}{k^6}(k_x^2 + 5k_y^2)|\hat{\psi}_k|^2 d\mathbf{k},$$ - Zonostrophy invariant.

Anizotropic cascades

Self-regulation loop in Rossby/drift turbulence



Balk, SN and Zakharov 1990

- Small-scale turbulence causes anomalous transport
- Negative feedback loop.
- Suppressed turbulence → suppressed transport across the jet

Barotropic governor in GFG

15 DECEMBER 1987 I. N. JAMES

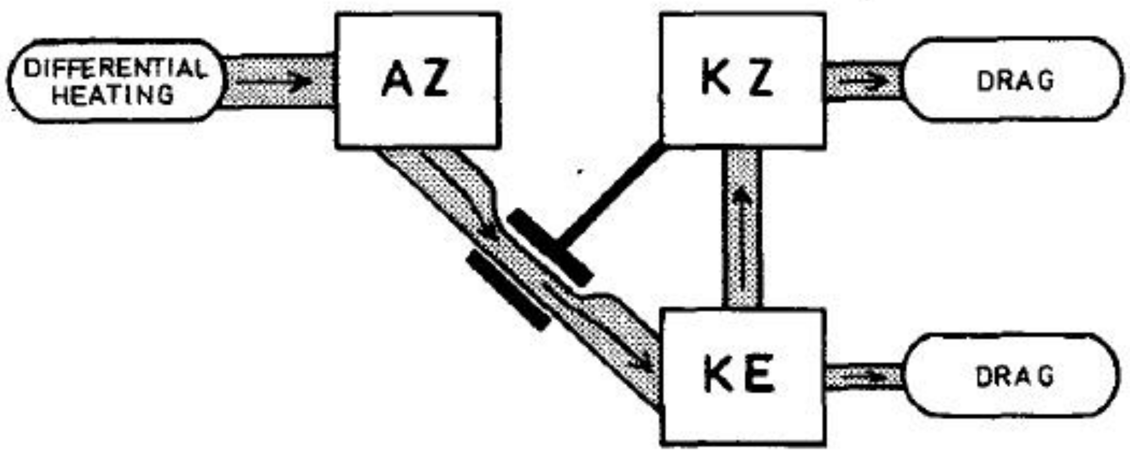
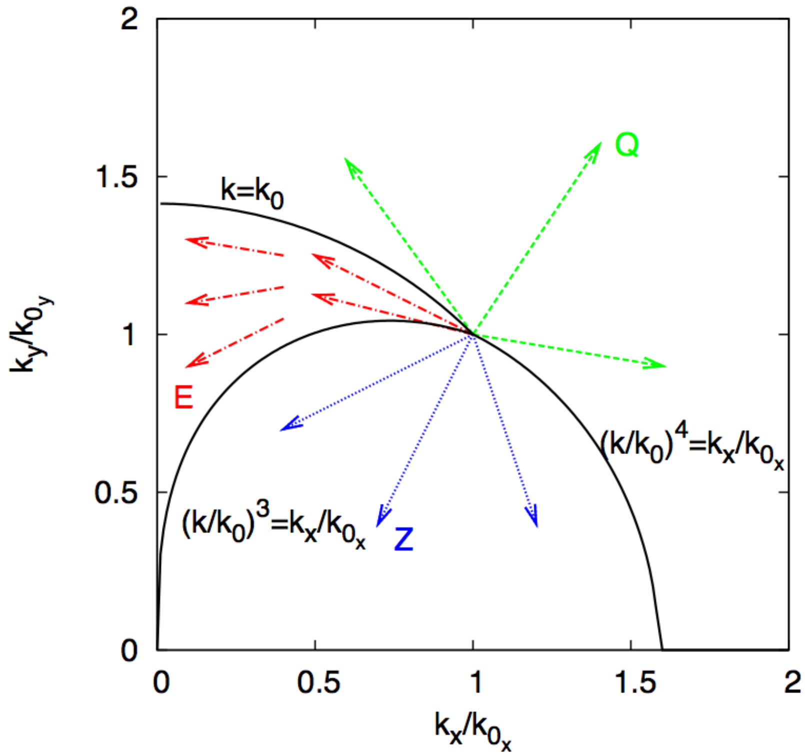


FIG. 1. Schematic illustration of the "barotropic governor," summarizing the effect of horizontal shears on baroclinic instability as postulated in JG. Energy conversion from available potential energy (AZ) to eddy kinetic energy (KE) results in momentum fluxes which increase the barotropic contribution to the zonal kinetic energy (KZ). As barotropic shears build up in the zonal flow, the baroclinic conversions are inhibited.

Mechanisms of zonal flow generation

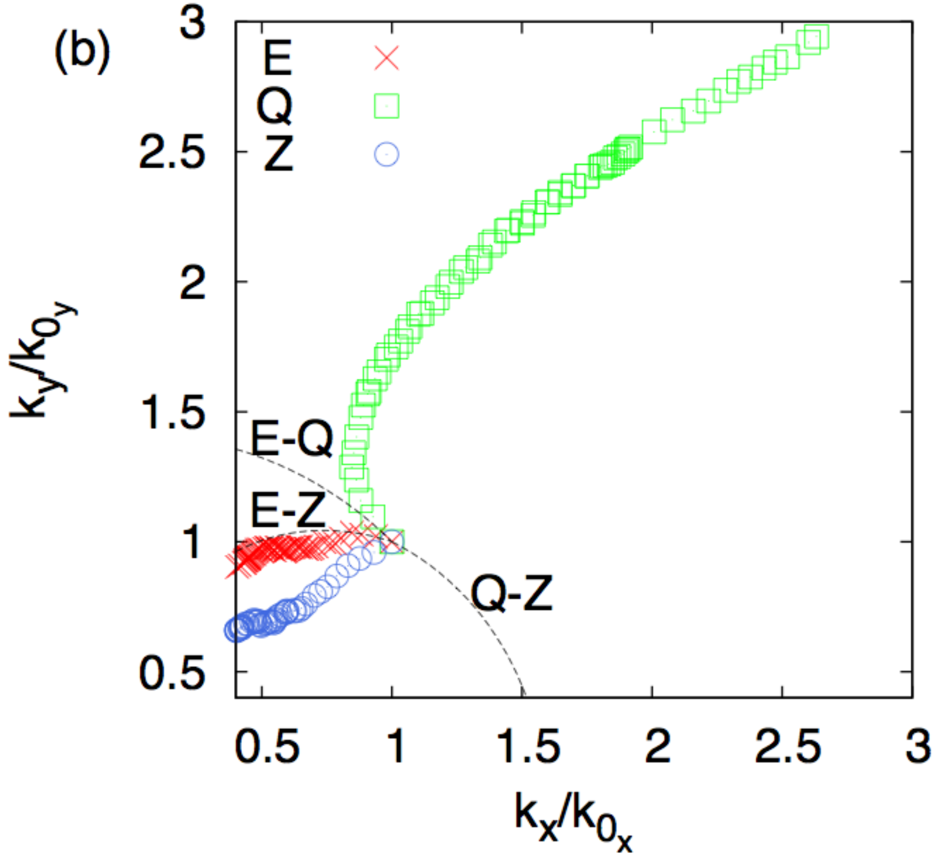
- Anizotropic inverse cascade (broadband initial condition)
- Modulational instability (narrowband initial condition)
- Cf. Benjamin-Fair Index in water wave theory and forecast

Mechanism 1: Anisotropic cascades of 3 invariants



Energy flows into the zonal flow sector

Triple cascade Rossby/Drift wave turbulence in numerics of unforced CHM



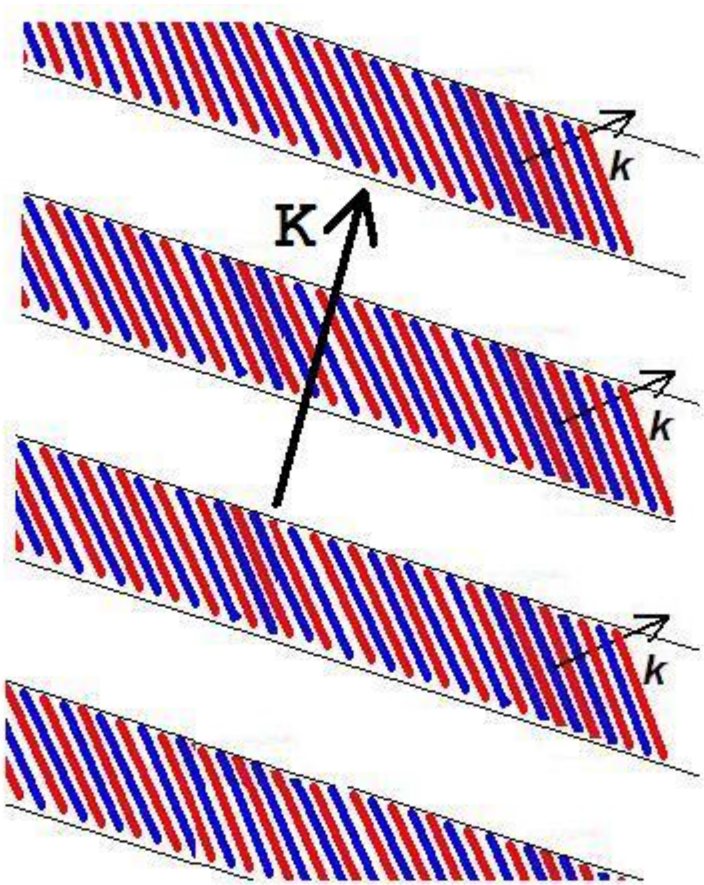
SN and B.Quinn, 2009.

Trajectories of the 3 centroids.

Fjortoft works well even for strong turbulence

Mechanism 2: Modulational Instability

Lorentz 1972, Gill 1973, Manin, Nazarenko, 1994; Manfroi, Young, 1999; Smolyakov et al, 2000; Connaughton, Nadiga, SN, Quinn, 2009.



- ***Cf. Benjamin-Fair Instability of water waves***

Modulational Instability

$$\psi_0(\mathbf{x},t) = \Psi_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \bar{\Psi}_0 e^{-i\mathbf{k}\cdot\mathbf{x}+i\omega t}$$

$$\omega(\mathbf{k}) = -\frac{\beta k_x}{k^2+F} \quad - \text{frequency of linear waves.}$$

- These waves are solutions of CHM equation for any amplitude. Are they stable? (Lorentz 1972, Gill 1973).

$$\psi(\mathbf{x},0) = \psi_0(\mathbf{x}) + \text{Ú}\psi_1(\mathbf{x}),$$

$$\psi_1(\mathbf{x}) = \psi_Z(\mathbf{x}) + \psi^+(\mathbf{x}) + \psi^-(\mathbf{x}) \quad - \text{perturbation.}$$

$$\psi_Z(\mathbf{x}) = a e^{i\mathbf{q}\cdot\mathbf{x}} + \bar{a} e^{-i\mathbf{q}\cdot\mathbf{x}} \quad - \text{zonal part} \quad \mathbf{q} = (0,q),$$

$$\psi^+(\mathbf{x}) = b^+ e^{i\mathbf{p}_+\cdot\mathbf{x}} + \bar{b}^+ e^{-i\mathbf{p}_+\cdot\mathbf{x}} \quad - \quad {}^{+}\text{satellite} \quad \mathbf{p}_+ = \mathbf{k}+\mathbf{q},$$

$$\psi^-(\mathbf{x}) = b^- e^{i\mathbf{p}_-\cdot\mathbf{x}} + \bar{b}^- e^{-i\mathbf{p}_-\cdot\mathbf{x}} \quad - \quad {}^{-}\text{satellite} \quad \mathbf{p}_- = \mathbf{k}-\mathbf{q}.$$

Instability dispersion relation

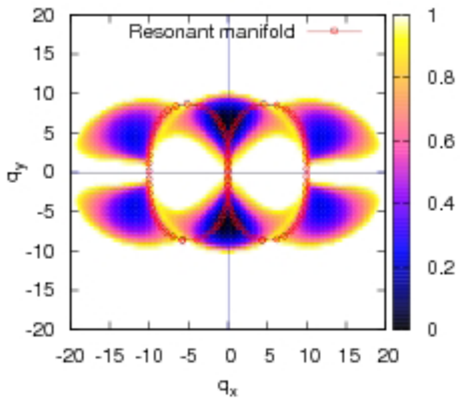
$$(q^2+F)\Omega+\beta q_x+\left|\Psi_0\right|^2\left|\mathbf{k}\times\mathbf{q}\right|^2(k^2-q^2)\left[\frac{p_+{}^2-k^2}{(p_+{}^2+F)(\Omega+\omega)+\beta p_{+x}}-\frac{p_-{}^2-k^2}{(p_-{}^2+F)(\Omega-\omega)+\beta p_{-x}}\right]=0$$

$M=\dfrac{\Psi_0 k^3}{\beta}$ – nonlinearity parameter.

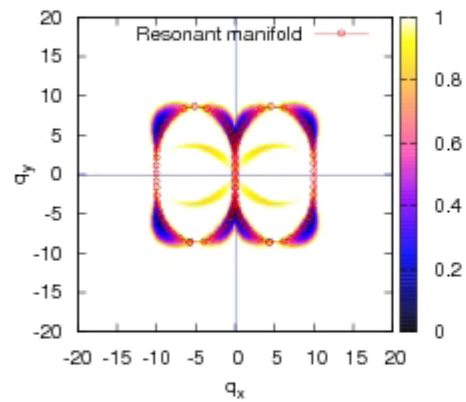
$M\to\infty$ – Euler limit (Rayleigh instabilty);

$M\to 0$ – weak monlinearity: resonant wave inetraction.

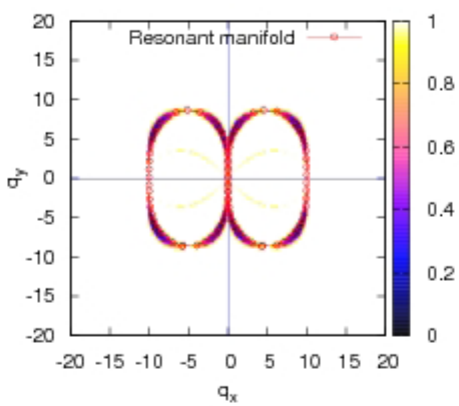
Structure of instability as a function of M



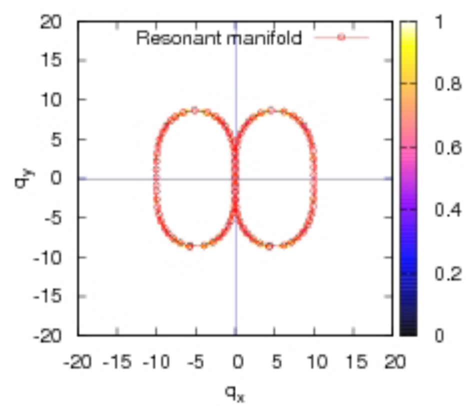
M=10



M=1



M=0.5



M=0.1

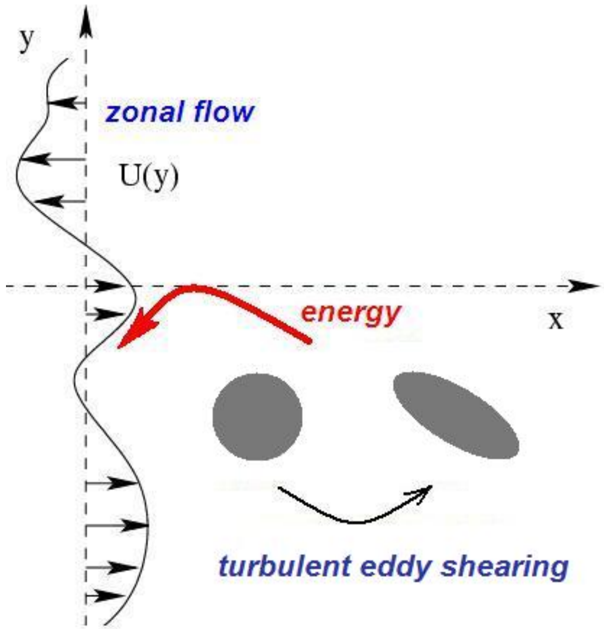
$$\mathbf{k} + \mathbf{k}_1 = \mathbf{k}_2,$$
$$\omega(\mathbf{k}) + \omega(\mathbf{k}_1) = \omega(\mathbf{k}_2)$$

Unstable region collapses onto the resonant curve. For small M the most unstable disturbance is not zonal.

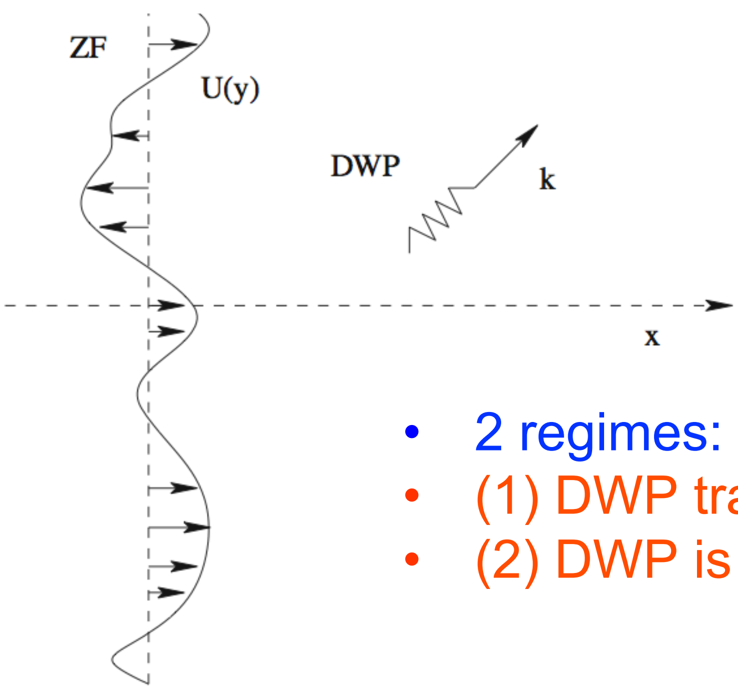
Feedback loop in Rossby turbulence

- Instability generates small-scale turbulence.
- Inverse cascade leads to energy condensation (into jets in presence of beta).
- Jets kill small-scale turbulence and saturate.

ZF-turbulence interaction

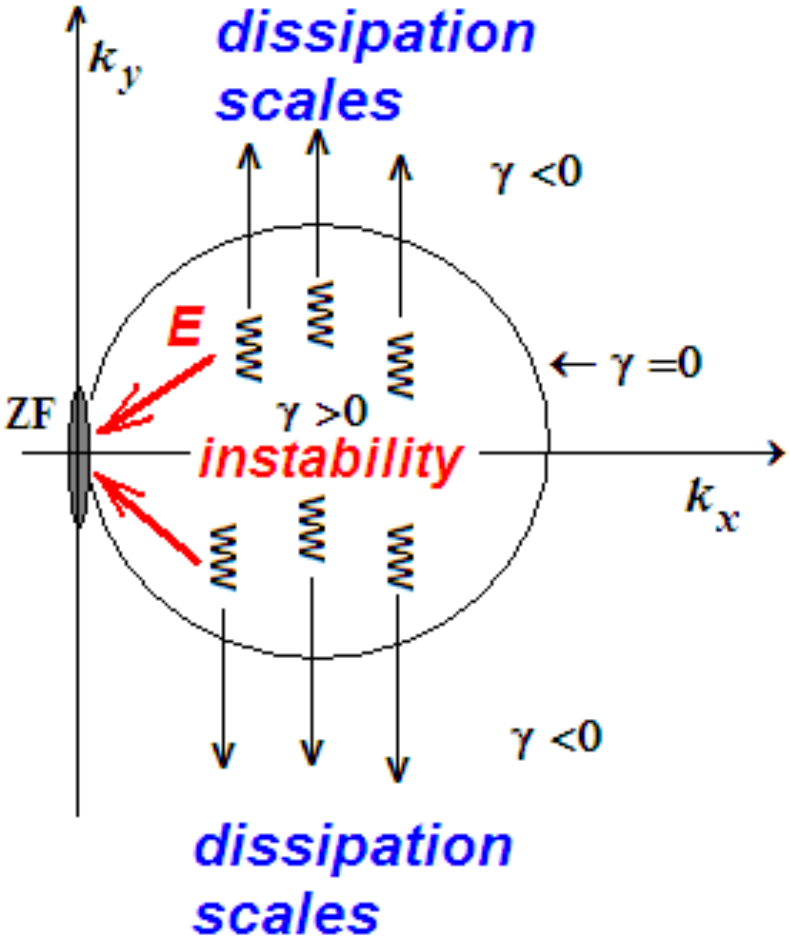


Victor P. Starr, *Physics of Negative Viscosity Phenomena* (McGraw Hill Book Co., New York 1968).



- 2 regimes:
- (1) DWP travels many ZF oscillations before is killed
- (2) DWP is killed within 1 ZF oscillation

Evolution in the k-space



- Energy of wave packet is partially transferred to ZF and partially dissipated at large ***k***'s.
- 2 regimes: random walk/ diffusion of wave packets in the k-space (*Balk, Nazarenko, Zakharov, 1990*),
- Coherent wave – modulational instability (*Manin, Nazarenko, 1994, Smolyakov et at, 2000*).

Kinetic equation for weak Rossby wave turbulence

(Longuet-Higgens &Gill, 1967)

$$\dot{n}_k = \int \; |V_{12k}|^2 \, \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \delta(\omega(\mathbf{k}_1) + \omega(\mathbf{k}_2) - \omega(\mathbf{k})) \times$$
$$[n(\mathbf{k}_1) n(\mathbf{k}_2) - 2n(\mathbf{k}) n(\mathbf{k}_1) \, \mathrm{sign}\,(k_x k_{1x})] \; d\mathbf{k}_1 d\mathbf{k}_2,$$

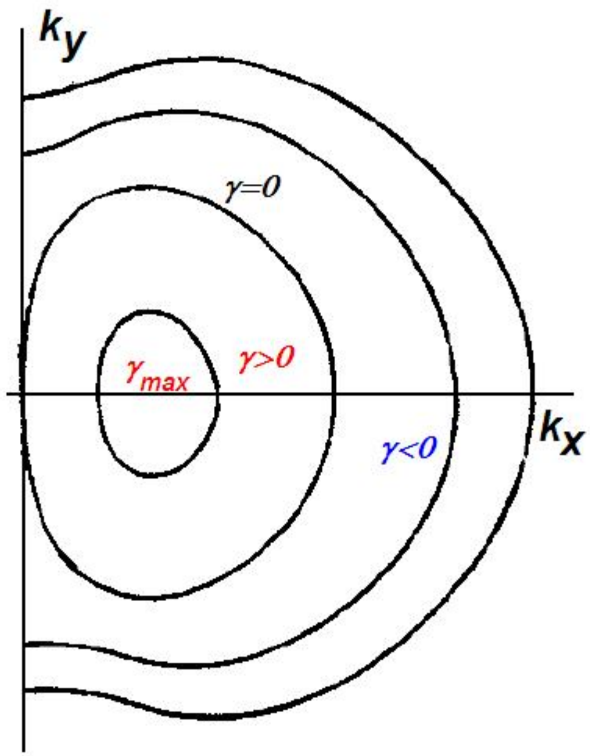
$\omega(\mathbf{k}) = -\beta k_x / k^2$, - frequency

$V_{12k} = |k_x k_{1x} k_{2x}|^{1/2} \left(\frac{k_{1y}}{k_1^2} + \frac{k_{2y}}{k_2^2} - \frac{k_y}{k^2} \right)$ - interaction coefficient

$n(\mathbf{k}) = k^4 |\hat{\psi}_k|^2 / (\beta k_x)$, - waveaction spectrum

Resonant three-wave interactions.

Forcing by an instability

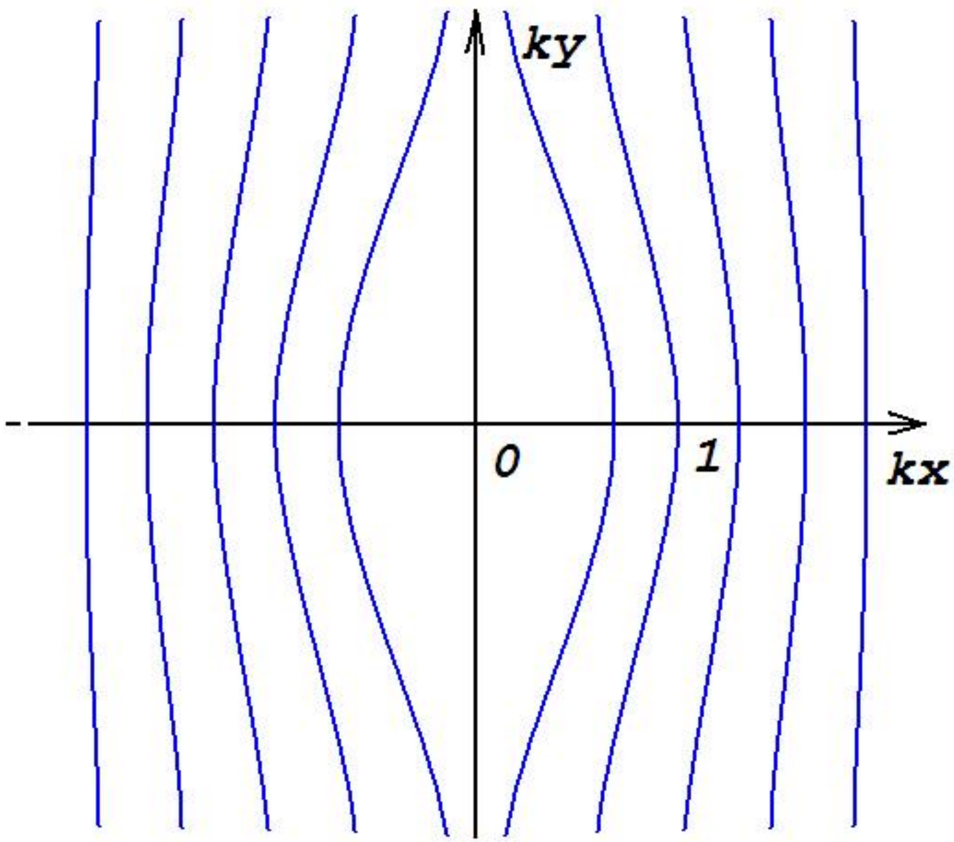


Baroclinic instability

- Maximum on the k_x-axis at $k\rho \sim 1$.
- $\gamma=0$ line crosses $k=0$ point.

Evolution of nonlocal Rossby wave turbulence:

retain only interaction with small k's and Taylor-expand the integrand of the wave-collision integral; integrate.



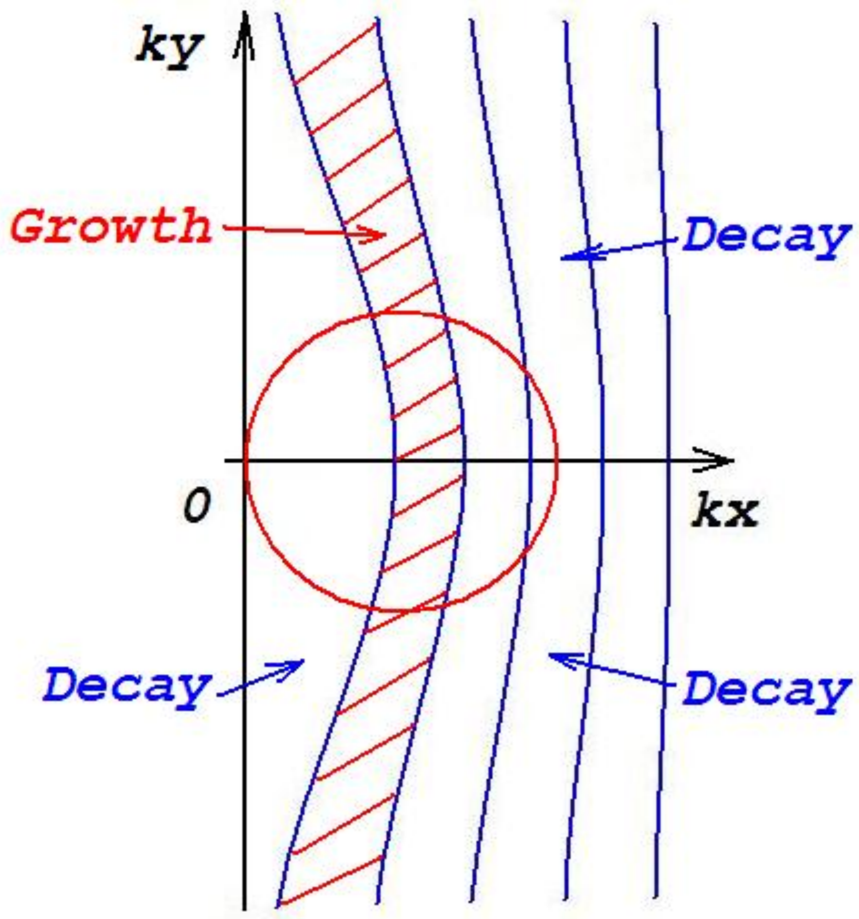
$$\frac{\partial n_k}{\partial t} = \frac{\partial \Omega_k}{\partial k_x} \frac{D}{Dk_y} S \frac{D}{Dk_y} n_k + \gamma_k n_k$$

- Diffusion along curves

$\Omega_k = \omega_k - \beta k_x = conts.$

- S ~ZF intensity

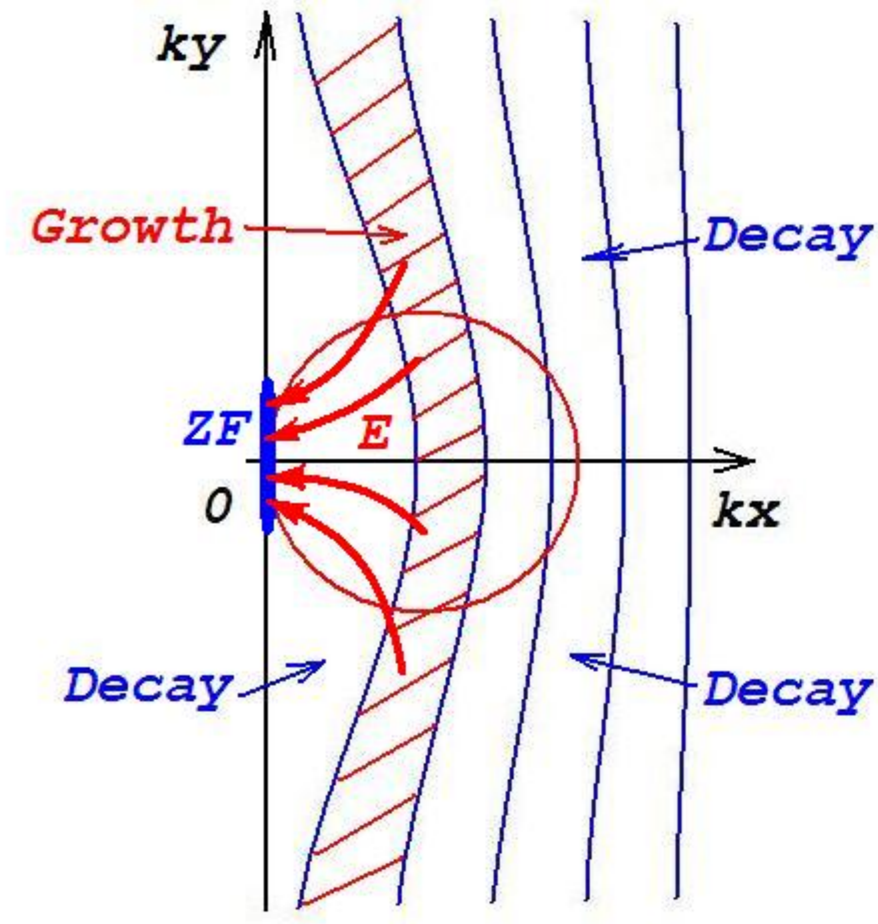
Initial evolution



$$\frac{\partial n_k}{\partial t} = \frac{\partial \Omega_k}{\partial k_x} \frac{D}{Dk_y} S \frac{D}{Dk_y} n_k + \gamma_k n_k$$

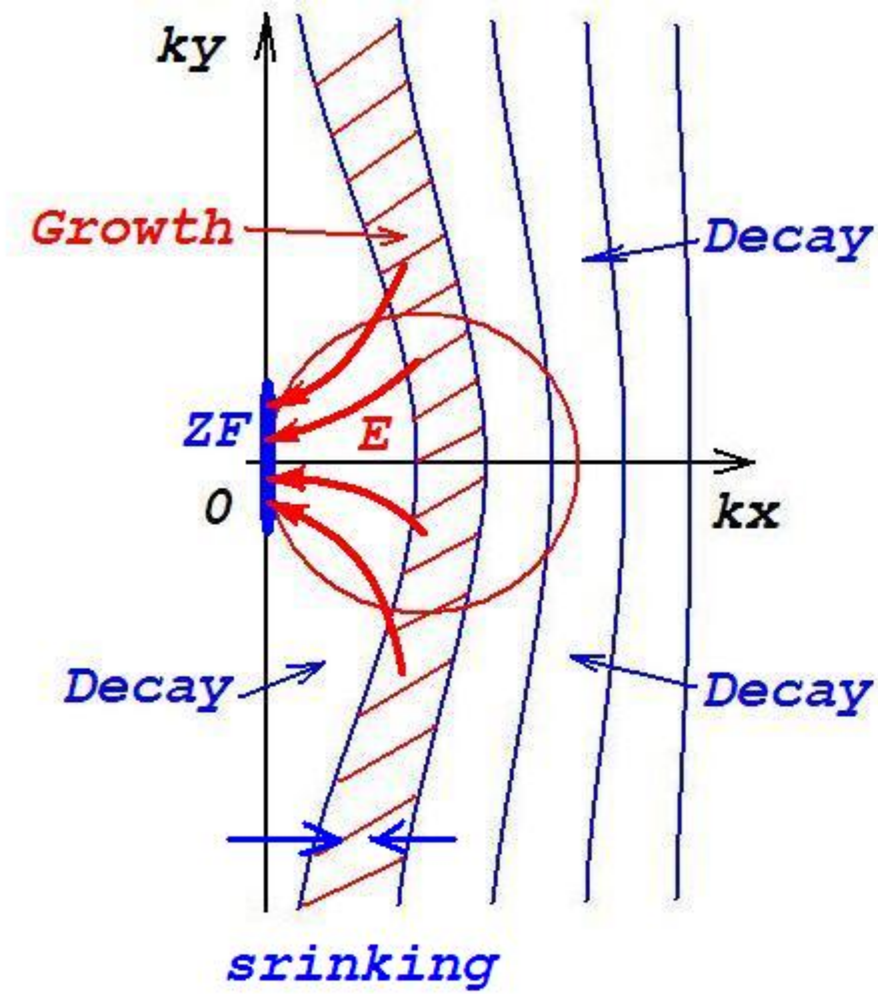
- Solve the eigenvalue problem at each curve.
- Max eigenvalue <0 → waves on this curve decay.
- Max eigenvalue >0 → waves on this curve grow.
- Growing curves pass through the instability scales

ZF growth



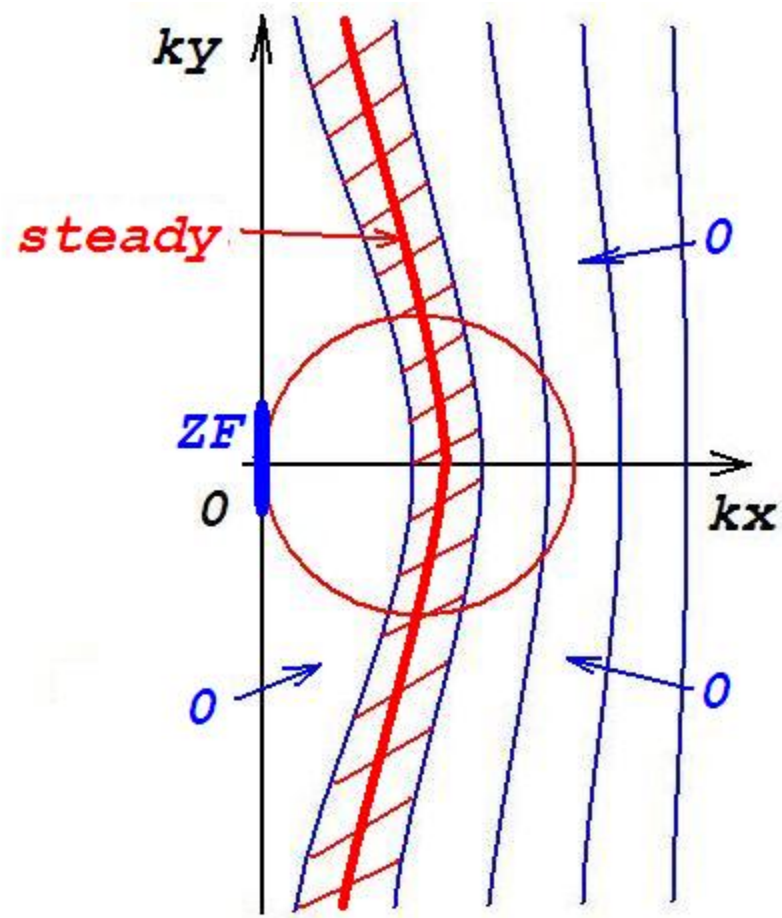
- Waves pass energy from the growing curves to ZF.
- ZF accelerates wave transfer to the dissipation scales via the increased diffusion coefficient.

Zonal Flow growth



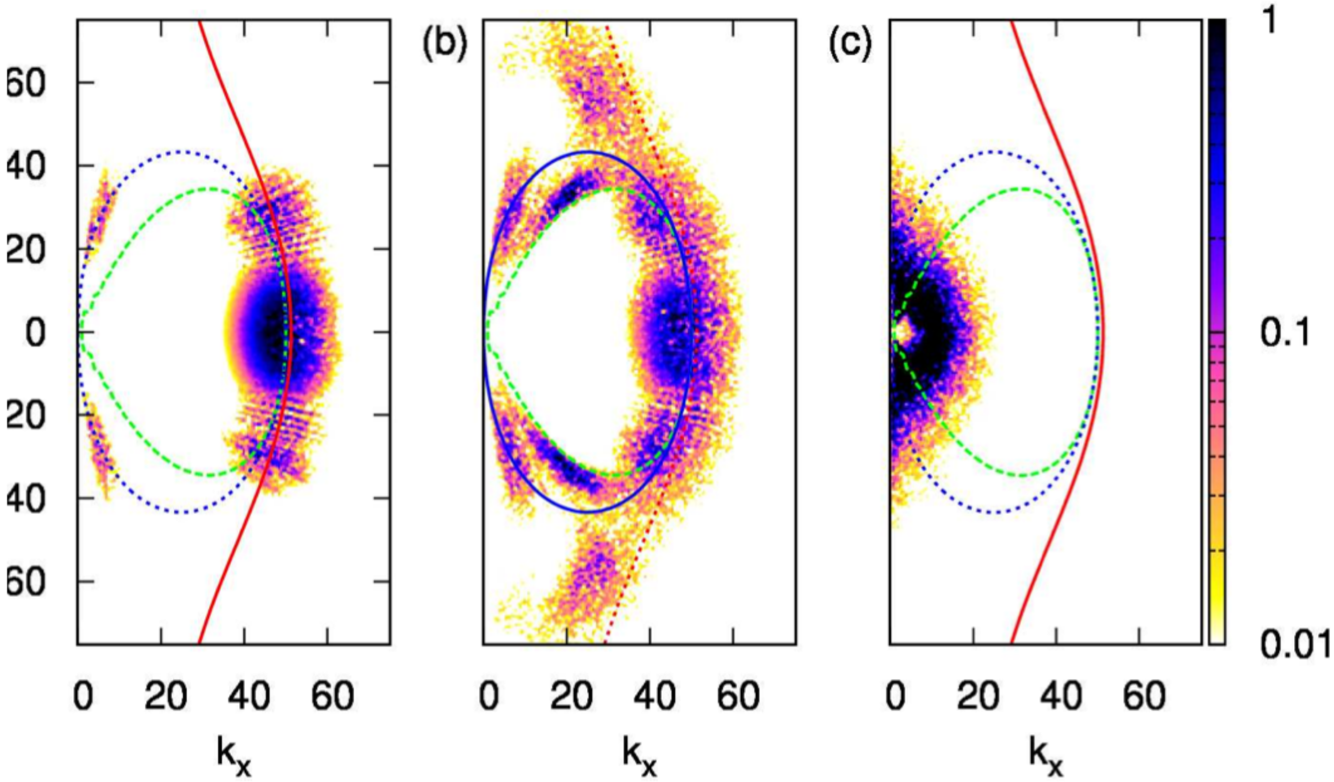
- Hence the growing region shrink.
- Rossby wave – Zonal Flow loop is closed!

Steady state



- Saturated ZF.
- Jet spectrum on a k-curve passing through the maximum of instability.
- Suppressed intermediate scales
- Balanced/correlated Waves and ZF

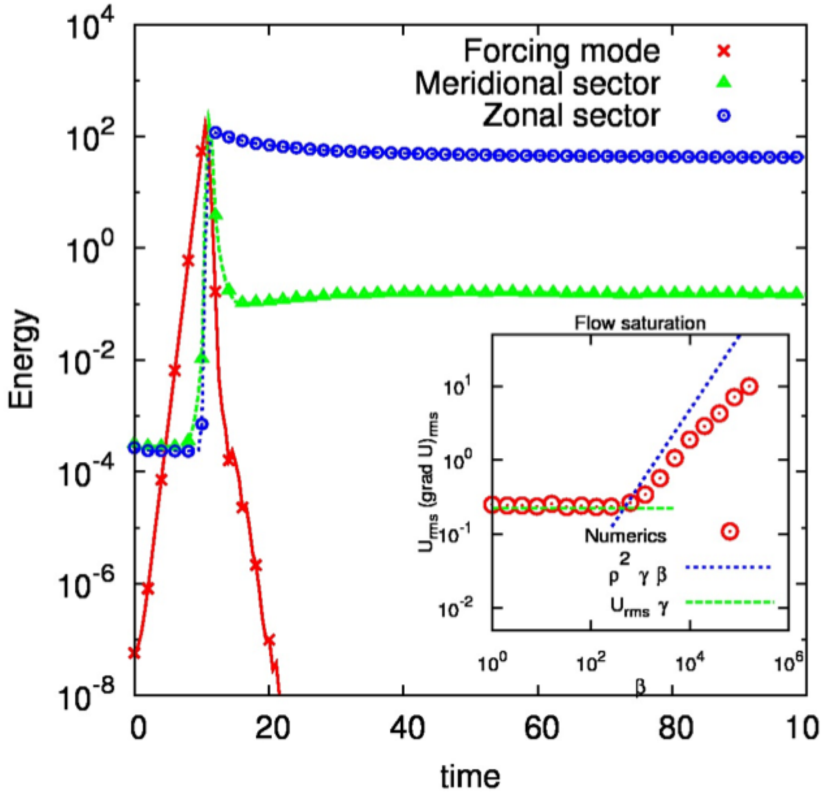
Numerics of instability-forced CHM



C.Connaughton, SN and B.Quinn, 2010.

- Zonal scales form.
- Small-scale turbulence is suppressed.

Numerics of instability-forced CHM



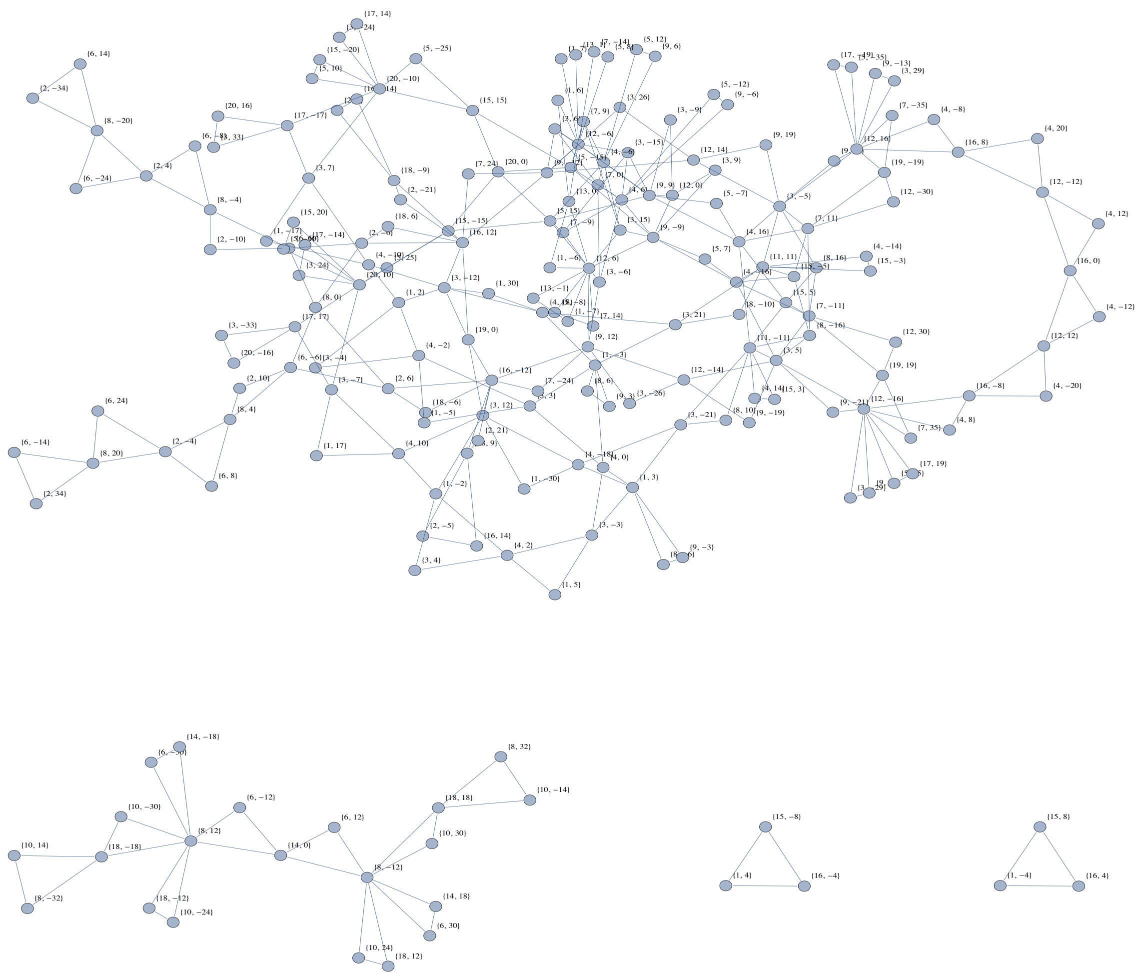
C.Connaughton, SN and B.Quinn, 2010.

Evolution in time of energies:
Read – zonal sector,
Green – off-zonal sector;
Blue – instability scales.

- Zonal scales form.
- Small-scale turbulence is suppressed.

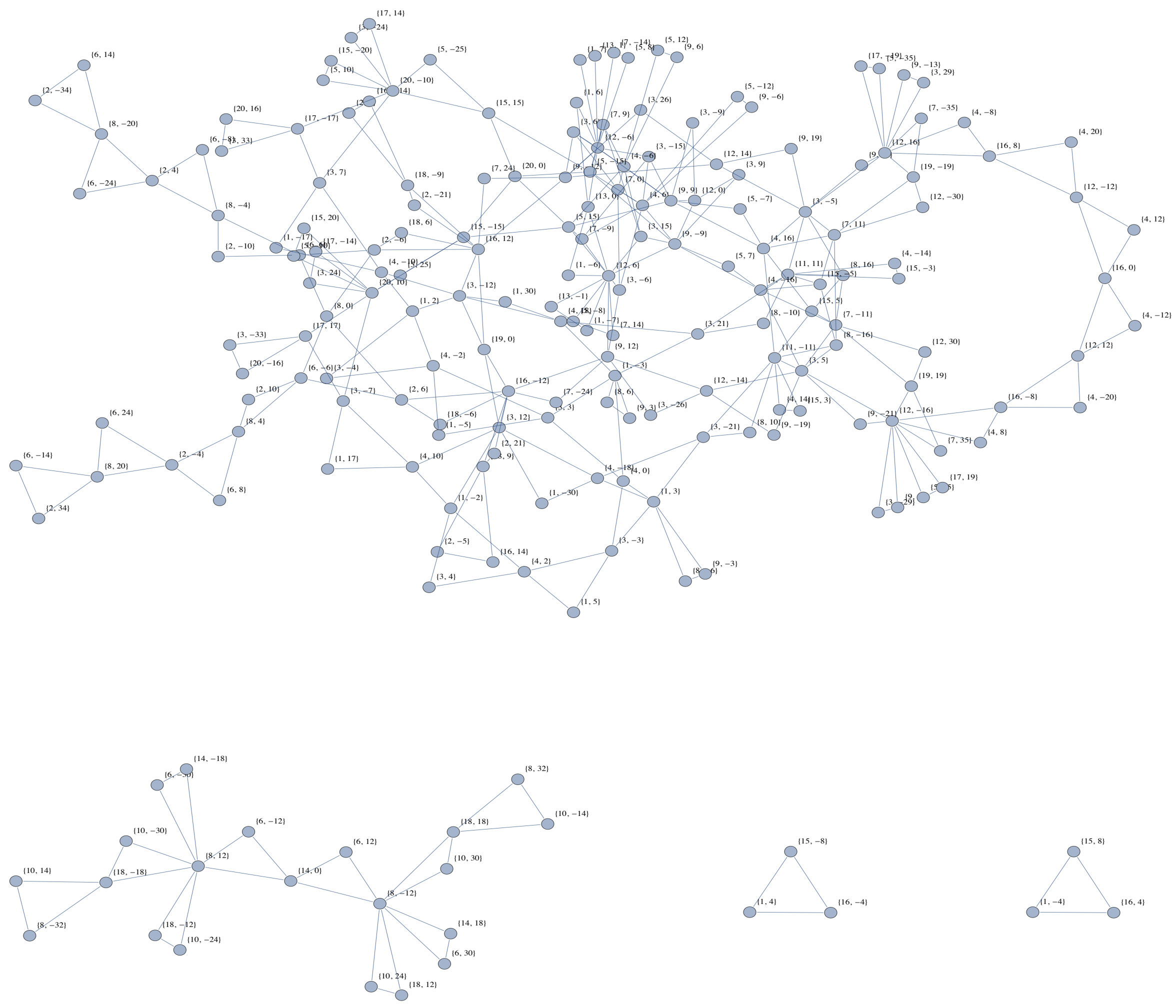
Discrete Rossby turbulence

- Discrete k-modes in finite domains
- Resonant triads get organised in independent finite-size clusters



Discrete Rossby wave turbulence

- Cluster with N waves and M triads has at least N-M quadratic invariants. How do they affect dynamics/statistics and the cascades?



Two-layer ocean model

- Coupled kinetic equations for the baroclinic and the barotropic mode.
- Anisotropy and nonlocality.
- Small-scale meridional baroclinic waves generate large-scale barotropic zonal motions.
- See Katie Harper’s poster.

Generalised plasma-specific models

- Modified Hassegawa-Mima equation (Connaughton, Hnat, Galagher and SN)
- Hassegawa-Wakatani model (Boss, Pushkarev and SN)

Conclusions

- CHM model is an excellent minimal model for both plasma and GFD turbulence
- ZF generation
- Turbulence and transport suppression by ZF: two regimes – wave and eddy dominated.
- Discrete Rossby wave turbulence: many remaining questions